International Journal of Dynamics and Control
https://doi.org/10.1007/s40435-024-01430-7

Control of a three-wheeled omnidirectional mobile robot via a mixed FTB/$\mathcal{H}_\infty$ approach

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Received: 25 January 2024 / Revised: 25 March 2024 / Accepted: 7 April 2024

Abstract
In this paper, the problem of guidance and motion control of mobile robots is addressed and solved within the novel framework of the mixed finite-time/$\mathcal{H}_\infty$ control theory of nonlinear quadratic systems (NLQSs). Starting from a NLQS describing the dynamics of omnidirectional mobile platforms, the main tasks performed for controlling in closed loop the motion of omnidirectional robots can be conveniently formulated as a mixed finite-time/$\mathcal{H}_\infty$ control problem. A robust motion controller, which can effectively rejects disturbances deviating the robot platform from a planned path, can be designed after choosing a linear state-feedback structure for the controller. The synthesis problem is solved through some sufficient conditions contemplating both norm-bounded disturbances and sets constraining initial and terminal conditions, together with a finite-time bound on the output transient. Therefore, for all the allowable uncertainties, in presence of nonzero initial conditions and exogenous disturbance inputs which are possible within an unstructured environment, the motion control tasks can be accomplished through optimal $\mathcal{H}_\infty$ performance by simultaneously guaranteeing that the NLQS, which governs in closed loop the robot platform, is finite-time bounded. Finally, the applicability and control performance of the design approach have been evaluated through numerical simulations.

Keywords Nonlinear quadratic systems · Omnidirectional robots · Planning · Guidance and motion control · Bounded disturbance · Finite-time boundedness · LMIs

1 Introduction

This paper investigates the performance of a novel methodology for controlling nonlinear quadratic systems (NLQSs), based on a mixed approach combining finite-time control (FTC) and $\mathcal{H}_\infty$ control theory.

As for FTC, finite-time stability (FTS) can be traced back to the papers [1–3]; roughly speaking, given a zero-input dynamical system, FTS requires that the state trajectories, starting from any initial condition belonging to a preassigned inner set surrounding the origin, remains confined into a given outer set in a finite-time interval. The study of FTS has seen a resurgence in interest over the past twenty-five years, and numerous references on the topic have been made available in the literature (see, for instance, the book by Amato et al. [4] and the bibliography therein).

While FTS looks at the trajectories behavior in the absence of inputs, the definition of finite-time boundedness (FTB), given in [5], requires that the state remains bounded whenever both nonzero initial conditions and the presence of external disturbances are simultaneously assumed; the initial state is still unknown and constrained to belong to a prespecified set (typically an ellipsoid), while the disturbance has to be the member of a well defined class of input signals. A sufficient condition for FTB has been given, for linear time-invariant (LTI) systems under disturbances generated by an exo-system, in [5] where the properties of quadratic Lyapunov functions have been exploited. More recently, through an approach based on operator theory, a necessary and sufficient condition for FTB has been provided in [6], by assuming that the disturbance is a square-integrable signal; moreover, such condition deals with the more general setting of linear time-varying systems.

Within FTC, the definition of FTB can be seen as the generalization of the concept of input–output (IO)-FTS (see

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Published online: 04 May 2024

[7, 8]) when the initial condition is nonzero; for this reason FTB is a strongest property than IO-FTS, since it guarantees state boundedness in the simultaneous presence of disturbance inputs and nonzero initial conditions.

To highlight the novelty of our contribution in the context of existing literature, we recall that, so far, the prevalent approach to FTC has been often referred to linear systems, e.g., see [9–13] and the book [14]. Some relevant applications to the context of NLQSs concern robotic control tasks [15], biomolecular systems [16, 17] and epidemiological processes [18]; to this regard, see also the recent paper by Nesci et al. [19] for further theoretical advancements.

In a different context, the standard $\mathcal{H}_\infty$ control problem is solved by minimizing the $\mathcal{H}_\infty$-norm of the operator between the disturbance and the system output, see [20]. In this framework, the role of the transients is discarded since the minimization of the output energy, which can be achieved through the optimization of the $\mathcal{H}_\infty$ norm, cannot guarantee further constraints on the output peaks over finite-time intervals. In [21], the position control of a robotic manipulator having a planar kinematic chain has been recently framed into the $\mathcal{H}_\infty$ control of time-varying polytopic NLQSs.

The combination of the above-mentioned control techniques has led to the definition of IO-FTS with a $\mathcal{H}_\infty$ bound, originally stated in [22] for (uncertain) linear time-invariant systems, in order to provide simultaneous constraints which guarantee that the system output is energy bounded and its instantaneous values are point-wise bounded over a finite interval of time. The control technique proposed in [22] has been tested in a real engineering application, namely the vibration control of vehicle active suspensions (see [23]), since the mixed design conditions allow to improve the ride comfort and safety of the vehicle, through an attenuation of the perturbations due to road asperities on the dynamics of the car body, in the presence of further constraints on the maximum allowable excursion of the suspension stroke and thresholds on the saturating force actuators.

In this work, we aim at controlling a three-wheeled omnidirectional mobile robot, using a mixed FTB/$\mathcal{H}_\infty$ approach. As shown in the following sections, the robot dynamics can be described by means of a nonlinear quadratic dynamical model. Since, the results developed in [22] are only applicable to the class of linear systems, a key contribution of this work is to extend the results provided in [22] to the design of a state-feedback controller for NLQSs. Furthermore, we also replace the IO-FTS requirement, used in [22], with a FTB one, which enables taking into account also a constraint on the initial conditions of the system. Therefore, in the following, the proposed methodology is referred as the mixed FTB/$\mathcal{H}_\infty$ approach which—over finite-time interval—will guarantee, in the presence of an unknown initial condition and exogenous disturbances, a point-wise bound on the system state together with the satisfaction of a quadratic-integral performance measure.

Indeed, whenever a FTC point of view is exploited to approach to the $\mathcal{H}_\infty$ control problem, a performance index, which can take into explicit account the robustness properties versus all the allowable initial conditions of the system state, can be straightforwardly considered. To this regard, the work [24] contemplates nonzero conditions in the definition of a $\mathcal{H}_\infty$ performance measure of the form [1]

$$\sup_{w \in \mathcal{L}_2^{n_w}(\Omega), x_0 \in \mathcal{X}_0} \left\{ \left[\frac{\|y(\cdot)\|_\Omega^2 + x^T(T) S x(T)}{\|w(\cdot)\|_\Omega^2 + x_0^T R x_0} \right]^{1/2} \right\} < \gamma , \tag{1}$$

where $\mathcal{L}_2^n(\Omega)$ denotes the set of square integrable vector-valued signals with n components defined in the set Ω, and, given $z(\cdot) \in \mathcal{L}_2^n(\Omega)$,

$$\|z(\cdot)\|_\Omega := \left[\int_\Omega z^T(t) z(t) \mathrm{d}t \right]^{1/2} \tag{2}$$

represents the classical 2-norm in $\mathcal{L}_2^n(\Omega)$. In (1), γ is a positive scalar, $x(T)$ is the state vector at the terminal time T, and x_0 is the initial state vector taken from $\mathcal{X}_0 \subset \mathbb{R}^n$ representing a set of allowable initial conditions; the matrices R and S weight the initial and terminal states, respectively.

The left hand side in (1) can be regarded as the induced norm of the linear operator generated by the closed-loop system, which maps the pair $(x(0), w)$ to $(x(T), y)$, see [24]. The $\mathcal{H}_\infty$ performance bound expressed by condition (1) implies—under uncertainty of the initial condition—a constraint on the sensitivity of the system response, in terms of both terminal state and output energy, to energy-bounded disturbance input $w(\cdot)$ and uncertainty in the initial conditions.

To reach our goal, we shall follow a machinery that is reminiscent of the papers [25, 26], where the authors proposed some sufficient conditions for the finite-time stabilization via linear state feedback of NLQSs in order to minimize a quadratic cost function. To this regard, the recent work [27] has dealt with finite-time boundedness and stabilization of NLQSs in the presence of disturbance inputs; some state-dependent conditions, which are based on annihilators (see [28]) and Finsler's lemma, have been provided therein.

A preliminary version of the theoretical results provided in the present paper has been presented in [29].

The reminder of this paper is as follows. Section 2 provides some preliminary definitions and the problem statement. In

[1] *Notation:* The symbol $\mathcal{L}_2^{n_w}(\Omega)$ denotes the subspace of vector-valued functions in $\mathbb{R}^{n_w}$ which are square-integrable over the time interval $\Omega := [0, T]$ with Euclidean vector norm $\|f\|_\Omega = [\int_\Omega \|f(\sigma)\|^2 \mathrm{d}\sigma]^{1/2}$

Sect. 3, the guidance and motion control of omnidirectional mobile robots is formulated within the framework of mixed FTB/$\mathcal{H}_\infty$ control of NLQSs. A sufficient condition for the existence of a mixed FTB/$\mathcal{H}_\infty$ controller, which represents the main contribution of this work, is reported in Sect. 4. The applicability and usefulness of the conceived design methodology are shown in Sect. 5, whereas the conclusions are reported in Sect. 6.

2 Mixed FTB/$\mathcal{H}_\infty$ control of NLQSs

2.1 Definition of nonlinear quadratic systems with disturbance input

Let us define a NLQS as

$$\dot{x} = Ax + B(x)x + Fu + N(x)u + Gw\,,\ x(0) = x_0 \tag{3a}$$
$$y = Cx\,, \tag{3b}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^p$ is the control input, $w(t) \in \mathbb{R}^{n_w}$ is an exogenous disturbance,

$$B(x) = \begin{pmatrix} B_1^T x & B_2^T x & \dots & B_n^T x \end{pmatrix}^T\,, \tag{4}$$

with $B_i \in \mathbb{R}^{n\times n}$, $i = 1, \dots, n$, and

$$N(x) = \begin{pmatrix} N_1^T x & N_2^T x & \dots & N_n^T x \end{pmatrix}^T\,,$$

with $N_i \in \mathbb{R}^{n\times m}$, $i = 1, \dots, n$.
With a time interval taken as $\Omega := [0, T]$, it assumed that the disturbance input $w(\cdot)$ belongs to the class of square-integrable signals

$$\mathcal{W} := \{w(t) \in \mathcal{L}_2^{n_w}(\Omega) : \|w(t)\|_\Omega^2 \le d\,,\ d > 0\}\,. \tag{5}$$

2.2 Problem statement

The mixed FTB/$\mathcal{H}_\infty$ control problem is stated here for NLQSs; a linear state feedback controller is adopted in the form

$$u(t) = Kx(t)\,, \tag{6}$$

where $K \in \mathbb{R}^{m\times n}$ is the control gain matrix. By interconnecting system (3) with the controller (6), the closed-loop system is obtained as

$$\dot{x} = A_{cl}x + B_{cl}(x)x + Gw \tag{7a}$$
$$y = Cx\,, \tag{7b}$$

where

$$A_{cl} := A + FK\,,\quad B_{cl}(x) := B(x) + N(x)K\,. \tag{8}$$

Preliminarily to the definition of the mixed FTB/$\mathcal{H}_\infty$ control problem, it is needed to recall from [5, 6] the definition of FTB.

Definition 1 (*Finite-time boundedness*) Given the time interval $\Omega := [0, T]$, some positive definite matrices Γ_0 and Γ, such that $\Gamma < \Gamma_0$, system (3), with $u(t) = 0$, is said to be finite-time bounded wrt $(\Omega, \Gamma_0, \Gamma)$ if, for any disturbance $w(\cdot) \in \mathcal{W}$, the following holds

$$x_0^T\Gamma_0 x_0 \le 1 \Rightarrow x^T(t)\Gamma x(t) < 1, \forall t \in \Omega.$$

$\diamondsuit$

Within the mixed FTB/$\mathcal{H}_\infty$ control theory, both the analysis and synthesis problems can exploit the following cost function

$$J(\Omega, R, S) := \|y(\cdot)\|_\Omega^2 + x^T(T)Sx(T) - \gamma^2\left(\|w(\cdot)\|_\Omega^2 + x_0^T Rx_0\right)\,, \tag{9}$$

where $\gamma > 0$ is the $\mathcal{H}_\infty$-performance gain. The cost function (9) defines a quadratic-integral performance index weighting the energies of the output and disturbance input, respectively. Moreover, the matrices R and S, which are positive definite, weight the effects that both the uncertainty of the initial conditions and of the terminal state have on the transient performance.

Through a formalization in terms of $\mathcal{L}_2$-gain, it is possible to extend the $\mathcal{H}_\infty$-norm control to NLQSs (see [25]). Therefore, given Ω and the scalar $\gamma > 0$, the existence of a state feedback control law in the form (6), such that

$$J(\Omega, R, S) < 0\,, \forall w(\cdot) \in \mathcal{W}\,, x_0 \in \mathcal{X}_0\,, \tag{10}$$

implies that

$$\sup_{w\in\mathcal{W}, x_0\in\mathcal{X}_0}\left\{\left[\frac{\|y(\cdot)\|_\Omega^2 + x^T(T)Sx(T)}{\|w(\cdot)\|_\Omega^2 + x_0^T Rx_0}\right]^{1/2}\right\} < \gamma.$$

The definition of the performance index in (10) is well-posed under the hypothesis that the NLQS exhibits FTB; therefore, in the presence of any exogenous disturbance $w(\cdot) \in \mathcal{W}$, over the interval Ω, the system state remains bounded for a prescribed set of initial conditions.

The ellipsoidal sets, which are induced by the matrices Γ_0 and Γ in Definition 1, are defined as

$$\mathcal{E}_0 := \{x \in \mathbb{R}^n \ : \ x^T\Gamma_0 x \leq 1, \ \Gamma_0 > 0\}\,, \tag{11a}$$
$$\mathcal{E}_\Omega := \{x \in \mathbb{R}^n \ : \ x^T\Gamma x < 1, \ 0 < \Gamma < \Gamma_0\}\,. \tag{11b}$$

To clarify the relationship between FTB and the $\mathcal{H}_\infty$-norm performance index (9), the following definition is needed.

Definition 2 Consider system (3) subject to a disturbance $w(\cdot) \in \mathcal{W}$. Given the time interval Ω, the cost index (9), a positive scalar γ, the ellipsoidal sets (11) and the positive definite matrices R and S, the controller (6) is said to be a mixed FTB/$\mathcal{H}_\infty$ controller for the NLQS (3) if the following hold:

(i) The closed-loop system (7) is FTB wrt $(\Gamma_0, \Gamma, \Omega)$;
(ii) The performance index (9) satisfies, for the closed-loop system (7),

$$J(\Omega, R, S) < 0, \forall w(\cdot) \in \mathcal{W}, x_0 \in \mathcal{E}_0.$$

$\Diamond$

Remark 1 The concept of mixed FTB/$\mathcal{H}_\infty$, which contemplates both nonzero initial conditions and $\mathcal{L}_2$ norm-bounded disturbances, is a novelty for NLQSs. Moreover, to extend prior results available only for linear systems, we propose a sufficient condition assuring simultaneously, for a NLQS under study, both a prescribed $\mathcal{H}_\infty$ performance index over finite time and FTB. Instead, in the case of mixed IO-FTS/$\mathcal{H}_\infty$ for linear systems, some distinct conditions, on $\mathcal{H}_\infty$ and IO-FTS, respectively, are separately invoked (see Amato et al. [14, p. 71]). $\triangle$

2.3 Some preliminary lemmas

Some technical lemmas, which are useful to derive the main result, are presented in what follows.

Lemma 1 *Consider the NLQS (3) subject to any disturbance input $w(\cdot) \in \mathcal{W}$. Given the time interval $\Omega := [0, T]$, the performance measure (9), a positive scalar γ, the ellipsoidal sets (11) and the positive definite matrices R and S, assume there exist a symmetric positive definite matrix Q, a positive scalar α and a matrix K, such that*

$$\begin{pmatrix} x \\ w \end{pmatrix}^T \begin{pmatrix} \Theta(x) & PG \\ G^T P & -\gamma^2 I \end{pmatrix} \begin{pmatrix} x \\ w \end{pmatrix} < 0\,, \quad \forall x \in \mathcal{E}_\Omega \tag{12a}$$
$$S < P < \gamma^2 R \tag{12b}$$
$$\Gamma < P < \Gamma_0 \tag{12c}$$
$$\left(\frac{1}{\lambda_{min}(Q)} + d\gamma^2\right) e^{\alpha T} \leq 1\,, \tag{12d}$$

where

$$\Theta(x) := PA_{cl} + A_{cl}^T P + PB_{cl}(x) + B_{cl}^T(x)P + C^T C\,, \tag{13}$$

$P = \Gamma_0^{\frac{1}{2}} Q^{-1} \Gamma_0^{\frac{1}{2}}$, *and $\lambda_{min}(\cdot)$ indicates the minimum eigenvalue of the argument. Then, the state feedback controller (6) is a mixed FTB/$\mathcal{H}_\infty$ controller for the NLQS (3).* ■

Proof Let us consider a quadratic Lyapunov function $v(x) = x^T Px$; its time derivative along the trajectories of the closed-loop system reads

$$\dot{v}(x) = x^T \left\{ PA_{cl} + A_{cl}^T P + PB_{cl}(x) + B_{cl}^T(x)P \right\} x + x^T PGw + w^T G^T Px\,. \tag{14}$$

The cost function (9), for any trajectory $x(\cdot)$ starting from $x_0 \in \mathcal{E}_0$, is such that

$$\begin{aligned} &J(\Omega, R, S) \\ &:= \int_\Omega \left(y(\tau)^T y(\tau) - \gamma^2 w^T(\tau) w(\tau) + \dot{v}(x(\tau)) \right) \mathrm{d}\tau \\ &\quad + x^T(T)Sx(T) - \gamma^2 x_0^T R x_0 - \int_\Omega \dot{v}(x(\tau))\mathrm{d}\tau \\ &= \int_\Omega \left(y(\tau)^T y(\tau) - \gamma^2 w^T(\tau) w(\tau) + \dot{v}(x(\tau)) \right) \mathrm{d}\tau + \\ &\quad + x^T(T)Sx(T) - \gamma^2 x_0^T R x_0 - x^T(T)Px(T) + x_0^T P x_0 \\ &= \int_\Omega \left(y(\tau)^T y(\tau) - \gamma^2 w^T(\tau) w(\tau) + \dot{v}(x(\tau)) \right) \mathrm{d}\tau \\ &\quad + x^T(T)\,(S - P)\,x(T) + x_0^T \left(P - \gamma^2 R\right) x_0 \\ &< \int_\Omega (y^T(\tau)y(\tau) - \gamma^2 w^T(\tau)w(\tau) + \dot{v}(x(\tau)))\mathrm{d}\tau\,, \end{aligned} \tag{15}$$

where the last inequality follows from the fact that the matrices $S - P$ and $P - \gamma^2 R$ are both negative definite in view of (12b).

From (15), a sufficient condition for $J(\cdot, \cdot, \cdot) < 0$ is

$$\dot{v}(x(t)) < -y^T(t)y(t) + \gamma^2 w(t)^T w(t), \ \forall t \in \Omega, \ w(\cdot) \in \mathcal{W}\,. \tag{16}$$

The satisfaction of condition ii) in Definition 2 can be inferred after recognizing that, from (14), condition (12a) implies (16) over Ω if any trajectory $x(\cdot)$ starting from $x_0 \in \mathcal{E}_0$ remains confined in the set $\mathcal{E}_\Omega$.

Now, we are ready to show that (12a) and (12d) assure condition i) of Definition 2. Since, in the first block at the left hand side of condition (12a), we have that $C^T C > 0$, it is readily seen that

$$\begin{pmatrix} PA_{cl} + A_{cl}^T P + PB_{cl}(x) + B_{cl}^T(x)P & PG \\ G^T P & -\gamma^2 I \end{pmatrix} < 0\,, \quad \forall x \in \mathcal{E}_\Omega\,. \tag{17}$$

By adding $-\alpha P$ to the first block at the LHS of (17), the above inequality implies

$$\begin{pmatrix} PA_{cl} + A_{cl}^T P + PB_{cl}(x) + B_{cl}^T(x)P - \alpha P & PG \\ G^T P & -\gamma^2 I \end{pmatrix} < 0\,, \quad \forall x \in \mathcal{E}_\Omega\,. \tag{18}$$

Condition (18) implies that

$$\dot{v}(x(t)) < \alpha v(x(t)) + \gamma^2 w(t)^T w(t)\,, \quad \forall t \in \Omega\,. \tag{19}$$

Given the candidate Lyapunov function, the induced ellipsoidal set is defined as

$$\mathcal{E} := \{x \in \mathbb{R}^n : x^T P x < 1\}\,. \tag{20}$$

Condition (12c) guarantees

$$\mathcal{E}_0 \subset \mathcal{E} \subset \mathcal{E}_\Omega\,. \tag{21}$$

Following similar arguments to those ones of the proof of Theorem 1 in [26], by contradiction, we shall show that any trajectory starting from $\mathcal{E}_0$ cannot exit $\mathcal{E}$ for any $t \in \Omega$.

Indeed, let us assume the existence a time instant $t^\star \in \Omega$ such that $x(t^\star)$ belongs to the boundary of $\mathcal{E}$, it follows that

$$x^T(t^\star)Px(t^\star) = 1\,. \tag{22}$$

From the inclusions $x(0) \in \mathcal{E}_0 \subset \mathcal{E}$ and continuity over time of the state trajectories, it is always possible to find a time instant $0 < \tilde{t} \le t^\star$ such that

(i) $x^T(\tilde{t})Px(\tilde{t}) = 1$;
(ii) $x(t) \in \mathcal{E}\ \forall t \in [0, \tilde{t}]$.

From (12c), condition (19) is fulfilled for all $t \in [0, \tilde{t}]$ if $\mathcal{E} \subset \mathcal{E}_\Omega$. By multiplying both sides of (19) by $e^{-\alpha t}$, we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}(e^{-\alpha t}v(x(t))) < e^{-\alpha t}\gamma^2 w^T(t)w(t);$$

after integrating the above inequality from 0 to t, with $t \in [0, \tilde{t}]$, it is readily obtained that

$$v(x) < e^{\alpha t}\left(v(x(0)) + \gamma^2 \int_0^t e^{-\alpha\sigma} w^T(\sigma)w(\sigma)\mathrm{d}\sigma\right)\,. \tag{23}$$

From conditions (23) and (12d), it follows, for all $t \in [0, \tilde{t}]$,

$$\begin{aligned} v(x(t)) &= x(t)^T P x(t) \\ &< e^{\alpha t}\left(x(0)^T P x(0) + \gamma^2 \int_0^t e^{-\alpha\sigma} w^T(\sigma)w(\sigma)\mathrm{d}\sigma\right) \\ &= e^{\alpha t}\left(x(0)^T \Gamma_0^{1/2} Q^{-1} \Gamma_0^{1/2} x(0) + \gamma^2 \int_0^t e^{-\alpha\sigma} w^T(\sigma)w(\sigma)\mathrm{d}\sigma\right) \\ &\le e^{\alpha t}\left(\lambda_{max}(Q^{-1})x(0)^T \Gamma_0 x(0) + \gamma^2 \int_0^t e^{-\alpha\sigma} w^T(\sigma)w(\sigma)\mathrm{d}\sigma\right) \\ &\le e^{\alpha T}\left(\lambda_{max}(Q^{-1}) + d\gamma^2\right) \le 1\,. \end{aligned} \tag{24}$$

The satisfaction of (24) at $t = \tilde{t}$ implies

$$x^T(\tilde{t})Px(\tilde{t}) < 1\,, \tag{25}$$

which contradicts (22). Therefore, a time point $t^\star \in \Omega$ satisfying (22) does not exist, and $x(t)$ cannot exit the set $\mathcal{E}$ for $t \in [0, T]$. The proof follows from the fact that $\mathcal{E}$ is a subset of $\mathcal{E}_\Omega$.

Although Lemma 1 provides a sufficient condition guaranteeing that a mixed $FTB/\mathcal{H}_\infty$ controller for the NLQS (3) exists, it cannot be exploited in practice, since the LHS of the LMI (12a) depends on x. The effort of the next section will be devoted to translate conditions (12) into a set of algebraic LMIs.

To this end, we need to recall the following lemmas.

Lemma 2 ([30]) *Consider the positive definite matrix $P \in \mathbb{R}^{n\times n}$, and a vector ν such that $\|\nu\| = 1$. Then, any point on the boundary of the ellipsoid (20) can be parameterized as $x = P^{-1/2}\Lambda\nu$, with Λ any matrix such that $\Lambda^T\Lambda = I$.* ■

Lemma 3 ([31]) *Given any scalar $\epsilon > 0$, a triplet of matrices of compatible dimensions Φ, Θ and Π, with $\Pi^T\Pi \le I$, then*

$$\Theta\Pi\Phi + \Phi^T\Pi^T\Theta^T \le \epsilon^{-1}\Theta\Theta^T + \epsilon\Phi^T\Phi.$$

■

3 Formulation of the control problem for omnidirectional mobile robots

3.1 NLQS modeling of three-wheeled robot platforms

In [32], the authors have derived the motion equations of omnidirectional mobile robot platforms consisting of three lateral orthogonal-wheel assemblies distributed at an equal distance from the center of gravity of the robot. Each assembly is configured to reproduce the behavior of an ideal spherical wheel. As an alternative, three mechanic wheels could be adopted without the necessity of complex wheel assemblies (see Fig. 1).

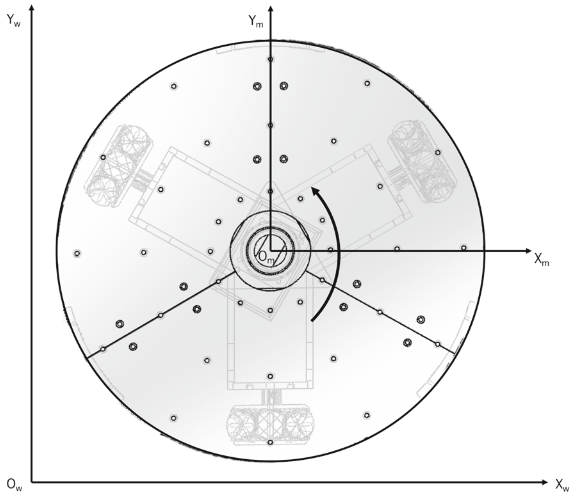

Fig. 1 Kinematics of the three-wheeled omnidirectional mobile robot

Through the on-board installation of an inertial measurement unit (IMU), it is possible to acquire all the feedback variables required for the implementation of a tracking control law (6) for the omnidirectional platform. Therefore, by adopting the mobile reference system in Fig. 1, the state vector can be taken as $x = \left(\dot{x}_m \; \dot{y}_m \; \dot{\phi}\right)^T$.

Starting from the motion equations expressed in the mobile reference system on the robot frame (see Watanabe et al. [32, p. 319]), the dynamics of a three-wheeled omnidirectional robot can be described through a NLQS (3), where the matrices in the state equations (3a) are given by

$$
\begin{aligned}
A &= \begin{pmatrix} a_1 & 0 & 0 & 0 \\ 0 & a_1 & 0 & 0 \\ 0 & 0 & a_3 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \\
B_1 &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -a_2/2 & 0 \\ 0 & -a_2/2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \\
B_2 &= \begin{pmatrix} 0 & 0 & a_2/2 & 0 \\ 0 & 0 & 0 & 0 \\ a_2/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad B_3 = 0_{4\times 4}, \\
N_1 &= 0_{4\times 3}, \quad N_2 = 0_{4\times 3}, \quad N_3 = 0_{4\times 3}, \\
G &= \begin{pmatrix} g \\ g \\ 0 \\ 0 \end{pmatrix}, F = \begin{pmatrix} -b_1 & -b_1 & 2b_1 \\ \sqrt{3}b_1 & -\sqrt{3}b_1 & 0 \\ b_2 & b_2 & b_2 \\ 0 & 0 & 0 \end{pmatrix}.
\end{aligned} \tag{26}
$$

with

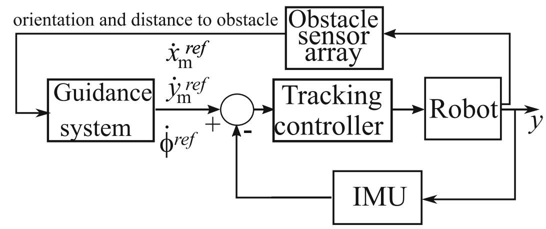

Fig. 2 Block scheme representation of the guidance and tracking control system of the omnidirectional robot

$$
\begin{aligned}
a_1 &= -\frac{3c}{3I_w + 2Mr^2}, \quad a_{2p} = \frac{2Mr^2}{3I_w + 2Mr^2}, \\
a_3 &= -\frac{3cL^2}{3I_w L^2 + I_v r^2}, \quad b_1 = \frac{kr}{3I_w + 2Mr^2}, \\
b_2 &= \frac{krL}{3I_w L^2 + I_v r^2}, \\
g &= \frac{1}{3I_w + 2Mr^2} \quad a_2 = 1 - a_{2p}.
\end{aligned} \tag{27}
$$

The parameter I_v is the moment of inertia of the robot, I_w is the moment of inertia of the wheel around the driving shaft, c the viscous friction factor of the wheel, m the mass of the robot, L the distance between any wheel assembly and the center of gravity of the robot, k the driving gain factor, r the radius of the wheel and $u_i, i = 1, \ldots, 3$, are the driving input torque on each wheel.

3.2 Guidance and tracking control system

The application to the quadratic model of a three-wheeled omnidirectional of the proposed mixed FTB/$\mathcal{H}_\infty$ control framework can help to evaluate its usefulness in a real case-study.

The main components and line signals of the closed-loop system implementing the guidance and tracking control of the robot are represented in the block diagram of Fig. 2. In an unstructured environment, the robot can proceed forward along a desired direction by keeping the cruise velocity; in the presence of an obstacle or in the case of an unwanted configuration of the robot, the guidance controller must update the set-point values of the rotational and translation velocities, which are sent to the tracking controller, to reconfigure the path of the robot platform. The presence of an obstacle along the forward direction of the path can be detected through an array of optical or ultrasonic sensors placed on the anterior portion of the robot frame.

3.3 Mixed FTB/$\mathcal{H}_\infty$ tracking controller

The control problem for the motion tracking of the omnidirectional robot can be conveniently formulated as a mixed FTB/$\mathcal{H}_\infty$ control problem.

Indeed, to effectively attenuate the impact of the exogeneous inputs and uncertainty on the initial conditions over finite time, and to robustly follow the planned path, the tracking controller has to compensate some bounded-energy disturbances, also when such perturbations are of short duration and large magnitude. In this case, to gain the desired robustness, the control performance has to be mandatorily optimized over the transient.

Both the robustness and accuracy of the tracking control performance can be optimized after defining a $\mathcal{H}_\infty$ cost function (9), in order to minimize the sensitivity of the tracking error to the energy-bounded disturbances perturbing the trajectory of the omnidirectional robot. Moreover, by specifying in (9) a set of non-null initial conditions of the kinematic variables, on which the mixed FTB/$\mathcal{H}_\infty$ control performance has to be guaranteed, it is possible to enlarge the operating range of the closed-loop control system.

By exploiting the results presented in Sect. 4, after recasting the quadratic model of a guidance and tracking control system of omnidirectional mobile robots in the form (7), it is possible to design a state-feedback tracking controller having the properties stated in Definition 2.

4 Mixed FTB/$\mathcal{H}_\infty$ controller design

According to Definition 2, the existence of a mixed FTB/$\mathcal{H}_\infty$ controller for NLQSs can be proved through the sufficient condition provided by the following Theorem.

Theorem 1 *Given the NLQS (3) subject to any disturbance $w(\cdot) \in \mathcal{W}$, the time interval Ω, the performance measure (9), a positive scalar γ, a pair of positive definite matrices R and S, and the admissible sets $\mathcal{E}_0$ and $\mathcal{E}_\Omega$ defined in (11), let $X = \Gamma_0^{-\frac{1}{2}} Q \Gamma_0^{-\frac{1}{2}}$, and assume there exist positive scalars α and ϵ, a matrix L and a positive definite matrix Q such that*

$$\begin{pmatrix} \Gamma & I \\ I & X \end{pmatrix} < 0 \tag{28a}$$

$$\begin{pmatrix} \Gamma_0 & I \\ I & X \end{pmatrix} > 0 \tag{28b}$$

$$\begin{pmatrix} S & I \\ I & X \end{pmatrix} < 0 \tag{28c}$$

$$\begin{pmatrix} \gamma^2 R & I \\ I & X \end{pmatrix} > 0 \tag{28d}$$

$$\left(\frac{1}{\lambda_{min}(Q)} + d\gamma^2 \right) e^{\alpha T} \leq 1 \tag{28e}$$

$$\begin{pmatrix} AX + FL + (AX+FL)^T + \epsilon I & G & XC^T & \Xi \\ G^T & -\gamma^2 I & 0 & 0 \\ CX & 0 & -I & 0 \\ \Xi^T & 0 & 0 & -\epsilon\widehat{\Gamma} \end{pmatrix} < 0\,, \tag{28f}$$

with $\Xi := \left((B_1 X + N_1 L)^T \, \ldots \, (B_n X + N_n L)^T \right)$ and $\widehat{\Gamma} = I \otimes \Gamma$; then, letting $K = LX^{-1}$, (6) is a mixed FTB/$\mathcal{H}_\infty$ control law.

Proof Starting again from the candidate Lyapunov function $v(x) = x^T P x$ and the set (20) induced by $v(x)$, Lemma 1 provides the main arguments used in this proof.

First, by exploiting the results in Lemmas 2 and 3 and after recognizing an affine structure in the involved matrix functions, we shall show that (28f) implies (12a).

To this aim, as first intermediate step, it is needed to show that (28a) and (28b) imply (21). Exploiting Schur complements (see Boyd et al. [33, p. 7]), from (28a) we obtain that $X^{-1} > \Gamma$; in the same way, we have that (28b) implies $X^{-1} < \Gamma_0$. From the inversion $P = X^{-1}$, inclusions (21) hold.

Now we prove that (28f) implies (12a). Condition (12a) is assured if

$$\begin{pmatrix} \Theta(x) & PG \\ G^T P & -\gamma^2 I \end{pmatrix} < 0\,, \quad \forall x \in \mathcal{E}_\Omega\,, \tag{29}$$

with $\Theta(x)$ given in (13). Through pre- and post-multiplying of the LHS in (29) by

$$\begin{pmatrix} P^{-1} & 0 \\ 0 & I \end{pmatrix}$$

then letting $X = P^{-1}$ and $KP^{-1} = L$ and applying Schur complements, condition (29) can be equivalently rewritten

$$\begin{pmatrix} AX+FL+(AX+FL)^T+\Psi x+x^T\Psi^T & G & XC^T \\ G^T & -\gamma^2 I & 0 \\ CX & 0 & -I \end{pmatrix} < 0\,, \quad \forall x \in \mathcal{E}_\Omega\,, \tag{30}$$

where $\Psi := \left((B_1 X + N_1 L)^T \, \ldots \, (B_n X + N_n L)^T \right)$.

Note that since $\mathcal{E}_\Omega$ is a convex set and the left hand side in (30) depends linearly on x, condition (30) holds for all x in the set $\mathcal{E}_\Omega$ if and only if it is satisfied on the boundary of $\mathcal{E}_\Omega$. Now, by virtue of Lemma 2, any point x on such boundary can be parameterized as $x = \Gamma^{-1/2} U v$, $U^T U = I$, $\|v\| = 1$, and therefore, for any positive ϵ, we have

$$\begin{aligned} \Psi x + x^T \Psi^T &= \Psi \Gamma^{-1/2} U v + v^T U^T \Gamma^{-1/2} \Psi^T \\ &\leq \epsilon^{-1} \Psi \Psi^T + \epsilon v^T U^T U v \\ &\leq \epsilon^{-1} \Psi \Psi^T + \epsilon I \end{aligned} \tag{31}$$

From (30) and (31), by exploiting Schur complement arguments, it is readily seen that condition (28f) implies (12a). Moreover, through further application of the Schur complements, the satisfaction of (12b) follows from (28c) and (28d). With $\mathcal{E} \subset \mathcal{E}_\Omega$ in force and from (28e), the proof follows.

Remark 2 Note that condition (28e) can be satisfied by imposing

$$Q \geq \frac{1}{e^{-\alpha T} - d\gamma^2} I.$$

For a fixed value of α, the conditions of Lemma 1 can be solved as a LMI optimization problem; a one-parameter search is used to find an optimal value for α. △

5 Control law design and simulation results

Thanks to the omnidirectionality of the robot platform, the commands which are updated by the guidance controller can be independently actuated through the following acts of motion:

- Orientation: the robot must rotate around its vertical axis in order to give to the mobile reference system the desired orientation.
- Linear motion: the robot is commanded to follow a straight line keeping the desired direction.

The task that we shall accomplish for the testing of the control performance—in closed loop—is the tracking of a straight line along a desired direction, which consists of a basic movement for the navigation in an unstructured environment.

Therefore, to robustly follow the desired direction by following a rectilinear path at cruise velocity, zero set-point values can be assumed for the linear transversal and rotational velocities, taking into account that the output components of the model consist of two translation velocities $\dot{x}_m$ and $\dot{y}_m$ with respect to the transversal x_m- and longitudinal y_m- axes, respectively, and the rotation velocity $\dot{\phi}$ about the z-axis of the robot platform.

We want the robot to move freely along the vertical axis, rejecting disturbances that would cause it to depart from its planned path. In order to maintain the planned direction, it is important to ensure that the controller is sufficiently robust to guarantee that the velocity $\dot{x}_m$ has a value sufficiently close to zero over finite-time.

The values of the physical parameters in (26)–(27) are taken as

$$I_v = 11.25\,\text{kgm}^2, \quad I_w = 0.02108\,\text{kgm}^2, \quad c = 5.983\,\text{kg/s}$$
$$M = 9.4\,\text{kg}, \quad L = 0.178\,\text{m}, \quad k = 1.0, \quad r = 0.0245\,\text{m},$$

and zero initial conditions are assumed.

The proposed framework enables to optimize the control action by prescribing a maximum amplitude of each component of $u(t)$; thus, the loop shaping method has been implemented. Define then the transfer function that weights the sensitivity function as

$$\frac{Y(s)}{U(s)} = \frac{s/M_{\max} + w_B}{s + w_B M_{\min}} \tag{32}$$

where $M_{\max} = 2$ dB is the high-frequency gain, $M_{\min} = 1/100$ dB is the low-frequency gain and $w_B = 1\,\text{rad/s}$ is the gain crossover frequency. The corresponding differential equations are

$$\begin{cases} \dot{\xi} = -w_B M_{\min}\xi + w_B(\frac{M_{\min}}{M_{\max}} + 1)\nu \\ z = \xi + \frac{1}{M_{\max}}\nu \end{cases} \tag{33}$$

From the series interconnection between (3) and (33) the augmented system reads

$$\begin{pmatrix} \dot{x} \\ \dot{\xi} \end{pmatrix} = \left(\begin{array}{c|c} A + B(x) & \emptyset \\ \hline w_B(\frac{M_{\min}}{M_{\max}} + 1)C & -w_B M_{\min} \end{array}\right) \begin{pmatrix} x \\ \xi \end{pmatrix} + \begin{pmatrix} F + N(x) \\ \hline \emptyset \end{pmatrix} u + \begin{pmatrix} G \\ \hline \emptyset \end{pmatrix} w \tag{34}$$

$$\begin{pmatrix} y \\ z \end{pmatrix} = \left(\begin{array}{c|c} C & \emptyset \\ \hline \frac{1}{M_{\max}}C & I \end{array}\right) \begin{pmatrix} x \\ \xi \end{pmatrix} \tag{35}$$

To accelerate the system response, the variable to be controlled was chosen to be an order of magnitude more than the gain crossover frequency

$$C = \begin{pmatrix} 10 & 0 & 0 & 0 \end{pmatrix},$$

A time bound $T = 100$ is chosen, with the matrices in (11) given by

$$\Gamma_0 = \begin{pmatrix} 95.19 & 0 & 0 & 0 & 0 \\ 0 & 95.19 & 0 & 0 & 0 \\ 0 & 0 & 95.19 & 0 & 0 \\ 0 & 0 & 0 & 95.19 & 0 \\ 0 & 0 & 0 & 0 & 95.19 \end{pmatrix},$$

$$\Gamma = \begin{pmatrix} 0.4 & 0 & 0 & 0 & 0 \\ 0 & 0.48 & 0 & 0 & 0 \\ 0 & 0 & 0.40 & 0 & 0 \\ 0 & 0 & 0 & 0.48 & 0 \\ 0 & 0 & 0 & 0 & 0.40 \end{pmatrix},$$

A bounded-energy disturbance belonging to the class of signal in (5), with $d = 1.5 \cdot 10^3$, is assumed to be in input to the control system. The input signal is a step having an unitary amplitude. Such perturbation can represent an external force

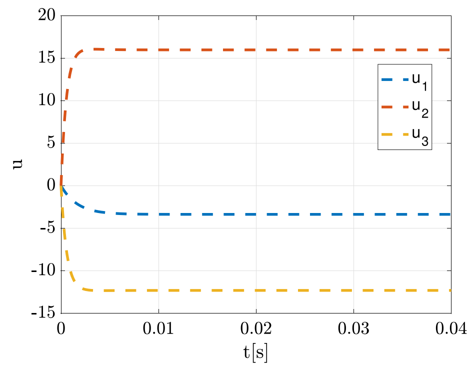

Fig. 3 Control Input of the closed-loop system with loop shaping on mixed H_∞/FTS

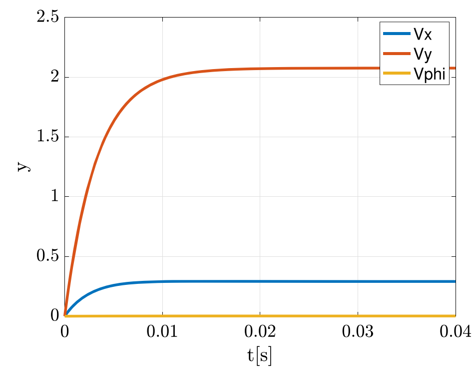

Fig. 4 Closed-loop system output velocity profiles with loop shaping on mixed H_∞/FTS

generating a lateral drift on the robot wheels, e.g., due to an alteration in the wheels grip and friction with the ground. This implies that the robot trajectory can be modified relatively to the rectilinear path.

The controller is designed on the basis of the result in Theorem 1. Therefore, a maximum allowable $\mathcal{H}_\infty$-performance gain is chosen as $\gamma = 3.1$, in order to guarantee robustness against disturbance input and possible uncertainties starting from non-null initial conditions. The matrices, which weight initial and terminal conditions, are chosen as

$$R = \begin{pmatrix} 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.5 \end{pmatrix} \cdot, \tag{36}$$

$$S = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 12 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 12 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \cdot 10^{-4} \tag{37}$$

With $\Omega = [0, 100]$, through the MOSEK optimization toolbox within the Matlab LMI environment, a solution for the mixed FTB/$\mathcal{H}_\infty$ controller is found, with

$$K = \begin{pmatrix} 21.64 & -53.94 & -108.26 & 0.47 & -10.16 \\ 44.98 & 66.72 & -108.96 & 0.48 & 10.16 \\ -66.55 & -12.99 & -108.63 & 0.48 & -0.13 \end{pmatrix},$$

$$Q = \begin{pmatrix} 19.81 & 0 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 & -0.19 \\ 0 & 0 & 19.84 & 0.03 & 0 \\ 0 & 0 & 0.03 & 19.84 & 0 \\ 0 & -0.19 & 0 & 0 & 20 \end{pmatrix}.$$

As it is shown in Figs. 3 and 4, by rejecting the disturbance, the output is able to give a reference-tracking result that is satisfactory, despite the velocity along x is slightly greater

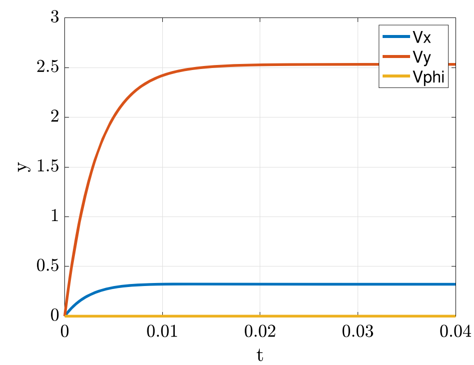

Fig. 5 Closed-loop system response with loop shaping on H_∞/FTS control and 5% parametric variation on a_1 and a_2

than zero. Therefore, the controller achieves the attenuation of the effect of the disturbance on the control system output.

By simulating the system response, in a set of tests generated from the combination of different initial conditions and disturbance inputs, the expected performance of closed-loop system has been validated.

In addition, a robustness analysis based on parametric variation was conducted. The velocities were calculated assuming a 5% variation in a_1 and a_2. Figure 5 demonstrates that the speed along x is unchanged, whereas the speed along y exhibits a deviation of 0.5. Figure 6 depicts essentially the same scenario, assuming a 10% variation for parameters a_1 and a_3 and a 5% variation for parameter a_2.

The simulation results demonstrate the benefit of combining $\mathcal{H}_\infty$ and FTC constraints. Through γ it is possible to optimize the control performance, over finite-time horizon, both in terms of the energy of the system output and its robustness versus $\mathcal{L}_2$ norm-bounded disturbances. Moreover, the loop shaping technique allows the simultaneous optimization of the control effort.

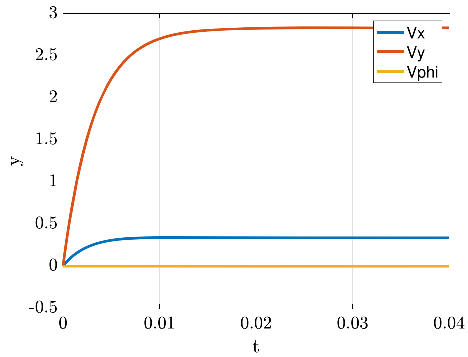

Fig. 6 Closed-loop system response with loop shaping on H_∞/FTS control and 10% parametric variation on a_1 and a_3 and 5% on a_2

6 Conclusions

In this work, an extension of the mixed FTC/$\mathcal{H}_\infty$ theory to the class of NLQSs has been provided; its concrete applicability has been evaluated in the context of the guidance/tracking control system design for omnidirectional mobile robots.

The main methodological contribution concerns a sufficient condition ensuring the mixed FTB/$\mathcal{H}_\infty$ closed-loop performance for a given NLQS under linear state feedback control. A mixed FTB/$\mathcal{H}_\infty$ law is designed through the solution of a LMIs optimization problem. The designed controller can guarantee, other than FTB, a prescribed $\mathcal{H}_\infty$ performance, which can be optimized—over finite-time horizon—on the basis of a versatile set of constraints, involving both bounds on the system output together with initial and terminal conditions, as shown by a design application concerning the closed-loop control of a omnidirectional mobile robot characterized by a three-wheeled frame. Indeed, the dynamic equations in the mobile reference system of such robot have been described through a NLQS.

The optimality and robustness of control performance, achievable in a closed-loop control scheme by the proposed design methodology, have been validated through numerical simulations. Through the robotic application, it has been possible to show the advantages provided by the proposed FTB/$\mathcal{H}_\infty$ control approach. Moreover, by complementing the design methodology with a loop shaping technique, an improved disturbance rejection has been achieved under the minimization of the control effort.

Acknowledgements None.

Author Contributions CC and AM were involved in the conceptualization; AM and FA contributed to the methodology; FN and DD were involved in the formal analysis and investigation; FN and DD performed the validation; FN curated the data; AM, CC and FN were involved in writing—original draft preparation. FA and CC assisted in writing—review and editing; AM and FA were involved in the supervision. All authors have read and approved the final version of the manuscript.

Funding No financial support to acknowledge. Open access funding provided by Università degli studi "Magna Graecia" di Catanzaro within the CRUI-CARE Agreement.

Data availability The source code and data presented in this work are available upon request to the corresponding author.

Declarations

Conflict of interest All authors declare that they have no conflict of interest to disclose.

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