Correction to: Rich dynamics of non-toxic phytoplankton, toxic phytoplankton and zooplankton system with multiple gestation delays

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In the original publication, Theorem 4.6 has been published incorrectly. The corrected theorem is given below:

Theorem 4.6 Suppose that system (3.2) satisfies the following conditions:

- (i) $k_2 - \frac{\beta_{21}k_1}{\alpha_1} > 0$ and/or $\frac{\gamma_1k_1}{\alpha_1} - \delta > 0$;
- (ii) $k_1 - \frac{\beta_{12}k_2}{\alpha_2} > 0$;
- (iii) $\gamma_1\hat{X}_1 - \delta - \gamma_2\hat{X}_2 > 0$;
- (iv) $k_2 - \beta_{21}\tilde{X}_1 - \gamma_2\tilde{Y} > 0$,

then system (3.2) is permanence.

Proof Let us consider the average Lyapunov function in the form $V(X_1, X_2, Y) = X_1^{\theta_1} X_2^{\theta_2} Y^{\theta_3}$ where each θ_i ($i = 1, 2, 3$) is assumed to be positive. In the interior of $\mathbb{R}_+^3$, we have

$$\begin{aligned} \frac{\dot{V}}{V} &= \psi(X_1, X_2, Y) = \theta_1 [k_1 - \alpha_1 X_1 - \beta_{12} X_2 - \gamma_1 Y] \\ &\quad + \theta_2 [k_2 - \alpha_2 X_2 - \beta_{21} X_1 - \gamma_2 Y] \\ &\quad + \theta_3 [\gamma_1 X_1 - \delta - \gamma_2 X_2]. \end{aligned}$$

To prove the permanence of the system, we shall have to show that $\psi(X_1, X_2, Y) > 0$, for all boundary equilibria of the system. The values of $\psi(X_1, X_2, Y)$, at the boundary equilibria E_0, E_1, E_2, E_3 and E_4 , are the following:

$$\begin{aligned} E_0 &: \theta_1 k_1 + \theta_2 k_2 - \theta_3 \delta. \\ E_1 &: \theta_2 \left(k_2 - \frac{\beta_{21}k_1}{\alpha_1} \right) + \theta_3 \left(\frac{\gamma_1 k_1}{\alpha_1} - \delta \right). \\ E_2 &: \theta_1 \left(k_1 - \frac{\beta_{12}k_2}{\alpha_2} \right) + \theta_3 \left(-\delta - \frac{\gamma_2 k_2}{\alpha_2} \right). \\ E_3 &: \theta_3 \left\{ \gamma_1 \hat{X}_1 - \delta - \gamma_2 \hat{X}_2 \right\}. \\ E_4 &: \theta_2 \left\{ k_2 - \beta_{21} \tilde{X}_1 - \gamma_2 \tilde{Y} \right\}. \end{aligned}$$

Now, $\psi(0, 0, 0) > 0$ is automatically satisfied for some $\theta_i > 0$ ($i = 1, 2, 3$). Also, if the inequalities (i) – (iv) hold, ψ is positive at E_1, E_2, E_3 and E_4 for some $\theta_i > 0$ ($i = 1, 2, 3$). Therefore, system (3.2) is permanence [1] if the conditions (i) – (iv) are fulfilled. Hence the theorem. $\square$

Reference

1. Freedman HI, Ruan S (1995) Uniform persistence in functional differential equations. J Differ Equ 115:173–192

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