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RESEARCH ARTICLE

The cost of proportional representations in electoral system design

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Abstract
We present an impossibility result concerning the design of dual vote electoral systems that meet three key conditions: proportional party representation, proportional local representation, and local accountability. By identifying the necessary number of compensatory seats to meet these three conditions in dual vote systems, we show that the number is not bounded in general; thus, it can be very costly to achieve the three conditions. When a cap is applied to the total seats, combined with a district-decentralization, semi-compensatory dual vote systems that distribute the limited compensatory seats to enhance proportionality become vulnerable to strategic manipulations. Specifically, when political parties seek to maximize their legislative representation, they might employ the strategy of creating 'decoy' party lists.

Keywords Electoral design · Voting · Dual vote system

JEL Classification C72 · D72

1 Introduction

Electoral systems serve as mechanisms that translate votes into seat allocations for political parties and candidates. A fundamental principle in these systems is the concept of proportional party representation, where the seat distribution mirrors the public's support for each party. The List Proportional Representation system epitomizes this principle. In this system, voters select their preferred party, leading to seat allocations that proportionally reflect the vote count. However, this system does not account for geographical representation. To address this, it's often combined with the local district system, where specific candidates represent distinct districts. Within this context, it's essential that seat distributions consider district populations, a concept we term as

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'proportional local representation'. Additionally, we emphasize 'local accountability', ensuring that parties with better performance in a district are duly rewarded; if party A receives fewer votes than party B in a district, it should not be given more seats than party B for that district.

In this paper, we examine dual vote systems, which integrate elements of both List Proportional Representation and local district systems. Within these systems, voters are given two ballots: one to select a candidate in their local district and another to choose a party from a designated list. A specific type of dual vote system, known as the Parallel system, lacks any correlation between district-level outcomes and party-list results, making it inherently challenging to achieve overall proportionality. To address this limitation, some countries have implemented compensatory mechanisms designed to recalibrate seat allocations, thereby enhancing proportional representation.[1] However, these compensatory strategies are not without pitfalls. They can sometimes demand additional seats to realize genuine proportionality. For instance, Germany's 20th lower house of parliament now holds 736 seats, overshooting the set quota by 138. This number has consistently risen since 2002, prompting the passage of a law in 2023 to restrict the total seat count.[2] In the 2020 legislative election, South Korea implemented a compensatory system to bolster party proportionality, yet set a ceiling of 300 seats. This led the nation's dominant parties to craft 'satellite parties' or 'decoy lists' as a tactical ploy to exploit the compensatory framework.[3] Interestingly, the use of decoy lists isn't unique to South Korea; other nations with analogous electoral systems have witnessed similar strategies.[4]

Our first theorem establishes the lower bound of additional seats required to meet three key objectives: proportional party representation, proportional local representation, and local accountability. Consequently, any dual vote electoral system aiming to satisfy these three criteria necessitates a number of compensatory seats exceeding this lower bound. The analysis reveals that there is no upper limit on the lower bound, making the pursuit of these objectives potentially expensive for all variants of dual vote electoral systems. Following this, we illustrate that when a cap is set on the overall seat count and compensatory seat allocation mechanisms are employed, the electoral system becomes susceptible to strategic manipulations, such as the use of 'decoy' party lists. In these circumstances, the resulting equilibrium mirrors the outcomes observed in a Parallel electoral system.

The study of apportionment in electoral design, particularly focusing on achieving both proportional party representation and proportional local representation, has been extensively explored in literature under the concept of "biproportional apportionment". The foundational work of Balinski and Demange (1989b) introduced the biproportional divisor methods, see also Balinski and Demange (1989a). Additionally, Demange (2012) delved into the challenge of achieving party proportionality, assuming a pre-determined allocation of seats to districts are distorted. Later, Demange

[1] Countries using compensatory systems include Germany, South Korea, Italy, Hungary, Bolivia, etc. For more details, see Farrell (2011).

[2] See Gadd and Walker (2022) for the election result.

[3] For a comprehensive analysis, consult (Hanelt 2020).

[4] For instance, the 2001 Italian general election saw the deployment of decoy lists. More can be found in Newell (2002).

(2013) examined different biproportional apportionment methods through the lens of a set of fairness axioms.

However, to our knowledge, previous studies have primarily centered around the single-vote system, where a vote for a candidate is simultaneously a vote for their party. This approach to achieving party-proportionality can be somewhat misleading. It primarily fails to account for scenarios where voters may back a candidate for reasons independent of their party affiliation. Our model addresses this oversight by explicitly incorporating a dual vote system, thereby broadening the scope to include the single-vote system as a special case within a more expansive framework.

On a different note, prior research has extensively delved into the implications of the integrality of seats. Various methodologies have been proposed to approximate ideal seat allocations, which inherently involve fractional seats, with a whole number of seats. In contrast, our model departs from this tradition. Instead of focusing on methods to achieve near-ideal proportional representation within the constraints of seat integrality, our approach deliberately omits the integrality constraint. We concentrate on analyzing the potential negative outcomes that arise when utilizing fractional shares. This aspect of our study is critical, as it underscores that the primary challenges in achieving accurate proportional representation are not necessarily tied to the restriction of allocating whole seats. However, it's important to acknowledge that this approach, while insightful, also highlights a limitation of our model, as it does not directly address the practical implications of seat integrality in real-world electoral systems.

2 Model

In our model, we consider n electoral districts, each indexed by i where $i \in N = \{1, \dots, n\}$, and m political parties, each indexed by j where $j \in M = \{1, \dots, m\}$. We employ v_{ij} to denote the vote count received by candidates from party j in district i, and f_{ij} to aggregate the votes cast for party j in the party-list vote within district i for each $(i, j) \in N \times M$. Summative notations $v_i = \sum_{j \in M} v_{ij}$ and $v_j = \sum_{i \in N} v_{ij}$ are used to represent the total votes in each district and for each party, respectively. Analogously, f_i and f_j are defined for the party-list vote. The overall voter turnout is denoted by v.

In a **dual vote electoral system**, the mappings $(v_{ij})_{(i,j) \in N \times M}$ and $(f_{ij})_{(i,j) \in N \times M}$ translate to $(s_{ij})_{(i,j) \in (N \cup \{0\}) \times M}$, where s_{ij} specifies the number of seats allocated to party j for district i. The term s_{0j} represents the compensatory seats assigned to party j via the party-list vote. The aggregate seat count is expressed as $x = x_C + x_L$, where $x_C = \sum_{j \in M} s_{0j}$ accounts for the total compensatory seats, and $x_L = \sum_{(i,j) \in N \times M} s_{ij}$ denotes the seats directly representing local districts.

Our model accommodates a variety of electoral systems, such as mixed-member proportional systems, additional member systems, and transferable vote systems. Central to its design are three key desiderata, often essential in electoral system design:

(a) Proportional party representation: This criterion ensures that the number of parliamentary seats allocated to each party is directly proportional to the votes received. Formally, it can be expressed as:

$$\sum_{i=0}^{n} s_{ij} = \frac{f_j}{v}(x_C + x_L), \text{ for each party } j \in M.$$

(b) Proportional local representation: This principle asserts that the number of seats representing a local district should be proportional to its population. Mathematically, it is expressed as:

$$\sum_{j=1}^{m} s_{ij} = \frac{v_i}{v} x_L, \text{ for each district } i \in N.$$

(c) Local accountability: This requirement posits that a party receiving fewer votes in a district should not win more seats than a party with more votes in that district, ensuring that parties must adequately represent their districts' interests. This requirement translates to a monotonicity constraint which can be expressed as:

$$(v_{ij} - v_{ij'})(s_{ij} - s_{ij'}) \geq 0, \text{ for each } i \in N \text{ and } j, \ j' \in M.$$

The objective is to determine whether these conditions can be simultaneously satisfied within a dual vote electoral system framework. This involves assessing the feasibility of a linear system where $(s_{ij})_{(i,j)\in(N\cup 0\times M)}$ are the variables subject to the aforementioned constraints.

A key challenge is that the feasibility of an integer solution for the linear system is not straightforward since the right-hand sides of conditions (a) and (b) are not integers. For instance, in an extreme case where $x_L = 1$ across two districts with positive voter turnout, it is impossible to allocate the seat without resorting to fractional seats. Allowing fractional seats, a straightforward solution for conditions (a) and (b) is:

$$s_{ij} = \frac{v_i f_j}{v^2} x_L, \text{ for each } (i, j) \in N \times M$$

and

$$s_{0j} = \frac{f_j}{v} x_C, \text{ for each } j \in M.$$

However, this solution might not fulfill the local accountability condition. In the following section, we demonstrate that a certain number of seats is necessary to simultaneously meet all three criteria.

3 Results

Theorem 1 *The minimum number of compensatory seats is* $\max_{j\in M}\{\frac{v_j - f_j}{f_j} x_L\}$ *for any dual vote system that satisfies the three conditions; proportional party representation, proportional local representation, and local accountability.*

Based on Theorem 1, the followings are established.

Corollary 1 *In the absence of split-ticket voting (i.e., where $f_j = v_j$ for all $j \in M$), no compensatory seats are necessary to fulfill the three conditions.*

Corollary 2 *There is no upper bound in the number of compensatory seats necessary under a dual vote system with the three conditions.*

It is important to note that the scenario described in Corollary 1, involving 'no split-ticket voting' essentially mirrors a single-vote system. Thus, this outcome is not a novel finding and aligns with the results observed in previous studies of single-vote systems, notably Balinski and Demange (1989b) and Demange (2012). In fact, without resorting to Theorem 1, one could deduce that setting $s_{ij} = \frac{v_{ij}}{v} x_L$ for all $(i, j) \in N \times M$ suffices as a solution to the linear system. However, our approach diverges by employing Farkas' lemma to reach the same conclusion, which presents an interesting alternative perspective on the problem.

It is a common practice that an electoral threshold is set and a party needs to exceed the threshold in the party-list vote to receive any compensatory seats. For instance, the threshold is 5% in Germany and 3% in South Korea. Theorem 1 still holds with a change that the minimum necessary seat is now

$$\min \left\{ \max_{j \in M} \frac{v_j - f_j}{f_j} x_L, \max_{j \in M} \frac{v_j - \alpha v}{\alpha v} x_L \right\},$$

where α is the electoral threshold.

For the proof of Theorem 1, which is provided in the appendix, we use the following known result in linear programming without a proof.

Lemma 1 (Farkas' alternative)[5] *Suppose that $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, $C \in \mathbb{R}^{l \times n}$, and $d \in \mathbb{R}^l$. Then, exactly one of the following statement is true.*

1. *there exists $x \in \mathbb{R}^n$ such that $Ax \leq b,\ Cx = d,\ x \geq 0$,*
2. *there exist $y \in \mathbb{R}^l$ and $z \in \mathbb{R}^m$ such that $z \geq 0,\ zA + yC \geq 0,\ z \cdot b + y \cdot d < 0$.*

The second system is called Farkas' alternative system. The idea of the proof is to identify the conditions on the variables in which the Farkas' alternative system becomes feasible.

Corollary 2 implies that it can be very costly to achieve the three conditions if there is a huge discrepancy between the popularity of individual candidates running for the local offices and the parties where the candidates belong. One natural remedy is to cap the total number of compensatory seats at a certain level.[6] However, our next result shows that capping the number of compensatory seats creates an opportunity for the parties to game the system. The most typical method of gaming the system is to split

[5] For the original version of the theorem, see Farkas (1902). The variation used here can be found in Border (2015). This theorem has been used in many economic applications; Crémer and McLean (1988), Nau and Mccardle (1991), and Border (2007).

[6] For instance, the number of compensatory seats is fixed at 30 in the general elections of South Korea. In Germany, while the current system does not cap the total number of seats, as the bigger size of the Bundestag becomes problematic, the government moves to limit the number of seats. See Cho (2020) and Brady (2020) for more details.

the party into two parties; one party runs for local offices and another only participates in the party-list vote. The latter is often called a decoy list.

By capping the total number of seats, while electoral systems do not guarantee proportional party representation anymore, the available compensatory seats should be allocated to minimize the discrepancy between proportion of seats won by a party and the popularity of the party in the party-list vote. The following linear system reflects the change.

(a′) (Semi-proportional party representation) The electoral vote system selects $(s_{ij})_{(i,j)\in N\times M}$ that minimizes $\max_{j\in M}\left[\frac{f_j}{v}x - \sum_{i=0}^{n} s_{ij}\right]$.

(b) (Proportional local representation) $\sum_{j=1}^{m} s_{ij} = \frac{v_i}{v}x_L$, for all $i \in N$.

(c) (Local accountability) $(v_{ij} - v_{ij'})(s_{ij} - s_{ij'}) \geq 0$, for each $i \in N$ and $j,\ j' \in M$.

Notice that any electoral system with (a′), (b), and (c) always selects the outcome that satisfies (a) if it is feasible.

In order to analyze the strategic behaviors of the parties, we assume that the objective of each party j is to maximize $\frac{\sum_{i=0}^{n} s_{ij}}{x}$. By creating a decoy list, the party j is split into two parties j_L and j_C: the party j_L runs only for the local offices and j_C only participates in the party-list vote. We assume that the party j_L receives the same amount of votes that the party j would have won from each district and the party j_C receives the same amount of votes the party j would have won from the party-list vote. We impose an additional assumption of district-decentralization, for our next result.

(d) (District-decentralization) A dual vote system is *district-decentralized* if $(s_{ij})_{j\in M}$ only depends on $(v_{ij})_{j\in M}$ for each $i \in N$.

District-decentralization condition, that was firstly studied by Demange (2012), mandates that the allocation of seats within a district remains unaffected by election results in other districts. This principle ensures that local election outcomes are not influenced by the performance of a party in regions outside of the district in question. Specifically, it prevents scenarios where seats might be taken away from locally elected candidates simply because their party performed poorly in other districts.

The following theorem shows that creating a decoy list is a dominant strategy under any electoral system that satisfies the conditions described above.

Theorem 2 *Suppose that a given electoral system satisfies (a′), (b), (c), and (d). If the total number of seats is fixed, splitting is a dominant strategy for every party.*

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Data availability We do not analyze or generate any datasets, because our work proceeds within a theoretical and mathematical approach.

Appendix A Proofs

A.1 Proof of Theorem 1

Proof We can rewrite the linear system by taking s_{0j} as free variables because they only appear in the proportional party representation condition and the modified system is as follows: for all $(i, j, j') \in N \times M \times M$,

$$\begin{aligned}\sum_{i=1}^{n} s_{ij} &\leq \frac{f_j}{v}(x_C + x_L),\\ \sum_{j=1}^{m} s_{ij} &= \frac{v_i}{v}x_L,\\ -(v_{ij} - v_{ij'})(s_{ij} - s_{ij'}) &\leq 0,\\ s_{ij} &\geq 0.\end{aligned}$$

The feasibility of the modified system is equivalent to the feasibility of the original system because we can set s_{0j} to the amount of the slack in the inequality constraint $\sum_{i=1}^{n} s_{ij} \leq \frac{f_j}{v}(x_C + x_L)$. The Farkas' alternative system is:

1. $y_j \geq 0, \forall j \in M$.
2. $z^i_{jj'} \geq 0, \forall (i, j, j') \in N \times M \times M$.
3. $y_j + \tilde{y}_i - \sum_{j'=1}^{m}(v_{ij} - v_{ij'})(z^i_{jj'} + z^i_{j'j}) \geq 0, \forall (i, j) \in N \times M$.
4. $\sum_{j=1}^{m} y_j \frac{f_j}{v}(x_C + x_L) + \sum_{i=1}^{n} \tilde{y}_i \frac{v_i}{v}x_L < 0, \forall i \in N$,

where each y_j corresponds to the equality constraint

$$\sum_{i=1}^{n} s_{ij} \leq \frac{f_j}{v}(x_C + x_L),$$

each $\tilde{y}_i$ corresponds to the equality constraint

$$\sum_{j=1}^{m} s_{ij} = \frac{v_i}{v}x_L,$$

and each $z^i_{jj'}$ corresponds to the inequality constraint

$$-(v_{ij} - v_{ij'})(s_{ij} - s_{ij'}) \leq 0.$$

1 and 2 come from the condition that dual variables corresponding the inequality constraints are positive. 3 comes from the condition that $zA + yC \geq 0$, and 4 comes from the condition that $z \cdot b + y \cdot d < 0$ in Lemma 1.

From 4,

$$\begin{aligned}
&\sum_{j=1}^{m} y_j \frac{f_j}{v}(x_C + x_L) + \sum_{i=1}^{n} \tilde{y}_i \frac{v_i}{v} x_L < 0 \\
&\iff \sum_{j=1}^{m} y_j f_j (x_C + x_L) + \sum_{i=1}^{n} \tilde{y}_i v_i x_L < 0 \\
&\iff \sum_{j=1}^{m} y_j f_j x_C + \sum_{j=1}^{m} y_j f_j x_L + \sum_{i=1}^{n} \tilde{y}_i v_i x_L < 0 \\
&\iff \sum_{j=1}^{m} y_j f_j x_C + \sum_{j=1}^{m} y_j \left(f_j - \sum_{i=1}^{n} v_{ij} + \sum_{i=1}^{n} v_{ij} \right) x_L + \sum_{i=1}^{n} \tilde{y}_i \sum_{j=1}^{m} v_{ij} x_L < 0 \\
&\iff \sum_{j=1}^{m} y_j f_j x_C + \sum_{j=1}^{m} y_j (f_j - v_j) x_L + \sum_{i=1}^{n} \sum_{j=1}^{m} v_{ij} (y_j + \tilde{y}_i) x_L < 0 \\
&\iff \sum_{j=1}^{m} y_j [f_j x_C + (f_j - v_j) x_L] + \sum_{i=1}^{n} \sum_{j=1}^{m} v_{ij} (y_j + \tilde{y}_i) x_L < 0. \qquad (1)
\end{aligned}$$

If we denote the left-hand side of (1) by $(*)$,

$$\begin{aligned}
(*) &\geq \sum_{j=1}^{m} y_j [f_j x_C + (f_j - v_j) x_L] + \sum_{i=1}^{n} \sum_{j=1}^{m} v_{ij} \left(\sum_{j'=1}^{m} (v_{ij} - v_{ij'})(z^i_{jj'} + z^i_{j'j}) \right) x_L \\
&= \sum_{j=1}^{m} y_j [f_j x_C + (f_j - v_j) x_L] + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{j'=1}^{m} v_{ij} (v_{ij} - v_{ij'})(z^i_{jj'} + z^i_{j'j}) x_L.
\end{aligned}$$

Notice that

$$v_{ij}(v_{ij} - v_{ij'})(z^i_{jj'} + z^i_{j'j}) + v_{ij'}(v_{i'} - v_{ij})(z^i_{j'j} + z^i_{jj'}) = (v_{ij} - v_{ij'})^2 (z^i_{jj'} + z^i_{j'j}),$$

thus,

$$2 \sum_{j=1}^{m} \sum_{j'=1}^{m} v_{ij}(v_{ij} - v_{ij'})(z^i_{jj'} + z^i_{j'j}) = \sum_{j=1}^{m} \sum_{j'=1}^{m} (v_{ij} - v_{ij'})^2 (z^i_{jj'} + z^i_{j'j}).$$

Therefore,

$$(*) \geq \sum_{j=1}^{m} y_j [f_j x_C + (f_j - v_j) x_L] + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{j'=1}^{m} (v_{ij} - v_{ij'})^2 (z^i_{jj'} + z^i_{j'j}) x_L. \qquad (2)$$

If $f_j x_C + (f_j - v_j)x_L \geq 0$ for all $j \in M$, then $(*)$ is always positive, thus, Farkas' alternative system is infeasible and the original problem is feasible. If $f_j x_C + (f_j - v_j)x_L < 0$ for some $j \in M$, then by setting $z^i_{\tilde{j}j'} = 0$ for all i, $\tilde{j}$, and j' and by setting y_i to an arbitrarily large number, Farkas' alternative system is feasible and the original system is not feasible. □

A.2 Proof of Theorem 2

Proof Suppose that a solution of the system (a′), (b), (c), and (d) is given by (s_{ij}). If $\sum_{i=0}^n s_{ij} > \frac{f_j}{v}x$, then $s_{0j} = 0$. Otherwise, it is possible to transfer some seats from s_{0j} to the party j', where $\frac{f_{j'}}{v}x - \sum_{i=0}^n s_{ij}$ is the greatest among parties to make $\max_{j \in M}\left[\frac{f_j}{v}x - \sum_{i=0}^n s_{ij}\right]$ smaller. For any $j, j' \in M$ such that s_{0j} and $s_{0j'}$ are positive, $\frac{f_j}{v}x - \sum_{i=0}^n s_{ij}$ and $\frac{f_{j'}}{v}x - \sum_{i=0}^n s_{ij'}$ must be the same. Otherwise, it is possible to transfer some seats around to make $\max_{j \in M}\left[\frac{f_j}{v}x - \sum_{i=0}^n s_{ij}\right]$ smaller. Thus, the solution is characterized by the following two statements. For some $A \geq 0$,

(1) for all $j \in M$ such that $\sum_{i=0}^n s_{ij} > \frac{f_j}{v}x$, $s_{0j} = 0$,
(2) for all $j \in M$ such that $\sum_{i=0}^n s_{ij} \leq \frac{f_j}{v}x$, $\frac{f_j}{v}x - \sum_{i=0}^n s_{ij} = A$.

Suppose that a party with $\sum_{i=0}^n s_{ij} > \frac{f_j}{v}x$ splits the party into j_C and j_L, where j_C is for the party-list vote and j_L only runs for the local seats. The district-decentralization property implies that, with the same set of candidates for local districts, the party j_L wins the $\sum_{i=1}^n s_{ij}$ number of seats. And the party j_C wins $B \geq 0$ number of seats and the equality holds at only knife-edge case by (2). This is an improvement for the party j, since $s_{0j} = 0$ without the split strategy by (1).

Suppose that a party with $\sum_{i=0}^n s_{ij} \leq \frac{f_j}{v}x$ splits the party into j_C and j_L. The party j_L wins the $\sum_{i=1}^n s_{ij}$ number of seats by the district-decentralization condition. Notice that if a party belongs to the group (1), it still belongs to the group (1) after another party splits. Suppose that the party j_C wins $\tilde{s}_{0j} < s_{0j}$. This implies that there exists j' such that $\frac{f_{j'}}{v}x - \sum_{i=0}^n s_{ij'}$ has increased since the number of parties in the group (1) does not decrease. This is a contradiction to (2). As the total number of seats is fixed, any increase in the number of seats is an improvement for the party. □

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