Optimal control model for criminal gang population in a limited-resource setting

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Abstract

In this present paper, the principles of optimal control theory is applied to a non-linear mathematical model for the population dynamics of criminal gangs with variability in the sub-population. To decrease (minimize) the progression rate of susceptible populations with no access to crime prevention programs from joining criminal gangs and increase (maximize) the rate of arrested and prosecution of criminals, we incorporate time-dependent control functions. These two functions represent the crime prevention strategy for the susceptible population and case finding control for the criminal gang population, in a limited-resource setting. Furthermore, we present a cost-effectiveness analysis for crime control intervention-related benefits to ascertain the most cost-effective and efficient optimal control strategy. The optimal control functions presented herein are solved by employing the Runge-Kutta Method of order four. Numerical results are demonstrated for different scenarios to exemplify the impact of the controls on the criminal gangs' population.

1 Introduction

1.1 Global crime

In 2011, the United Nations Office on Drugs and Crime (UNODC) estimated the total number of annual deaths by homicides to be 468,000 globally. Over a third (36%) of this occurrence happened on the African continent. Similarly in 2011, gender-based violence affect most women worldwide and poses a severe threat to mankind, particularly the peaceful coexistence human beings in the society [1]. According to the UNODC, the global male homicide rate is put at 9.7% as against the female homicide rate 2.7% in 2013, with the highest in the Americas (29.3 per 100,000 males). The increase in the degrees of homicide are linked to organized crime and gangs as coordinated wrongdoing in the Americas is seen to be more than other regions whose homicide rates are put together at 4.5 per 100,000 males. It is troubling to report that 43% of all homicide victims globally are young males aged 15–29 [2]. According to the UNODC homicide report in 2019, the Americas continue to report high homicide rates among young men with a homicide rate for men aged 18–19 estimated at 46 per 100,000. By contrast, Europe has seen a decrease in the homicide rate by 63% and 38% since 2002 and 1990 respectively. Similarly, there is a huge decline rate (36%) in Asia since 1990 [3]. In 2021, it was reported that Latin America witnessed a relatively high unrest in 2019 while the novel coronavirus (COVID-19) ripped through the region in 2020, upending everything from commercial trade to the operations of local gangs and transnational criminal organizations. It is, however, important to note that the

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pandemic may have impacted levels of violence in some of the countries in the region [4]. Consequently, it is therefore important to take the issues of crime seriously if we are prepared to make the world a better place.

1.2 Crime in Africa

African nations suffer from poorly-resourced criminal justice systems by having the world's least favorable police–and judge-to-population ratios. This eventually affects prosecution and sentencing rates; regardless of whether the security agencies perform optimally, criminals are considerably less liable to be penalized than offenders in other parts of the world. Africa has consistently been at the forefront of global statistics on crime: out of 437,000 deaths caused by intentional homicide globally in 2012, 31% happened in Africa, consequently, suggesting that Africa has a high homicide rate among the countries of the world [5]. Conflicts in recent times focus on mentally destroying the people through extreme cruelty and brutality as the young ones are often at the receiving end. Among the Angolan children interviewed by the UNODC in 2015, two-thirds of these children had witnessed people murdered in cold blood. Similarly, 56% of Rwanda's children during the genocide had witnessed other children murder people, while 80% had lost immediate family members. While it is harsh to affirm that survivors of brutality are mechanistically destined to visit it upon others, exposure to brutality has been discovered by crime analysts to be a typical component in the upbringings of offenders. All over the world, teenagers and young adults (males) involve in criminal activities more than their female counterparts, and Africa's young population of about 43% falls within this pool of potential offenders. Sadly, most of these young populations are either out of school or unemployed [6]. According to the South African Police report on crime statistics in 2021, contact crimes (sexual offenses, murder) and all other categories of assault registered a 60.6% increase, compared to the corresponding period of the previous year [7]. It is, however, important to note that crime statistics in Africa during the pandemic took different dimensions and irregular variation to the crime trends.

1.3 Crime in Nigeria

According to the National Bureau of Statistics (NBS), a total of 125,790 and 134,663 criminal cases in 2016 [8] and 2017 [9] respectively were reported across Nigeria. These high numbers could be attributed to the causative factors of criminal behaviors. Specifically, peer pressure, parental imitation, and neglect, hereditary or natural factor, poor education, media violence, poverty, child abuse, etc [10]. Nigeria has many children identified as out-of-school, homeless, in poverty, orphaned, or single-parent, especially those born

out-of-wedlock. The current trend of the Almajiri system in the North; the Alaye or Area Boys in the South-West, particularly the Lagos-Ibadan axis; the Yandaba in Kano are all indications of the breakdown of community norms and value systems. These groups of delinquent youths have many things in common: they are mostly unemployed or not engaged; homeless, poor, and poorly educated, if at all. They are mostly found in slums, on the streets, and under the bridges of cities, and tend to be a complete nuisance [10]. Research has shown that these poorly managed youths were susceptible to contracting the COVID-19 [11,12].

Furthermore, popular gangs whose members are usually teens and young adults are beginning to emerge. One of such is the *yahoo boys* gang which has spread across Nigeria. These groups are usually independent but engage in diverse criminal activities which includes internet scam, hacking bank accounts, money ritual amongst others. Unidentified criminal gangs periodically perpetrate other criminal evils which include but are not limited to robbery, rape, kidnapping, cattle rustling, banditry, militancy, and other criminal activities that may result in the loss of lives of residents' loved ones and breadwinners [13]. Another contributing factor to Nigeria crime rate is the unemployment among the youth population. It is estimated to have grown from 12.6% to 21.5% between 2012 and 2015 as it is believed to have contributed significantly to the increase in social vices and insecurity. One more important reason is the lack of political will of previous administration to address the problems of poverty, unemployment, and inequitable distribution of wealth among the country [14]. This is observed to have compounded the problem and make it more complicated for new administration to manage.

The aforementioned factors must have triggered the recent report by the Nigeria Police Force–a total number of 1210 stolen vehicles and 1075 rape cases were recorded in 2017. In 2020, the National Bureau of Statistics (NBS) reported a total number 1890 and 1173 of trafficked persons in 2017 and 2018 respectively with a minimal decrease in 2019 which is stood at 1152 [15]. Nigeria needs an holistic approach which encompasses scientific and non-scientific measures in order to end this rising insecurity as this research will explore the former in addressing the criminal issues in one of the most populous countries of the world.

1.4 Paper organization

This paper is arranged in the following ways to address the sociological problem discussed earlier from the mathematical perspective. Section 2 presents the formulation of the optimal control model. The stability analysis and the numerical simulation of the optimal control model are considered in Sects. 3 and 4 receptively. The cost-effective analysis is

also presented in Sect. 5 while the discussion and conclusion are in Sects. 6 and 7 respectively.

2 Optimal control model

2.1 Model formulation

Several mathematical techniques have been employed to study the dynamics of epidemic models from optimal control perspective, see [16–19]. More recently, mathematical modeling have been used to study the dynamics of criminal gangs, see, [13,20–36]. As it can rarely be found in the literature, our interest is to investigate the impact of optimal control strategy on the dynamics of the age-structured criminal gangs, in a limited-resource setting. This idea is informed by the previous work in [37]. We are set to answer this open research question in this paper by properly modifying the deterministic mathematical models for the dynamics of the criminal gang as studied in [13]. Consider the age-structured model in [13], while drawing inspiration from the sociological and criminological works in [37–40], we introduce two time dependent control functions, $u_1(t)$ and $u_2(t)$ to all the age-structured classes.

The time-dependent control function: $u_1(t)$, is bounded Lebesgue integrable. The function $u_1(t)$ is a control that supports enlightenment campaign programs and empowerment programs (door-to-door, community square talks, television and radio jingles, print media, youth empowerment, access to loan, and other prevention strategies of crime as listed in [41] report) by government agencies and non-governmental organization (NGO)'s during the criminally active period. The expression $1 - u_1(t)$ denotes the drive that supports the enlightenment campaign programs and empowerment programs during the criminally active period. The control $u_1(t) \rightarrow 1$ indicates the impact intensive enlightenment campaigns and consistent empowerment programs will have on gang formation and their activities in the community. In other words, it implies the enlightenment campaign and consistent empowerment programs that prevent the susceptible individuals (S_2 and S_3) from joining the gang classes (G_1, G_2, and G_3). The control $u_1(t) \rightarrow 0$ indicates that the enlightenment campaign and empowerment program are loose and no longer effective. Hence, there will be high progression rate of susceptible individuals (S_2 and S_3) to the gang classes (G_1, G_2 and G_3).

The time-dependent control function: $u_2(t)$ is bounded Lebesgue integrable. The function $u_2(t)$ is a case-finding control that represents the proportion of initiated persons who are arrested, law-enforced, and imprisoned in the correctional center for correction and the prevention of close interaction with susceptible individuals, in a criminally active population. The term $1 + u_2(t)$ represents the effort that sustains the

police hunting at criminals and correctional policy that 'holds down' the convicts serving a sentencing term for proper correction.

Here, we minimize the objective functional

$$
G(u_1, u_2) = \int_0^{t_{final}} \Big(G_1(t) + G_2(t) + G_3(t) - C_1(t) \\
- C_2(t) - C_3(t) + \frac{B_1}{2} u_1^2(t) + \frac{B_2}{2} u_2^2(t) \Big) \, \mathrm{d}t
$$

(2.1)

In (2.1), we minimize the population with delinquent behavior in classes G_1 and G_2 through the NGOs and police hunting strategies. Also, we minimize the population with delinquent behavior in class G_3 through police hunting strategies only. In the same manner, we desire to increase the effort that takes criminals from the street for appropriate corrections in the correctional centers, this is of great interest in this work as well. Thus, we desire to minimize functional (2.1) that implies a trade-off required in minimizing the population exhibiting status offenses (eg., tobacco consumption, curfew violation, having delinquent friends, alcohol consumption, underage voting, assault, breaking, and illegal use of banned drugs, etc) in class G_1, and non-status offenses (murder, robbery, illegal sale of drugs, human trafficking, money laundering, kidnapping, illegal smuggling of goods, assassinations, weapon, and drug trafficking, rape among, etc) in classes G_2 and G_3 respectively while increasing the number of arrested, prosecuted and isolated individuals for corrections, with minimal associated cost in their respective classes C_1, C_2 and C_3.

The constants, B_1 and B_2, are the positive weights for balancing cost factors in the functional (2.1). We work with the notion that the $B_2 > B_1$ because the cost associated with control u_2 involves searching for criminal gangs, picking them for arrest, and prosecuting at the court of competent jurisdiction, then sentencing them for correction may require a huge cost. Hence the government will have more funds to use in this situation and citizens selfless effort is also involved in this regard. The expenses linked with control u_1 entails government and NGOs embarking on enlightenment and empowerment programs only.

The t_{final} is the final time of the intervention. We note that the set $U = \{(u_1, u_2)$ is measurable and $0 \le u_1, u_2 \le 1$ for $t \in [0, t_{final}]\}$ is the control set. Thus,

$$
G(u_1^*, u_2^*) = \min\{G(u_1^*, u_2^*) : u_1^*, u_2^* \in U\}, \qquad (2.2)
$$

where the set U can further be expressed as

$$
U = \{(u_1, u_2) \in L^1(0, t_{final}) \times L^1(0, t_{final}) | a_i \le u_i \le b_i\}.
$$

(2.3)

where a_1, a_2, b_1, b_2 are fixed constants such that $a_1 > 0, a_2 > 0, b_1 > 0, b_2 > 0$.

We note that the convention used in setting up the functional (2.1) is exclusively subject to the specific application and goal to be considered. This implies that different types of the functional (2.1) could be defined. Furthermore, it is well established that optimal control results are tied to the optimal control model, how the controls are used in the model and how the associated objective functional is defined [42] As a result, this will genuinely guide us on how the controls presented herein are being placed in the constraints to be formulated. The constraint equations subjected to the objective function $G(u_1, u_2)$ is given by

change in the prosecuted and law-enforced gang members (individual within the age bracket 18 - above years) serving jail terms with respect to time t; N is the total population at any given time t. The parameters are as well defined in the Table 1.

Furthermore, the terms $(1 - u_1)\beta_1(1 + \sigma_1)G_1 S_2$, $(1 - u_1)\beta_2(1 + \sigma_2)G_2 S_2$, $(1 - u_1)\beta_3(1 + \sigma_3)G_3 S_3$ in (2.4) represent the effort $(1 - u_1)$ required to sustain the enlightenment campaign programs and empowerment programs in classes S_2 and S_3, respectively, during the criminally active period. Also, the terms $(1 + u_2)\gamma_1 G_1$, $(1 + u_2)\gamma_2 G_2$, $(1 + u_2)\gamma_3 G_3$ in (2.4) represent the effort $(1 + u_2)$ required to acquire the rates $\gamma_1, \gamma_2, \gamma_3$ at which gang members in classes G_1, G_2, G_3 are

$$
\left.
\begin{aligned}
S_1' &= \Lambda - (\mu + \alpha_1)S_1 \\
S_2' &= \alpha_1 S_1 - (\mu + \alpha_2)S_2 - (1 - u_1)\beta_1(1 + \sigma_1)G_1 S_2 - (1 - u_1)\beta_2(1 + \sigma_2)G_2 S_2 + \tau_1 C_1 \\
S_3' &= \alpha_2 S_2 - \mu S_3 - (1 - u_1)\beta_3(1 + \sigma_3)G_3 S_3 + \tau_2 C_3 \\
G_1' &= (1 - u_1)\beta_1(1 + \sigma_1)G_1 S_2 + (1 - u_1)p\beta_2(1 + \sigma_2)G_2 S_2 - ((1 + u_2)\gamma_1 + \mu + \delta_1)G_1 \\
G_2' &= (1 - u_1)(1 - p)\beta_2(1 + \sigma_2)G_2 S_2 - ((1 + u_2)\gamma_2 + \alpha_3 + \mu + \delta_2)G_2 \\
G_3' &= (1 - u_1)\beta_3(1 + \sigma_3)G_3 S_3 - ((1 + u_2)\gamma_3 + \mu + \delta_3)G_3 + \alpha_3 G_2 \\
C_1' &= (1 + u_2)\gamma_1 G_1 - (\mu + \delta_4 + \tau_1)C_1 \\
C_2' &= (1 + u_2)\gamma_2 G_2 - (\mu + \delta_5 + \alpha_4)C_2 \\
C_3' &= (1 + u_2)\gamma_3 G_3 + \alpha_4 C_2 - (\mu + \delta_6 + \tau_2)C_3.
\end{aligned}
\right\}
\tag{2.4}
$$

In system (2.4), S_1', represents the rate of change in susceptible individuals within the age bracket 0–7 years (they are resistant to crime) with respect to time t; S_2', means the rate of change in susceptible individuals within the age bracket 8–17 years (they are exposed to delinquent behaviors from their peers) with respect to time t; S_3', represents the rate of change in susceptible individuals within the age bracket 18–above years (they are expose to delinquent behaviors from their peers) with respect to time t; G_1', implies the rate of change in criminal gang population (within the age bracket 8–17 years) with status offenses (eg., tobacco consumption, alcohol consumption, assault, breaking and entering, etc [43,44]), with respect to time t; G_2', represents the rate of change in criminal gang population (within the age bracket 8–17 years) that exhibit capital offenses (murder), with respect to time t; G_3', represents the rate of change in criminal gang population (within the age bracket 18–above years) with non-status offenses (eg., murder, robbery, fraud, illegal sale of drugs, human trafficking, money laundering, kidnapping, illegal smuggling of goods, weapon and drug trafficking, domestic violence, rape, etc) [45], with respect to time t; C_1', means the rate of change in the individuals under rehabilitation (individual within the age bracket 8–17 years) with respect to time t; C_2', implies the rate of change in the individuals in government remand homes (individual within the age bracket 8–17 years) with respect to time t; C_3', represents the rate of

being law-enforced (arrested, prosecuted and sent to the correctional center C_3) by police during the criminally active period. The bounds are u_1: $0 \leq u_1 \leq 1$ following in [16,42] and u_2: $0 \leq u_2 \leq \frac{1}{\gamma_1 + \gamma_2 + \gamma_3}$ for numerical simulation as implied from [42]. We have chosen that of u_2 because of the rates of enforcement and prosecution in classes G_1, G_2, G_3 influences the level of control that should be enforced to get criminal gangs corrected in the correctional centers C_1, C_2, C_3.

3 Analysis of the optimal control functions

It follows from the work in [46] that the necessary conditions that an optimal pair must satisfy comes from Pontryagin's Maximum Principles (PMP). This principle converts (2.1, 2.2, 2.4), into the problem of minimizing a Hamiltonian (H), pointwisely with respect to the controls, $u_1(t)$ and $u_2(t)$:

$$
H = G_1(t) + G_2(t) + G_3(t) - C_1(t) - C_2(t) - C_3(t)
$$
$$
+ \frac{B_1}{2}u_1^2(t) + \frac{B_2}{2}u_2^2(t) + \sum_{i=1}^{9} \lambda_i^* f_i, \tag{3.1}
$$

Table 1 Description of parameters

Parameters	Interpretation
Λ	Recruitment (birth-rate) into the susceptible class S_1
μ	Natural death-rate
α_1	Rate at which individuals in class S_1 aged-into the class S_2
α_2	Rate at which individuals in class S_2 aged-into the class S_3
α_3	Rate at which individuals in class G_2 aged-into the class G_3
α_4	Rate at which individuals in class C_2 aged-into the class C_3
β_1	Initiation rate for individuals between classes (S_2, G_1)
β_2	Initiation rate for individuals between classes (S_2, G_2)
β_3	Initiation rate for individuals between classes (S_3, G_3)
σ_1	Imitation rate for individuals between classes (S_2, G_1)
σ_2	Imitation rate for individuals between classes (S_2, G_2)
σ_3	Imitation rate for individuals between classes (S_3, G_3)
p	Fraction of delinquent behavior progression
γ_1	Transfer rate of individuals in class G_1 to rehabilitation center C_1
γ_2	Arrested and law-enforced rate of individual in class G_2
γ_3	Arrested and law-enforced rate of individual in class G_3
τ_1	Release rate from the rehabilitation center
τ_2	Release rate from the prison
δ_1	Death rate due to consequences of criminal activities in classes G_1
δ_2	Death rate due to consequences of criminal activities in class G_2
δ_3	Death rate due to consequences of criminal activities in class G_3
δ_4	Death rate due to correctional measures and attacks on rehabilitation centers in class C_1
δ_5	Death rate due to correctional measures and attacks on remand homes in class C_2
δ_6	Death rate due to correctional measures and to attacks on prisons in class C_3
u_1	First time-dependent control function
u_2	Second time-dependent control function

where f_i, $i = 1, \ldots, 9$ is the right hand side (RHS) of the system of DEs of the $i - th$ state variable. Since the work of [18] guarantee the existence of optimal control model, then PMP can be used and invoke the existence result for the control pairs from the work in [18]. The function H is formed by allowing each of the adjoint variables to correspond to each of the state variables accordingly and combining the results with the objective functional leading to

Hence, we state the following Theorem:

Theorem 3.1 *There exists an optimal control pair $u_1^*(t)$, $u_2^*(t)$ and the corresponding solutions $S_1^*, S_2^*, S_3^*, G_1^*, G_2^*, G_3^*, C_1^*, C_2^*, C_3^*$ that minimizes $G(u_1(t), u_2(t))$ over U. Additionally, there exists adjoint functions (co-state variable): $\lambda_1^*, \lambda_2^*, \lambda_3^*, \lambda_4^*, \lambda_5^*, \lambda_6^*, \lambda_7^*, \lambda_8^*,$ and λ_9^* satisfying*

$$
\left.
\begin{aligned}
H = {} & G_1(t) + G_2(t) + G_3(t) - C_1(t) - C_2(t) - C_3(t) + \frac{B_1}{2}u_1^2(t) + \frac{B_2}{2}u_2^2(t) \\
& + \lambda_1^*\left[\Lambda - (\mu + \alpha_1)S_1\right] \\
& + \lambda_2^*\left[\alpha_1 S_1 - (\mu + \alpha_2)S_2 - (1 - u_1)\beta_1(1 + \sigma_1)G_1 S_2 - (1 - u_1)\beta_2(1 + \sigma_2)G_2 S_2 + \tau_1 C_1\right] \\
& + \lambda_3^*\left[\alpha_2 S_2 - \mu S_3 - (1 - u_1)\beta_3(1 + \sigma_3)G_3 S_3 + \tau_2 C_3\right] \\
& + \lambda_4^*\left[(1 - u_1)\beta_1(1 + \sigma_1)G_1 S_2 + (1 - u_1)p\beta_2(1 + \sigma_2)G_2 S_2 - ((1 + u_2)\gamma_1 + \mu + \delta_1)G_1\right] \\
& + \lambda_5^*\left[(1 - u_1)(1 - p)\beta_2(1 + \sigma_2)G_2 S_2 - ((1 + u_2)\gamma_2 + \alpha_3 + \mu + \delta_2)G_2\right] \\
& + \lambda_6^*\left[(1 - u_1)\beta_3(1 + \sigma_3)G_3 S_3 - ((1 + u_2)\gamma_3 + \mu + \delta_3)G_3 + \alpha_3 G_2\right] \\
& + \lambda_7^*\left[(1 + u_2)\gamma_1 G_1 - (\mu + \delta_4 + \tau_1)C_1\right] \\
& + \lambda_8^*\left[(1 + u_2)\gamma_2 G_2 - (\mu + \delta_5 + \alpha_4)C_2\right] \\
& + \lambda_9^*\left[(1 + u_2)\gamma_3 G_3 + \alpha_4 C_2 - (\mu + \delta_6 + \tau_2)C_3\right].
\end{aligned}
\right\}
\tag{3.2}
$$

$$\frac{d\lambda_1^*}{dt} = -\left(-\lambda_1^*[\mu + \alpha_1] + \lambda_2^*[\alpha_1]\right),$$

$$\frac{d\lambda_2^*}{dt} = -\left(\lambda_2^*\left[-(\mu + \alpha_2) - (1 - u_1)\beta_1(1 + \sigma_1)G_1^* - (1 - u_1)\beta_2(1 + \sigma_2)G_2^*\right] + \right.$$
$$\left. + \lambda_3^*[\alpha_2] + \lambda_4^*\left[(1 - u_1)\beta_1(1 + \sigma_1)G_1^* + (1 - u_1)p\beta_2(1 + \sigma_2)G_2^*\right] + \right.$$
$$\left. + \lambda_5^*\left[(1 - u_1)(1 - p)\beta_2(1 + \sigma_2)G_2^*\right]\right),$$

$$\frac{d\lambda_3^*}{dt} = -\left(\lambda_3^*\left[-\mu - (1 - u_1)\beta_3(1 + \sigma_3)G_3^*\right]\right.$$
$$\left. + \lambda_6^*\left[(1 - u_1)\beta_3(1 + \sigma_3)G_3^*\right]\right),$$

$$\frac{d\lambda_4^*}{dt} = -\left(1 + \lambda_2^*\left[-(1 - u_1)\beta_1(1 + \sigma_1)S_2^*\right]\right.$$
$$\left. + \lambda_4^*\left[(1 - u_1)\beta_1(1 + \sigma_1)S_2^* - ((1 + u_2)\gamma_1 + \mu + \delta_1)\right] + \lambda_7^*\left[(1 + u_2)\gamma_1\right]\right),$$

$$\frac{d\lambda_5^*}{dt} = -\left(1 + \lambda_2^*\left[-(1 - u_1)\beta_2(1 + \sigma_2)S_2^*+\right] + \lambda_4^*\left[(1 - u_1)p\beta_2(1 + \sigma_2)S_2^*\right]\right.$$
$$\left. + \lambda_5^*\left[(1 - u_1)(1 - p)\beta_2(1 + \sigma_2)S_2^* - ((1 + u_2)\gamma_2 + \alpha_3 + \mu + \delta_2)\right]\right.$$
$$\left. + \lambda_6^*[\alpha_3] + \lambda_8^*\left[(1 + u_2)\gamma_2\right]\right),$$

$$\frac{d\lambda_6^*}{dt} = -\left(1 + \lambda_3^*\left[-(1 - u_1)\beta_3(1 + \sigma_3)S_3^*\right]\right.$$
$$\left. + \lambda_6^*\left[(1 - u_1)\beta_3(1 + \sigma_3)S_3^* - ((1 + u_2)\gamma_3 + \mu + \delta_3)\right] + \lambda_9^*\left[(1 + u_2)\gamma_3\right]\right),$$

$$\frac{d\lambda_7^*}{dt} = -\left(-1 + \lambda_2^*[\tau_1] + \lambda_7^*\left[-(\mu + \delta_4 + \tau_1)\right]\right),$$

$$\frac{d\lambda_8^*}{dt} = -\left(-1 + \lambda_8^*\left[-(\mu + \delta_5 + \alpha_4)\right] + \lambda_9^*[\alpha_4]\right),$$

$$\frac{d\lambda_9^*}{dt} = -\left(-1 + \lambda_3^*[\tau_2] + \lambda_9^*\left[-(\mu + \delta_6 + \tau_2)\right]\right).$$

$$(3.3)$$

Further simplifying (3.3) yields

$$\frac{d\lambda_1^*}{dt} = \lambda_1^*[\mu + \alpha_1] - \lambda_2^*[\alpha_1],$$

$$\frac{d\lambda_2^*}{dt} = \lambda_2^*\left[(\mu + \alpha_2) + (1 - u_1)\beta_1(1 + \sigma_1)G_1^* + (1 - u_1)\beta_2(1 + \sigma_2)G_2^*\right]$$
$$- \lambda_3^*[\alpha_2] - \lambda_4^*\left[(1 - u_1)\beta_1(1 + \sigma_1)G_1^* + (1 - u_1)p\beta_2(1 + \sigma_2)G_2^*\right]$$
$$- \lambda_5^*\left[(1 - u_1)(1 - p)\beta_2(1 + \sigma_2)G_2^*\right],$$

$$\frac{d\lambda_3^*}{dt} = \lambda_3^*\left[\mu + (1 - u_1)\beta_3(1 + \sigma_3)G_3^*\right] - \lambda_6^*\left[(1 - u_1)\beta_3(1 + \sigma_3)G_3^*\right],$$

$$\frac{d\lambda_4^*}{dt} = -1 + \lambda_2^*\left[(1 - u_1)\beta_1(1 + \sigma_1)S_2^*\right]$$
$$- \lambda_4^*\left[(1 - u_1)\beta_1(1 + \sigma_1)S_2^* - ((1 + u_2)\gamma_1 + \mu + \delta_1)\right] - \lambda_7^*\left[(1 + u_2)\gamma_1\right],$$

$$\frac{d\lambda_5^*}{dt} = -1 + \lambda_2^*\left[(1 - u_1)\beta_2(1 + \sigma_2)S_2^*\right] - \lambda_4^*\left[(1 - u_1)p\beta_2(1 + \sigma_2)S_2^*\right]$$
$$- \lambda_5^*\left[(1 - u_1)(1 - p)\beta_2(1 + \sigma_2)S_2^* - ((1 + u_2)\gamma_2 + \alpha_3 + \mu + \delta_2)\right]$$
$$- \lambda_6^*[\alpha_3] - \lambda_8^*\left[(1 + u_2)\gamma_2\right],$$

$$\frac{d\lambda_6^*}{dt} = -1 + \lambda_3^*\left[(1 - u_1)\beta_3(1 + \sigma_3)S_3^*\right]$$
$$- \lambda_6^*\left[(1 - u_1)\beta_3(1 + \sigma_3)S_3^* - ((1 + u_2)\gamma_3 + \mu + \delta_3)\right] - \lambda_9^*\left[(1 + u_2)\gamma_3\right],$$

$$\frac{d\lambda_7^*}{dt} = 1 - \lambda_2^*[\tau_1] + \lambda_7^*\left[(\mu + \delta_4 + \tau_1)\right],$$

$$\frac{d\lambda_8^*}{dt} = 1 + \lambda_8^*\left[(\mu + \delta_5 + \alpha_4)\right] - \lambda_9^*[\alpha_4],$$

$$\frac{d\lambda_9^*}{dt} = 1 - \lambda_3^*[\tau_2] + \lambda_9^*\left[(\mu + \delta_6 + \tau_2)\right]$$

$$(3.4)$$

with transversality conditions

$$\lambda_i^*(t_{final}) = 0, \quad i = 1, \ldots, 9, \tag{3.5}$$

and $N^* = S_1^* + S_2^* + S_3^* + G_1^* + G_2^* + G_3^* + C_1^* + C_2^* + C_3^*$.

Furthermore, the following characterization holds:

$$\left.\begin{aligned}
u_1^*(t) &= min\left(max\left(a_1, \frac{1}{B_1}T_1\right), b_1\right), \\
u_2^*(t) &= min\left(max\left(a_2, \frac{1}{B_2}T_2\right), b_2\right).
\end{aligned}\right\} \tag{3.6}$$

where

$$\left.\begin{aligned}
T_1 &= \beta_1(1+\sigma_1)G_1^*(\lambda_2^* - \lambda_4^*) + \beta_2(1+\sigma_2)G_2^*(\lambda_2^* - \lambda_5^*) \\
&\quad + p\beta_2(1+\sigma_2)G_2^*(\lambda_5^* - \lambda_4^*) + \beta_1(1+\sigma_1)S_1^*(\lambda_2^* - \lambda_4^*) + \beta_2(1+\sigma_2)S_2^*(\lambda_2^* - \lambda_5^*) \\
&\quad + p\beta_2(1+\sigma_2)S_2^*(\lambda_5^* - \lambda_4^*) + \beta_3(1+\sigma_3)S_3^*(\lambda_3^* - \lambda_6^*) + \beta_3(1+\sigma_3)G_3^*(\lambda_3^* - \lambda_6^*), \\
T_2 &= \gamma_1(\lambda_4^* - \lambda_7^*) + \gamma_2(\lambda_5^* - \lambda_8^*) + \gamma_3(\lambda_6^* - \lambda_9^*).
\end{aligned}\right\} \tag{3.7}$$

The proof of the theorem (3.1) is given thus:

Proof It follows from the Corollary 4.1 in [18], that the convexity of the integrand of the objective functional G with respect to (u_1, u_2) ensures the existence of the control pair (u_1, u_2), and the Lipschitz property of the state system with regards to $S_1, S_2, S_3, G_1, G_2, G_3, C_1, C_2, C_3$. Through the PMP, the adjoint equations and transversality conditions are established, then we have:

$$\left.\begin{aligned}
\frac{d\lambda_1^*}{dt} &= -\frac{\partial H}{\partial S_1}, \quad \lambda_1^*(t_{final}) = 0, \\
\frac{d\lambda_2^*}{dt} &= -\frac{\partial H}{\partial S_2}, \quad \lambda_2^*(t_{final}) = 0, \\
\frac{d\lambda_3^*}{dt} &= -\frac{\partial H}{\partial S_3}, \quad \lambda_3^*(t_{final}) = 0, \\
\frac{d\lambda_4^*}{dt} &= -\frac{\partial H}{\partial S_4}, \quad \lambda_4^*(t_{final}) = 0, \\
\frac{d\lambda_5^*}{dt} &= -\frac{\partial H}{\partial S_5}, \quad \lambda_5(t_{final}) = 0, \\
\frac{d\lambda_6^*}{dt} &= -\frac{\partial H}{\partial S_6}, \quad \lambda_6^*(t_{final}) = 0, \\
\frac{d\lambda_7^*}{dt} &= -\frac{\partial H}{\partial S_7}, \quad \lambda_7^*(t_{final}) = 0, \\
\frac{d\lambda_8^*}{dt} &= -\frac{\partial H}{\partial S_8}, \quad \lambda_8^*(t_{final}) = 0, \\
\frac{d\lambda_9^*}{dt} &= -\frac{\partial H}{\partial S_9}, \quad \lambda_9^*(t_{final}) = 0.
\end{aligned}\right\} \tag{3.8}$$

We put into consideration the optimality conditions

$$\frac{\partial H}{\partial u_1} = 0, \quad \text{and} \quad \frac{\partial H}{\partial u_2} = 0, \tag{3.9}$$

and the control pair is determined, subject to $S_1, S_2, S_3, G_1, G_2, G_3, C_1, C_2, C_3$. Having obtained the characterization in (3.6), then we have for the u_1^*

$$\frac{\partial H}{\partial u_1} = B_1 u_1 + T_1 = 0 \tag{3.10}$$

which implies that

$$u_1^*(t) = \frac{1}{B_1}T_1 \tag{3.11}$$

on the set $\{t : a_1 < u_1^*(t) < b_1\}$. Similarly for u_2^*, we have

$$\frac{\partial H}{\partial u_2} = B_2 u_2 + T_2 = 0 \tag{3.12}$$

which implies that

$$u_2^*(t) = \frac{1}{B_2}T_2, \tag{3.13}$$

over the set $\{t : a_2 < u_2^*(t) < b_2\}$.

It is important to note that a few limitations ought to be forced on the period up to t_f, to guarantee the uniqueness of the optimality system. Such limitations can be attributed to the contrary time orientation of (2.4), (3.4), (3.5) since the variables $S_1, S_2, S_3, G_1, G_2, G_3, C_1, C_2, C_3$ have starting values while the adjoint equations have output values [17,47].

4 Numerical simulation

In this present section, the optimal strategy for effective criminal gang control, consisting of controls that supports enlightenment campaign programs and empowerment

Table 2 Description of variables

Optimal control parameters	Interpretation	Value/Baseline (range)	References
u_1	First control	$[0-1]$	Implied from [42]
u_2	Second control	$\left[0 - \dfrac{1}{\gamma_1 + \gamma_2 + \gamma_3}\right]$	Implied from [42]

programs (which includes door-to-door, community square talks, television and radio jingles, print media, youth empowerment, access to loan, and other prevention strategies of crime) and a case finding control that represent the fraction of initiated persons who are arrested, law-enforced and imprisoned in the correctional center for proper correction and prevention of contacts with susceptible individuals. The system is solved by employing the Runge–Kutta scheme, implemented in a forward-backward sweep fashion. The control functions baseline range is given on Table 2 while the parameter values are given in Table 3. We take $N = 100,000$. The value for N is so chosen because most sociological and criminological data on crime incidence in human population are reported in 100,000 s.

Table 3 Parameters and values

Parameters	Value/Baseline (range)	References
Λ	3,623,682 [3,000,000–4,000,000]	[48,49]
μ	0.0185 [0.0181–0.0188]year^{-1}	[48,49]
α_1	0.105[0.0000000001–0.1]year^{-1}	Assumed
α_2	0.005[0–1]year^{-1}	[13]
α_3	0.311[0–1]year^{-1}	[13]
α_4	0.324[0–1]year^{-1}	[13]
β_1	0.010[0–1]year^{-1}	[13]
β_2	0.009[0–1]year^{-1}	[13]
β_3	0.001[0–1]year^{-1}	[13]
σ_1	0.318[0–1]year^{-1}	[13]
σ_2	0.311[0–1]year^{-1}	[13]
σ_3	0.853[0–1]year^{-1}	[13]
p	0.1[0–1]year^{-1}	Assumed
γ_1	0.338[0–1]year^{-1}	[13]
γ_2	0.173[0–1]year^{-1}	[13]
γ_3	0.292[0–1]year^{-1}	[13]
τ_1	0.030[0–1]year^{-1}	[13]
τ_2	0.329[0–1]year^{-1}	[13]
δ_1	0.342[0–1]year^{-1}	[13]
δ_2	0.310[0–1]year^{-1}	[13]
δ_3	0.291[0–1]year^{-1}	[13]
δ_4	0.405[0–1]year^{-1}	[13]
δ_5	0.482[0–1]year^{-1}	[13]
δ_6	0.395[0–1]year^{-1}	[13]

From Fig. 1, the u_1 and u_2 are plotted where the contact rates β_1, β_2 and β_3 were increased. In Fig. 1, changing contact rates β_1, β_2 and β_3 does not have a significant effect on the two values of the control u_2, however, it does have a significant effect on the two results of the control u_1; for the values of contact rates β_1, β_2 and β_3, we observe in (a) that the control u_1 experienced a fleeting increase (and decrease) for about 1.5 years but remained close to the lower bound for the remaining part of the 10 years of the simulation. This is indicative of the fact that there is a small proportion of susceptible population gaining from the campaign program and that criminal gang's activities have a severe negative impact on the campaign or the intervention is no longer needed. In (b), the control u_2 remains in the upper bound for almost 6-year of the simulation before experiencing a drop to the lower bound for the remaining 4-year period to minimize the number of criminal gangs in the population.

Figure 2 represents the susceptible class S_2 when we had an increase in the contact rates. From Fig. 2 a, we observe that an increase in the contact rate does not impact the susceptible population. Interestingly, Fig. 2a implies a significant drop in the susceptible population who would have proceeded to join criminal gangs G_1 and G_2. Hence, using the first strategy u_1 in class S_2, we were able to prevent 2945/100,000 S_2 susceptible individuals from being initiated by criminal gangs G_1 and G_2 at the end of 10 years period.
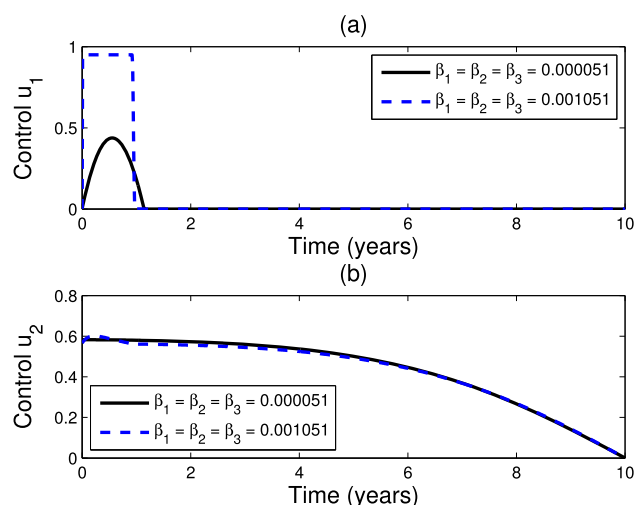

Fig. 1 Optimal control strategies when β_1, β_2, and β_3 are varied

Fig. 2 Population S_2 and S_3 when β_1, β_2, and β_3 are varied
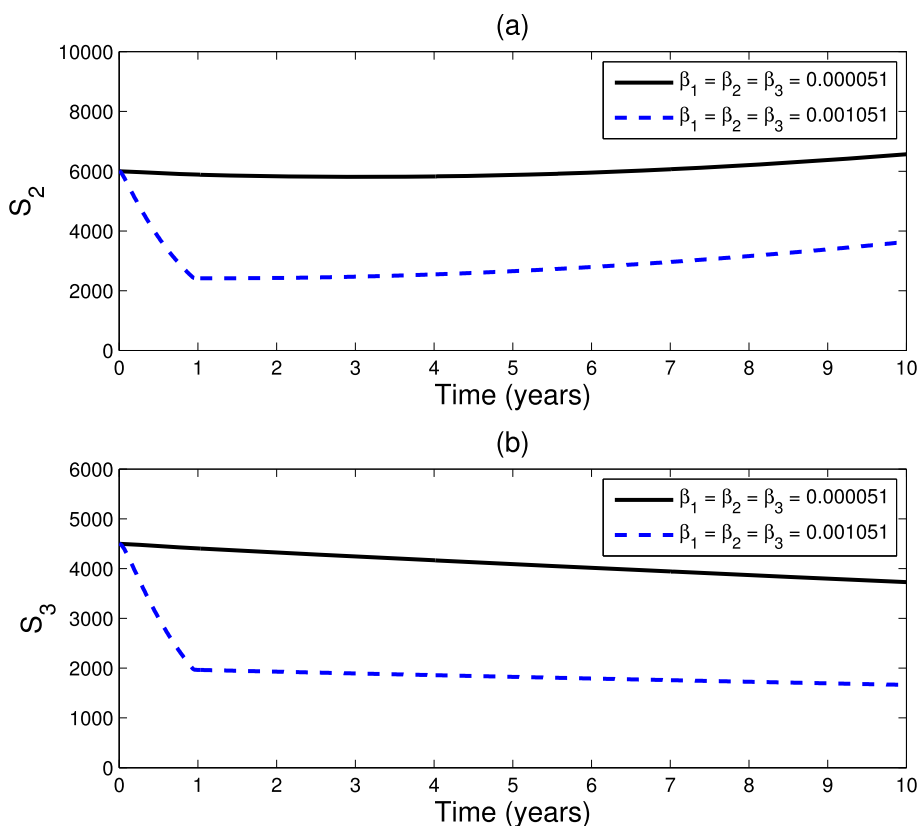

Figure 2b implies the susceptible class S_3 when we had an increase in the contact rates. From Fig. 2b, we observe that an increase in the contact rate does not impact the susceptible population. Hence, Fig. 2b implies a significant drop in the susceptible population who would have proceeded to join criminal gangs G_3. Thus, using the first strategy u_1 on class S_3, we were able to prevent 2071/100,000 S_2 susceptible individuals from being initiated by criminal gangs G_3 at the end of 10 years period.

Summarily, the study suggests that if the first strategy u_1 is sustained throughout the 10 years or more, we can prevent more susceptible individuals in both the adolescent and adult populations from being initiated into gangs.

From Fig. 3, the controls functions are plotted while the arrest and prosecution rates γ_1, γ_2, and γ_3 were increased from 0.1 to 1. In Fig. 3, changing the arrest and prosecution rates γ_1, γ_2, and γ_3 have a significant effect on control u_2 and the control u_1 respectively; for the values of the arrest and prosecution rates γ_1, γ_2, and γ_3, we observe in (a) that the control u_1 at $\gamma_1 = \gamma_2 = \gamma_3 = 0.1$ stayed close to the lower bound for the 10-year period. Then, u_1 at $\gamma_1 = \gamma_2 = \gamma_3 = 1$ experienced a momentary increase from about 0 to about 1.8 years but dropped down to the lower bound for the remaining part of the 10-year period of simulation. Also, for the values of the arrest and prosecution rates γ_1, γ_2, and γ_3
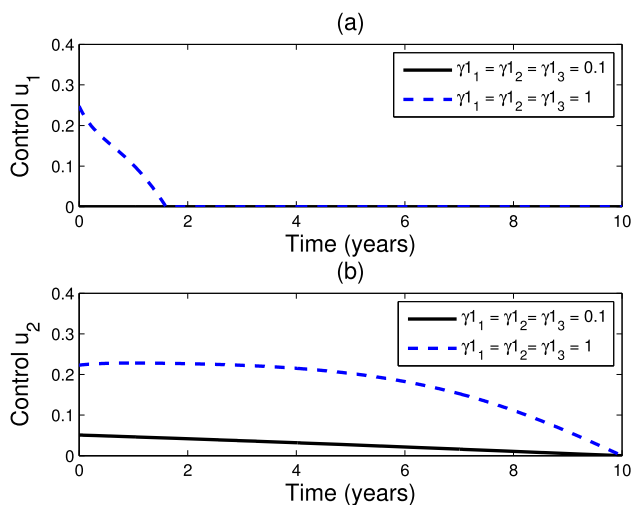

Fig. 3 Optimal control strategies when γ_1, γ_2, and γ_3 are varied

at different levels, we observe in (b) that the u_2 stayed close to the upper bound for almost 6-year period before dropping to the lower bound at $\gamma_1 = \gamma_2 = \gamma_3 = 1$ while the control u_2 stayed close to the lower bound for almost 10-year period at $\gamma_1 = \gamma_2 = \gamma_3 = 0.1$.

Summarily, an increase in the arrest and prosecution rates γ_1, γ_2, and γ_3 can sustain the control strategy for a

longer time in the population thereby suggesting a significant clampdown (minimization) on the number of criminal gangs in the population.

Figures 4 and 5 represents the criminal gang classes G_1, G_2 and G_3 and correctional center classes C_1, C_2 and C_3 when we had an increase in the rates of arrest and prosecution. From Fig. 4, we observe that an increase in the arrest and prosecution rates, decreases the gang population. Interestingly, Fig. 4 shows a decrease in the gang population has resulted into increase in the correctional center population as evident in Fig. 5. By this, we mean that alot of criminals are taken a way from the society through arrested and prosecution that leads to sentencing or remanding period.

Thus, using the strategy u_2 in class G_1, we can reduce 5,460/100,000 G_1 criminals from the population at the end of 10 years; using this strategy u_2 in class G_2, we were able to reduce 209/100,000 G_2 criminals from the population at the end of 10 years period; using this strategy u_2 in class G_3, we were able to reduce 6,748/100,000 G_3 criminals at the end of 10 years in the population.

Hence, using the strategy u_2 in class C_1, we were able to increase the population of the correctional center C_1 by 4,575/100,000 at the end of 10 years; using the strategy u_2 in class C_2, we were able to increase the population of the correctional center C_2 by 171/100,000 at the end of 10 years;

using the strategy u_2 in class C_3, we were able to increase the population of the correctional center C_3 by 5,724/100,000 at the end of 10 years using this strategy u_2 in class G_3.

In summary, we stressed that by adopting the second strategy u_2, we can reduce criminals from the street for appropriate corrections in the correctional centers C_1, C_2 and C_3 at the end of 10 years in the population.

Remark $u_1(t)$: The values of cases averted (and implemented) are obtained by just taking the difference in the quantity of susceptible (and gang) population when both contact rates are utilized; $u_2(t)$: the difference in the number of gang (and inmates) population when arrest and prosecution rates are utilized.

Furthermore, considering the criminal gang model (2.4) with and without controls, we note that the total number of susceptible population (S_2 and S_3) prevented from being initiated into criminal gang population ((G_1 and G_2) and (G_3)) respectively in Fig. 6; total number of criminal gang population ((G_1 and G_2) and (G_3)) taken away (arrested and prosecuted) from the society to the correctional centers (C_1, C_2, G_3) Fig. 7. To be precised further, applying the control u_1 on the criminal gang model, we observed that we were able to avert almost 3980/100,000 new gang initiations in the susceptible adolescent population (S_2) and 2831/100,000 new gang

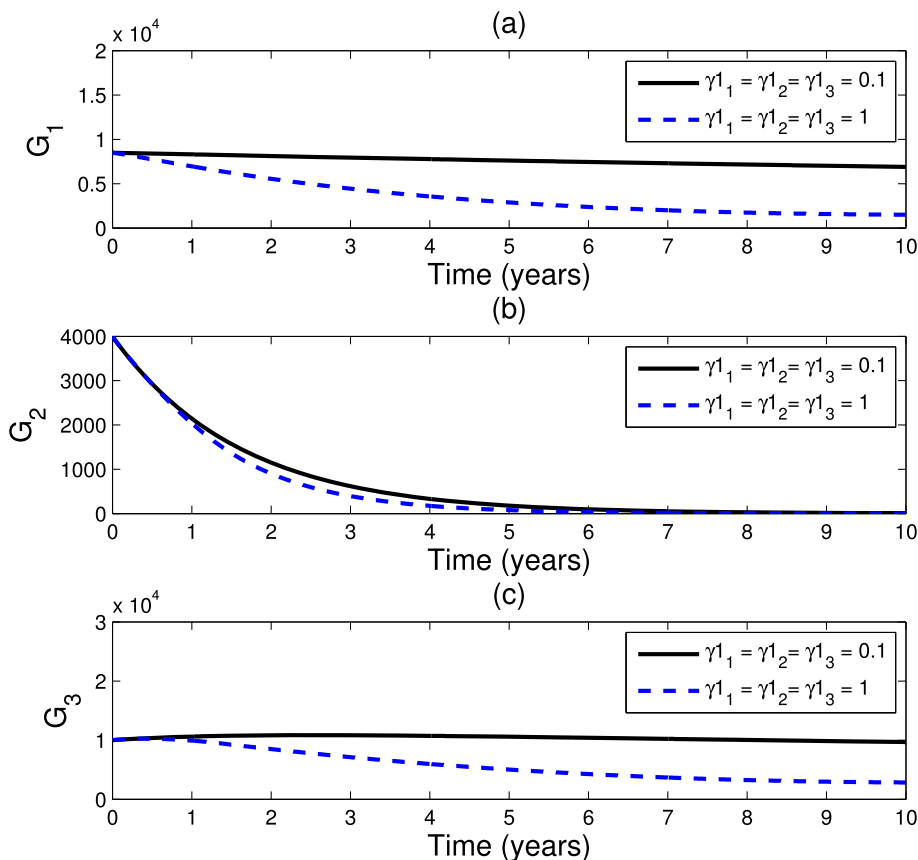**Fig. 4** Population G_1, G_2 and G_3 when γ_1, γ_2, and γ_3 are varied

Fig. 5 Population C_1, C_2 and C_3 when γ_1, γ_2, and γ_3 are varied
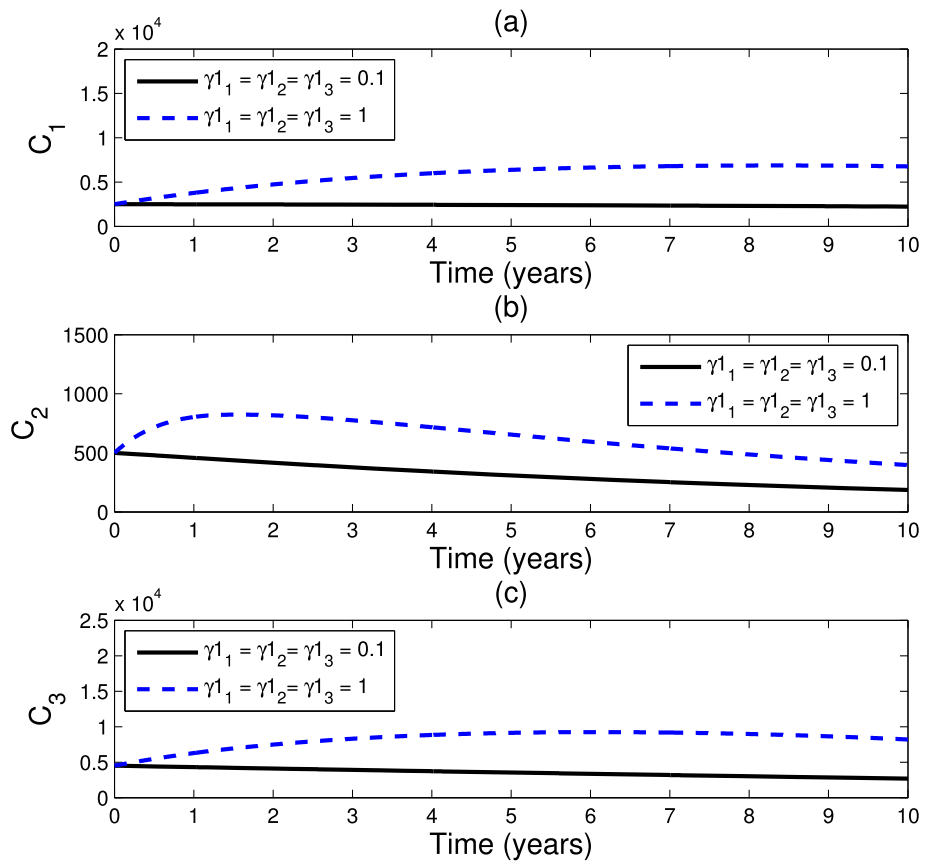

Fig. 6 Susceptible population (S_2 and S_3) with control ($u_1 \neq 0$) and without control ($u_1 = u_2 = 0$)
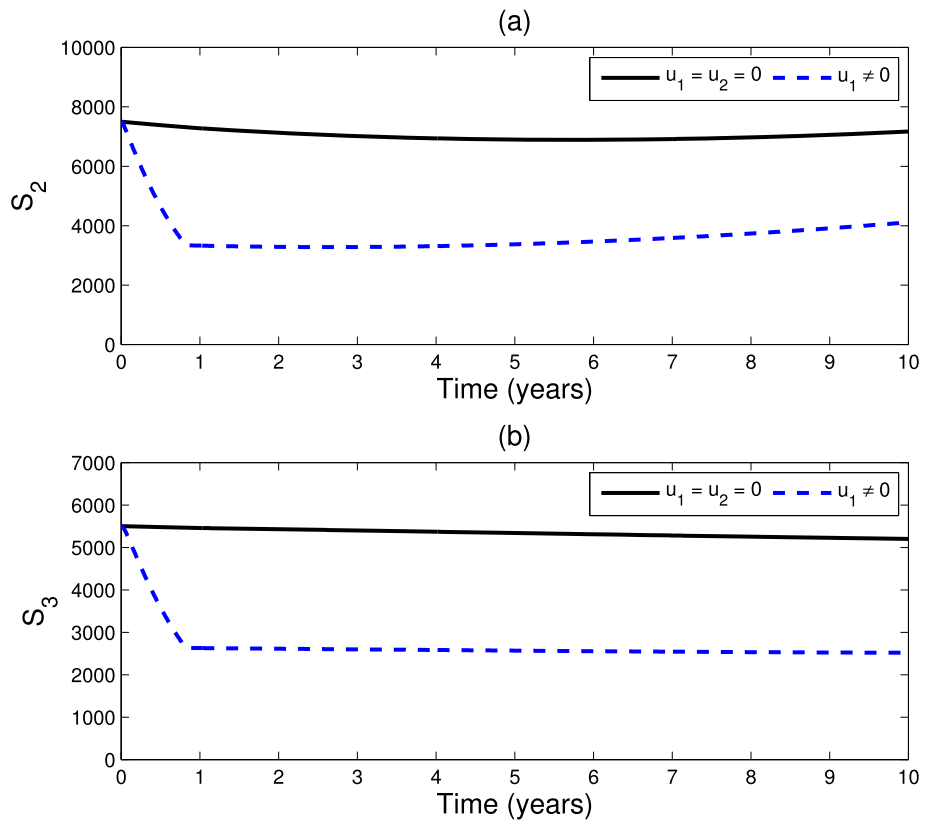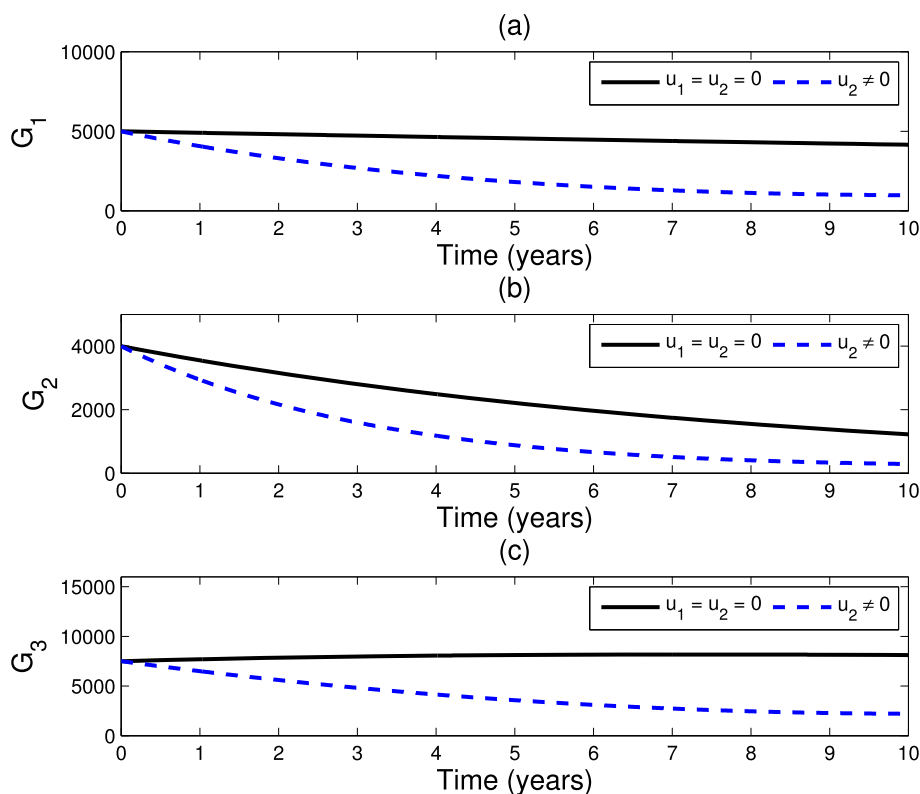
Fig. 7 Criminal gang population (G_1, G_2 and G_3) with control ($u_2 \neq 0$) and without control ($u_1 = u_2 = 0$)

initiations in susceptible adult population (S_3). Interestingly, we have established that the control u_1 is more effective in the susceptible adolescent population (S_2) compared to the susceptible adult population (S_3). Also, applying the control u_2, we have arrested and prosecuted more criminals by reducing their population and isolating/sentencing them in/to the correctional centers. That is, we have prevented a total of 3,203/100,000 new criminal cases in the adolescent population (G_1), a total of 1319/100,000 new criminal cases in the adolescent population (G_2) and a total of 5864/100,000 new criminal cases in criminal gang (adult) population (G_3). Evidently, the control u_2 is more effective in adult criminal gang population than the adolescent criminal gang population. This is quite instructive of the direction and concentration of the limited-resources, as the control u_1 that deals with the preventive approach is more effective in the adolescent susceptible population while the control u_2 that deals with curative approach is more effective in the adult criminal gang population.

5 Cost-effectiveness analysis

This section presents a cost-effectiveness analysis for crime-fighting and control interventions to justify the expenses of the optimal control model strategies $u_1(t)$ and $u_2(t)$ respectively, and both simultaneously. This approach has

been explored in [20,50] for some mathematical models. For our age-structured optimal control model, we carry out the quantitative analysis by comparing the differences among the criminal incidence (effect of contact rates and arrest and prosecution rates) outcomes, and enormous costs of these interventions; achieved by calculating the average cost-effectiveness ratio (ACER) and incremental cost-effectiveness ratio (ICER), which is defined as the cost per crime control outcome.

We assume here that, the cost of implementing case finding control that represents the fraction of initiated persons who are arrested, law-enforced, and imprisoned in the correctional center for adequate correction and prevention from interacting with susceptible individuals is a lot higher than the cost of executing the control that supports enlightenment campaign programs and empowerment programs. That is, we assume the weight constants $B_1 = 50$, $B_2 = 100$. We apply the cost functions $\frac{1}{2}B_1u_1(t)$, $\frac{1}{2}B_2u_2(t)$, over time, to calculate the total cost for the strategies that were executed.

Strategy M_1: Cost of implementing the control ($u_1(t)$) that supports enlightenment campaign programs and empowerment programs

We calculate the total number of gang initiations averted and the total cost of the strategies employed as seen in Table 4. We have shown in the previous section that by implementing the optimal control strategy $u_1(t)$, we prevented:

- 2,945/100,000 S_2 from criminal gang (G_1 and G_2) initiations,
- 2,071/100,000 S_3 from criminal gang (G_3) initiations.

Strategy M_2: Case finding control ($u_2(t)$) that represent the proportion of initiated persons who are law-enforced and imprisoned in the correctional center

We calculate the total number of criminals convicted and the total cost of the strategies applied as seen in Tables 4. We have shown in the previous section that by implementing the optimal control strategy $u_2(t)$, we increased the number convicted criminals by sentencing:

- 4575/100,000 G_1 to the correctional center C_1,
- 171/100,000 G_2 to correctional center C_2,
- 5724/100,000 G_3 to correctional center C_3.

Strategy M_3: Cost of implementing the control ($u_1(t)$) and cost of implementing the control ($u_2(t)$) simultaneously

We calculate the total number of criminal incidents averted, and the total cost of the strategies applied as seen in Tables 4. This has been stated in Strategy A and Strategy B respectively.

Implementation of the strategies

As a matter of necessity, we establish the most cost-effective strategy among the M_1, M_2 and M_3 strategies considered in this section. To accomplish this, the cost-effectiveness analysis is obtained by computing the ACER and ICER, adopting the idea in [20,50]. The outcomes from the mathematical simulation executed, Table 4 gives the ranking of M_1, M_2 and M_3 strategies.

$$ACER(M_1) = \frac{50}{5,016} = 0.00997$$

$$ACER(M_2) = \frac{100}{10,470} = 0.00955$$

$$ICER(M_1) = \frac{50}{5,016} = 0.00997$$

$$ICER(M_2) = \frac{100 - 50}{10,470 - 5,016} = 0.00916$$

From ICER (M_1) and ICER(M_2), we notice a cost saving of 0.00917 for procedure M_2 than procedure M_1. This infers that procedure M_1 unequivocally overshadow procedure M_2, showing that procedure M_1 is more costly and less effective compared to procedure M_2. Consequently, procedure M_1 is taken out from subsequent ICER calculations, as evident in Table 5. Now, we compare procedures M_2 and M_3.

$$ICER(M_2) = \frac{100}{10,470} = 0.00955$$

$$ICER(M_3) = \frac{150 - 100}{15,486 - 10,470} = 0.00997$$

Comparing procedures M_2 and M_3, we see that ICER(M_3) > ICER(M_2), showing that procedure M_3 strongly dominated procedure M_2 and is more costly and less effective compared to procedure M_2. Consequently, procedure M_2 (arrest and sentencing rates) has the least ICER and is the most cost-effective of all the control strategies for crime prevention and control in this study. This additionally corroborates with the result of the ACER technique in Table 4 that procedure M_2 is the most cost-effective strategy. It is also deserving of note to see that this strategy averts more criminal incidences and reduces crime rates than any other control strategy implemented. This is so because the severity of arresting and punishing criminals serves as a deterrent to others in the society.

The results from the control model indicate how a cost-effective analysis could help policymakers gain further insight into the study of crime by considering the cost of implementing the model parameters, in particular, contact rates (β_1, β_2, β_3); arrest and prosecution rates (γ_1, γ_2, γ_3).

Table 4 Criminal incidence strategies

Strategy	Total criminal incidence averted	Total cost	ACER	ICER
$M_1 : u_1(t)$	5,016	50	0.00997	0.00997
$M_2 : u_2(t)$	10,470	100	0.00955	0.00917
$M_3 : u_1(t) \& u_2(t)$	15,486	150	0.00969	0.00955

Table 5 Criminal incidence strategies

Strategy	Total criminal incidence averted	Total cost	ACER	ICER
$M_2 : u_2(t)$	10,470	100	0.00955	0.00955
$M_3 : u_1(t) \& u_2(t)$	15,486	150	0.00969	0.00997

Conclusively, this research result suggests that the crime control programs that adopt these strategies explained herein can help reduce new gang initiations and criminal activities.

6 Disscusion

Based on the results obtained from this new approach for studying the dynamics of criminal gangs, the initiation rate between susceptible individuals and gang members is a determining factor in controlling the activities of criminals in a limited-resource setting. By reducing the initiation rates through empowerment and enlightenment programs among these individuals, we can potentially decrease the initiation rate and thus minimize the gang population.

We performed a further study (cost-effectiveness analysis) on the optimal control model by considering three strategies (M_1, M_2, M_3). Consequently, we established that the singular implementation of strategy M_1 (empowerment and enlightenment campaign program) is more costly and less effective among the three strategies considered. Thus, the implication of this is that the implementation of strategy M_2 (the arrest and sentencing criminals for corrective measure) and strategy M_3 (the simultaneous implementation of strategies M_1 and M_2), are less costly and more effective than the singular implementation of strategy M_1 (the empowerment and enlightenment campaign strategy) only.

7 Conclusion

A criminal gang optimal control model is proposed and studied herein. The main contribution of this work, as it can rarely be found in the literature, is the formulation of optimal control models that would help during crime fighting and control in a limited-resource setting. In addition, we have introduced cost-effective analysis to help us gain further insight on the best strategy for handling the activities of criminal gangs in a limited-resource setting. We hereby note that the major limitation of the study is our inability to validate this current work with real-life data due to lack of access to primary data that will fit into all the variables. However, we rely on secondary data from reputable literature such as a recent publication in Ibrahim et al. [13], United Nations [48] and World Bank [49].

The result from the current study has shown that if the Nigerian government, non-governmental organizations, religious bodies and other stakeholders as highlighted in Sect. 2 can intensify effort in lifting a reasonable number of Nigerians from poverty through empowerment programs and job creation; reducing out of school children by taking these adolescents from the street for proper education and engagement,

then we could have a society devoid of rancor and acrimony. Interestingly, our research result is in agreement with the postulate in [14] which has extensively identified unemployment as a root cause of crime in Nigeria and possible solutions.

We have also shown that the least costly and most effective strategy is the arrest and sentencing of criminals for corrective measures in limited-resource setting. This strategy is observed to be more realistic in combating crime since the criminal activities is on the inside and there is inadequate funding for security personals and institutions. The criminal elements then capitalize on this by take advantage of the loop holes in the system. Becker's economic theory of criminal behavior is also corroborates this assertion by stating that, potential criminals are economically rational and respond significantly to the deterring incentives by the criminal justice system. Thus, if we are immediately interested in combating crime in Nigeria, the government should invest more in security by providing sophisticated weapons and infrastructure for the police and correctional center staff as evident in the cost-effective analysis. The arrest and sentencing criminals for corrective measures strategy, as observed to be the most effective strategy in this study, is currently being tested and seen to be effective in some African countries such as Rwanda. Could the Nigerian situation be an exception?

In a society like Nigeria, being the most populous nation in Africa, the principal intervention as evident in this research is seen to be adequate funding for the police, judiciary and correctional center staff since the majority of the arrested and prosecuted criminals are detained and managed by these individuals/institutions. Specifically, the welfare of the staff of these institutions must be of necessity so that criminal elements do not explore the loop holes to entice and corrupt the government personnel. When the well-being of the staff become the government's top priority, the personnel will be motivated to use and work with the available resources (weapon and instruments) for crime-fighting and correction).

Declarations

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