Influence of tolerances on error estimation in P3R and 4R planar mechanisms

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Abstract

Four-bar linkages form a primitive configuration of many mechanisms. The output of these mechanisms deviates from the desired one due to many factors including tolerance on links. For satisfactory application, the mechanism performance needs to be characterized. This article presents a treatment on the synthesis and analysis of linear-input P3R (1-Prismatic, 3-Revolute) and rotary-input 4R (4-Revolute) planar mechanism configurations, and the estimation of mechanical error under the influence of link tolerances. A detailed methodology is presented for error estimation as part of performance analysis for each mechanism. As an illustrative case, mechanisms are considered to operate under identical output generation conditions and comparative performance evaluation is carried out. Effect of link proportion is studied to investigate the mechanism behavior related to mechanical error. Comparison of error provides the basis for selecting the mechanism that gives better performance. The proposed methodology demonstrates generalized characterization of the mechanical error sources with validation.

Keywords Planar mechanism · Link tolerances · Mechanical error · Comparison of error · Validation

List of symbols

l_1, l_2, l_3 and l_4	Link Lengths: input link, coupler link, output link and fixed link, respectively	E_{m1}, E_{m2}, E_{m3} and E_{m4}	Total mechanical error on link 1, link 2, link 3 and Link 4, respectively, by modified Taylor series approximation
θ_1, θ_3	Angular displacement at input and output link, respectively	Ψ	Internal angle between Link 1 and Link 4
A, B, C, D, E, F, G, H and I	Coefficient of the design parameters	l_{t1}, l_{t2}, l_{t3} and l_{t4}	Total link Lengths on link 1 to 4 after deviations
$\Delta l_1, \Delta l_2, \Delta l_3$ and Δl_4	Deviation of the link lengths link 1 to 4, respectively	K	Proportion factor
		E_{d1}, E_{d2}, E_{d3} and E_{d4}	Total mechanical error on link 1, link 2, link 3 and Link 4, respectively, by geometrical method

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1 Introduction

Four-bar linkages are commonly used in machines and mechanisms for achieving the desired path, function and motion generation task. Four bar mechanisms and its inversion are used in variety of applications as in biomedical equipment, automobiles, machine linkages, etc. The revolute and prismatic joint configurations are used as a part of planar and parallel manipulators. Various complex mechanisms are studied with four-link chain as primitive configurations.

The performance characteristics, namely positioning accuracy, structural and mechanical errors, depend on the kind of input condition of the mechanism [1, 2]. These mechanisms employ rotary or linear actuators for desired task. The selection of particular actuator is governed by functional requirements such as input range, position, velocity and accelerations of output link as well as force required at output link. Therefore, comparative performance analysis is important for selection of linear or rotary actuation. The primary performance features for such analysis are the positional error due to the structural and mechanical parameters. The structural error is the inability in reaching the desired function or position due to the link dimension after dimensional synthesis of a specific configuration. For few precision positions, the structural error becomes zero. The mechanical error occurs due to the deviations of link dimensions from the designed dimensions on account of design constraints, manufacturing process limitations and subsequent assembly. The factors affecting the performance of manipulators include clearance, backlash, drive error, assembly error, deflection and expansion of linkages, etc., found in the literature. As link tolerances are inevitable, the mechanism does have positional deviations. The exact tolerance after design and manufacturing is possible to quantify and thus it is possible to estimate mechanical error. The attempt is presented herewith to investigate the mechanical error due to quantified link tolerances in general.

Many researchers have performed structural and mechanical error analysis for path, motion and function generation problems in four-link mechanisms [3–6]. Dubowsky et al. and Erkaya et al. [7] discussed the effect of clearance and link flexibility on stresses in joints of high speed and slider-crank mechanisms under dynamic conditions. Sharfi and Smith [8] investigated the link-length tolerance and joint clearance in the multi-link mechanism. K. L. Ting et al. [9] presented modeling and analysis of joint clearances and resulting uncertainty in revolute as well as prismatic pairs in multi-loop mechanisms. Flores [10] established a general methodology for quantifying the effects of kinematic position errors due to manufacturing and assembly tolerance for open as well as closed chain planar mechanisms. Ting, et al. described an approach to determine the effects of joint clearance on position and orientation deviation in linkages, based on N-bar rotatability laws concept and evolutionary optimization techniques [11]. The problems of positional error, performance quality, transmission angle of open and closed loop mechanisms, using various techniques such as stochastic approach [3, 12–14], probabilistic model [15], loop closure method [16], genetic algorithm [17] are found to be employed in the literature. The probability approach is used to determine the uncertainty effect of link tolerance and joint clearance in serial, planar and spatial robots [18, 19]. Zhang and Xianmin [20] and Chen, et al. [21] studied the

influence of clearance in joints of planar 3-RRR and 4-RRR mechanisms, wherein comparison between the two mechanisms based on position deviation is presented. Jawale and Thorat [22] analyzed the 4R, 2-serial, and P3R mechanisms and influence of clearances and backlash is presented. Tian et al. and Flores et al. [23–26] investigated the dynamics of multi-body mechanical systems by analytical, numerical and experimental techniques and considered imperfect clearance, friction, link flexibility and geometric imperfections effects of clearances in lubricated joints. Zhan et al. applied a hybrid method of first-order second moment (FOSM) technique for multiple uncertainty in planar parallel manipulator like manufacturing tolerances, input errors and joint clearances, while carrying out motion reliability analysis [27]. Erkaya discussed the effect of joint clearances in robotic system and kinematic and dynamic analysis of trajectory of end-effector is carried out to know the motion sensitivity [28]. Cammarata and Erkaya investigated the dynamic behavior in a 3D slider-crank mechanism and RRR spherical parallel manipulator under the effect of with and without clearance joints using computational and elastostatic approach, respectively [29]. The effect of clearance sizes and driving speeds results are determined and verified by ADAMS simulations software [30]. Erkaya proposed the ANFIS approach algorithm to optimize the trajectory parameters of a walking mechanism with joint clearance and compared the obtained results by analytical approach [31].

Erkaya et al. evaluated kinematic and dynamic performance analysis of the four-bar and slider joint mechanism under effect of joint clearance [32, 33]. Tsai and Lai evaluated kinematic position and accuracy analysis of the multi-link mechanism using wrench screw method. The obtained results with joint clearances are compared with that of the ideal mechanism [34]. Jawale and Thorat analyzed the position accuracy of the serial and closed chain manipulator [35]. Li et al. presented model for angular errors due to joint clearances in multi-loop structures. By using optimization method and geometric method, the angular errors are determined. The results were verified using Monte Carlo simulations [36]. Tsai and Lai analyzed the position errors and transmission quality of the mechanism affects due to joint clearances [37]. Wu and Rao applied the interval approach for the modeling of tolerances and clearances and the fuzzy error analysis of mechanisms. Further, comparison of the interval number approach and traditional approach is presented out [38]. Zhang and Han proposed a method based on reliability which considers the randomness of the link lengths and estimates the error within a sphere of radius equal to the estimated error through saddle point approximation [39].

Jaiswal and Jawale et.al evaluated mechanical error under the influence of line tolerance in four bar revolute joint mechanisms [40, 41]. The extensive work presented in the

literature was found to cater the effect on characteristic of mechanism output.

The effect of tolerances, specifically on revolute joint (4R) mechanisms, is seen to be considered in most of the literature cited above. The effect of clearance contributes to randomness in the behavior of the mechanism, whereas tolerances are presumed to give cumulative effect on mechanism performance. In mechanical design and manufacturing sciences, tolerances on machine components are stacked giving cumulative end values. In closed chain mechanisms, it leads to mechanical error but the rule is not established. The influence of tolerance needs to be investigated in mechanisms. The attempt is made herewith to uncover the variation in error pattern under the influence of link known tolerance. The mechanisms providing limited range of motion are of special use in automation application. The task of position generation, function generation and rigid body guidance are accomplished through use of mechanisms by applying reversible rotary actuation or linear actuation. The drive-specific behavior of the mechanism needs to be analyzed for performance features like mechanical error under the influence of link tolerance. The error estimation carried out by authors Jaiswal and Jawale [40, 41] is extended further to four bar prismatic input (P3R) mechanism. Investigation of effect of link proportions on error pattern through generalized formulation is also aimed in the work presented herewith. Further, comparative performance evaluation is targeted which may lead to select a configuration for a given application. The extent of sensitivity to the deviations on account of tolerances is the aim of present work. The present work considers the exact tolerance on the links, which is important for estimating the mechanical error rather than range of the error at the coupler positions, which use the probabilistic analysis of dimensional deviations on the links to a specified problem threshold. The traditional reliability-based methods are complex and thus are insufficient to consider randomness in link length variation or mechanical error estimation.

The major limitations of such methods lie in estimation of the exact error for continuous trajectory. The proposed method in this paper enables to estimate mechanical error at any set of finite coupler positions which is important for performance analysis over a trajectory.

The work presented in this paper is organized as follows—Sect. 2 deals with methodology and Sect. 3 deals with kinematic synthesis and mechanical error formulation of 4R and P3R mechanisms. Section 4 deals with formulations for exact error prediction for both configurations. Section 5 presents concept of identical operating condition for possible comparative study, with demonstration of sample case. Also, considerations for various link proportions are dealt in this section. Section 6 provides results and discussions followed by comparative error analysis for both mechanism

configurations in Sect. 7. Conclusions and future scope are discussed in Sect. 8.

2 Methodology

The P3R and 4R mechanism are investigated for mechanical error herewith using traditional approximate estimation method at initial treatment. Error propagation method is well known statistical technique based on first-order Taylor's series approximation for error analysis. The literature reports this method being used for mechanical error analysis; with probable approximate estimations. This method with small error in terms of known link deviation takes into account input variables, leading to a prediction of maximum and total deviation. From the literature, it is found that the effect of clearances and deviations on link lengths does not have straightforward correlation with the positional error in case of closed chain mechanisms; anticipating the error to vary with change in local actuation specific input variables. This is the motive behind not considering the input as in Gaussian distribution but to use small error in terms of known link deviation.

The exact error estimation method is also tailored herewith, which involves known fixed values or constraints that are used to find an exact solution. This method is used with known independent variable (tolerances in this case) and the final value of deviations is obtained. The analytical treatment gives an exact and pointed deviation. This is the strength of this technique, suitable for analyzing the mechanical error in individual configuration. The deviations of all links in mechanisms are precisely carried on to contribute in the final estimation.

For each mechanism, synthesis equation is developed, and a mechanical error equation is obtained by considering link tolerance as variable for possible drive inputs (i.e., rotary and linear actuation) for approximation and exact estimation, respectively.

Effect of link proportions on error is evaluated. An equivalence relation between the inputs under identical output conditions is established for both mechanisms by analytical and CAD approaches. The effect of link tolerance is analyzed as specific error to account for theoretical and actual conditions. Finally, the performance of the two mechanisms with rotary and linear actuations is compared.

The methodology applied for analyzing the error is described in flowchart (Fig. 1). The formulation was developed for a manipulator analysis through approximate approach and exact approach. The validation of the results for these methods is carried out through CAD model. The deviations in end effector position under the influence of link tolerance are estimated through both methods on 4R and P3R configurations, in order to obtain link proportions. Later on, effect of coupler

Fig. 1 Flowchart

position on error is estimated. Comparison of the error for 4R and P3R mechanisms is carried out.

3 Kinematic synthesis of 4R and P3R mechanism

3.1 Characteristic equation—4R mechanism

Consider a 4R mechanism in a rectangular coordinate system as shown in Fig. 2. Taking design parameters $l_1, l_2, l_3, l_4, \theta_1$, as inputs and θ_3 as the output [1, 41], the derivation of displacement equations is written as,

For point A:

$$x_2 = l_1 \cdot \cos \theta_1 \quad (1)$$

$$y_2 = l_1 \cdot \sin \theta_1 \quad (2)$$

For point B:

$$x_3 = -l_4 + l_3 \cdot \cos \theta_3 \quad (3)$$

Fig. 2 4R mechanism configuration

$$y_3 = l_3 \cdot \sin \theta_3 \quad (4)$$

The distance between the point A and point B is fixed and equal to l_2

$$(x_2 - x_3)^2 + (y_2 - y_3)^2 = l_2^2 \quad (5)$$

Substituting Eqs. (1) to (4) in Eq. (5),

$$(l_1 \cos \theta_1 + l_4 - l_3 \cos \theta_3)^2 + (l_1 \sin \theta_1 - l_3 \sin \theta_3)^2 = l_2^2 \quad (6)$$

On simplifying Eq. (6) in trigonometric form,

$$A \sin \theta_3 + B \cos \theta_3 = C \quad (7)$$

$$A = \sin \theta_1 \quad (8)$$

$$B = \frac{l_4}{l_1} + \cos \theta_1 \quad (9)$$

$$C = \frac{l_4}{l_3} \cos \theta_1 + \frac{l_1^2 - l_2^2 + l_3^2 + l_4^2}{2l_1l_3} \quad (10)$$

$$\tan \frac{\theta_3}{2} = \frac{A \pm \sqrt{A^2 + B^2 - C^2}}{B + C} \quad (11)$$

Equation (11) is used to obtain the output link angle of the 4R mechanism.

3.2 Characteristic equation—P3R mechanism

Consider a planar four-link mechanism with a linear actuation (P3R) as shown in Fig. 3. The equation can be obtained from position analysis of the configuration to determine the input and output angles due to varying input displacement of linear drive motor or actuator. Let instantaneous leg length v be given as

$$v = (l_1 + l_2) + \delta \quad (12)$$

where, δ =displacement of actuator.

Fig. 3 P3R mechanism configuration

Using cosine theorem,

$$\psi = a \cos \left(\frac{v^2 + l_4^2 - l_3^2}{2l_4v} \right) \quad (13)$$

$$\theta_3 = a \cos \left(\frac{l_3^2 + l_4^2 - v^2}{2l_3l_4} \right) \quad (14)$$

$$\theta_1 = 180 - \psi \quad (15)$$

Equation (14) is used to determine the output link angle of the P3R mechanism.

3.3 Mechanical error—approximation approach

Consider a 4R planar mechanism with constant length parameters l_1, l_2, l_3, l_4 transforming a motion defined by an input variable (θ_1) and output variable (θ_3). The primary condition of loop closure for existence of four-bar mechanism may be written in general form as

$$f(l_1, l_2, l_3, l_4, \dots, \theta_i, \theta_o) = 0 \quad (16)$$

Each independent parameter inaccuracy is going to affect the performance of the mechanism. Thus, the aggregate estimated error (E_m) is a combination of the partial derivatives of the error function F with respect to the individual independent parameters, as given by

$$E_m = - \sum_{i=1}^n \frac{\partial f / \partial l_i}{\partial f / \partial \theta_3} \Delta l_i \quad (17)$$

This total error will cause a deviation in output link angle as,

$$E_m = \Delta \theta_3 \quad (18)$$

Re-arranging Eq. (7), the deviation at the output link, i.e., mechanical error in the mechanism, is estimated as:

$$D \sin \theta_3 + E \cos \theta_3 = F \quad (19)$$

$$D = 2l_1l_3 \sin \theta_1 \quad (20)$$

$$E = 2l_3l_4 + 2l_1l_3 \cos \theta_1 \quad (21)$$

$$F = 2l_1l_4 \cos \theta_1 + l_1^2 - l_2^2 + l_3^2 + l_4^2 \quad (22)$$

Deviation Δl in the link lengths l_1, l_2, l_3 and l_4 will modify the coefficients D, E , and F by $\Delta D, \Delta E$ and ΔF , respectively. Deviation in each link i (Δl_i) will produce separate errors in D, E , and F , governed by an error $\Delta \theta_3$ in the output. The total

mechanical error of the linkage (E_m) will be the sum of the separate errors.

Error due to deviations $\Delta l_1, \Delta l_2, \Delta l_3$ and Δl_4 will be given as

$$E_{m1} = 2 \frac{l_4 \cos \theta_1 + l_1 - l_3 \cos(\theta_1 - \theta_3)}{D \cos \theta_3 - E \sin \theta_3} \Delta l_1 \quad (23)$$

$$E_{m2} = \frac{-2l_2}{D \cos \theta_3 - E \sin \theta_3} \Delta l_2 \quad (24)$$

$$E_{m3} = -2 \frac{l_4 \cos \theta_1 + l_3 - l_1 \cos \theta_3}{D \cos \theta_3 - E \sin \theta_3} \Delta l_3 \quad (25)$$

$$E_{m4} = 2 \frac{l_1 \cos \theta_1 + l_4 - l_3 \cos \theta_3}{D \cos \theta_3 - E \sin \theta_3} \Delta l_4 \quad (26)$$

The maximum and root-mean-square error, respectively, are given as

$$E_{\max} = |E_{m1}| + |E_{m2}| + |E_{m3}| + |E_{m4}| \quad (27)$$

$$E_{\text{Total}} = E_{m1} + E_{m2} + E_{m3} + E_{m4} \quad (28)$$

The above equations are used to determine the mechanical error for 4R and P3R mechanism due to link error [41].

4 Kinematic synthesis of 4R and P3R mechanism—exact estimation approach

4.1 Characteristic equation—4R mechanism

Consider a four-bar mechanism as shown in Fig. 4, with O_2 and O_4 as fixed pivots, the input to link 1 is θ_1 . The output angle θ_3 can be found from component θ_{31} and θ_{32} . Consider triangles O_2O_4A and AO_4B , with distance between A and O_4 as l_r such that

$$l_r^2 = l_1^2 + l_4^2 - 2l_1l_4 \cos(\psi) \quad (29)$$

Fig. 4 Geometric analysis of 4R mechanism

$$l_r = \sqrt{l_1^2 + l_4^2 - 2l_1l_4 \cos(\psi)} \quad (30)$$

The output angle θ_4 can be written as,

$$\theta_3 = \theta_{31} + \theta_{32} \quad (31)$$

where,

$$\theta_{31} = a \cos \left(\frac{l_r^2 + l_4^2 - l_1^2}{2l_r l_4} \right) \quad (32a)$$

and

$$\theta_{32} = a \cos \left(\frac{l_r^2 + l_3^2 - l_2^2}{2l_r l_3} \right) \quad (32b)$$

Similarly, considering tolerances Δl_1 , Δl_2 , Δl_3 and Δl_4 on links l_1 , l_2 , l_3 , and l_4 , respectively, as shown in Fig. 5, the above equations can be rewritten as:

$$l_{t1} = l_1 + \Delta l_1, l_{t2} = l_2 + \Delta l_2, l_{t3} = l_3 + \Delta l_3 \text{ and } l_{t4} = l_4 + \Delta l_4 \quad (33)$$

$$l_{tr}^2 = l_{t1}^2 + l_{t4}^2 - 2l_{t1}l_{t4} \cos(\psi) \quad (34)$$

$$l_{tr} = \sqrt{l_{t1}^2 + l_{t4}^2 - 2l_{t1}l_{t4} \cos(\psi)} \quad (35)$$

$$(\theta_{31})_t = a \cos \left(\frac{l_{tr}^2 + l_{t4}^2 - l_{t1}^2}{2l_{tr}l_{t4}} \right) \quad (36)$$

$$(\theta_{32})_t = a \cos \left(\frac{l_{tr}^2 + l_{t3}^2 - l_{t2}^2}{2l_{tr}l_{t3}} \right) \quad (37)$$

$$(\theta_3)_t = (\theta_{31})_t + (\theta_{32})_t \quad (38)$$

Fig. 5 4R mechanism with link tolerances

Equation (38) is used to obtain the output angle of the 4R mechanism with tolerances by the exact estimation approach.

4.2 Characteristic equation—P3R mechanism

Considering the P3R configurations with tolerances $\Delta l_1, \Delta l_2, \Delta l_3$ and Δl_4 on the links l_1, l_2, l_3 , and l_4 , respectively, the link lengths are given as

$$l_{t1} = l_1 + \Delta l_1, l_{t2} = l_2 + \Delta l_2, l_{t3} = l_3 + \Delta l_3 \text{ and } l_{t4} = l_4 + \Delta l_4 \quad (39)$$

The modified mechanism is as shown in Fig. 6. The instantaneous leg length v_t will be given as—

$$v_t = (l_{t1} + l_{t2}) + \delta \quad (40)$$

$$(\psi)_t = a \cos \left(\frac{(v_t)^2 + (l_{t4})^2 - (l_{t3})^2}{2(l_{t1} + (l_{t2} + \delta))l_{t4}} \right) \quad (41)$$

$$(\theta_3)_t = a \cos \left(\frac{l_{t4}^2 + l_{t3}^2 - (v_t)^2}{2l_{t3}l_{t4}} \right) \quad (42)$$

$$(\theta_1)_t = 180 - (\psi)_t \quad (43)$$

4.3 Mechanical error estimation—exact estimation approach

The geometrical formulation for position analysis of mechanism forms a part of—exact estimation approach. To find the error at individual link, single link is considered to have tolerance at a time; and other links without tolerance. Error due to deviation on link 1, 2, 3 and 4, respectively, with tolerance conditions is given as under:

Error due to link 1,

Fig. 6 P3R mechanism with link tolerances

$$l_{t1} = l_1 + \Delta l_1, l_{t2} = l_2, l_{t1} = l_3 \text{ and } l_{t4} = l_4$$

$$E_{d1} = (\theta_3) - (\theta_3)_t \quad (44)$$

Errors due to deviation on link 2:

$$l_{t1} = l_1, l_{t2} = l_2 + \Delta l_2, l_{t1} = l_3 \text{ and } l_{t4} = l_4$$

$$E_{d2} = (\theta_3) - (\theta_3)_t \quad (45)$$

Errors due to deviation on link 3:

$$l_{t1} = l_1, l_{t2} = l_2, l_{t1} = l_3 + \Delta l_3 \text{ and } l_{t4} = l_4$$

$$E_{d3} = (\theta_3) - (\theta_3)_t \quad (46)$$

Errors due to deviation on link 4:

$$l_{t1} = l_1, l_{t2} = l_2, l_{t1} = l_3 \text{ and } l_{t4} = l_4 + \Delta l_4$$

$$E_{d4} = (\theta_3) - (\theta_3)_t \quad (47)$$

Combined error due to deviation on all links:

$$l_{t1} = l_1 + \Delta l_1, l_{t2} = l_2 + \Delta l_2, l_{t3} = l_3 + \Delta l_3 \text{ and } l_{t4} = l_4 + \Delta l_4$$

$$(E_{Total})_d = E_{d1} + E_{d2} + E_{d3} + E_{d4} \quad (48)$$

$$(E_{max})_d = |E_{d1}| + |E_{d2}| + |E_{d3}| + |E_{d4}| \quad (49)$$

5 Case study—generalized proportions and identical operating conditions

The mechanical error in four-bar 4R and P3R mechanisms of a given configuration is a function of tolerances on link dimensions and actuator input [1]. Both mechanisms are subjected to equivalent output generation conditions, and the required inputs are calculated.

5.1 Equivalent condition between 4R and P3R mechanism using analytical approach

Figure 7a–b shows the graphical representation of 4R and P3R mechanisms. Link 1 in 4R and P3R mechanisms is given rotary and linear inputs, respectively. Link 3 generates a common path at output for both mechanisms, i.e., both mechanisms are subjected to identical operating conditions as seen in Fig. 7c–d. Link 3 and 4 are identical links (output and fixed link) in both mechanisms. Figure 7b–d shows initial and final condition of P3R

Fig. 7 4R (a, c) and P3R (b, d) mechanisms under identical operating conditions

mechanism with zero and maximum input displacement, respectively; Fig. 7a–c represents equivalent 4R mechanism for output identical to P3R mechanism. For minimum and maximum linear displacement (δ) in P3R mechanism, the range of angular displacement (θ_1) for 4R mechanism is obtained.

In Fig. 7d, the value of input link angle θ_1 and output link angle θ_3 is obtained by Eqs. (14) and (15), respectively. Substituting the value of A , B and C in Eq. (7), and rearranging the expressions as in Appendix-I,

$$\sin \theta_1 \cdot \sin \theta_3 + \left(\frac{l_4}{l_1} + \cos \theta_1 \right) \cdot \cos \theta_3 = \frac{l_4}{l_3} \cos \theta_1 + \frac{l_1^2 - l_2^2 + l_3^2 + l_4^2}{2l_1l_3} \quad (50)$$

The final expression is,

$$G \cdot \sin \theta_1 + H \cdot \cos \theta_1 = I \quad (51)$$

where,

$$G = \sin \theta_3 \quad (52)$$

$$H = \cos \theta_3 - \frac{l_4}{l_3} \quad (53)$$

$$I = \frac{l_1^2 - l_2^2 + l_3^2 + l_4^2}{2l_1l_3} - \frac{l_4}{l_1} \cdot \cos \theta_3 \quad (54)$$

$$\theta_1 = 2 \cdot a \tan \left(\frac{G \pm \sqrt{G^2 + H^2 - I^2}}{H + I} \right) \quad (55)$$

This equation provides input angle of 4R mechanism.

5.2 P3R mechanism proportions

Effect of link proportions on behavior of the P3R and 4R mechanism is important for generality in applying formulation given in above sections. Initial P3R configuration is devised with the link proportions expressed in terms of link length l_4 as—

$$\begin{aligned} l_3 &= 1.5 \times l_4 \\ l_1 = l_2 &= \delta = (K \times l_3)/3 \end{aligned} \quad (56)$$

whereas K is proportion factor within limiting constraint of P3R configuration, such that.

$$0.6 < K < 1.5$$

On using K , desired variation in configuration is obtained. Amongst these proportions, selecting any sample proportion factor K for P3R configuration, say 0.9 and assuming fixed link l_4 as 100 units, the link proportions can be given as

$$\begin{aligned} l_3 &= 1.5 \times l_4 = 150; \\ l_1 = l_2 &= \delta = (K \times l_3)/3 = 45; \end{aligned} \quad (57)$$

For all values of K , a set of dimensions is generated for further analysis.

5.3 4R mechanism proportions

Now, to have the equivalent mechanism, 4R configuration is obtained for each variation in P3R mechanism. Assigning the fixed link l_4 of identical length as in P3R and using $\nu = (l_1 + l_2 + \delta)$ of P3R configuration, the proportions for 4R mechanism identified as—

$$\begin{aligned} l_3 &= 1.5 \times l_4 \\ l_1 = l_2 &= ((\nu)/0.9)/2 \end{aligned} \quad (58)$$

Hereby using division factor 0.9, non-singular 4R configuration is ensured. Similarly, the link proportions for 4R mechanisms will be given as

$$\begin{aligned} l_4 &= 100; \\ l_3 &= 1.5 \times l_4 = 150; \\ l_1 = l_2 &= ((\nu)/0.9)/2 = 75 \end{aligned} \quad (59)$$

The maximum linear displacement for the P3R mechanism is 45 for $K=0.9$. Taking intermediate values between 0 and 45, the values of output angle θ_3 are determined, which are then used to calculate input angle θ_1 for the 4R mechanism. Thus, for given value of total linear displacement (ν) in a P3R mechanism, the input angle (θ_1) for a 4R mechanism can be determined under the same output generation condition. Subsequent coupler positions, $P1$ – $P11$,

Fig. 8 CAD model for identical operating conditions of P3R and 4R mechanism

Table 1 Equivalent input conditions for P3R and 4R mechanism

Linear Input Displacement in P3R mechanism (ν)	Position of (common) output	Angular Input Displacement in 4R mechanism (θ_1 deg.)
90	$P1$	22.7
94.5	$P2$	28.16
99	$P3$	33.46
103.5	$P4$	38.67
108	$P5$	43.83
112.5	$P6$	48.98
117	$P7$	54.17
121.5	$P8$	59.43
126	$P9$	64.83
130.5	$P10$	70.45
135	$P11$	76.39

are obtained on account of the linear displacement of actuator. Therefore, an equivalence relation between input linear displacement and angular displacement for a specific output characteristic curve can be determined. The 4R mechanism in Fig. 8 is O_2Q1P1O_4 and equivalent P3R mechanism is O_2P1O_4 . Length O_2P1 indicates distance (ν) between the linear displacement of actuator (δ) and link length ($l_1 + l_2$) such that $\nu = l_1 + l_2 + \delta$. Similarly all other equivalent P3R mechanisms are obtained for respective 4R mechanism from $P1$ to $P11$. The equivalent range of inputs in both mechanisms thus can be established as given in Table 1.

The equivalent input condition obtained for positions $P1$ and $P11$ by analytical approach is cross-checked using CAD modeling approach as seen in Fig. 8.

6 Results and discussion

The mechanical error analysis of 4R and P3R mechanisms is carried out for a tolerance range. The notional values for range of tolerance, which could be obtained from manufacturing of the links, are considered for analysis. The estimated deviations are expressed as specific error, which is defined and applied herewith as the ratio of positional deviation and tolerance on links at that instance. This has made it possible to assess normalized tolerance effect in general.

6.1 Effect of link proportion on error—P3R mechanism

Error estimation of the P3R mechanism is carried out for various link proportions of the mechanism resulting from Eq. 56–57. The proportions of l_1 , l_2 and l_3 are expressed in relation with fixed link l_4 . For the proportion $l_3 = 1.5 \times l_4$ and displacement δ as $(K \times l_3)/3$ varying between $0.6 < K < 1.5$, respectively, the error is estimated for individual link

tolerances. The results are as expressed in Fig. 9a–d. Variation in error is observed for the increase in length of link 1 and link 2. For tolerance on individual links, fairly constant error for various positions is noticed at proportions with $K=0.9$. As link proportions shift toward the extreme values of output angle, the error values deviate away. The change in link proportions results in change in error pattern for individual link tolerances. The effect of tolerances on each individual link is estimated and represented.

6.2 Effect of link proportion on error—4R mechanism

Error estimation of the 4R mechanism is carried out for various link proportions of the mechanism equivalent to P3R mechanism, resulting from Eq. 58–59. The link 3 and link 4 have identical lengths as in P3R, and lengths of link 1 and link 2 are obtained for identical coupler position of P3R mechanism. The proportions of link 1, link 2 and link 3 are expressed in relation with fixed link l_4 . Taking these link proportions for 4R mechanism, the error is estimated for individual link tolerances. The results are as expressed in

(a) Error pattern on Link - 1

(b) Error pattern on Link - 2

(c) Error pattern on Link - 3

(d) Error pattern on Link - 4

Fig. 9 (a–d). Error estimations due to change in link proportions—P3R Mechanism

Fig. 10 (a–d) Error estimations due to change in link proportions—4R Mechanism

Fig. 10a–d. The convergence of the error pattern is seen for link—1 error. Variation in error is observed for the change in length of link 2. For tolerance on any link, fairly uniform error for various positions is noticed at proportions corresponding to $K=0.9$. As link proportions shift toward the minimum and maximum output angle in 4R configuration, the error values deviate away. The variation in link proportions results in change of error pattern for individual link tolerances.

The individualized error pattern is noted for respective variation in link proportion (K) of links in P3R and 4R mechanism and the uniform generalized estimations are not obtained. The subsequent error estimations are presented herewith for the proportion which presents approximately uniform error over various coupler positions, i.e., $K=0.9$.

6.3 Errors in P3R mechanism

Considering tolerance on link 1, link 2, link 3 and link 4, respectively, the specific error is estimated for P3R mechanism for any link proportions ($K=0.6$ to 1.5) as represented

Fig. 11 Specific error of P3R mechanism with tolerance

in Fig. 11. Same results are observed for any set of link proportions and any coupler position.

The estimation against change in positions P1 to P11, the specific error due to tolerances on links 1 and 2 is found to be identical and above zero line, represented as overlapped

Fig. 12 Specific error of P3R mechanism with positions

Fig. 13 Influence of links tolerances on specific error

lines. However, for the links 3 and 4, it is below zero line with little difference in the magnitude. The effect of link tolerance on specific error at any tolerance value over coupler position is as represented in Fig. 12a, b.

6.4 Error estimations in 4R mechanism—approximation and exact estimation approach

The effect of increment in link tolerance on specific mechanical error is represented in Fig. 13. The specific error shows linear variation throughout the tolerance change. For any position of output link (i.e., $P1$ to $P11$) and any link proportions ($K=0.6$ to 1.5), the specific error is uniform and constant for equivalent 4R mechanism.

The error deviation with respect to output link position is shown in Fig. 14(a-b). The specific error due to tolerances on links 1, 2, 3 and 4 is a function of the input link orientation for achieving positions $P1$ to $P11$.

The result with approximation approach, i.e., Fig. 14a is verified through geometric formulation of exact estimation approach, as seen in Fig. 14b.

The change in tolerance sign (i.e., -0.001 to -0.1) results in reversing the error pattern for all links in both mechanisms, with sign reversed; without any change in magnitude of values and position [40].

Fig. 14 Specific error of 4R mechanism with positions

6.5 Validation of estimations

The estimations from the approximation and exact estimation approach is carried out as given in above sections. The estimations through the exact and approximation are giving most identical and conforming values. One of the estimations is presented below for P3R as well as 4R mechanisms, for link proportions with $K=0.9$, at position P6, and tolerance of 0.1. The corresponding values are given in Table 2, showing the conformance. The same is also found by using CAD approach with the use of 2D drafting tool. The CAD estimation plotted with and without tolerances is shown in Fig. 15a, b. The estimations are conforming up to three decimal places.

Similarly, one of the estimation values for P3R mechanism is given in Table 3. The displacement δ of the actuator is 22.5 units, with link lengths l_1 and l_2 each 45 units as shown in Fig. 16a, b.

7 Comparative error analysis in 4R and P3R mechanism

Figure 17a, b shows the comparison between 4R and P3R mechanisms under the effect of link tolerance. The plot represents error deviation against coupler position and as well as link proportions. The maximum error in P3R mechanism

Table 2 Specific error estimation for 4R mechanism by CAD approach

Total error at $K=0.9$, Position = (P6), tolerance (t)=0.1		
Approximation approach	Exact estimation approach	CAD approach
0.1308	0.130	0.130

Fig. 15 Specific error estimation for 4R mechanism by CAD approach

Table 3 Specific error estimation for P3R mechanism by CAD approach

Total error at $K=0.9$, Position = (P6), tolerance (t)=0.1		
Approximation approach	Exact estimation approach	CAD approach
0.7148	0.714	0.710

is higher than that in 4R mechanism. The maximum mechanical error is function of coupler position, as link proportion in P3R mechanism increases, the specific error reduces for coupler position P1, reversing the findings toward position P11. The error plot is fairly uniform for 4R mechanism, as seen in Fig. 17a. The total error bearing the compensatory effect of individual link errors represents the minimum and more realistic error representation, as in Fig. 17b. Identical error plots are obtained through geometrical analysis of exact estimation approach, thus verifying and validating the approximation approach results.

Thus, from performance point of view for identical output generation conditions at given tolerance, 4R mechanism is better in regards to mechanical error as compared to P3R mechanism.

8 Conclusions

The aim of the work carried out in this paper was to develop generalized formulation using modified Taylor series approximation method for positional error estimation in P3R and 4R planar manipulator configurations and subsequent validation of the approach using geometric approach and CAD model; followed by comparison of error in two configurations. This paper presents a detailed methodology for calculating mechanical errors in 4R and P3R mechanisms

Fig. 16 Specific error estimation for P3R mechanism by CAD approach

Fig. 17 (a–d) Comparative error curve between P3R and 4R mechanism

by considering the influence of tolerances on the link dimensions. The formulation for mechanical error estimation in both configurations is developed. Traditional approximation approach is extended for analyzing mechanical error in P3R configuration. Error for individual link tolerance is estimated. The generality of applying the formulation is attempted, by analyzing various link proportions. The error variation pattern is investigated, leading to identification of a link ratio to have least error variation.

Geometric analysis for P3R and 4R configurations is devised as exact estimation approach; further used to verify and validate resulting error through approximation approach. Identical operating conditions are established between the two configurations. The methodology for the comparative evaluation of specific mechanical error in an active revolute and prismatic joint mechanism is presented. For the sample case of operational equivalence between these mechanisms, the method of error estimation is demonstrated. The

estimated values are verified using CAD approach and generality of solution is checked.

Major contribution of the work lies in arriving at link proportions so as to have the minimal error, expressed as specific error, which shall help designer to select the link proportions.

The following generalized conclusions are drawn on the basis of this study:

1. The mechanical error varies from position to position throughout the range of operation i.e., positions P1 to P11 for both mechanisms. Traditionally, the error is suspected to have same over entire workspace.
2. The mechanical error depends on the link proportions. The link ratio can be identified to give least error variation pattern in P3R and 4R configurations.
3. Comparative evaluation of mechanical error under link tolerance in 4R and P3R mechanisms shows that 4R

configuration is robust against P3R configuration under identical operating conditions. Thus, it is shown here, that revolute joint actuation becomes more advisable over prismatic actuation of robotic manipulators from positional error sensitivity perspective.

The established methodology and characterized performance will be useful in analyzing parallel mechanisms with rotary and linear actuation. The future scope lies in extending the approaches and methodology presented in this paper to study other issues related to mechanisms performance.

Appendix I

On substituting the values of A , B and C in Eq. (7) and rearranging the expressions,

$$\sin \theta_1 \cdot \sin \theta_3 + \left(\frac{l_4}{l_1} + \cos \theta_1 \right) \cdot \cos \theta_3 = \frac{l_4}{l_3} \cos \theta_1 + \frac{l_1^2 - l_2^2 + l_3^2 + l_4^2}{2l_1 l_3}$$

$$\sin \theta_1 \cdot \sin \theta_3 + \frac{l_4}{l_1} \cdot \cos \theta_3 + \cos \theta_1 \cdot \cos \theta_3 - \frac{l_4}{l_3} \cos \theta_1 - \frac{l_1^2 - l_2^2 + l_3^2 + l_4^2}{2l_1 l_3}$$

$$\sin \theta_1 \cdot \sin \theta_3 + \cos \theta_1 \cdot \left(\cos \theta_3 - \frac{l_4}{l_3} \right) = \frac{l_1^2 - l_2^2 + l_3^2 + l_4^2}{2l_1 l_3} - \left(\frac{l_4}{l_1} \cdot \cos \theta_3 \right)$$

$$G \cdot \sin \theta_1 + H \cdot \cos \theta_1 = I$$

$$G = \sin \theta_3$$

$$H = \cos \theta_3 - \frac{l_4}{l_3}$$

$$I = \frac{l_1^2 - l_2^2 + l_3^2 + l_4^2}{2l_1 l_3} - \frac{l_4}{l_1} \cdot \cos \theta_3$$

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Declarations

Conflict of interest The author declares that he has no conflict of interest.

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