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Existence results for sequential fractional integro-differential equations with impulsive conditions

P. Karthikeyan[1] · S. Poornima[1]

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Abstract
In this paper, we investigate the existence and uniqueness results for sequential fractional integro-differential equations with impulsive conditions. The nonlinear term contains the integral terms which are used to represent in the thermal conductivity of the material problems. Our methods are based on the fixed point theorems such as Banach fixed point theorem and Krasnoselskii's. We obtain the sufficient conditions of the existence of solutions. Examples are given to illustrate the results.

Keywords Sequential fractional differential equations · Impulses · Existence · Uniqueness

Mathematics Subject Classification 34A08 · 34A37 · 34A12

1 Introduction

Fractional calculus is a mathematical branch investigating the properties of derivatives and integrals of non-integer orders. In the past two decades, fractional calculus has been a research focus and attracted the attention of many researchers all over the world. More than that, fractional calculus is more and more widely used in various disciplines, especially in fluid mechanics, physics, signal processing, materials science, electrochemistry, biology and so on. It is due to the further development of fractional calculus theory itself.

In recent years, fractional differential equations have been widely used in the mathematical modeling of real-world phenomena. These applications have motivated many researchers in the field of differential equations to investigate fractional differential equations with different fractional derivatives [1–5].

The dynamics of populations subject to abrupt changes, as well as other phenomena like harvesting, diseases, and other phenomena, have all been described using impulsive differential equations. In [6], Karthikeyan et al. discussed the existence results for impulsive fractional integro-differential equations involving integral boundary conditions. In [7], the authors studied almost sectorial operators on Ψ—Hilfer derivative fractional impulsive integro-differential equations. Renusumrit et al. [8] investigated the existence and stability results for impulsive fractional integro-differential equations involving the ABC derivative under integral boundary condition.

Sequential fractional differential equations have also received considerable attention. The concept of this kind of equations was assumed in the book [9], which includes an in-depth investigation of a particular class of sequential differential equations. Fascinated by this kind of problem, various authors have examined these equations covering numerous fractional derivative types [10–12]. In mathematical physics, there are many significant boundary value problems corresponding to the real-life problem. Specifically, in the studies of vibrations of a membrane, vibrations of a structure one have to solve a homogeneous boundary value problem. Recently, the existence of solutions to boundary value problems and boundary condition has received a great deal of interest; see reference [13–16].

In [17], the authors have analyzed the following nonlinear fractional integro-differential equations

$$\begin{aligned} &{}^{c}D^{\alpha}({}^{c}D^{\beta})\upsilon(t) = f(t, \upsilon(t), \Phi\upsilon(t), \psi\upsilon(t)),\ (0 < t < 1),\\ &\upsilon(1) = \upsilon(0) = \upsilon^{'}(1) = 0, \end{aligned}$$

✉ P. Karthikeyan
pkarthisvc@gmail.com

S. Poornima
spoorni26@gmail.com

1 Department of Mathematics, Sri Vasavi College, Erode 638 316, India

where $\alpha \in (1,2]$, $\beta \in (0,1]$, $f:[0,1]\times\mathbb{R}^3 \to \mathbb{R}$ is continuous and

$$\varphi\upsilon(t)=\int_0^t \gamma(t,s)u(s)\mathrm{d}s,\ \ \psi\upsilon(t)=\int_0^t \lambda(t,s)u(s)\mathrm{d}s$$

where $\gamma,\ \lambda:[0,1]\times[0,1]\to[0,+\infty)$ are such that with $\varphi^*=\sup_{t\in[0,1]}(\int_0^t \lambda(t,s)\mathrm{d}s)<\infty$, and $\psi^*=\sup_{t\in[0,1]}(\int_0^t \gamma(t,s)\mathrm{d}s)<\infty$.

In [18], the authors have examined the new existence results for nonlinear fractional integro-differential equations

$$\begin{aligned}&{}^cD^{\alpha}({}^cD^{\beta})\upsilon(t)=f(t,\upsilon(t),\Phi\upsilon(t),\psi\upsilon(t)),\ t\in[0,1]\\ &\upsilon(0)=\upsilon(1)=({}^cD^{\beta})\upsilon(1)=({}^cD^{\beta})\upsilon(0)=0,\end{aligned}$$

where $\alpha \in (1,2]$, $\beta \in (0,2]$, $f:[0,1]\times\mathbb{R}^3\to\mathbb{R}$ is continuous and

$$\Phi\upsilon(t)=\int_0^t \lambda(t,s)\upsilon(s)\mathrm{d}s,\ \ \psi\upsilon(t)=\int_0^t \delta(t,s)\upsilon(s)\mathrm{d}s$$

where $\lambda,\ \delta:[0,1]\times[0,1]\to[0,+\infty)$ are such that with $\phi^*=\sup_{t\in[0,1]}(\int_0^t \lambda(t,s)\mathrm{d}s)<\infty$, and $\psi^*=\sup_{t\in[0,1]}(\int_0^t \gamma(t,s)\mathrm{d}s)<\infty$.

In [19], the authors have discussed the following sequential fractional differential equations with three-point boundary conditions,

$$\begin{aligned}&{}^cD^{\alpha}(D+\lambda)x(t)=f(t,x(t)),\ 0<t<1,\ 1<\alpha\le 2,\\ &x(0)=0,x^{'}(0)=0,x(1)=\beta x(\eta),\ 0<\eta<1.\end{aligned}$$

By using the Banach's contraction principle and Krasnoselskii's fixed point theorem, they proved the existence and uniqueness Results.

In [20], the authors have studied the following nonlinear implicit neutral fractional differential equations with finite delay and impulses

$$\begin{aligned}{}^cD^{\alpha}_{t_k}[y(t)-\phi(t,y_t)]&=f(t,y_t,{}^cD^{\alpha}_{t_k}y(t)),\\ &\text{for each } t\in(t_k,t_{k+1}],\\ &k=0,...,m,\ 0<\alpha\le 1,\\ \Delta y|_{t=t_k}&=I_k(y_{t_k^-}),\ k=1,..m\\ y(t)&=\varphi,\ t\in[-r,0],\ r>0,\end{aligned}$$

where ${}^cD^{\alpha}_{t_k}$ is the Caputo fractional derivative, $f:[0,T]\times PC([0,T],\mathbb{R})\times\mathbb{R}\to\mathbb{R}$, $\phi:[0,T]\times PC([0,T],\mathbb{R})\to\mathbb{R}$ are given functions with $\phi(0,\varphi)=0$, $I_k:PC([0,T],\mathbb{R})\to\mathbb{R}$, $\varphi\in PC([0,T],\mathbb{R})$, $0=t_0<t_1<...<t_m<t_{m+1}=T$.

Inspired by the above works, we consider the sequential fractional integro-differential equations with impulsive conditions of the form:

$$\begin{aligned}{}^cD^{\varpi}({}^cD^{\vartheta})\upsilon(\mathrm{t})&=\check{f}(\mathrm{t},\upsilon(\mathrm{t}),\Psi\upsilon(\mathrm{t}),\chi\upsilon(\mathrm{t})),\ \mathrm{t}\neq\mathrm{t}_{\Bbbk},\\ &\mathrm{t}\in[0,1]=\Theta\\ \Delta\upsilon|_{\mathrm{t}=\mathrm{t}_{\Bbbk}}&=\check{I}_{\Bbbk}(\upsilon_{\mathrm{t}_{\Bbbk}^-}),\ \Bbbk=1,..m\\ \upsilon(0)=\upsilon(1)&=({}^cD^{\vartheta})\upsilon(1)=({}^cD^{\vartheta})\upsilon(0)=0,\end{aligned}\qquad(1.1)$$

$\varpi\in(1,2]$, $\vartheta\in(0,2]$, ${}^cD^{\varpi}$, ${}^cD^{\vartheta}$ are the Caputo fractional derivatives, $\check{f}:\Theta\times PC(\Theta,\mathbb{R})\times\mathbb{R}\to\mathbb{R}$, $\check{I}_{\Bbbk}:PC(\Theta,\mathbb{R})\times\mathbb{R}\to\mathbb{R}$

$$\Psi\upsilon(\mathrm{t})=\int_0^{\mathrm{t}}\alpha(\mathrm{t},\ell)\upsilon(\ell)\mathrm{d}\ell$$

$$\chi\upsilon(\mathrm{t})=\int_0^{\mathrm{t}}\beta(\mathrm{t},\ell)\upsilon(\ell)\mathrm{d}\ell$$

where $\alpha,\beta:\Theta\times\Theta\to[0,+\infty)$ with $\Psi^*=\sup|\int_0^{\mathrm{t}}\alpha(\mathrm{t},\ell)\mathrm{d}\ell|<\infty$, $\chi^*=\sup|\int_0^{\mathrm{t}}\beta(\mathrm{t},\ell)\mathrm{d}\ell|<\infty$. $\Delta\upsilon(\mathrm{t}_{\Bbbk})=\upsilon(\mathrm{t}_{\Bbbk}^+)-\upsilon(\mathrm{t}_{\Bbbk}^-)$ denotes the jump of υ at $\mathrm{t}=\mathrm{t}_{\Bbbk}$, $\upsilon(\mathrm{t}_{\Bbbk}^+)$ and $\upsilon(\mathrm{t}_{\Bbbk}^-)$ represent the right and left limits of $\upsilon(\mathrm{t})$ at $\mathrm{t}=\mathrm{t}_{\Bbbk}$ respectively, $\Bbbk=1,2,...,m$.

This paper is organized as follows: In Sect. 2, we introduce some preliminaries about fractional calculus, lemma and definitions. In Sect. 3, we present two main results: the first one is based on the Krasnoselskii's the fixed point theorem, and the second one is based on Banach contraction principle. The results are illustrated by two examples in the last section.

2 Preliminaries

We introduce few definitions, notations, and lemmas of fractional calculus.

Definition 2.1 [21] The fractional integral of order $\alpha>0$ with the lower limit zero for a function f can be defined as

$$I^{\alpha}f(\mathrm{t})=\frac{1}{\Gamma(\alpha)}\int_0^{\mathrm{t}}(\mathrm{t}-s)^{\alpha-1}f(s)\mathrm{d}s,$$

where $\Gamma(.)$ is the Gamma function.

Definition 2.2 [21] The Caputo derivative of order $\alpha>0$ with the lower limit zero for a function f can be defined as

$${}^cD^{\alpha}f(\mathrm{t})=\frac{1}{\Gamma(n-\alpha)}\int_0^{\mathrm{t}}(\mathrm{t}-s)^{n-\alpha-1}f^{(n)}(s)\mathrm{d}s,$$

where $\mathrm{t}>0,\ 0\le n-1<\alpha<n,\ n\in\mathbb{N}$.

Lemma 2.3 *[21] Let $\alpha,\beta\le 0$; then, the following relation holds:*

$$I^{\alpha}\mathrm{t}^{\beta}=\frac{\Gamma(\beta+1)}{\Gamma(\alpha+\beta+1)}\mathrm{t}^{\alpha+\beta}.$$

Lemma 2.4 [21] *Let $n \in \mathbb{N}$ and $n-1<\alpha<n$. If f is a continuous function, then we have*

$$I^{\alpha c}D^{\alpha} f(t) = f(t) + a_0 + a_1 t + a_2 t^2 + ... + a_{n-1}t^{n-1}.$$

Theorem 2.5 [8] *Let M be a convex, closed, bounded and nonempty subset of a Banach space X. Let A and B be two operators such that*

1. *$Ax + By \in M$ whenever $x, y \in M$*
2. *A is continuous and compact*
3. *B is a contraction mapping.*

Then, there exists $z \in M$ such that $z = Az + Bz$.

Lemma 2.6 *Let $\alpha \in (1,2]$, $\beta \in (0,2]$ and $h \in C(\Theta, \mathbb{R})$. A function x is a solution of the fractional integral equation*

$$\begin{aligned} x(t) &= \frac{1}{\Gamma(\alpha+\beta)} \sum_{i=1}^{\Bbbk} \int_{t_{i-1}}^{t_i} (t_i - s)^{\alpha+\beta-1} h(s)\mathrm{d}s \\ &+ \frac{1}{\Gamma(\alpha+\beta)} \int_{t_{\Bbbk}}^{t} (t-s)^{\alpha+\beta-1} h(s)\mathrm{d}s \\ &- \sum_{i=1}^{\Bbbk} \frac{t_i^{\beta+1} - t_i}{\Gamma(\alpha)\Gamma(\beta+2)} \int_0^1 (1-s)^{\alpha-1} h(s)\mathrm{d}s \\ &- \frac{t^{\beta+1} - t}{\Gamma(\alpha)\Gamma(\beta+2)} \int_0^1 (1-s)^{\alpha-1} h(s)\mathrm{d}s \\ &- \sum_{i=1}^{\Bbbk} \frac{t_i}{\Gamma(\alpha+\beta)} \int_0^1 (1-s)^{\alpha+\beta-1} h(s)\mathrm{d}s \\ &- \frac{t}{\Gamma(\alpha+\beta)} \int_0^1 (1-s)^{\alpha+\beta-1} h(s)\mathrm{d}s \\ &+ \sum_{i=1}^{\Bbbk} \check{I}_i(x_{t_i^-}), \end{aligned}$$

$\Bbbk = 1, ..m$, if and only if x is a solution of the following fractional problem

$$\begin{aligned} {}^cD^{\alpha}({}^cD^{\beta})x(t) &= h(t), \ t \in \Theta \\ \Delta x|_{t=t_{\Bbbk}} &= \check{I}_{\Bbbk}(x_{t_{\Bbbk}^-}), \ \Bbbk = 1, ..m \\ x(0) = x(1) &= ({}^cD^{\beta})x(0) = ({}^cD^{\beta})x(1) = 0. \end{aligned}$$

Proof By applying Lemma 2.4, we get

$$\begin{aligned} {}^cD^{\alpha}x(t) &= I^{\alpha}h(t) + a_0 + a_1 t, \\ x(t) &= I^{\alpha+\beta}h(t) + I^{\beta}a_0 + I^{\beta}a_1 t + a_2 + a_3 t, \end{aligned}$$

where $a_0, a_1, a_2, a_3 \in \mathbb{R}$. So

$$\begin{aligned} x(t) &= \frac{1}{\Gamma(\alpha+\beta)} \int_0^t (t-s)^{\alpha+\beta-1} h(s)\mathrm{d}s \\ &+ \frac{t^{\beta}}{\Gamma(\beta+1)} a_0 + \frac{t^{\beta+1}}{\Gamma(\beta+2)} a_1 + a_2 + a_3 t. \end{aligned}$$

And by using ${}^cD^{\alpha}x(0) = x(0) = 0$, we attain $a_0 = 0$ and $a_2 = 0$. As a result of ${}^cD^{\alpha}x(1) = 0$, we have

$$a_1 = -\frac{1}{\Gamma(\alpha)} \int_0^1 (1-s)^{\alpha-1} h(s)\mathrm{d}s.$$

Now, we use $x(1) = 0$ to get

$$\begin{aligned} a_3 &= -\frac{1}{\Gamma(\alpha+\beta)} \int_0^1 (1-s)^{\alpha+\beta-1} h(s)\mathrm{d}s \\ &+ \frac{1}{\Gamma(\alpha)\Gamma(\beta+2)} \int_0^1 (1-s)^{\alpha-1} h(s)\mathrm{d}s. \end{aligned}$$

By substituting the value of a_0, a_1, a_2, a_3, we obtain

$$\begin{aligned} x(t) &= \frac{1}{\Gamma(\alpha+\beta)} \int_0^t (t-s)^{\alpha+\beta-1} h(s)\mathrm{d}s \\ &- \frac{t^{\beta+1} - t}{\Gamma(\alpha)\Gamma(\beta+2)} \int_0^1 (1-s)^{\alpha-1} h(s)\mathrm{d}s \\ &- \frac{t}{\Gamma(\alpha+\beta)} \int_0^1 (1-s)^{\alpha+\beta-1} h(s)\mathrm{d}s. \end{aligned}$$

If $t \in (t_1, t_2]$,

$$\begin{aligned} x(t) &= x(t_1^+) + \frac{1}{\Gamma(\alpha+\beta)} \int_{t_1}^t (t-s)^{\alpha+\beta-1} h(s)\mathrm{d}s \\ &- \frac{t^{\beta+1} - t}{\Gamma(\alpha)\Gamma(\beta+2)} \int_0^1 (1-s)^{\alpha-1} h(s)\mathrm{d}s \\ &- \frac{t}{\Gamma(\alpha+\beta)} \int_0^1 (1-s)^{\alpha+\beta-1} h(s)\mathrm{d}s \\ &= \Delta x|_{t=t_1} + x(t_1^-) + \frac{1}{\Gamma(\alpha+\beta)} \int_{t_1}^t (t-s)^{\alpha+\beta-1} h(s)\mathrm{d}s \\ &- \frac{t^{\beta+1} - t}{\Gamma(\alpha)\Gamma(\beta+2)} \int_0^1 (1-s)^{\alpha-1} h(s)\mathrm{d}s \\ &- \frac{t}{\Gamma(\alpha+\beta)} \int_0^1 (1-s)^{\alpha+\beta-1} h(s)\mathrm{d}s \\ &= \check{I}_1(x_{t_1^-}) + \frac{1}{\Gamma(\alpha+\beta)} \int_0^{t_1} (t_1-s)^{\alpha+\beta-1} h(s)\mathrm{d}s \\ &- \frac{t_1^{\beta+1} - t_1}{\Gamma(\alpha)\Gamma(\beta+2)} \int_0^1 (1-s)^{\alpha-1} h(s)\mathrm{d}s \\ &- \frac{t_1}{\Gamma(\alpha+\beta)} \int_0^1 (1-s)^{\alpha+\beta-1} h(s)\mathrm{d}s+ \\ &+ \frac{1}{\Gamma(\alpha+\beta)} \int_{t_1}^t (t-s)^{\alpha+\beta-1} h(s)\mathrm{d}s \\ &- \frac{t^{\beta+1} - t}{\Gamma(\alpha)\Gamma(\beta+2)} \int_0^1 (1-s)^{\alpha-1} h(s)\mathrm{d}s \end{aligned}$$

$$- \frac{t}{\Gamma(\alpha+\beta)} \int_0^1 (1-s)^{\alpha+\beta-1} h(s) \mathrm{d}s.$$

If $t \in (t_2, t_3]$,

$$\begin{aligned}
x(t) &= x(t_2^+) + \frac{1}{\Gamma(\alpha+\beta)} \int_{t_2}^t (t-s)^{\alpha+\beta-1} h(s) \mathrm{d}s \\
&\quad - \frac{t^{\beta+1}-t}{\Gamma(\alpha)\Gamma(\beta+2)} \int_0^1 (1-s)^{\alpha-1} h(s) \mathrm{d}s \\
&\quad - \frac{t}{\Gamma(\alpha+\beta)} \int_0^1 (1-s)^{\alpha+\beta-1} h(s) \mathrm{d}s \\
&= \Delta x|_{t=t_2} + x(t_2^-) + \frac{1}{\Gamma(\alpha+\beta)} \int_{t_2}^t (t-s)^{\alpha+\beta-1} h(s) \mathrm{d}s \\
&\quad - \frac{t^{\beta+1}-t}{\Gamma(\alpha)\Gamma(\beta+2)} \int_0^1 (1-s)^{\alpha-1} h(s) \mathrm{d}s \\
&\quad - \frac{t}{\Gamma(\alpha+\beta)} \int_0^1 (1-s)^{\alpha+\beta-1} h(s) \mathrm{d}s \\
&= \check{I}_2(x_{t_2^-}) + \Bigg[\check{I}_1(x_{t_1^-}) + \frac{1}{\Gamma(\alpha+\beta)} \int_0^{t_1} (t_1-s)^{\alpha+\beta-1} h(s) \mathrm{d}s \\
&\quad - \frac{t_1^{\beta+1}-t_1}{\Gamma(\alpha)\Gamma(\beta+2)} \int_0^1 (1-s)^{\alpha-1} h(s) \mathrm{d}s \\
&\quad - \frac{t_1}{\Gamma(\alpha+\beta)} \int_0^1 (1-s)^{\alpha+\beta-1} h(s) \mathrm{d}s \\
&\quad + \frac{1}{\Gamma(\alpha+\beta)} \int_{t_1}^{t_2} (t_2-s)^{\alpha+\beta-1} h(s) \mathrm{d}s \\
&\quad - \frac{t_2^{\beta+1}-t_2}{\Gamma(\alpha)\Gamma(\beta+2)} \int_0^1 (1-s)^{\alpha-1} h(s) \mathrm{d}s \\
&\quad - \frac{t_2}{\Gamma(\alpha+\beta)} \int_0^1 (1-s)^{\alpha+\beta-1} h(s) \mathrm{d}s \Bigg] \\
&\quad + \frac{1}{\Gamma(\alpha+\beta)} \int_{t_2}^t (t-s)^{\alpha+\beta-1} h(s) \mathrm{d}s \\
&\quad - \frac{t^{\beta+1}-t}{\Gamma(\alpha)\Gamma(\beta+2)} \int_0^1 (1-s)^{\alpha-1} h(s) \mathrm{d}s \\
&\quad - \frac{t}{\Gamma(\alpha+\beta)} \int_0^1 (1-s)^{\alpha+\beta-1} h(s) \mathrm{d}s.
\end{aligned}$$

Repeating this process, the solution $x(t)$, for $t \in (t_{\Bbbk}, t_{\Bbbk+1}]$, where $\Bbbk = 1, ..., m$ could be expressed as

$$\begin{aligned}
x(t) &= \frac{1}{\Gamma(\alpha+\beta)} \sum_{i=1}^{\Bbbk} \int_{t_{i-1}}^{t_i} (t_i-s)^{\alpha+\beta-1} h(s) \mathrm{d}s \\
&\quad + \frac{1}{\Gamma(\alpha+\beta)} \int_{t_{\Bbbk}}^t (t-s)^{\alpha+\beta-1} h(s) \mathrm{d}s \\
&\quad - \sum_{i=1}^{\Bbbk} \frac{t_i^{\beta+1}-t_i}{\Gamma(\alpha)\Gamma(\beta+2)} \int_0^1 (1-s)^{\alpha-1} h(s) \mathrm{d}s \\
&\quad - \frac{t^{\beta+1}-t}{\Gamma(\alpha)\Gamma(\beta+2)} \int_0^1 (1-s)^{\alpha-1} h(s) \mathrm{d}s \\
&\quad - \sum_{i=1}^{\Bbbk} \frac{t_i}{\Gamma(\alpha+\beta)} \int_0^1 (1-s)^{\alpha+\beta-1} h(s) \mathrm{d}s \\
&\quad - \frac{t}{\Gamma(\alpha+\beta)} \int_0^1 (1-s)^{\alpha+\beta-1} h(s) \mathrm{d}s \\
&\quad + \sum_{i=1}^{\Bbbk} \check{I}_i(x_{t_i^-}).
\end{aligned}$$

Conversely, by direct estimations, we get the desired result. □

3 Main results

Consider the Banach space $\check{X}$ of all continuous functions from $\Theta \to \mathbb{R}$ endowed with the norms $\|\mathfrak{y}\| = \sup\{|\mathfrak{y}(\mathfrak{t})| : \mathfrak{t} \in \Theta\}$. Also consider the space

$$PC(\Theta, \mathbb{R}) = \{\mathfrak{y} : \Theta \to \mathbb{R} : \mathfrak{y} \in C(\mathfrak{t}_{\Bbbk}, \mathfrak{t}_{\Bbbk+1}], \mathbb{R}),$$
$\Bbbk = 0, ..., m$ and there exist
$\mathfrak{y}(\mathfrak{t}_{\Bbbk}^-)$ and $\mathfrak{y}(\mathfrak{t}_{\Bbbk}^+)$, $\Bbbk = 0, ..., m$ with $\mathfrak{y}(\mathfrak{t}_{\Bbbk}^-) = \mathfrak{y}(\mathfrak{t}_{\Bbbk}^+)\}$,

$\|\mathfrak{y}\|_{\upsilon} = \sup_{\mathfrak{t}\in\Theta}\left(\frac{|\mathfrak{y}(\mathfrak{t})|}{e^{\upsilon \mathfrak{t}}}\right)$ where $\upsilon > \frac{(1+\Phi^*+\varphi^*)}{(\Gamma(\varpi+\vartheta))}\|\sigma\|$, and $\sigma \in C(\Theta; [0, \infty)) \cap L^1(\Theta; [0, \infty))$.
The following hypotheses are needed to prove the main results:
(H_1) For all $\mathfrak{t} \in \Theta$ and $\upsilon_1, \upsilon_2, \upsilon_3, \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3 \in \mathbb{R}$, we have

$$\begin{aligned}
&|\check{f}(\mathfrak{t}, \upsilon_1, \upsilon_2, \upsilon_3) - \check{f}(\mathfrak{t}, \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3)| \\
&\quad \le \sigma(\mathfrak{t})(|\upsilon_1 - \mathfrak{y}_1| + |\upsilon_2 - \mathfrak{y}_2| + |\upsilon_3 - \mathfrak{y}_3|)
\end{aligned}$$

with $\sigma \in C(\Theta; [0, \infty))$
(H_2) $|\check{f}(\mathfrak{t}, \upsilon, \mathfrak{y}, z)| \le \theta(\mathfrak{t}), \forall(\mathfrak{t}, \upsilon, \mathfrak{y}, z) \in \Theta \times \mathbb{R}^3$ with $\theta \in C(\Theta; \mathbb{R}^+)$.
(H_3) There exists a constant $l > 0$ such that

$$|\check{I}_{\Bbbk}(\upsilon) - \check{I}_{\Bbbk}(\mathfrak{y})| \le l|\upsilon - \mathfrak{y}|$$

and $|\check{I}_{\Bbbk}(\upsilon)| \le l\theta(\mathfrak{t})$.
(H_4) For all $\mathfrak{t} \in \Theta$ and $\upsilon_1, \upsilon_2, \upsilon_3, \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3 \in \mathbb{R}$, we have $|\check{f}(\mathfrak{t}, \upsilon_1, \upsilon_2, \upsilon_3) - \check{f}(\mathfrak{t}, \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3| \le \sigma(\mathfrak{t})|\upsilon_1 - \mathfrak{y}_1| + |\upsilon_2 - \mathfrak{y}_2| + |\upsilon_3 - \mathfrak{y}_3|$ with $\sigma(\mathfrak{t}) \in L^1(\Theta; [0, \infty))$ and
(H_5) There exists a constant $l > 0$ such that $|\check{I}_{\Bbbk}(\upsilon) - \check{I}_{\Bbbk}(\mathfrak{y})| \le l|\upsilon - \mathfrak{y}|$.

Theorem 3.1 *Assume that hypotheses $(H_1) - (H_3)$ are satisfied. Then, problem (1.1) has at least one solution.*

Proof Let the ball $B_{\mathfrak{r}} = \{\mathfrak{y} \in \check{X} : \|\mathfrak{y}\|_{\upsilon} \le \mathfrak{r}\}$ with

$$\mathfrak{r} \ge \frac{(m+1)\|\theta\|}{\upsilon}\left(\frac{2(e^{\upsilon}-1)}{\Gamma(\varpi)\Gamma(\vartheta+2)} + \frac{(e^{\upsilon})}{\Gamma(\varpi+\vartheta)}\right) + ml.$$

Define the operators $\digamma_1, \digamma_2$ on $B_{\mathfrak{r}}$, where

$$\digamma_{1\upsilon(\mathrm{t})} = \frac{1}{\Gamma(\varpi+\vartheta)} \sum_{i=1}^{\Bbbk} \int_{\mathrm{t}_{i-1}}^{\mathrm{t}_i} (\mathrm{t}_i-\ell)^{\varpi+\vartheta-1} \check{f}(\mathrm{t}, \upsilon(\mathrm{t}), \Psi\upsilon(\mathrm{t}), \chi\upsilon(\mathrm{t}))d\ell + \frac{1}{\Gamma(\varpi+\vartheta)} \int_{\mathrm{t}_{\Bbbk}}^{\mathrm{t}} (\mathrm{t}-\ell)^{\varpi+\vartheta-1} \check{f}(\mathrm{t}, \upsilon(\mathrm{t}), \Psi\upsilon(\mathrm{t}), \chi\upsilon(\mathrm{t}))d\ell.$$

$$\digamma_{2\upsilon(\mathrm{t})} = \sum_{0<\mathrm{t}_{\Bbbk}<\mathrm{t}} \check{I}_i(\mathfrak{y}_{\mathrm{t}_i^-}) - \sum_{i=1}^{\Bbbk} \frac{\mathrm{t}_i^{\vartheta+1}-\mathrm{t}_i}{\Gamma(\varpi)\Gamma(\vartheta+2)} \int_0^1 (1-\ell)^{\varpi-1} \check{f}(\mathrm{t}, \upsilon(\mathrm{t}), \Psi\upsilon(\mathrm{t}), \chi\upsilon(\mathrm{t}))d\ell - \frac{\mathrm{t}^{\vartheta+1}-\mathrm{t}}{\Gamma(\varpi)\Gamma(\vartheta+2)} \int_0^1 (1-\ell)^{\varpi-1} \check{f}(\mathrm{t}, \upsilon(\mathrm{t}), \Psi\upsilon(\mathrm{t}), \chi\upsilon(\mathrm{t}))d\ell - \sum_{i=1}^{\Bbbk} \frac{\mathrm{t}_i}{\Gamma(\varpi+\vartheta)} \int_0^1 (1-\ell)^{\varpi+\vartheta-1} \check{f}(\mathrm{t}, \upsilon(\mathrm{t}), \Psi\upsilon(\mathrm{t}), \chi\upsilon(\mathrm{t}))d\ell - \frac{\mathrm{t}}{\Gamma(\varpi+\vartheta)} \int_0^1 (1-\ell)^{\varpi+\vartheta-1} \check{f}(\mathrm{t}, \upsilon(\mathrm{t}), \Psi\upsilon(\mathrm{t}), \chi\upsilon(\mathrm{t}))d\ell.$$

For $\upsilon, \mathfrak{y} \in B_{\mathfrak{r}}$, we have

$$\begin{aligned}
\|\digamma_{1\upsilon(\mathrm{t})}\| &\le \sup_{\mathrm{t}\in\Theta} \frac{1}{e^{v\mathrm{t}}} \left| \frac{1}{\Gamma(\varpi+\vartheta)} \sum_{i=1}^{\Bbbk} \int_{\mathrm{t}_{i-1}}^{\mathrm{t}_i} (\mathrm{t}_i-\ell)^{\varpi+\vartheta-1} \check{f}(\mathrm{t}, \upsilon(\mathrm{t}), \Psi\upsilon(\mathrm{t}), \chi\upsilon(\mathrm{t}))d\ell + \frac{1}{\Gamma(\varpi+\vartheta)} \int_{\mathrm{t}_{\Bbbk}}^{\mathrm{t}} (\mathrm{t}-\ell)^{\varpi+\vartheta-1} \check{f}(\mathrm{t}, \upsilon(\mathrm{t}), \Psi\upsilon(\mathrm{t}), \chi\upsilon(\mathrm{t}))d\ell \right| \\
&\le \sup_{\mathrm{t}\in\Theta} \frac{1}{e^{v\mathrm{t}}} \left\{ \frac{1}{\Gamma(\varpi+\vartheta)} \sum_{i=1}^{\Bbbk} \int_{\mathrm{t}_{i-1}}^{\mathrm{t}_i} (\mathrm{t}_i-\ell)^{\varpi+\vartheta-1} |\theta(\ell)|d\ell + \frac{1}{\Gamma(\varpi+\vartheta)} \int_{\mathrm{t}_{\Bbbk}}^{\mathrm{t}} (\mathrm{t}-\ell)^{\varpi+\vartheta-1} |\theta(\ell)|d\ell \right\} \\
&\le \sup_{\mathrm{t}\in\Theta} \frac{1}{e^{v\mathrm{t}}} \left\{ \frac{1}{\Gamma(\varpi+\vartheta)} \sum_{i=1}^{\Bbbk} \int_{\mathrm{t}_{i-1}}^{\mathrm{t}_i} (\mathrm{t}_i-\ell)^{\varpi+\vartheta-1} |\theta(\ell)|d\ell + \frac{1}{\Gamma(\varpi+\vartheta)} \int_{\mathrm{t}_{\Bbbk}}^{\mathrm{t}} (\mathrm{t}-\ell)^{\varpi+\vartheta-1} |\theta(\ell)|d\ell \right\} \\
&\le \sup_{\mathrm{t}\in\Theta} \frac{1}{e^{v\mathrm{t}}} \left\{ \frac{1}{\Gamma(\varpi+\vartheta)} \sum_{i=1}^{\Bbbk} \int_{\mathrm{t}_{i-1}}^{\mathrm{t}_i} (\mathrm{t}_i-\ell)^{\varpi+\vartheta-1} \frac{|\theta(\ell)|e^{v\ell}}{e^{v\ell}}d\ell + \frac{1}{\Gamma(\varpi+\vartheta)} \int_{\mathrm{t}_{\Bbbk}}^{\mathrm{t}} (\mathrm{t}-\ell)^{\varpi+\vartheta-1} \frac{|\theta(\ell)|e^{v\ell}}{e^{v\ell}}d\ell \right\} \\
&\le \sup_{\mathrm{t}\in\Theta} \frac{\|\theta\|_v}{e^{v\mathrm{t}}\Gamma(\varpi+\vartheta)} \left\{ \sum_{i=1}^{\Bbbk} \int_{\mathrm{t}_{i-1}}^{\mathrm{t}_i} (\mathrm{t}_i-\ell)^{\varpi+\vartheta-1} e^{v\ell}d\ell + \int_{\mathrm{t}_{\Bbbk}}^{\mathrm{t}} (\mathrm{t}-\ell)^{\varpi+\vartheta-1} e^{v\ell}d\ell \right\} \\
&\le \frac{(m+1)\|\theta\|_v}{v\Gamma(\varpi+\vartheta)}.
\end{aligned}$$

$$\begin{aligned}
\|\digamma_{2\mathfrak{y}(\mathrm{t})}\| &\le \sup_{\mathrm{t}\in\Theta} \frac{1}{e^{v\mathrm{t}}} \Bigg\{ \Bigg| \sum_{0<\mathrm{t}_{\Bbbk}<\mathrm{t}} \check{I}_i(\mathfrak{y}_{\mathrm{t}_i^-}) - \sum_{i=1}^{\Bbbk} \frac{\mathrm{t}_i^{\vartheta+1}-\mathrm{t}_i}{\Gamma(\varpi)\Gamma(\vartheta+2)} \int_0^1 (1-\ell)^{\varpi-1} \check{f}(\mathrm{t}, \mathfrak{y}(\mathrm{t}), \Psi\mathfrak{y}(\mathrm{t}), \chi\mathfrak{y}(\mathrm{t}))d\ell \\
&\quad - \frac{\mathrm{t}^{\vartheta+1}-\mathrm{t}}{\Gamma(\varpi)\Gamma(\vartheta+2)} \int_0^1 (1-\ell)^{\varpi-1} \check{f}(\mathrm{t}, \mathfrak{y}(\mathrm{t}), \Psi\mathfrak{y}(\mathrm{t}), \chi\mathfrak{y}(\mathrm{t}))d\ell - \sum_{i=1}^{\Bbbk} \frac{\mathrm{t}_i}{\Gamma(\varpi+\vartheta)} \int_0^1 (1-\ell)^{\varpi+\vartheta-1} \check{f}(\mathrm{t}, \mathfrak{y}(\mathrm{t}), \Psi\mathfrak{y}(\mathrm{t}), \chi\mathfrak{y}(\mathrm{t}))d\ell \\
&\quad - \frac{\mathrm{t}}{\Gamma(\varpi+\vartheta)} \int_0^1 (1-\ell)^{\varpi+\vartheta-1} \check{f}(\mathrm{t}, \mathfrak{y}(\mathrm{t}), \Psi\mathfrak{y}(\mathrm{t}), \chi\mathfrak{y}(\mathrm{t}))d\ell \Bigg| \Bigg\} \\
&\le \sup_{\mathrm{t}\in\Theta} \frac{1}{e^{v\mathrm{t}}} \left[ml|\theta(\mathrm{t})| + \frac{2m}{\Gamma(\varpi)\Gamma(\vartheta+2)} \int_0^1 (1-\ell)^{\varpi-1} |\theta(\ell)|d\ell + \frac{2}{\Gamma(\varpi)\Gamma(\vartheta+2)} \int_0^1 (1-\ell)^{\varpi-1} |\theta(\ell)|d\ell \right. \\
&\quad \left. + \frac{m}{\Gamma(\varpi+\vartheta)} \int_0^1 (1-\ell)^{\varpi+\vartheta-1} |\theta(\ell)|d\ell + \frac{1}{\Gamma(\varpi+\vartheta)} \int_0^1 (1-\ell)^{\varpi+\vartheta-1} |\theta(\ell)|d\ell \right] \\
&\le \sup_{\mathrm{t}\in\Theta} \frac{1}{e^{v\mathrm{t}}} \left[ml|\theta(\mathrm{t})| + \frac{2(m+1)}{\Gamma(\varpi)\Gamma(\vartheta+2)} \int_0^1 (1-\ell)^{\varpi-1} \frac{|\theta(\ell)|}{e^{v\ell}} e^{v\ell}d\ell + \frac{m+1}{\Gamma(\varpi+\vartheta)} \int_0^1 (1-\ell)^{\varpi+\vartheta-1} \frac{|\theta(\ell)|}{e^{v\ell}} e^{v\ell}d\ell \right] \\
&\le ml\|\theta(\mathrm{t})\|_v + \sup_{\mathrm{t}\in\Theta} \frac{\|\theta(\ell)\|_v}{e^{v\mathrm{t}}} \left[\frac{2(m+1)}{\Gamma(\varpi)\Gamma(\vartheta+2)} \int_0^1 (1-\ell)^{\varpi-1} e^{v\ell}d\ell + \frac{m+1}{\Gamma(\varpi+\vartheta)} \int_0^1 (1-\ell)^{\varpi+\vartheta-1} e^{v\ell}d\ell \right] \\
&\le ml\|\theta(\mathrm{t})\|_v + \sup_{\mathrm{t}\in\Theta} \frac{\|\theta(\ell)\|_v}{e^{v\mathrm{t}}} \left[\frac{2(m+1)}{\Gamma(\varpi)\Gamma(\vartheta+2)} \int_0^1 e^{v\ell}d\ell + \frac{m+1}{\Gamma(\varpi+\vartheta)} \int_0^1 e^{v\ell}d\ell \right] \\
&\le \|\theta(\mathrm{t})\|_v \left[ml + \sup_{\mathrm{t}\in\Theta} \frac{1}{ve^{v\mathrm{t}}} \right.
\end{aligned}$$

$$\left(\frac{2(m+1)(e^{v}-1)}{\Gamma(\varpi)\Gamma(\vartheta+2)}+\frac{(m+1)(e^{v}-1)}{\Gamma(\varpi+\vartheta)}\right)\Bigg]$$
$$\leq \|\theta(t)\|_{v}\left[ml+(m+1)\frac{(e^{v}-1)}{v}\right.$$
$$\left.\left(\frac{2}{\Gamma(\varpi)\Gamma(\vartheta+2)}+\frac{1}{\Gamma(\varpi+\vartheta)}\right)\right].$$

Therefore,

$$\|F_1\upsilon+F_2\mathfrak{y}\| \leq \|\theta\|_{v}$$
$$\left[\frac{(m+1)}{v}\left(\frac{2(e^{v}-1)}{\Gamma(\varpi)\Gamma(\vartheta+2)}+\frac{e^{v}}{\Gamma(\varpi+\vartheta)}\right)+ml\right].$$

Then $F_1\upsilon+F_2\mathfrak{y}\in B_{\mathfrak{r}}$.
Now, we prove that F_1 is a contraction. For $\upsilon,\mathfrak{y}\in B_{\mathfrak{r}}$, we have

$$\|F_{1\mathfrak{y}(\mathfrak{t})}-F_{1\upsilon(\mathfrak{t})}\|$$
$$\leq \sup_{\mathfrak{t}\in\Theta}\frac{1}{\Gamma(\varpi+\vartheta)e^{v\mathfrak{t}}}\sum_{0<\mathfrak{t}_{\Bbbk}<\mathfrak{t}}\int_{\mathfrak{t}_{i-1}}^{\mathfrak{t}_i}(\mathfrak{t}_i-\ell)^{\varpi+\vartheta-1}$$
$$|\check{f}(\ell,\mathfrak{y}(\ell),\Psi\mathfrak{y}(\ell),\chi\mathfrak{y}(\ell))$$
$$-\check{f}(\ell,\upsilon(\ell),\Psi\upsilon(\ell),\chi\upsilon(\ell))|d\ell$$
$$+\sup_{\mathfrak{t}\in\Theta}\frac{1}{\Gamma(\varpi+\vartheta)e^{v\mathfrak{t}}}\int_{\mathfrak{t}_{\Bbbk}}^{\mathfrak{t}}(\mathfrak{t}-\ell)^{\varpi+\vartheta-1}$$
$$|\check{f}(\ell,\mathfrak{y}(\ell),\Psi\mathfrak{y}(\ell),\chi\mathfrak{y}(\ell))-\check{f}(\ell,\upsilon(\ell),\Psi\upsilon(\ell),\chi\upsilon(\ell))|d\ell$$
$$\leq \sup_{\mathfrak{t}\in\Theta}\frac{1}{\Gamma(\varpi+\vartheta)e^{v\mathfrak{t}}}$$
$$\sum_{0<\mathfrak{t}_{\Bbbk}<\mathfrak{t}}\int_{\mathfrak{t}_{i-1}}^{\mathfrak{t}_i}(\mathfrak{t}_i-\ell)^{\varpi+\vartheta-1}\sigma(\ell)(|\mathfrak{y}(\ell)-\upsilon(\ell)|$$
$$+|\Psi\mathfrak{y}(\ell)-\Psi\upsilon(\ell)|+|\chi\mathfrak{y}(\ell)-\chi\upsilon(\ell)|)d\ell$$
$$+\sup_{\mathfrak{t}\in\Theta}\frac{1}{\Gamma(\varpi+\vartheta)e^{v\mathfrak{t}}}$$
$$\int_{\mathfrak{t}_{\Bbbk}}^{\mathfrak{t}}(\mathfrak{t}-\ell)^{\varpi+\vartheta-1}\sigma(\ell)(|\mathfrak{y}(\ell)-\upsilon(\ell)|$$
$$+|\Psi\mathfrak{y}(\ell)-\Psi\upsilon(\ell)|+|\chi\mathfrak{y}(\ell)-\chi\upsilon(\ell)|)d\ell$$
$$\leq \sup_{\mathfrak{t}\in\Theta}\frac{\|\sigma\|}{\Gamma(\varpi+\vartheta)e^{v\mathfrak{t}}}$$
$$\sum_{0<\mathfrak{t}_{\Bbbk}<\mathfrak{t}}\int_{\mathfrak{t}_{i-1}}^{\mathfrak{t}_i}(\mathfrak{t}_i-\ell)^{\varpi+\vartheta-1}(e^{v\ell})(1+\Psi^*+\chi^*)\|\mathfrak{y}-\upsilon\|d\ell$$
$$+\sup_{\mathfrak{t}\in\Theta}\frac{\|\sigma\|}{\Gamma(\varpi+\vartheta)e^{v\mathfrak{t}}}$$
$$\int_{\mathfrak{t}_{\Bbbk}}^{\mathfrak{t}}(\mathfrak{t}-\ell)^{\varpi+\vartheta-1}(e^{v\ell})(1+\Psi^*+\chi^*)\|\mathfrak{y}-\upsilon\|d\ell$$
$$\leq \sup_{\mathfrak{t}\in\Theta}\frac{\|\sigma\|(m+1)(1+\Psi^*+\chi^*)}{v\Gamma(\varpi+\vartheta)}\frac{e^{v\mathfrak{t}}-1}{e^{v\mathfrak{t}}}\|\mathfrak{y}-\upsilon\|d\ell$$
$$\leq \frac{\|\sigma\|(m+1)(1+\Psi^*+\chi^*)}{v\Gamma(\varpi+\vartheta)}\|\mathfrak{y}-\upsilon\|_{v}d\ell.$$

We arrive at the conclusion that F_1 is a contraction.
Next, we have to show that F_2 is continuous and compact. $\check{f}$ is continuous that implies that the operator F_2 is continuous. Also, F_2 is bounded uniformly on $B_{\mathfrak{r}}$ as

$$\|F_2\mathfrak{y}\| \leq \|\theta(\mathfrak{t})\|_{v}\left[ml+(m+1)\frac{(e^{v}-1)}{v}\right.$$
$$\left.\left(\frac{2}{\Gamma(\varpi)\Gamma(\vartheta+2)}+\frac{1)}{\Gamma(\varpi+\vartheta)}\right)\right].$$

Assume $0\leq\mathfrak{t}_1<\mathfrak{t}_2\leq 1$. We get

$$|F_2\mathfrak{y}(\mathfrak{t}_2)-F_2\mathfrak{y}(\mathfrak{t}_1)|$$
$$\leq \frac{|\mathfrak{t}_2^{\vartheta+1}-\mathfrak{t}_1^{\vartheta+1}|+|\mathfrak{t}_2-\mathfrak{t}_1|}{\Gamma(\varpi)\Gamma(\vartheta+2)}$$
$$\int_0^{\mathfrak{t}}(1-\ell)^{\varpi-1}|\check{f}(\ell,\upsilon(\ell),\Psi\upsilon(\ell),\chi\upsilon(\ell))|d\ell$$
$$+\frac{|\mathfrak{t}_2-\mathfrak{t}_1|}{\Gamma(\varpi+\vartheta)}\int_0^{\mathfrak{t}}(1-\ell)^{\varpi-1}$$
$$|\check{f}(\ell,\upsilon(\ell),\Psi\upsilon(\ell),\chi\upsilon(\ell))|d\ell.$$

Then, $|F_2\mathfrak{y}(\mathfrak{t}_2)-F_2\mathfrak{y}(\mathfrak{t}_1)|\to 0$, as $\mathfrak{t}_1\to\mathfrak{t}_2$ independently from $\mathfrak{y}\in B_{\mathfrak{r}}$. This proves the operator F_2 is relatively compact on $B_{\mathfrak{r}}$. Thus, by the Arzela Ascoli theorem, we obtain that F_2 is compact on $B_{\mathfrak{r}}$. By the Krasnoselskii's fixed point theorem, problem (1.1) has at least one solution on $B_{\mathfrak{r}}$. ◻

Theorem 3.2 *Suppose that $\check{f}:\Theta\times\mathbb{R}^3\to\mathbb{R}$ is a continuous function satisfying hypotheses (H_4) and (H_5), then there exists a unique solution for problem (1.1) under the following condition: $\mathfrak{r}_1<1$,*
where

$$\mathfrak{r}_1=2(m+1)(1+\Psi^*+\chi^*)\sigma^*\left(\frac{1}{\Gamma(\varpi+\vartheta)}\right.$$
$$\left.+\frac{1}{\Gamma(\varpi)\Gamma(\vartheta+2)}\right)+ml$$

with $\sigma^*=\int_0^1\sigma(\mathfrak{t})d\mathfrak{t}$.

Proof Define the operator $F:\check{X}\to\check{X}$ by

$$F\upsilon(\mathfrak{t})=\frac{1}{\Gamma(\varpi+\vartheta)}$$
$$\sum_{i=1}^{\Bbbk}\int_{\mathfrak{t}_{i-1}}^{\mathfrak{t}_i}(\mathfrak{t}_i-\ell)^{\varpi+\vartheta-1}\check{f}(\ell,\upsilon(\ell),\Psi\upsilon(\ell),\chi\upsilon(\ell))d\ell$$
$$+\frac{1}{\Gamma(\varpi+\vartheta)}\int_{\mathfrak{t}_{\Bbbk}}^{\mathfrak{t}}(\mathfrak{t}-\ell)^{\varpi+\vartheta-1}\check{f}(\ell,\upsilon(\ell),\Psi\upsilon(\ell),\chi\upsilon(\ell))d\ell$$
$$-\sum_{i=1}^{\Bbbk}\frac{\mathfrak{t}_i^{\vartheta+1}-\mathfrak{t}_i}{\Gamma(\varpi)\Gamma(\vartheta+2)}$$
$$\int_0^1(1-\ell)^{\varpi-1}\check{f}(\ell,\upsilon(\ell),\Psi\upsilon(\ell),\chi\upsilon(\ell))d\ell$$

$$
\begin{aligned}
&-\frac{\mathfrak{t}^{\vartheta+1}-\mathfrak{t}}{\Gamma(\varpi)\Gamma(\vartheta+2)}\\
&\int_0^1 (1-\ell)^{\varpi-1}\check{f}(\ell, \upsilon(\ell), \Psi\upsilon(\ell), \chi\upsilon(\ell))d\ell\\
&-\sum_{i=1}^{\mathbb{k}}\frac{\mathfrak{t}_{\mathbb{k}}}{\Gamma(\varpi+\vartheta)}\\
&\int_0^1 (1-\ell)^{\varpi+\vartheta-1}\check{f}(\ell, \upsilon(\ell), \Psi\upsilon(\ell), \chi\upsilon(\ell))d\ell\\
&-\frac{\mathfrak{t}}{\Gamma(\varpi+\vartheta)}\\
&\int_0^1 (1-\ell)^{\varpi+\vartheta-1}\check{f}(\ell, \upsilon(\ell), \Psi\upsilon(\ell), \chi\upsilon(\ell))d\ell+\sum_{i=1}^{\mathbb{k}}\check{I}_i(\upsilon_{\mathfrak{t}_{\mathbb{k}}^-}).
\end{aligned}
$$

Setting $\sup|\check{f}(\mathfrak{t},0,0,0)| = P$.
Consider the set $B_{\mathfrak{r}} = \{\upsilon \in \check{X} : \|\upsilon\| \leq \mathfrak{r}\}$, where $\mathfrak{r} \geq \dfrac{\mathfrak{r}_2}{1-\mathfrak{r}_1}$, with

$$
\mathfrak{r}_2 = 2P(m+1)\left(\frac{1}{\Gamma(\varpi+\vartheta)}+\frac{1}{\Gamma(\varpi)\Gamma(\vartheta+2)}\right).
$$

For each $\mathfrak{t} \in \Theta$ and $\upsilon \in B_{\mathfrak{r}}$, we have

$$
\begin{aligned}
|F\upsilon(\mathfrak{t})| \leq\ & \frac{1}{\Gamma(\varpi+\vartheta)}\\
&\sum_{i=1}^{\mathbb{k}}\int_{\mathfrak{t}_{i-1}}^{\mathfrak{t}_i}(\mathfrak{t}_i-\ell)^{\varpi+\vartheta-1}|\check{f}(\ell, \upsilon(\ell), \Psi\upsilon(\ell), \chi\upsilon(\ell))|d\ell\\
&+\frac{1}{\Gamma(\varpi+\vartheta)}\\
&\int_{\mathfrak{t}_{\mathbb{k}}}^{\mathfrak{t}}(\mathfrak{t}-\ell)^{\varpi+\vartheta-1}|\check{f}(\ell, \upsilon(\ell), \Psi\upsilon(\ell), \chi\upsilon(\ell))|d\ell\\
&+\sum_{i=1}^{\mathbb{k}}\frac{\mathfrak{t}_i^{\vartheta+1}-\mathfrak{t}_i}{\Gamma(\varpi)\Gamma(\vartheta+2)}\\
&\int_0^1(1-\ell)^{\varpi-1}|\check{f}(\ell, \upsilon(\ell), \Psi\upsilon(\ell), \chi\upsilon(\ell))|d\ell\\
&+\frac{\mathfrak{t}^{\vartheta+1}-\mathfrak{t}}{\Gamma(\varpi)\Gamma(\vartheta+2)}\\
&\int_0^1(1-\ell)^{\varpi-1}|\check{f}(\ell, \upsilon(\ell), \Psi\upsilon(\ell), \chi\upsilon(\ell))|d\ell\\
&+\sum_{i=1}^{\mathbb{k}}\frac{\mathfrak{t}_{\mathbb{k}}}{\Gamma(\varpi+\vartheta)}\\
&\int_0^1(1-\ell)^{\varpi+\vartheta-1}|\check{f}(\ell, \upsilon(\ell), \Psi\upsilon(\ell), \chi\upsilon(\ell))|d\ell\\
&+\frac{\mathfrak{t}}{\Gamma(\varpi+\vartheta)}\\
&\int_0^1(1-\ell)^{\varpi+\vartheta-1}|\check{f}(\ell, \upsilon(\ell), \Psi\upsilon(\ell), \chi\upsilon(\ell))|d\ell\\
&+\sum_{i=1}^{\mathbb{k}}|\check{I}_i(\upsilon_{t_{\mathbb{k}}^-})|\\
\leq\ & \frac{1}{\Gamma(\varpi+\vartheta)}\\
&\sum_{i=1}^{\mathbb{k}}\int_{\mathfrak{t}_{i-1}}^{\mathfrak{t}_i}(\mathfrak{t}_i-\ell)^{\varpi+\vartheta-1}|\check{f}(\ell, \upsilon(\ell), \Psi\upsilon(\ell), \chi\upsilon(\ell))\\
&-\check{f}(\ell,0,0,0)+\check{f}(\ell,0,0,0)|d\ell\\
&+\frac{1}{\Gamma(\varpi+\vartheta)}\\
&\int_{\mathfrak{t}_{\mathbb{k}}}^{\mathfrak{t}}(\mathfrak{t}-\ell)^{\varpi+\vartheta-1}|\check{f}(\ell, \upsilon(\ell), \Psi\upsilon(\ell), \chi\upsilon(\ell))\\
&-\check{f}(\ell,0,0,0)+\check{f}(\ell,0,0,0)|d\ell\\
&+\sum_{i=1}^{\mathbb{k}}\frac{\mathfrak{t}_i^{\vartheta+1}-\mathfrak{t}_i}{\Gamma(\varpi)\Gamma(\vartheta+2)}\\
&\int_0^1(1-\ell)^{\varpi-1}|\check{f}(\ell, \upsilon(\ell), \Psi\upsilon(\ell), \chi\upsilon(\ell))\\
&-\check{f}(\ell,0,0,0)+\check{f}(\ell,0,0,0)|d\ell\\
&+\frac{\mathfrak{t}^{\vartheta+1}-\mathfrak{t}}{\Gamma(\varpi)\Gamma(\vartheta+2)}\\
&\int_0^1(1-\ell)^{\varpi-1}|\check{f}(\ell, \upsilon(\ell), \Psi\upsilon(\ell), \chi\upsilon(\ell))\\
&-\check{f}(\ell,0,0,0)+\check{f}(\ell,0,0,0)|d\ell\\
&+\sum_{i=1}^{\mathbb{k}}\frac{\mathfrak{t}_{\mathbb{k}}}{\Gamma(\varpi+\vartheta)}\\
&\int_0^1(1-\ell)^{\varpi+\vartheta-1}|\check{f}(\ell, \upsilon(\ell), \Psi\upsilon(\ell), \chi\upsilon(\ell))\\
&-\check{f}(\ell,0,0,0)+\check{f}(\ell,0,0,0)|d\ell\\
&+\frac{\mathfrak{t}}{\Gamma(\varpi+\vartheta)}\int_0^1(1-\ell)^{\varpi+\vartheta-1}|\check{f}(\ell, \upsilon(\ell), \Psi\upsilon(\ell), \chi\upsilon(\ell))\\
&-\check{f}(\ell,0,0,0)+\check{f}(\ell,0,0,0)|d\ell+\sum_{i=1}^{\mathbb{k}}|\check{I}_i(\upsilon_{\mathfrak{t}_{\mathbb{k}}^-})|\\
\leq\ & \frac{1}{\Gamma(\varpi+\vartheta)}\sum_{i=1}^{\mathbb{k}}\int_{\mathfrak{t}_{i-1}}^{\mathfrak{t}_i}(\mathfrak{t}_i-\ell)^{\varpi+\vartheta-1}(\sigma(\ell)(|\upsilon(\ell)|\\
&+|\Psi\upsilon(\ell)|+|\chi\upsilon(\ell)|)+P)d\ell\\
&+\frac{1}{\Gamma(\varpi+\vartheta)}\\
&\int_{\mathfrak{t}_{\mathbb{k}}}^{\mathfrak{t}}(\mathfrak{t}-\ell)^{\varpi+\vartheta-1}(\sigma(\ell)(|\upsilon(\ell)|\\
&+|\Psi\upsilon(\ell)|+|\chi\upsilon(\ell)|)+P)d\ell\\
&+\frac{2m}{\Gamma(\varpi)\Gamma(\vartheta+2)}\\
&\int_0^1(1-\ell)^{\varpi-1}(\sigma(\ell)(|\upsilon(\ell)|\\
&+|\Psi\upsilon(\ell)|+|\chi\upsilon(\ell)|)+P)d\ell\\
&+\frac{2}{\Gamma(\varpi)\Gamma(\vartheta+2)}\int_0^1(1-\ell)^{\varpi-1}(\sigma(\ell)(|\upsilon(\ell)|\\
&+|\Psi\upsilon(\ell)|+|\chi\upsilon(\ell)|)+P)d\ell\\
&+\frac{m}{\Gamma(\varpi+\vartheta)}\int_0^1(1-\ell)^{\varpi+\vartheta-1}(\sigma(\ell)(|\upsilon(\ell)|\\
&+|\Psi\upsilon(\ell)|+|\chi\upsilon(\ell)|)+P)d\ell
\end{aligned}
$$

$$+\frac{1}{\Gamma(\varpi+\vartheta)}\int_0^1(1-\ell)^{\varpi+\vartheta-1}(\sigma(\ell)(|\upsilon(\ell)|+|\Psi\upsilon(\ell)| + |\chi\upsilon(\ell)|)+P)\mathrm{d}\ell+ml\|\upsilon\|$$
$$\leq \frac{m}{\Gamma(\varpi+\vartheta)}(1+\Psi^*+\chi^*)\|\upsilon\|\int_0^1\sigma(\ell)\mathrm{d}\ell+\frac{P}{\Gamma(\varpi+\vartheta)}\sum_{i=1}^{\Bbbk}\int_{\mathfrak{t}_{i-1}}^{\mathfrak{t}_i}(\mathfrak{t}_i-\ell)^{\varpi+\vartheta-1}\mathrm{d}\ell$$
$$+\frac{1}{\Gamma(\varpi+\vartheta)}(1+\Psi^*+\chi^*)\|\upsilon\|\int_0^1\sigma(\ell)\mathrm{d}\ell+\frac{P}{\Gamma(\varpi+\vartheta)}\int_{\mathfrak{t}_{\Bbbk}}^{\mathfrak{t}}(\mathfrak{t}-\ell)^{\varpi+\vartheta-1}\mathrm{d}\ell$$
$$+\frac{2m}{\Gamma(\varpi)\Gamma(\vartheta+2)}(1+\Psi^*+\chi^*)\|\upsilon\|\int_0^1\sigma(\ell)\mathrm{d}\ell$$
$$+\frac{2mP}{\Gamma(\varpi)\Gamma(\vartheta+2)}\int_0^1(1-\ell)^{\varpi-1}\mathrm{d}\ell$$
$$+\frac{2}{\Gamma(\varpi)\Gamma(\vartheta+2)}(1+\Psi^*+\chi^*)\|\upsilon\|\int_0^1\sigma(\ell)\mathrm{d}\ell$$
$$+\frac{2P}{\Gamma(\varpi)\Gamma(\vartheta+2)}\int_0^1(1-\ell)^{\varpi-1}\mathrm{d}\ell$$
$$+\frac{m}{\Gamma(\varpi+\vartheta)}(1+\Psi^*+\chi^*)\|\upsilon\|\int_0^1\sigma(\ell)\mathrm{d}\ell$$
$$+\frac{P}{\Gamma(\varpi+\vartheta)}\int_0^1(1-\ell)^{\varpi+\vartheta-1}\mathrm{d}\ell$$
$$+\frac{1}{\Gamma(\varpi+\vartheta)}(1+\Psi^*+\chi^*)\|\upsilon\|\int_0^1\sigma(\ell)\mathrm{d}\ell$$
$$+\frac{P}{\Gamma(\varpi+\vartheta)}\int_0^1(1-\ell)^{\varpi+\vartheta-1}\mathrm{d}\ell+ml\|\upsilon\|$$
$$\leq\left\{2(m+1)(1+\Psi^*+\chi^*)\sigma^*\left[\frac{1}{\Gamma(\varpi+\vartheta)}+\frac{1}{\Gamma(\varpi)\Gamma(\vartheta+2)}\right]+ml\right\}\|\upsilon\|$$
$$+2P(m+1)\left[\frac{1}{\Gamma(\varpi+\vartheta)}+\frac{1}{\Gamma(\varpi)\Gamma(\vartheta+2)}\right].$$

Then,

$$\|\digamma\upsilon\|\leq\mathfrak{r}.$$

Therefore,

$$\digamma B_{\mathfrak{r}}\subseteq B_{\mathfrak{r}}.$$

Next, we have to prove that $\digamma$ is a contraction mapping. For $\upsilon,\mathfrak{y}\in B_{\mathfrak{r}}$, we have

$$|\digamma\upsilon(\mathfrak{t})-\digamma\mathfrak{y}(\mathfrak{t})|$$
$$\leq\frac{1}{\Gamma(\varpi+\vartheta)}\sum_{i=1}^{\Bbbk}\int_{\mathfrak{t}_{i-1}}^{\mathfrak{t}_i}(\mathfrak{t}_i-\ell)^{\varpi+\vartheta-1}|\check{f}(\ell,\upsilon(\ell),\Psi\upsilon(\ell),\chi\upsilon(\ell)) - \check{f}(\ell,\mathfrak{y}(\ell),\Psi\mathfrak{y}(\ell),\chi\mathfrak{y}(\ell))|\mathrm{d}\ell$$
$$+\frac{1}{\Gamma(\varpi+\vartheta)}\int_{\mathfrak{t}_{\Bbbk}}^{\mathfrak{t}}(\mathfrak{t}-\ell)^{\varpi+\vartheta-1}|\check{f}(\ell,\upsilon(\ell),\Psi\upsilon(\ell),\chi\upsilon(\ell)) - \check{f}(\ell,\mathfrak{y}(\ell),\Psi\mathfrak{y}(\ell),\chi\mathfrak{y}(\ell))|\mathrm{d}\ell$$
$$+\sum_{i=1}^{\Bbbk}\frac{\mathfrak{t}_i^{\vartheta+1}-\mathfrak{t}_i}{\Gamma(\varpi)\Gamma(\vartheta+2)}\int_0^1(1-\ell)^{\varpi-1}|\check{f}(\ell,\upsilon(\ell),\Psi\upsilon(\ell),\chi\upsilon(\ell)) - \check{f}(\ell,\mathfrak{y}(\ell),\Psi\mathfrak{y}(\ell),\chi\mathfrak{y}(\ell))|\mathrm{d}\ell$$
$$+\frac{\mathfrak{t}^{\vartheta+1}-\mathfrak{t}}{\Gamma(\varpi)\Gamma(\vartheta+2)}\int_0^1(1-\ell)^{\varpi-1}|\check{f}(\ell,\upsilon(\ell),\Psi\upsilon(\ell),\chi\upsilon(\ell)) - \check{f}(\ell,\mathfrak{y}(\ell),\Psi\mathfrak{y}(\ell),\chi\mathfrak{y}(\ell))|\mathrm{d}\ell$$
$$+\sum_{i=1}^{\Bbbk}\frac{\mathfrak{t}_{\Bbbk}}{\Gamma(\varpi+\vartheta)}\int_0^1(1-\ell)^{\varpi+\vartheta-1}|\check{f}(\ell,\upsilon(\ell),\Psi\upsilon(\ell),\chi\upsilon(\ell)) - \check{f}(\ell,\mathfrak{y}(\ell),\Psi\mathfrak{y}(\ell),\chi\mathfrak{y}(\ell))|\mathrm{d}\ell$$
$$+\frac{\mathfrak{t}}{\Gamma(\varpi+\vartheta)}\int_0^1(1-\ell)^{\varpi+\vartheta-1}|\check{f}(\ell,\upsilon(\ell),\Psi\upsilon(\ell),\chi\upsilon(\ell)) - \check{f}(\ell,\mathfrak{y}(\ell),\Psi\mathfrak{y}(\ell),\chi\mathfrak{y}(\ell))|\mathrm{d}\ell$$
$$+\sum_{i=1}^{\Bbbk}|\check{I}_i(\upsilon_{\mathfrak{t}_{\Bbbk}^-})-\check{I}_i(\mathfrak{y}_{\mathfrak{t}_{\Bbbk}^-})|$$
$$\leq\frac{1}{\Gamma(\varpi+\vartheta)}\sum_{i=1}^{\Bbbk}\int_{\mathfrak{t}_{i-1}}^{\mathfrak{t}_i}(\mathfrak{t}_i-\ell)^{\varpi+\vartheta-1}\sigma(\ell)(1+\Psi^*+\chi^*)\|\upsilon-\mathfrak{y}\|\mathrm{d}\ell$$
$$+\frac{1}{\Gamma(\varpi+\vartheta)}\int_{\mathfrak{t}_{\Bbbk}}^{\mathfrak{t}}(\mathfrak{t}-\ell)^{\varpi+\vartheta-1}\sigma(\ell)(1+\Psi^*+\chi^*)\|\upsilon-\mathfrak{y}\|\mathrm{d}\ell$$
$$+\frac{2m}{\Gamma(\varpi)\Gamma(\vartheta+2)}\int_0^1(1-\ell)^{\varpi-1}\sigma(\ell)(1+\Psi^*+\chi^*)\|\upsilon-\mathfrak{y}\|\mathrm{d}\ell$$
$$+\frac{2}{\Gamma(\varpi)\Gamma(\vartheta+2)}\int_0^1(1-\ell)^{\varpi-1}\sigma(\ell)(1+\Psi^*+\chi^*)\|\upsilon-\mathfrak{y}\|\mathrm{d}\ell$$

$$+\frac{m}{\Gamma(\varpi+\vartheta)}$$
$$\int_0^1 (1-\ell)^{\varpi+\vartheta-1}\sigma(\ell)(1+\Psi^*+\chi^*)\|\upsilon-\mathfrak{y}\|d\ell$$
$$+\frac{1}{\Gamma(\varpi+\vartheta)}$$
$$\int_0^1 (1-\ell)^{\varpi+\vartheta-1}\sigma(\ell)(1+\Psi^*+\chi^*)\|\upsilon-\mathfrak{y}\|d\ell$$
$$+ml\|\upsilon-\mathfrak{y}\|$$
$$\leq \left[\frac{2(m+1)}{\Gamma(\varpi+\vartheta)}(1+\Psi^*+\chi^*)\sigma^* + \frac{2(m+1)}{\Gamma(\varpi)\Gamma(\vartheta+2)}(1+\Psi^*+\chi^*)\sigma^*+ml\right]\|\upsilon-\mathfrak{y}\|$$
$$\leq \left\{2(m+1)(1+\Psi^*+\chi^*)\sigma^* \left[\frac{1}{\Gamma(\varpi+\vartheta)}+\frac{1}{\Gamma(\varpi)\Gamma(\vartheta+2)}\right]+ml\right\}\|\upsilon-\mathfrak{y}\|.$$

Since, $\mathfrak{r}_1 < 1$, then $\digamma$ is a contraction. Therefore, system (1.1) has a unique solution. □

4 Examples

We provide two examples in this section to demonstrate the applicability of our findings.

Example 4.1 Assume the following problem:

$$^cD^{\frac{17}{11}}(^cD^{\frac{16}{11}})\upsilon(t) = \frac{t^3}{400}\left(\frac{|\upsilon(t)e^{-t}|}{|1+\upsilon(t)|} + \int_0^t \frac{(t+\ell)^3|\upsilon(\ell)|\cos(\ell)+\sin(\ell)}{400+(1+|\upsilon(\ell)|)}\right)$$
$$\Delta\upsilon|_{t=\frac{1}{2}} = \frac{|\upsilon(\frac{1^-}{2})|}{10+|\upsilon(\frac{1^-}{2})|}$$
$$\upsilon(0)=\upsilon(1)={}^c D\pi^{16}\upsilon(0)={}^c D\pi^{16}\upsilon(1)=0,\ t\in\Theta.$$

Here $\vartheta=\frac{16}{11}$, $\varpi=\frac{17}{11}$, $m=1$

$$\check{f}(t,\upsilon,\mathfrak{y},z) = \frac{t^3}{400}\left(\frac{|\upsilon(t)|e^{-t}}{1+|\upsilon(t)|}+\frac{|\mathfrak{y}(t)|\cos(t)}{1+|\mathfrak{y}(t)|}+\frac{|z(t)|\sin(t)}{1+|z(t)|}\right)$$
$$\check{I}_1(\upsilon) = \frac{|\upsilon|}{1+|\upsilon|}$$
$$|\check{I}_1(\upsilon)-\check{I}_1(\upsilon)| = \left|\frac{|\upsilon|}{1+|\upsilon|}-\frac{|\mathfrak{y}|}{1+|\mathfrak{y}|}\right| \leq \frac{1}{10}\|\upsilon-\mathfrak{y}\|$$

$$\alpha(t,\ell)=\beta(t,\ell)=\frac{(t+\ell)^3}{400}$$

$$\sigma(t)=\frac{t^3}{400},\ \theta(t)=\frac{3t^3}{400}\ \&\ l=\frac{1}{10}.$$

It follows that

$$\Psi* = \chi* = \frac{15}{600},$$
$$\sigma* = \frac{1}{1600}.$$

Theorem 3.1 allows us to conclude that there is at least one solution to the given problem.

Example 4.2 Take the following problem:

$$^cD^{\frac{10}{7}}(^cD^{\frac{11}{7}})\upsilon(t) = \frac{t^2}{200}\left(\frac{1}{1+|\upsilon(t)|}+\frac{1}{100}\int_0^t t^4 s^3\upsilon(s)d\ell\right),\ t\in\Theta$$
$$\Delta\upsilon|_{t=\frac{1}{2}} = \frac{|\upsilon(\frac{1^-}{2})|}{10+|\upsilon(\frac{1^-}{2})|}$$
$$\upsilon(0)=\upsilon(1)={}^c D^{\frac{10}{7}}\upsilon(0)={}^c D^{\frac{10}{7}}\upsilon(1)=0$$

here $\vartheta=\frac{11}{7}$, $\varpi=\frac{10}{7}$, $m=1$

$$\check{f}(t,\upsilon,\mathfrak{y},z) = \frac{t^3}{400}\left(\frac{1}{1+|\upsilon(t)|}+\mathfrak{y}(t)+z(t)\right)$$
$$\check{I}_1(\upsilon) = \frac{|\upsilon|}{1+|\upsilon|}$$
$$|\check{I}_1(\upsilon)-\check{I}_1(\upsilon)| = \left|\frac{|\upsilon|}{1+|\upsilon|}-\frac{|\mathfrak{y}|}{1+|\mathfrak{y}|}\right| \leq \frac{1}{10}\|\upsilon-\mathfrak{y}\|$$

$$\alpha(t,\ell)=\beta(t,\ell)=\frac{t^4\ell^3}{200},\ \&\ l=\frac{1}{10}$$

$$\sigma(t)=\frac{t^2}{200}\ \&\ \sigma*=\frac{1}{600}$$

$$r_1 = 2(m+1)(1+\Psi^*+\chi^*)\sigma^*\left(\frac{1}{\Gamma(\varpi+\vartheta)}+\frac{1}{\Gamma(\varpi)\Gamma(\vartheta+2)}\right)+ml$$
$$\leq 2(1+1)\left(1+\frac{1}{800}+\frac{1}{800}\right)$$
$$\frac{1}{600}\left(\frac{1}{\Gamma(\frac{10}{7}+\frac{11}{7})}+\frac{1}{\Gamma(\frac{10}{7})\Gamma(\frac{11}{7}+2)}\right)+\frac{1}{10}$$
$$\approx 0.105433 < 1.$$

We conclude from Theorem 3.2 that the given problem has an unique solution.

5 Conclusion

In this paper, we have investigated sequential fractional integro-differential equations with impulsive conditions.. We proved the existence and uniqueness of solutions by using the Banach contraction mapping principle and Krasnoselskii's fixed point theorem. Examples are given to highlight the main results. Furthermore, this can be extended with delay conditions over the infinite interval.

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