Economic Theory Bulletin (2023) 11:81–91
https://doi.org/10.1007/s40505-022-00241-3

RESEARCH ARTICLE

Losses from cross-holdings in a duopoly with convex cost and strategic input price determination

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Received: 27 June 2022 / Accepted: 26 December 2022 / Published online: 24 January 2023

Abstract
It is well-known that positive output externality on the outside firms is the reason for unprofitable passive cross-holding, which refers to a situation where a producer holds non-controlling shares in rival firms. Considering a final goods market with Cournot duopoly, where cross-holdings do not create positive output externality on the outside firms, we show that cross-holdings can be unprofitable under strategic input price determination by an input supplier if the final goods are produced with decreasing returns to scale technologies. Our results hold under symmetric and asymmetric cross-holdings. We show that cross-holdings can be unprofitable also under Bertrand duopoly in the final goods market. Thus, we provide a new reason for unprofitable passive cross-holdings. We also show the implications of a higher product differentiation on the profits and welfare.

Keywords Convex cost · Cross-holdings · Profit · Vertical structure

JEL Classification D43 · L10 · L12 · L13

I would like to thank an anonymous referee and the editor of this journal for helpful comments and suggestions. The usual disclaimer applies.

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1 Introduction

Passive cross-holding, which has grown significantly in recent decades, refers to a situation where a firm holds non-controlling shares in rival firms. For example, it can be found in automobile (Alley 1997), IT (Gilo et al. 2006), telecommunications (Brito et al. 2014), and banking industries (Azar et al. 2022).

In a symmetric-cost Cournot oligopoly with homogeneous products, Reynolds and Snapp (1986) show that cross-holdings reduce the total outputs of the firms and increase the industry profits. However, it is not immediate from their analysis whether the firms involved in cross-holdings will benefit from it.

In an oligopoly with homogeneous products, Reitman (1994) shows that cross-holding between two firms *is not profitable if the number of firms in the industry is at least three*. The reason for this result is similar to the reason for an unprofitable merger in Salant et al. (1983). If a firm holds shares in a rival firm, the firms involved in cross-holding reduce their total outputs, which allow the firms not involved in the cross-holding to increase their outputs. Hence, cross-holdings among some firms create a positive output externality on the outside firms and may make the cross-holdings unprofitable.[1] This result suggests that cross-holdings in a duopoly will always be profitable, since there will be no positive output externality on the outside firms.

Using a final goods market with Cournot duopoly, thus eliminating the possibility of a positive output externality on the outside firms, we provide a new reason for unprofitable cross-holdings. We show that passive cross-holdings among the final goods producers can be unprofitable under strategic input price determination by an input supplier if the final goods are produced with decreasing returns to scale technologies, which show soft capacity constraints as mentioned in Cabon-Dhersin and Drouhin (2020). We show this result for symmetric and asymmetric cross-holdings. We show that cross-holdings can be unprofitable even if there is Bertrand competition in the final goods market. We also show the implications of a higher product differentiation on the profits and welfare.

The reason for unprofitable cross-holdings in our analysis is as follows. On the one hand, given the input price, cross-holdings reduce the total outputs of the final goods producers, which tend to increase the profits of the final goods producers. We call it *output effect*. On the other hand, cross holdings make the input demand curve more inelastic and increase the input prices, which tend to reduce the profits of the final goods producers. We call it *input price effect*. The input price effect can dominate the output effect to reduce the profits of the final goods producers following cross-holdings.

As a corollary, our result suggests that a complete cooperation among the final goods producers is not profitable under strategic input price determination by an input supplier if the final goods producers use decreasing returns to scale technologies. Thus, we provide a new reason for an unprofitable merger in a duopoly market, and our reason is different from Fershtman and Gandal (1994) and Brod and Shivakumar (1999), where cooperation in the product market between the duopolists may be unprofitable

[1] This finding has generated an interest to see the ways firms engaged in cross-holdings can increase their profits (see, e.g. Farrell and Shapiro 1990, for cost asymmetry, Wang 1994, for market uncertainty, Li et al. 2015, for entry deterrence, and Ghosh and Morita 2017, for knowledge transfer).

if they compete in R&D investments prior production. In contrast, our result is driven by the input price effect following cooperation among the final goods producers.[2]

Our paper is related to Symeonidis (2008), which considers cross-holdings in a vertical structure with firm-specific input suppliers and constant returns to scale technologies in the final goods market and finds cross-holdings are profitable. Mukherjee (2010) extends Symeonidis (2008) with an industry-wide input supplier. He also considers constant returns to scale technologies for the final goods producers and finds cooperation among the final goods producers profitable. Hence, our paper shows the importance of the technologies used by the final goods producers for profitable cross-holdings among the final goods producers.

Chen et al. (2021) look at the welfare implications of cross-holdings in a vertical structure but don't examine the profitability of cross-holdings. Further, unlike Symeonidis (2008) and our paper, which consider cross-holdings among the final goods producers only, Chen et al. (2021) consider cross-holdings among the final goods producers as well as among the input supplier and the final goods producers. Hence, Symeonidis (2008) and our paper are more applicable in industries with no cross-holdings between the input suppliers and the final goods producers. Labour unions, which supply workers to firms but don't hold shares in firms, will be a natural candidate for such an input supplier.

Shuai et al. (2022) show that cross-holding between two final goods producers may not be profitable provided there are *at least three final goods producers*. They show it under a vertical structure with constant returns to scale technologies in the final goods market and under no vertical structure with decreasing returns to scale technologies in the final goods market. In their paper, if there is a vertical structure, cross-holding does not affect the weighted input prices faced by the participating firms, and they are equal to the input prices faced by the outside firms. Hence, the reason for an unprofitable cross-holding in their analysis is the *positive output externality* on the outside firms, as in Reitman (1994) and Ghosh and Morita (2017). In contrast, we don't have the positive output externality on the outside firms. Strategic input price determination and the convex costs of the final goods producers are the reasons for our result.[3]

Although our main focus is on the profitability of cross-holdings, we also show their welfare implications. In our analysis, cross-holding is always welfare reducing, unlike Mukherjee (2010) and Chen et al. (2021). This happens since cross-holdings in our analysis create a collusive behaviour among the final goods producers and increase the input prices.

[2] There is a literature considering downstream mergers in vertically related markets (see, e.g., Lommerud et al. 2005, 2006, and Mukherjee and Zhao 2016). However, unlike our paper, those papers consider more than two firms. Therefore, the positive output externality on the outside firms is relevant in those papers.

[3] As a representative sample of the recently growing literature on passive cross-holdings without strategic input price determination, one may look at Banerjee and Mukherjee (2010), Shelegia and Spiegel (2012), Ghosh and Morita (2017), Brito, et al. (2014, 2019), Shy and Stenbacka (2020), López and Vives (2019), Vives (2020), Backus et al. (2021), Ma et al. (2021), Leonardos, et al. (2021), and Ma and Zeng (2021). See Breshnahan and Salop (1986), Reynolds and Snapp (1986), Farrell and Shapiro (1990), Mukhopadhyay et al. (1999) and Mukherjee and Sengupta (2001), for some early contributions on passive cross-holdings without strategic input price determination.

The remainder of the paper is organised as follows. Section 2 describes the model and shows the results under Cournot competition. Section 3 concludes. The Online Appendix shows the implications of Bertrand competition.

2 The model and the results

Consider an industry with an input supplier (called firm I) and two final goods producers (called firm 1 and firm 2). The input supplier produces a critical input and the final goods producers use this input to produce final goods. Assume that the marginal cost of input production is c and the final goods producers compete like Cournot duopolists. We will show the implications of Bertrand competition in the Online Appendix.

Assume that the inverse demand function for the ith firm is $P_i = 1 - q_i - \gamma q_j$, where q_i and P_i are respectively the output and price of the ith firm and q_j is the output of the jth firm, with $i, j = 1, 2, i \neq j$. The term $\gamma \in [0, 1]$ shows the degree of product differentiation. $\gamma = 0$ implies the products of firms 1 and 2 are isolated and $\gamma = 1$ implies the products of firms 1 and 2 are perfect substitutes. In this section, we concentrate on $\gamma \in (0, 1]$. This demand structure is derived from a representative consumer's utility function $U = q_1 + q_2 - \frac{q_1^2 + q_2^2 + 2\gamma q_1 q_2}{2}$, which is widely used in the literature (see, e.g., Singh and Vives 1984).[4]

We assume that each final goods producer requires $\sqrt{L_i}$ units of inputs to produce q_i units of the final goods, i.e., the production technology for the final goods is $q_i = \sqrt{L_i}$ or $q_i^2 = L_i$, where $i = 1, 2$. Hence, q_i^2 units of inputs are required to produce q_i units of the final goods.

To show our point in the simplest way, we will first focus on symmetric cross-holdings in Sect. 2.1, where each final goods producer holds $\alpha \in [0, 0.5]$ fraction of shares in the rival firm. We will examine how the total profits of firms 1 and 2 change with respect to α. This case will show the incentive for symmetric cross-holdings between firms 1 and 2. We will then show in Sect. 2.2 how the total profits of firms 1 and 2 change when only one final goods producer changes its shares in the rival firm.

2.1 Symmetric cross-holdings

Consider the following game. Given the cross-holdings, in stage 1, firm I determines the input price, w. In stage 2, firms 1 and 2 produce their outputs like Cournot duopolists and the profits are realised. We solve the game through backward induction.

Given the input price w, firms 1 and 2 determine their outputs to maximise the following expressions respectively:

$$\underset{q_1}{Max}(1 - \alpha)(1 - q_1 - \gamma q_2 - w q_1) q_1 + \alpha(1 - q_2 - \gamma q_1 - w q_2) q_2 \quad (1)$$

[4] Both the inverse demand function and the direct demand function are well behaved if the quadratic utility function is strictly concave. If the quadratic utility function is not *strictly* concave, the direct demand function need not be well defined. For our case of substitute goods with symmetric product differentiation, $\gamma \in (0, 1)$ implies that the quadratic utility function is strictly concave. See Amir et al. (2017) for micro foundation of the linear demand function.

$$\underset{q_2}{Max}\, \alpha(1 - q_1 - \gamma q_2 - wq_1)q_1 + (1 - \alpha)(1 - q_2 - \gamma q_1 - wq_2)q_2. \tag{2}$$

The equilibrium outputs are

$$q_1^* = q_2^* = \frac{1 - \alpha}{\gamma + 2(1 + w)(1 - \alpha)} \tag{3}$$

Lemma 1 *For a given input price, cross-holdings reduce the total outputs of firms 1 and 2.*

Proof We get $\frac{\partial q^*}{\partial \alpha} = -\frac{2\gamma}{(\gamma+2(1+w)(1-\alpha))^2} < 0$, where $q^* = q_1^* + q_2^*$.

Lemma 1 is the well-known effect of cross-holdings (Reynolds and Snapp 1986), where cross-holdings create the anti-competitive effect and reduce the total outputs.

The total input demand is $L = L_1 + L_2 = q_1^{*2} + q_2^{*2} = 2\left(\frac{1-\alpha}{\gamma+2(1+w)(1-\alpha)}\right)^2$. Firm I determines the input price to maximise the following expression:

$$\underset{w}{Max}\, 2(w - c)\left(\frac{1 - \alpha}{\gamma + 2(1 + w)(1 - \alpha)}\right)^2. \tag{4}$$

The equilibrium input price can be found as

$$w^* = \frac{2(1 - \alpha)(1 + 2c) + \gamma}{2(1 - \alpha)}. \tag{5}$$

Lemma 2 *A higher α increases the input price.*

Proof We get $\frac{\partial w^*}{\partial \alpha} = \frac{\gamma}{2(1-\alpha)^2} > 0$.

Higher cross-holdings make the input demand curve more inelastic and increase the input price.[5] This is in contrast to the extant literature where higher cross-holdings (or cooperation) among the final goods producers either do not affect the input price (Mukherjee 2010) or do not affect the weighted input price (Shuai et al. 2022).[6] This is due to the constant returns to scale technologies considered in those papers. Although Chen et al. (2021) found higher cross-holdings increase the input prices, it happens due to the cross-holdings between the input supplier and the final goods producer, which is not the case in our paper.

Lemmas 1 and 2 show the main trade-off for our analysis. On the one hand, cross-holdings reduce the total outputs in the final goods market, which tend to increase the

[5] The elasticity of the input demand curve is $\varepsilon_L = -\frac{\partial L}{\partial w}\frac{w}{L} = \frac{4w(1-\alpha)}{\gamma+2(1+w)(1-\alpha)}$ and $\frac{\partial \varepsilon_L}{\partial \alpha} = -\frac{4\gamma w}{(\gamma+2(1+w)(1-\alpha))^2} < 0$.

[6] Mukherjee (2010) found that a higher product market cooperation may increase or decrease the input price under bargaining between the input supplier and the final goods producers but the input price is unaffected by product market cooperation when the input supplier alone sets the input price, which is similar to our case.

profits of the final goods producers. On the other hand, cross-holdings increase the input prices, which tend to reduce the profits of the final goods producers.

Given the equilibrium input price, the equilibrium outputs of firms 1 and 2 are $q_1^* = q_2^* = \frac{1-\alpha}{2(2(1-\alpha)(1+c)+\gamma)}$ and the corresponding equilibrium profits are $\pi_1^* = \pi_2^* = \frac{(1-\alpha)(4+\gamma+4c(1-\alpha)-2\alpha(2-\gamma))}{8(2(1-\alpha)(1+c)+\gamma)^2}$. Hence, the total profits of firms 1 and 2 are

$$\pi^* = \pi_1^* + \pi_2^* = \frac{(1-\alpha)(4+\gamma+4c(1-\alpha)-2\alpha(2-\gamma))}{4(2(1-\alpha)(1+c)+\gamma)^2}. \tag{6}$$

Proposition 1 *Symmetric cross-holdings reduce the total profits of firms 1 and 2 compared to no cross-holding.*

Proof We get $\frac{\partial \pi^*}{\partial \alpha} = -\frac{\gamma(2(1-\alpha)(1+c)+\gamma(4\alpha-1))}{4(2(1-\alpha)(1+c)+\gamma)^3} < 0.$

Proposition 1 shows that the input price effect dominates the output effect and makes the cross-holding unprofitable.

If $\alpha = \frac{1}{2}$, there is a complete cooperation between firms 1 and 2. In this situation, the total profits of firms 1 and 2 are $\pi^*(\alpha = \frac{1}{2}) = \frac{1}{4(1+\gamma+c)}$, which is less than the total profits of firms 1 and 2 under no cross-holding, which is $\pi^*(\alpha = 0) = \frac{4+\gamma+4c}{4(2+\gamma+2c)^2}$. Hence, we get the following corollary immediately.

Corollary 1 *Complete cooperation between firms 1 and 2 is not profitable compared to non-cooperation.*

It can be shown that a higher product differentiation, i.e., a lower γ, increases profit of the ith firm, since $\frac{\partial \pi_i^*}{\partial \gamma} = \frac{-(1-\alpha)(2(1+c)(1-\alpha)(3-2\alpha)+(1+2\alpha)\gamma)}{8(2(1+c)(1-\alpha)+\gamma)^3} < 0$, $i = 1, 2$. Due to the symmetry, it then implies that a higher product differentiation increases the total profits of the firms.

2.1.1 Welfare implications

Now we show the welfare implications of cross-holdings considered above. Given the utility function, we get the equilibrium welfare as

$$W^* = U^* - c(L_1^* + L_2^*) = \frac{(1-\alpha)((7+6c)(1-\alpha)+(3+\alpha)\gamma)}{4(2(1+c)(1-\alpha)+\gamma)^2}.$$

We further get $\frac{\partial W^*}{\partial \alpha} = -\frac{\gamma((1-\alpha)(3+2c)+\gamma(1+\alpha))}{2(2(1+c)(1-\alpha)+\gamma)^3} < 0$. Hence, symmetric cross-holdings reduce welfare compared to no cross-holding. Collusive behaviour and higher input prices following cross-holdings reduce welfare under symmetric cross-holdings compared to no cross-holding.

It can be shown that a higher product differentiation increases welfare, since $\frac{\partial W^*}{\partial \gamma} = \frac{-(1-\alpha)(2(1-\alpha)(4+c(3-\alpha)-\alpha)+(3+\alpha)\gamma)}{4(2(1+c)(1-\alpha)+\gamma)^3} < 0.$

2.2 Unilateral change in cross-holding

The previous subsection shows that symmetric cross-holdings are not profitable. Now we want to show that cross-holdings will be unprofitable even if only one firm increases its shares in the rival firm, as considered, e.g., in Reitman (1994), Ghosh and Morita (2017), Ma et al. (2021) and Chen et al. (2021). To show this result for any combinations of shareholdings, assume that firm 1 holds $\alpha \in [0, \ 0.5]$ fraction of shares in firm 2 and firm 2 holds $\beta \in [0, \ 0.5]$ fraction of shares in firm 1. Without loss of generality, we will show how the total profits of firms 1 and 2 change with respect to α. The result will be similar if we have considered a change in β.

Consider the following game. Given the cross-holdings, in stage 1, firm I determines the input prices, w_1 and w_2, for firms 1 and 2 respectively.[7] In stage 2, firms 1 and 2 produce their outputs like Cournot duopolists and the profits are realised. We solve the game through backward induction.

Given the input prices w_1 and w_2, firms 1 and 2 determine their outputs to maximise the following expressions respectively:

$$\underset{q_1}{Max}(1-\beta)(1-q_1-\gamma q_2-w_1 q_1)q_1+\alpha(1-q_2-\gamma q_1-w_2 q_2)q_2 \tag{7}$$

$$\underset{q_2}{Max}\ \beta(1-q_1-\gamma q_2-w q_1)q_1+(1-\alpha)(1-q_2-\gamma q_1-w_2 q_2)q_2. \tag{8}$$

The equilibrium outputs are

$$q_1^*=\frac{(1-\alpha)(2(1+w_2)(1-\beta)-\gamma(1+\alpha-\beta))}{4(1+w_1)(1+w_2)(1-\alpha)(1-\beta)-\gamma^2\left(1-(\alpha-\beta)^2\right)} \tag{9}$$

$$q_2^*=\frac{(1-\beta)(2(1+w_1)(1-\alpha)-\gamma(1+\beta-\alpha))}{4(1+w_1)(1+w_2)(1-\alpha)(1-\beta)-\gamma^2\left(1-(\alpha-\beta)^2\right)}. \tag{10}$$

We find for the given input prices, $\frac{\partial q^*}{\partial\alpha}=-\frac{\gamma V}{\left(4(1+w_1)(1+w_2)(1-\alpha)(1-\beta)-\gamma^2\left(1-(\alpha-\beta)^2\right)\right)^2}$, where

$$\begin{aligned}V=&\gamma^2\left((1-\alpha)^2+2(1+\alpha)\beta-3\beta^2\right)+4(1+w_1)(1+w_2)(1-\beta)\left((1-\alpha)^2+\beta-\beta^2\right)\\&-2\gamma(2+w_1+w_2)(1-\beta)\left((-1+\alpha)^2+2\beta-\beta^2\right).\end{aligned}$$

The sign of $\frac{\partial q^*}{\partial\alpha}$ will be opposite of the sign of V. We get $\frac{\partial V}{\partial w_i}>0$, $i=1, 2$, and $\frac{\partial V}{\partial\gamma}<0$. Hence, V reaches minimum at $w_1=w_2=0$ and $\gamma=1$, and we get $V(w_1=w_2=0, \gamma=1)=(1-\alpha-\beta)^2>0$, implying $\frac{\partial q^*}{\partial\alpha}<0$. This is similar to Lemma 1, i.e., a higher cross-holding reduces the total outputs of firms 1 and 2 for the given input prices.

[7] Since we are considering asymmetric cross-holdings, it may be natural to consider the case of input price discrimination. The previous subsection could also consider w_1 and w_2. However, given the symmetry of firms 1 and 2 considered in that subsection, the equilibrium values of w_1 and w_2 will be equal to the equilibrium input price found there.

Given the input demand $L_1 = q_1^{*2}$ and $L_2 = q_2^{*2}$, the input supplier determines w_1 and w_2 to maximise $(w_1 - c)L_1 + (w_2 - c)L_2$. We find the equilibrium input prices as

$$w_1^* = \frac{1}{2}\left(2 + 4c + \gamma + \frac{\gamma\beta}{1-\alpha}\right) \quad \text{and} \quad w_2^* = \frac{1}{2}\left(2 + 4c + \gamma + \frac{\gamma\alpha}{1-\beta}\right). \tag{11}$$

It immediately follows from (11) that $\frac{\partial w_1^*}{\partial \alpha} > 0$ as long as $\beta > 0$, and $\frac{\partial w_2^*}{\partial \alpha} > 0$. This is similar to Lemma 2.

It is worth pointing out that $\frac{\partial w_1^*}{\partial \alpha} = 0$ for $\beta = 0$, i.e., if only firm 1 holds shares in firm 2. The reason for this is as follows. If $\beta = 0$, the equilibrium outputs for the given input prices are $q_1^* = \frac{2(1+w_2)-\gamma(1+\alpha)}{4(1+w_1)(1+w_2)-\gamma^2(1+\alpha)}$, $q_2^* = \frac{2(1+w_1)-\gamma}{4(1+w_1)(1+w_2)-\gamma^2(1+\alpha)}$ and the equilibrium input prices are $w_1^* = \frac{1}{2}(2(1+2c)+\gamma)$ and $w_2^* = \frac{1}{2}(2(1+2c)+\gamma(1+\alpha))$. Since firm 1 holds shares in firm 2, it produces less than firm 2 and therefore, demands less inputs than firm 2. It is immediate from the above expressions that, for the same input prices, the difference in the outputs of firms 1 and 2 is weighted by $\gamma\alpha$. Hence, to keep the same input demand from firms 1 and 2, the input supplier charges $\frac{\gamma\alpha}{2}$ more to firm 2. The input supplier charges firm 1 the input price that it would charge to firm 1 in the absence of cross-holding and increases the input price for firm 2 by $\frac{\gamma\alpha}{2}$. With these input prices, we get the equilibrium outputs of firms 1 and 2 as $q_1^* = q_2^* = \frac{1}{4+4c+\gamma(2+\alpha)}$.

Given the input prices, the total profits of firms 1 and 2 are

$$\pi^* = \frac{(1-\alpha)(1-\beta)\begin{pmatrix} 8(1-\alpha)(1-\beta) \\ +8c(1-\alpha)(1-\beta) \\ +\gamma\left(2+\alpha-3\alpha^2+\beta+2\alpha\beta-3\beta^2\right) \end{pmatrix}}{2\left(-(1-\alpha)(4+4c+\gamma(2+\alpha)) + (4+4c+\gamma-2(2+2c+\gamma)\alpha)\beta + \gamma\beta^2\right)^2}.$$

We get

$$\frac{\partial \pi^*}{\partial \alpha} = \frac{\gamma(1-\beta)\left((1-\alpha)^2 + \beta(1-\beta)\right)H}{2\left(4(1+c)(1-\alpha)(1-\beta) + \gamma\left(2-\alpha-\alpha^2-\beta-\beta^2+2\alpha\beta\right)\right)^3}, \tag{12}$$

where $H = \left(-4\left(1-\alpha\right)(1-\beta) - 4c(1-\alpha)(1-\beta) + \gamma\left(2-\alpha\left(5-3\alpha\right) - \beta(5 - 2\alpha) + 3\beta^2\right)\right)$. The sign of $\frac{\partial \pi^*}{\partial \alpha}$ is similar to the sign of H, $\frac{\partial H}{\partial c} < 0$ and $H|_{c=0} = \left[-4(1-\alpha)(1-\beta) + \gamma(2-\alpha(5-3\alpha) - 5\beta + 2\alpha\beta + 3\beta^2)\right] < 0$. Hence, $\frac{\partial \pi^*}{\partial \alpha} < 0$, i.e., a higher unilateral cross-holding by firm 1 reduces the total profits of firms 1 and 2.

Since $\frac{\partial \pi^*}{\partial \alpha} < 0$ for any combinations of $\alpha \in [0, 0.5]$ and $\beta \in [0, 0.5]$, and firms 1 and 2 are symmetric in all respects except for the values of α and β, the above analysis will suggest that we will also get $\frac{\partial \pi^*}{\partial \beta} < 0$. Hence, we get $\pi^*(\alpha = 0, \beta = 0) > \pi^*(\alpha > 0, \beta = 0) > \pi^*(\alpha > 0, \beta > 0)$. As an example, for $c = 0$ and $\gamma = 1$, we get $\pi^*|_{\alpha=0,\beta=0} = \frac{5}{36} > \pi^*|_{\alpha=0.1,\beta=0} = \frac{515}{3721} > \pi^*|_{\alpha=0.1,\beta=0.1} = \frac{27}{196}$.

We get the following result from the above discussion.

Proposition 2 *Unilateral cross-holding by a final goods producer in the rival firm reduces the total profits of the final goods producers compared to no cross-holding.*

2.2.1 Welfare implications

If $\alpha \in [0,\ 0.5]$ and $\beta \in [0,\ 0.5]$, we get the equilibrium welfare as

$$W^* = \frac{(1-\alpha)(1-\beta)\begin{pmatrix}(1-\alpha)(7+6c+\gamma(3+2\alpha)) \\ -(7+6c+\gamma-(7+6c+3\gamma)\alpha)\beta-2\gamma\beta^2\end{pmatrix}}{\left(4(1+c)(1-\alpha)(1-\beta)+\gamma\left(2-\alpha-\alpha^2-\beta-\beta^2+2\alpha\beta\right)\right)^2},$$

and $\frac{\partial W^*}{\partial \alpha} = -\frac{\begin{pmatrix}2\gamma(1-\beta)\left((1-\alpha)^2+\beta(1-\beta)\right) \\ \left((3+2c)(1-\alpha)(1-\beta)+\gamma\left(1-\alpha^2-\beta^2+\alpha\beta\right)\right)\end{pmatrix}}{\left(4(1+c)(1-\alpha)(1-\beta)+\gamma\left(2-\alpha-\alpha^2-\beta-\beta^2+2\alpha\beta\right)\right)^3} < 0$. Hence, a higher unilateral cross-holding reduces welfare compared to no cross-holding.

Like the case of symmetric cross-holdings, it can be shown that a higher product differentiation increases the profit of each firm and therefore, the total profits of the firms, and welfare under asymmetric cross-holdings.[8]

3 Conclusion

The common wisdom suggests that cross-holdings may not be profitable if it creates positive output externality on the outside firms. We provide a new reason for unprofitable cross-holdings. In a Cournot duopoly model, where cross-holdings do not create positive output externality on the outside firms, we show that cross-holdings can be unprofitable under strategic input price determination by an input supplier when the final goods are produced with decreasing returns to scale technologies. Our results hold under symmetric and asymmetric cross-holdings. We also show that cross-holdings reduce welfare in our analysis. Further, a higher product differentiation increases the profits of the firms and welfare.

We show in the Online Appendix that unprofitable cross-holdings also occur under Bertrand competition in the final goods market. If either the marginal cost of input production or product differentiation in the final goods market is sufficiently high, cross-holdings between the final goods producers are unprofitable if there is a Bertrand duopoly in the final goods market. Like the case of Cournot competition, cross-holdings also reduce welfare. However, unlike the case of Cournot competition, a higher product differentiation may reduce the profits of the firms and welfare.

Supplementary Information The online version contains supplementary material available at https://doi.org/10.1007/s40505-022-00241-3.

[8] Since the expressions are complicated, I don't show them here.

Data Availability statement No new data were created or analyzed in this study. Data sharing is not applicable to this article.

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