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TECHNICAL PAPER

Optimization of sequential grinding process in a fuzzy environment using genetic algorithms

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Abstract

The paper presents the methodology of optimization of the sequential grinding process with the application of fuzzy logic for the definition of objectives and constraints imposed on the machining process. The presented method includes the succession of several subsequent operations and the dimensional and shape inaccuracies between them. The use of the fuzzy sets theory enabled the definition of not only the space of expectable solutions, but also the space of acceptable solutions for which the goals and limits imposed on the grinding process are partially met. The presented methodology was used to optimize the process of sequential grinding of small ceramic elements (corundum ceramics with Al_2O_3 content of 92–99%.) The definition of fuzzy objective and constraints in the process of sequential grinding of small ceramics elements was proposed. The influence of the speed of the rotary grinding table and the machining allowance on the deviation of the flatness and height of the grinding elements and the value of the component of the normal grinding force were determined. Using the developed relationships, the definition of fuzzy objectives and constraints defined in the process output parameter space was transferred to the process parameter set space. In such a defined space, the optimization process was carried out using the genetic algorithm. The analysis of the impact of the applied t-norm functions used for aggregation of the fuzzy objective and constraints on the obtained results was performed. It was shown that in the case of sequential grinding of small ceramic elements, the use of minimum t-norm for an aggregation of grinding objective and constraints allows to achieve the highest process efficiency.

Keywords Grinding processes · Dimension and shape accuracy · Process efficiency · Genetic algorithm · Fuzzy optimization

List of symbols

a_e	Total allowance (μm)
a_i	Allowance for the ith grinding wheel (μm)
C	Process constraint
D	Diameter of ground elements, mm
F_n	Normal component of the grinding force (N)
G	Process objective
Q_p	Grinding efficiency (pcs/s)
Q_v	Grinding volume efficiency (mm^3/s)
T_o	Distance between consecutive elements on the rotary table perimeter (mm)
v_w	Speed of the grinder rotary table (mm/s)
V_w	Volume of removed material (mm^3/pcs)
X	Set of process input parameters
y_{acc}	Accepted value of the process output variable
y_{exp}	Expected value of the process output variable
Y	Set of process output parameters
Δh	Workpiece height deviation, μm
Δp	Workpiece flatness deviation, μm
μ_D	Fuzzy decision
μ_{Fn}	Degree of fulfillment of the constraint imposed on the normal component of the grinding force
μ_{Qp}	Degree of fulfillment of the objective imposed on the grinding efficiency
$\mu_{\Delta h}$	Degree of fulfillment of the constraint imposed on the workpiece height deviation
$\mu_{\Delta p}$	Degree of fulfillment of the constraint imposed on the workpiece flatness deviation

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1 Introduction

Grinding processes are one of the final operations in the manufacturing process. They determine the accuracy of the shape, dimensions and quality of the machined surface. The result of the grinding process is affected by many factors related to the properties of the grinding tool and workpiece as well as the parameters and conditions of grinding process [1]. The influence of these factors on the grinding results is often characterized by a complex mechanism of their cumulative effects.

In grinding processes, material is removed by abrasive grains located on the grinding wheel active surface. These grains are characterized by a varied shape and random distribution on the surface of the active abrasive tool. The interactions in the contact area of abrasive grain with the workpiece determine the results of the grinding process. Studies carried out in the works [2] indicate the interactions variability in the contact area occurring with the change in grinding parameters. In addition, material separation efficiency also depends on the shape of the abrasive grains [3, 4].

In addition, the topography of the active surface of the abrasive tool changes during grinding as a result of abrasive wear and micro-crushing of the surface of abrasive grains and as a result of crushing of whole grains as well as the abrasion of the grinding wheel's surface with the workpiece. Therefore, the grinding process is characterized by significant randomness.

In the grinding processes of brittle materials, which include aluminum oxide ceramics, two types of material separation can be distinguished: ductile type and brittle type. In the first of these, the material is removed as a result of plastic–elastic material separation. In the second, material removal occurs as a result of cracking and separation of material. Conducting the grinding process with small cutting depth allows the process to be run in the ductile regime, which ensures a higher quality of the treated surface [5, 6].

Selection of parameters and conditions of the grinding process is a frequent demand of manufacturing companies as this operation often corresponds to the final accuracy of the shape and dimensions of the machined elements. The issues concerning optimization of machining processes presented in the literature [7–9] assume the search for process parameters allowing to obtain: minimum production cost, maximal production rate, finest possible surface quality. Limitations in the grinding process are most often associated with: thermal damage of the ground surface, wear of the grinding wheel, stiffness of the grinder-workpiece-grinding tool system. Determination of the relation of the grinding process parameters on the above values was conducted using analytical methods [10, 11] or neural networks [12, 13]. Gradient methods, nonlinear programming methods and evolutionary algorithms were used to solve optimization tasks [9, 14]. It was demonstrated in the work [15] that the use of evolutionary algorithms to solve the task of grinding processes optimization gives the best results.

The multitude of factors affecting the outcome of the machining process as well as its complexity of mechanisms of cumulation of the effects of their interactions leads to the application of procedures effective in the processing of these types of data. The use of fuzzy logic methods allows for ambiguity and uncertainty in the description of the analyzed phenomena. The paper [16] presents the use of the fuzzy logic methods in the procedure for parameters selection of the surface grinding process. The developed approach enabled the determination of a set of optimal design variables in order to achieve a set of desired process variables. Abbas [17] proposed a method of the optimization under uncertainty in machining processes (abrasive water jet machining, abrasive water jet and ultrasonic machining). The method applied to machining parameters optimization takes into account the variability of the process parameters and their effect on the variability of the machining results. Chiang [18] developed a gray-based fuzzy algorithm which simplifies the optimization procedure for the complicated performance characteristics. Rahul et al. [19] described an integrated optimizing path combining satisfaction function, fuzzy inference system and Taguchi method for machining performance optimization for electro-discharge machining of Inconel alloys.

So far, the methods of grinding processes optimization have assumed crisp definitions of objectives and constraints. In this approach, the transition from the set of acceptable processing parameters to the set of unacceptable parameters takes place in a stepwise manner. The variability of the results of grinding process resulting from the randomness of phenomena occurring in the grinding zone is in some contradiction with this approach. A more natural approach would be an introduction of graduation determining the membership of grinding parameters to the set of assumed objectives and constraints. In addition, many optimization criteria are contradictory (e.g., maximum processing efficiency, high quality of processed surface). For a sharp transition from the set of permissible parameters to the set of unacceptable parameters, the degree of fulfillment of the constraints and objective of the grinding process is purely zero–one. For contradictory constraints, when the set of grinding parameters allowing to fulfill all constraints is empty, the lack of graduation in the evaluation of the fulfillment of optimization criteria makes it difficult to evaluate the solutions obtained by optimization.

The introduction of graduation in the description of objectives and constraints cannot be recognized by the application of crisp set theory and enforces the application of fuzzy sets theory [20–22]. The basics of decision making in a fuzzy environment are described in the study by Belman and Zadeh [23]. Since then, the issues related to decision-making methodology in a fuzzy environment have been of an interest to a number of researchers. Tang [24] presents an overview of theories and methods related to fuzzy optimization, classification of fuzzy modeling and optimization, and methods of solving them.

The multitude of factors affecting the result of grinding process and the variability of their effect during the process induces the use of fuzzy logic methods in the definition of machining objectives and constraints. This is particularly important in sequential grinding with the cumulative impact of individual operations inaccuracy on the result of the process.

The paper presents the assumptions and results of the optimization of the grinding process using fuzzy logic to the definition of objectives and constraints imposed on the example of the sequential grinding of small ceramic elements. On the basis of experimental research, the variability of the values of parameters describing the machining accuracy was determined, which was included in the definition of the constraints membership functions. The assumptions of the optimization process also include the sequentiality of the process and the associated fact of propagation of machining inaccuracies between successive machining zones. An objective function has been defined that allows determining the impact of the degree of fulfillment of individual objectives and processing constraints on the result of the fuzzy decision.

Experimental study was carried out to determine the relationships allowing the transfer of objectives and constraints of the grinding process from the output variable space into the grinding parameters space. Process parameters assuring maximization of fuzzy decision using a genetic algorithm were optimized. An analysis of the impact of t-norms used for an aggregation of fuzzy objectives and constraints on optimization results was performed.

2 Basis of making fuzzy decision in fuzzy environment

Fuzzy decisions making requires defining the fuzzy objectives and constraints [22]. The definition of fuzzy objectives requires the determination of the membership function assigning each element from the process output set Y the value in the range [0, 1] informing about the degree of membership μ of the element y to the given objective G or the constraint C:

$$\mu_{\mathrm{G}}(y) : Y \to [0, 1] \tag{1}$$

$$\mu_{\mathrm{C}}(y) : Y \to [0, 1] \tag{2}$$

The objectives and constraints imposed on the machining result mainly from the technological requirements (e.g., the expected value of the surface roughness parameter Sa, the acceptable tolerance of the shape and dimensions) and the economic requirements (e.g., high grinding performance, small grinding cost). In the case of the machining, the membership functions determining the degree of membership of the process output y to a given objective or constraint can be interpreted as functions determining the satisfaction level of the machining result.

In the machining, the definition of objectives and constraints is usually made in the output parameters space. The optimization of machining process input parameters allowing to achieve machining objectives and constraints requires the transfer of objective definitions and constraints from the process output space Y to the process input parameters space X. This requires determination of the function f:

$$f : X \to Y, \quad y = f(x) \tag{3}$$

being a model of the machining process. With this assumption, it is possible to transfer the definition of the fuzzy objectives and constraints into the space of the adjustable machining parameters:

$$\mu'_{\mathrm{G}}(x) = \mu_{\mathrm{G}}\left[f(x)\right], \quad \forall x \in X \tag{4}$$

$$\mu'_{\mathrm{C}}(x) = \mu_{\mathrm{C}}\left[f(x)\right], \quad \forall x \in X \tag{5}$$

The decision concerning selection of the appropriate machining parameters is formulated with a fuzzy set D, obtained as a result of an aggregation of fuzzy sets of n objectives and m constraints. The assumption that the fuzzy decision should provide both the fulfillment of objectives and constraints requires the use of t-norms as operators of fuzzy sets aggregation:

$$\mu_{\mathrm{D}}(x) = \mu'_{\mathrm{G}_1}(x) \odot \ldots \odot \mu'_{\mathrm{G}_n}(x) \odot \mu'_{\mathrm{C}_1}(x) \odot \ldots \odot \mu'_{\mathrm{C}_m}(x) \tag{6}$$

where $\odot$ a is t-norm operator.

The fuzzy decision μ_{D} defines the degree of fulfillment of fuzzy objectives and constraints. The type of t-norm applied significantly affects the aggregated fuzzy decision.

The need to determine the optimal machining parameters to ensure the expected quality of the process requires the determination of the non-fuzzy values from the fuzzy decision. Because the fuzzy decision determines the degree of fulfillment of particular fuzzy objectives and constraints, the optimal set of machining parameters is the set x^*, for which:

$$\mu_D(x^*) = \max_{x \in X} \mu_D(x) \tag{7}$$

This method provides a selection of machining parameters for maximizing the degree of fulfillment of machining objectives and constraints.

3 Assumptions of the optimization of the process of sequential grinding of small ceramic elements

The process of sequential grinding of small ceramic elements involves the implementation of a series of grinding operations in one pass. The total machining allowance a_e is divided between grinding wheels located on the periphery of the rotary table with the ceramic elements placed on it (Fig. 1). The workpieces move along with the table, passing under the successive grinding wheels, where the calculated allowance is removed. Ceramic elements, placed on the rotary table, are ground and smoothed successively.

The purpose of grinding process is to remove relatively large machining allowances resulting from the geometric inaccuracy of ceramic elements obtained in the sintering process. The grinding process is carried out using the grinding wheels inclined at an angle α and β with respect to the plane of workpieces' track (Fig. 2).

The method, due to the appropriate kinematics, allows accurate and efficient grinding of ceramic elements. The inclination of the conical grinding wheel increases the grinding path and thus reduces the removal speed of the allowance [25].

During the grinding process, the elements are fixed on the surface of the rotary table; and during the finishing process, they are lifted and flexibly pressed to the active surfaces of the smoothing and polishing wheel.

The assumed objectives in the grinding processes are mainly based on economic requirements and assumptions for assuring the quality requirements of ground parts. The constraints imposed on the grinding process are most often attributed to the properties of the technological devices and the properties of the tool itself, as well as an impact of the grinding parameters on the process quality.

The basic economic criterion of the processing is the volume efficiency of the process. The volume efficiency Q_v of the ceramic elements grinding process in the automatic cycle is determined by the relationship:

$$Q_v = V_w \cdot Q_p = \frac{\pi \cdot d^2}{4} \cdot a_e \cdot \frac{v_W}{T_o} \tag{8}$$

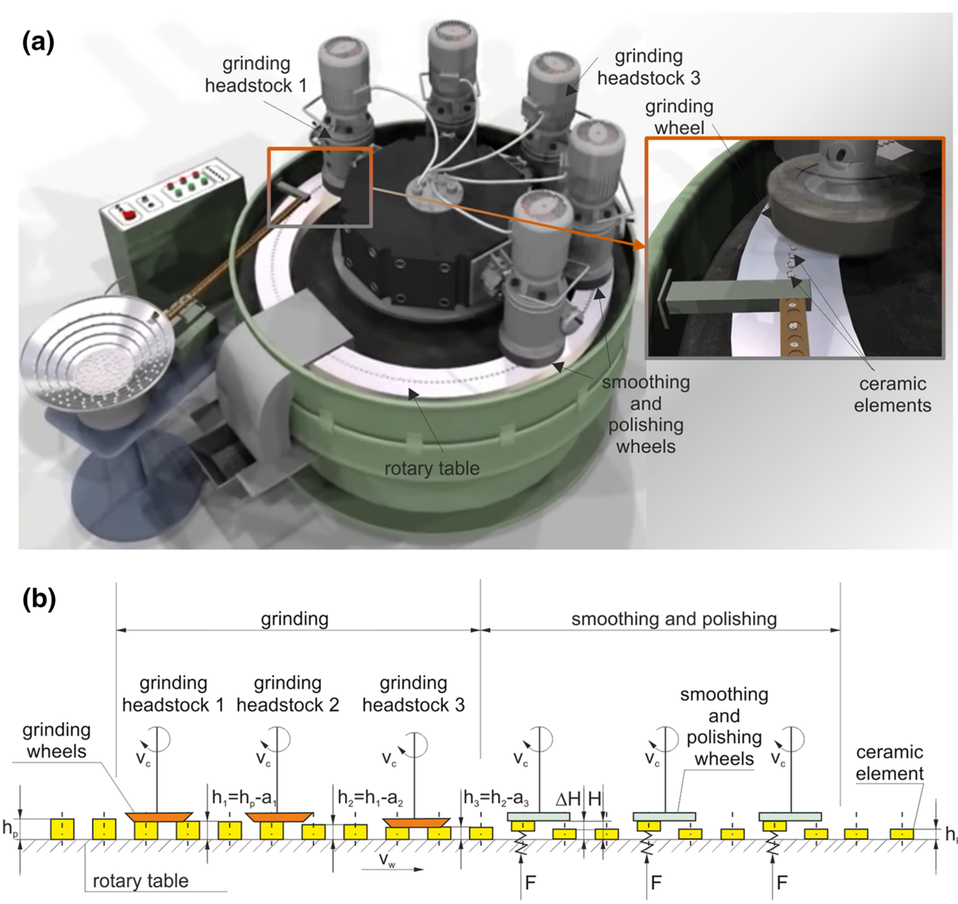

Fig. 1 Integrated machining of ceramic elements in automatic cycle: process visualization (**a**) and general scheme of the method (**b**)

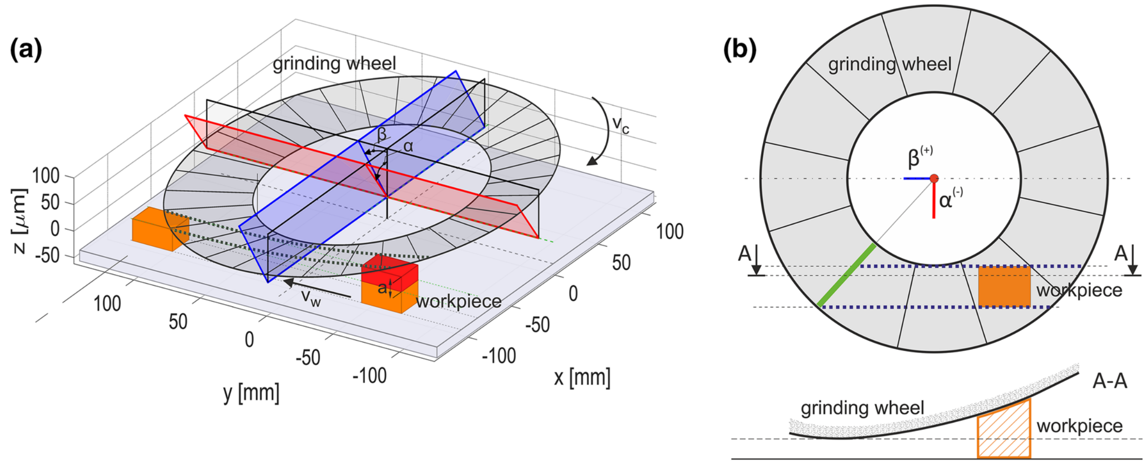

Fig. 2 Illustration of the grinding method with the face of a conical grinding wheel: 3D view (**a**) and projection on a plane perpendicular to the z axis (**b**) [25]

where N—the number of grinding wheels; $a_e = \sum_{i=1}^{N} a_i$; a_i—the allowance for the ith grinding wheel, mm.

It may be concluded from the above relationship that it is necessary to increase the speed of the grinder rotary table to increase the volume efficiency of the process. Thus, the criterion of the highest speed of the grinder rotary table can be accepted as the ultimate objective of optimization, which is at the same time a controllable parameter of the grinding process.

The grinding objective fuzzy membership function μ_G was defined in the form of a trapezoidal function, defined by the relationship:

$$\mu_G(y) = \left(\frac{|y - y_{acc}|}{y_{exp} - y_{acc}} \right) \cdot z_1 + z_2 \tag{9}$$

$$z_1 = \begin{cases} 1 & \text{if } y_{acc} < y < y_{exp} \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

$$z_2 = \begin{cases} 1 & \text{if } y \geq y_{exp} \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

where y_{acc}—the accepted value of the process output variable, y_{exp}—the expected value of the output variable, z_1, z_2—auxiliary variables.

An exemplary representation of the grinding objective fuzzy membership function $\mu_G(Q_p)$ and $\mu_G(v_w)$ is shown in Fig. 3.

Based on relationship (8), the grinding objective fuzzy membership function μ_G was transformed into the space of the grinding process parameters X (Fig. 3b):

$$\mu_G'(v_w) = \mu_G[f(v_w)], \forall v_w \in X. \tag{12}$$

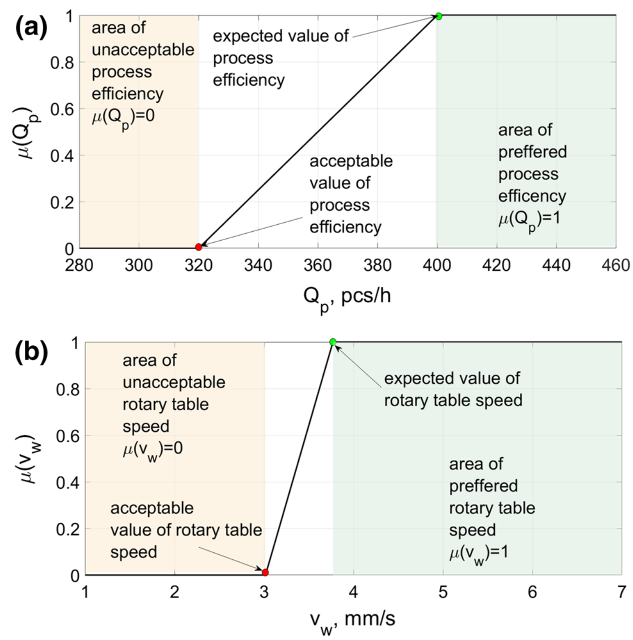

Fig. 3 The grinding objective fuzzy membership function for: $Q_{p\text{-}exp} = 400$ pcs/h, $Q_{p\text{-}acc} = 320$ pcs/h in the space of output parameters Y (**a**) and input parameters X (**b**)

$$f(v_w) = \frac{4 \cdot T_o}{\pi \cdot d^2} \cdot \frac{Q_p}{\sum_{i=1}^{N} a_i} \tag{13}$$

The selection of grinding parameters should ensure obtaining the assumed grinding efficiency without the negative effects on the dimensions and shape accuracy of the workpieces and the quality of their surface.

The possible range of parameter values depends on the constraints resulting from the properties of the grinder, the workpiece and the grinding wheel. Because the grinding power required to remove the total allowance a_e is small in the analyzed grinding process, there is no constraint associated with the power of the grinder. There are constraints related to the accuracy of the shape and dimensions of the workpieces. The machined ceramic elements work as a sliding cooperating part, and proper tightness of connections must be ensured. Accordingly, the processing parameters should ensure the fulfillment of the constraints related to the value of acceptable and expected deviation of height Δh and deviation of flatness Δp of the workpiece. The acceptable and expected value of the parameters results from the requirements for dimension and shape tolerance of the product and the variation of the machining results.

The expected value y_{exp} and the acceptable y_{acc} of the output values of the process can be determined in accordance with the equation:

$$y_{\mathrm{acc}} = y_{\mathrm{UTL}} - n \cdot \bar{\sigma}_{\mathrm{R}} \tag{14}$$

$$y_{\mathrm{exp}} = y_{\mathrm{UTL}} - m \cdot \bar{\sigma}_{\mathrm{R}}, \quad \text{where} \quad m > n \tag{15}$$

where y_{UTL}—upper tolerance limit for variable y, $\bar{\sigma}_{\mathrm{R}}$—the mean standard deviation of variable y.

The difference between the acceptable value y_{acc} and the expected value y_{exp} of the parameter allows for taking into account the shift of the mean value of the output value of the process as a result of affecting the process of random factors by a value equal to $(m - n) \cdot \sigma_{\bar{\mathrm{R}}}$ (Fig. 4).

The acceptable and expected values of the output parameters determined on the basis of relationships 12 and 13 may also include expectations regarding the values of the qualitative indicators. For example, for $n = 3$ and $m = 5$ the expected value of the C_{pk} capability index will be 1.66 and the permissible value will be 1.

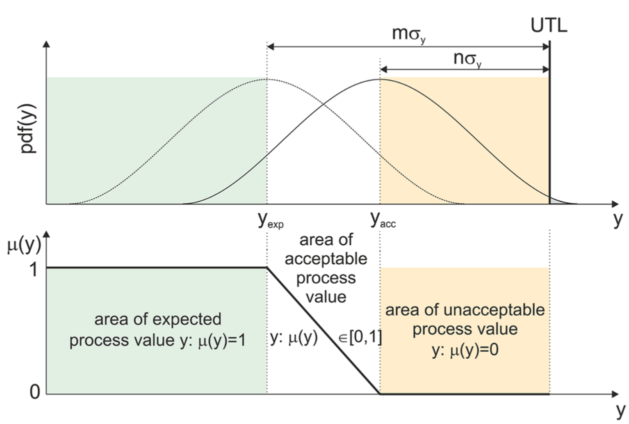

Fig. 4 An example of determination of the membership function of the fuzzy constraint, including information about the variability of the variable

The process of grinding of the brittle ceramic elements is fraught with the risk of fracture micro-cracking of the ground surface. To ensure a suitable surface quality of the cooperating elements, the restriction of the normal component of the grinding force F_n is required.

The membership functions of the constraints related to the deviation of height $\mu_{C1}(\Delta h)$, the deviation of flatness $\mu_{C2}(\Delta p)$ and normal component of the grinding force $\mu_{C3}(F_n)$ are defined as trapezoidal functions:

$$\mu_C(y) = \left(\frac{\left| y - y_{\mathrm{acc}} \right|}{y_{\mathrm{acc}} - y_{\mathrm{exp}}} \right) \cdot z_1 + z_2 \tag{16}$$

$$z_1 = \begin{cases} 1 & \text{if } y_{\mathrm{acc}} < y < y_{\mathrm{exp}} \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

$$z_2 = \begin{cases} 1 & \text{if } y \geq y_{\mathrm{exp}} \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

Fuzzy constraints are defined in the space of grinding output parameters Y. Determination of the optimum values of grinding parameters satisfying these constraints requires the transfer of the membership functions of fuzzy constraints to the space of grinding input parameters X:

$$\mu'_{C1}(v_{\mathrm{w}}, a) = \mu_{C1}\left[f_{\Delta h}(v_{\mathrm{w}}, a) \right], \quad \forall v_{\mathrm{w}}, a \in X \tag{19}$$

$$\mu'_{C2}(v_{\mathrm{w}}, a) = \mu_{C2}\left[f_{\Delta p}(v_{\mathrm{w}}, a) \right], \quad \forall v_{\mathrm{w}}, a \in X \tag{20}$$

$$\mu'_{C3}(v_{\mathrm{w}}, a) = \mu_{C3}\left[f_{\mathrm{Fn}}(v_{\mathrm{w}}, a) \right], \quad \forall v_{\mathrm{w}}, a \in X \tag{21}$$

The functions $f_{\Delta h}$, $f_{\Delta p}$ and f_{Fn} determining the influence of grinding parameters on the process output values were obtained by the experiment described in Sect. 4.

In the optimization process of the grinding parameters, it was assumed that the next grinding wheel removes the allowance increased by the value of the deviation of height resulting from deformation of the former grinder headstock. Thus, the resulting value of the deviation of the height on the subsequent headstocks is defined by:

$$\begin{aligned} \Delta h_1 &= f_{\Delta h}(a_1, v_{\mathrm{w}}) \\ \Delta h_2 &= f_{\Delta h}(a_2 + \Delta h_1, v_{\mathrm{w}}) \\ &\vdots \\ \Delta h_N &= f_{\Delta h}(a_N + \Delta h_{N-1}, v_{\mathrm{w}}) \end{aligned} \tag{22}$$

Where Δh_i—ceramic element height deviation on the ith grinding wheel, $f_{\Delta h}$—function determining the relationship between the grinding parameters and the deviation of grinding elements height.

Schematically, the above assumptions for the process of sequential grinding of small ceramics elements fuzzy optimization are shown in Fig. 5.

An increase in grinding efficiency (increase in the speed of the rotary table v_w) is possible due to appropriate division of the total grinding allowance $a_e = [a_1, a_2, a_3]$ among the grinding wheels. The fuzzy constraint of the normal grinding force was imposed on all grinding wheels, whereas the fuzzy constraints of the deviation of height and flatness were imposed on the last grinding wheel in the sequential grinding operation. An assessment of the degree of fulfillment of the fuzzy objective and constraints is made by an

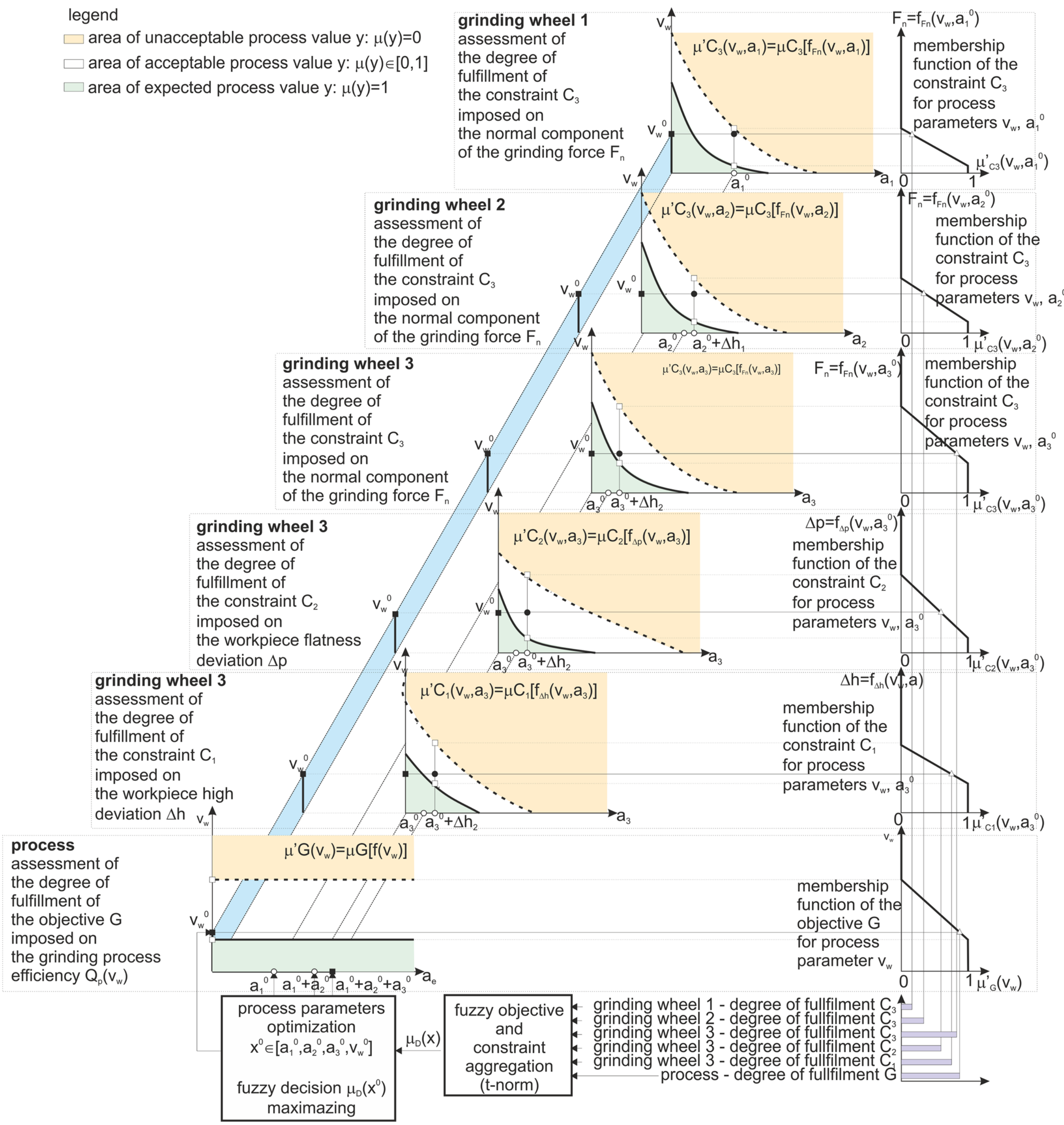

Fig. 5 General scheme of fuzzy optimization method in application for the optimization of parameters of sequential grinding of small ceramic elements

aggregation of fuzzy membership functions according to Eq. 6.

The use of following t-norms was assumed in the aggregation of fuzzy membership function:

- minimum: $t_{\min}(a, b) = \min(a, b)$;;
- product: $t_{\text{prod}}(a, b) = a \cdot b$;;
- Łukasiewicz t-norm:

$t_{\text{Luk}}(a, b) = \max(0, a + b - 1)$;
- Hamacher product:

$$t_{\text{H}}(a, b) = \begin{cases} 0 & \text{if } a = b = 0 \\ \frac{a \cdot b}{a + b - a \cdot b} & \text{otherwise} \end{cases}.$$

The grinding process parameters that maximize a fuzzy decision were found using a genetic algorithm. A detailed description of the results of the simulation procedure and their discussion is provided in Sect. 5.

4 Experimental study

Selection of the optimum values of grinding parameters requires the determination of the relationship between the grinding input parameters (ith grinding wheel allowance a_i and speed of the grinder rotary table v_w), and selected grinding output parameters (flatness deviation Δp and height deviation Δh of ground elements, and the normal component of the grinding force F_n). The study was carried out on an automatic grinder AU-16 for small ceramic elements (Fig. 6).

The grinding parameters and conditions are summarized in Table 1. The ground workpiece is made of corundum ceramics with Al_2O_3 content of 92–99%. The workpiece diameter was 15.8 mm, and the height was 3.8 mm. The mechanical and physical properties of the material are summarized in Table 2.

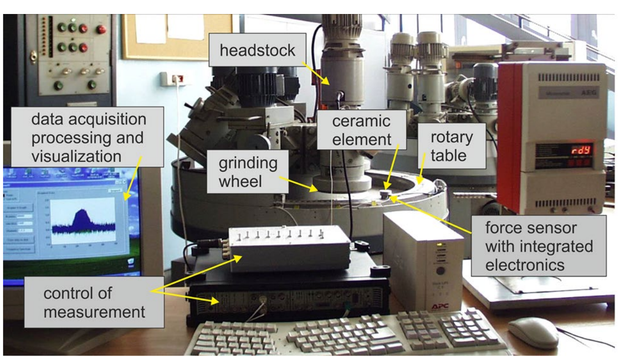

Fig. 6 General view of the test stand

Table 1 Grinding process parameters

Properties	Unit	Value
Grinding wheel	–	S3020 200×85×30×4 SD 125/100 BT
Grinding speed, v_c	m/s	29.5 m/s
Allowance, a	µm	50, 100, 200
Table speed, v_w	mm/s	4, 5, 7
Grinding condition	–	Wet
Grinding fluid	–	Water

Due to the fact that the active surface of the grinding wheels is inclined in relation to the plane of the rotary table, a long grinding path has been obtained. The removal speed of the allowance decreases along the grinding path, which is advantageous due to the smaller thermal effects in the machining zone. Water cooling was used in the process due to the fact that in addition to the cooling functions it had a positive effect on the condition of the active surface of the diamond wheels. The active surface retained its ability to last longer, and there was no excessive sticking of the waste products of the machining process.

Workpieces were pressed by the grinding forces to the profiled table edge as a result of the tilting of the grinding wheel at angles α and β (Fig. 2). Cooling with water ensured efficient washing out of the machining products from the clamping zone.

The quality of the ground elements was evaluated by the following devices: L-GAGE LG5 laser sensor for height deviation measurement (sensing window range 1.5 mm, sensing beam wavelength 650 nm, response speed slow) and PIK-1A analyzer for flatness deviation measurement (sensor type: inductive, measuring tip length 45 mm). During the grinding process, values of the force components were measured using a piezoelectric force sensor with integrated electronics 9602A by Kistler. The measurement data were registered at 10 kHz frequency using a 16-bit measurement card.

Table 2 Mechanical and physical properties of ground elements

Properties	Unit	Value
Density	g/cm^3	3.4–3.8
Flexural strength	MPa	250–300
Impact strength	kJ/m^2	4
E-modulus	GPa	220–300
Hardness (Knoop, 100 g)	GPa	20
Specific heat capacity	J/kg/K	850
Thermal conductivity	W/m/K	27.8–34.9

The experiment plan and the results are summarized in Tables 3, 4 and 5. The standard deviation of the measurement results was determined based on the range of the variable values, taking into account the correction factor d_2^* [26, 27].

The model describing the influence of grinding parameters on the output values of the process was formulated as follows:

$$y = f\left(a, v_{\mathrm{w}}\right) = k_0 \cdot a^{k_1} \cdot v_{\mathrm{w}}^{k_2} \tag{23}$$

where k_i—equation coefficients.

The task of matching the model to the experimental data was reduced to finding assessments of parameters a and b minimizing the sum of residual squares. The equation coefficients are based on the classic method of least squares. The degree of correctness of the description of experimental data was also determined by the regression function, determining for each

Table 3 Influence of grinding parameters on the workpiece height deviation Δh

No.	v_{w} (mm/s)	a (μm)	Δh (μm)			$\overline{\Delta h}$ (μm)	$\sigma_{\Delta h}$
			1	2	3		
1	4	50	4.1	4.4	3.1	3.87	0.68
2	5	50	10.1	10.2	12.6	10.97	1.31
3	7	50	14.5	16.1	16.0	15.53	0.84
4	4	100	7.9	9.1	7.9	8.30	0.63
5	5	100	16.2	16.4	14.3	15.63	1.10
6	7	100	21.6	24.8	22.4	22.93	1.67
7	4	200	16.1	19.2	18.4	17.90	1.62
8	5	200	25.1	25.7	26.3	25.70	0.63
9	7	200	34.8	32.7	28.5	32.00	3.30
						$\bar{\sigma}_{\Delta h}$	1.31

Table 4 Influence of grinding parameters on the workpiece flatness deviation Δp

No.	v_{w} (mm/s)	a (μm)	Δp (μm)			$\overline{\Delta p}$ (μm)	$\sigma_{\Delta p}$
			1	2	3		
1	4	50	1.49	1.73	1.49	1.57	0.13
2	5	50	1.47	1.62	1.84	1.64	0.19
3	7	50	2.71	2.73	2.64	2.69	0.05
4	4	100	1.74	2.10	2.02	1.95	0.19
5	5	100	2.12	2.46	2.33	2.30	0.18
6	7	100	3.50	3.42	2.86	3.26	0.33
7	4	200	3.02	2.77	2.81	2.87	0.13
8	5	200	4.08	3.81	3.83	3.91	0.14
9	7	200	4.92	5.51	5.60	5.34	0.36
						$\bar{\sigma}_{\Delta p}$	0.19

Table 5 Influence of grinding parameters on the normal component of grinding force *Fn*

No.	v_{w} (mm/s)	a (μm)	F_n (N)			$\bar{F}_n$ (N)	σ_{Fn}
			1	2	3		
1	4	50	15.2	13.8	11.5	13.50	1.94
2	5	50	15.9	19.2	20.1	18.40	2.20
3	7	50	25.7	28.4	29.6	27.90	2.04
4	4	100	20.1	18.9	22.5	20.50	1.88
5	5	100	28.1	26.2	30.2	28.17	2.09
6	7	100	33.1	30.9	35.4	33.13	2.35
7	4	200	35.5	32.8	31.7	33.33	1.99
8	5	200	33.1	35.0	38.9	35.67	3.03
9	7	200	35.6	42.1	39.2	38.97	3.40
						$\bar{\sigma}_{\mathrm{Fn}}$	2.33

of the models the square of the relationship between the data obtained in the experiment y_i and the response of the model $\hat{y}_i$:

$$R^2 = 1 - \frac{\mathrm{SSE}}{\mathrm{SST}} \tag{24}$$

where $\mathrm{SSE} = \sum_{i=1}^{N} \left(y_i - \hat{y}_i\right)^2$—the square of the difference between the values obtained in the experiment $\hat{y}_i$ and as a result of modeling $\hat{y}_i$; $\mathrm{SST} = \sum_{i=1}^{N} \left(y_i - \bar{y}\right)^2$—square difference between the values obtained in the experiment y_i and the mean value $\bar{y}$.

As a result of approximation of experimental data using the above relationship, the models allowing to transfer of the fuzzy constraints definition into input grinding parameters space were obtained (Fig. 7).

The analysis of regression coefficients indicates a greater influence of the speed of rotation of the rotary table v_{w} than the value of the allowance a on the value of the monitored output variables (height and flatness deviation as well as normal component of the grinding force). Increasing the value of the allowance, in the analyzed grinding method, increases the length of the grinding zone, which causes that the effect of the increase in the allowance on the normal component of the grinding force is smaller than the influence of the feed speed of the workpiece. The increase in the value of the component of the normal grinding force is one of the main factors affecting the size of the deformation of the machining system, and hence the increase in the deviations of the height and flatness of the workpieces.

The obtained models have a good fit for the experimental data. The average value of determination factor R^2 for the models is 0.93. Higher impact of the grinder rotary table speed v_{w} on the output value of the process parameters is noticeable in each of the developed models.

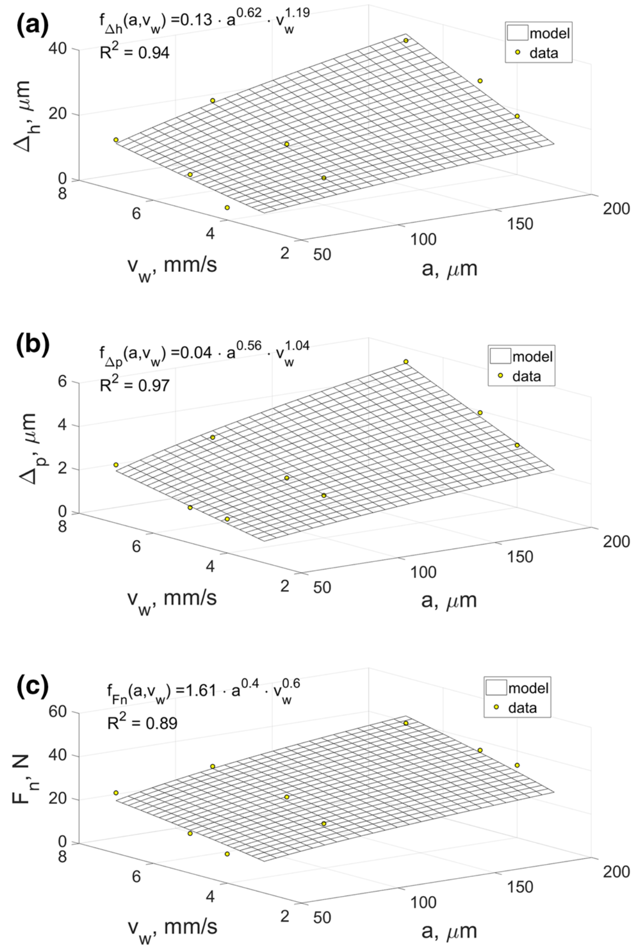

Fig. 7 Models determining the effect of processing parameters on the workpiece height deviation (**a**), workpiece flatness deviation (**b**), normal component of the grinding force (**c**)

5 Fuzzy optimization of process parameters

The genetic algorithm was used to optimize the parameters of sequential grinding of small ceramic elements. The scheme of the optimization method is shown in Fig. 8.

The range of the initial population values corresponded to the range of input process parameters used in the experiment. A constraint-dependent crossover operator was used.

Grinding parameters $x = [a_1, a_2, a_3, v_{\mathrm{w}}]$ were optimized to maximize the value of fuzzy decision $\mu_{\mathrm{D}}(x)$. The optimization was made in the MATLAB 2016b environment, for the following genetic optimization parameters:

- initial population size 500;
- generations 20;
- function tolerance 1e−10;
- constraint tolerance 1e−10.

The results of the optimization were analyzed for two cases, differing in the grinding objectives values:

- *Case 1*: $Q_{\mathrm{p-acc}} = 230$ pcs/h, $Q_{\mathrm{p\_exp}} = 286$ pcs/h,
- *Case 2*: $Q_{\mathrm{p\_acc}} = 275$ pcs/h, $Q_{\mathrm{p\_exp}} = 400$ pcs/h.

In both cases, the constraints related to the accuracy of the dimensions and shape of the workpieces and the surface quality were assumed at the same level and determined on the basis of Eqs. 12 and 13 (for $\Delta h_{\mathrm{UTL}} = 14$ µm, $\Delta p_{\mathrm{UTL}} = 2.4$ µm, $F_{n\_\mathrm{UTL}} = 42$ N, $n = 3$, $m = 5$):

- deviation of height: $\Delta h_{\mathrm{acc}} = 10.08$ µm, $\Delta h_{\mathrm{exp}} = 7.46$ µm,
- deviation of flatness: $\Delta p_{\mathrm{acc}} = 1.84$ µm, $\Delta p_{\mathrm{exp}} = 1.46$ µm,
- normal component of grinding force: $F_{n\mathrm{-acc}} = 35$ N, $F_{n\mathrm{-exp}} = 30.33$ N.

The fuzzy decision $\mu_{\mathrm{D}}(x)$ values in subsequent generations of genetic algorithm, determined for particular t-norms

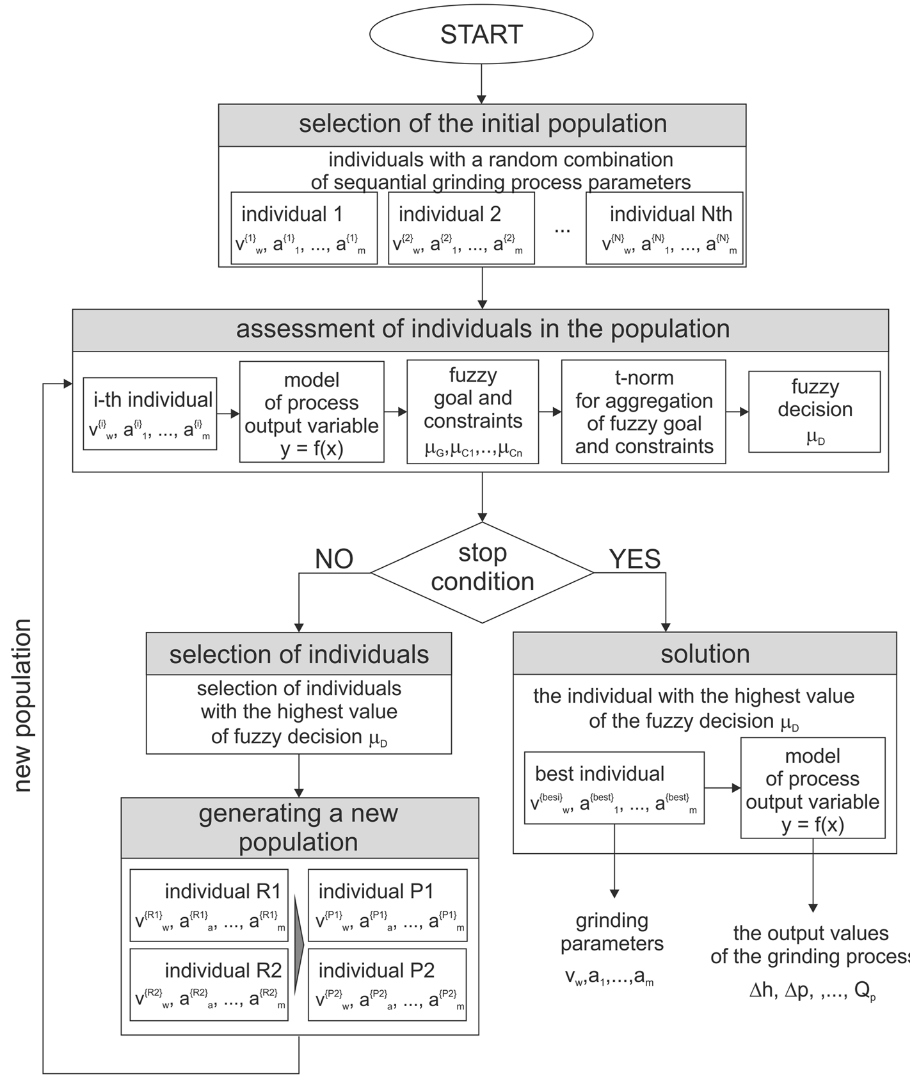

Fig. 8 Scheme of the procedure for optimizing the fuzzy process of sequential grinding using a genetic algorithm

used for an aggregation of fuzzy objectives and constraints, are presented in Fig. 9.

The fuzzy decision values in successive generations of genetic algorithms indicate that increasing number of generations of genetic algorithm above a 16th does not significantly improve the obtained results. An increase in the requirements concerning grinding efficiency results in a decrease in the degree of the fuzzy objective and constraints fulfillment and thus in reduction in the fuzzy decision values (Fig. 9). This is the result of the consensus between contradictory requirements imposed on the grinding process. The highest value of the fuzzy decision is achieved in the case of using of minimum function (Fig. 9a). This is a consequence of the fact that in the case of the minimum function use, the fuzzy decision value is only determined by the lowest degree of the fulfillment of any objective or fuzzy constraint. This leads to the maximization of the degree of fulfillment of individual objectives and constraints (Table 6).

In the case of a large discrepancy between objectives and constraints imposed on the grinding process, the application of the Łukasiewicz t-norm for an aggregation of objectives and fuzzy constraints causes that the value of the fuzzy decision for any values of the grinding parameters equals 0 (Fig. 9c, Table 6). In effect, it is impossible to evaluate the genetic algorithm optimization results.

For lower requirements imposed on the grinding process efficiency, values of the degree of fulfillment of fuzzy objectives and constraints (Table 6) and grinding parameters (Table 7) obtained for product, Hamacher product and Łukasiewicz t-norms are comparable. An application of the Łukasiewicz t-norms for the aggregation leads to the search for grinding parameters which allow to fulfill in a full degree the highest number of fuzzy objectives and constraints. (The value of the fuzzy membership function equals 1.) In the case when higher grinding requirements were imposed on the process efficiency, it is not possible to obtain in full degree all the fuzzy objectives

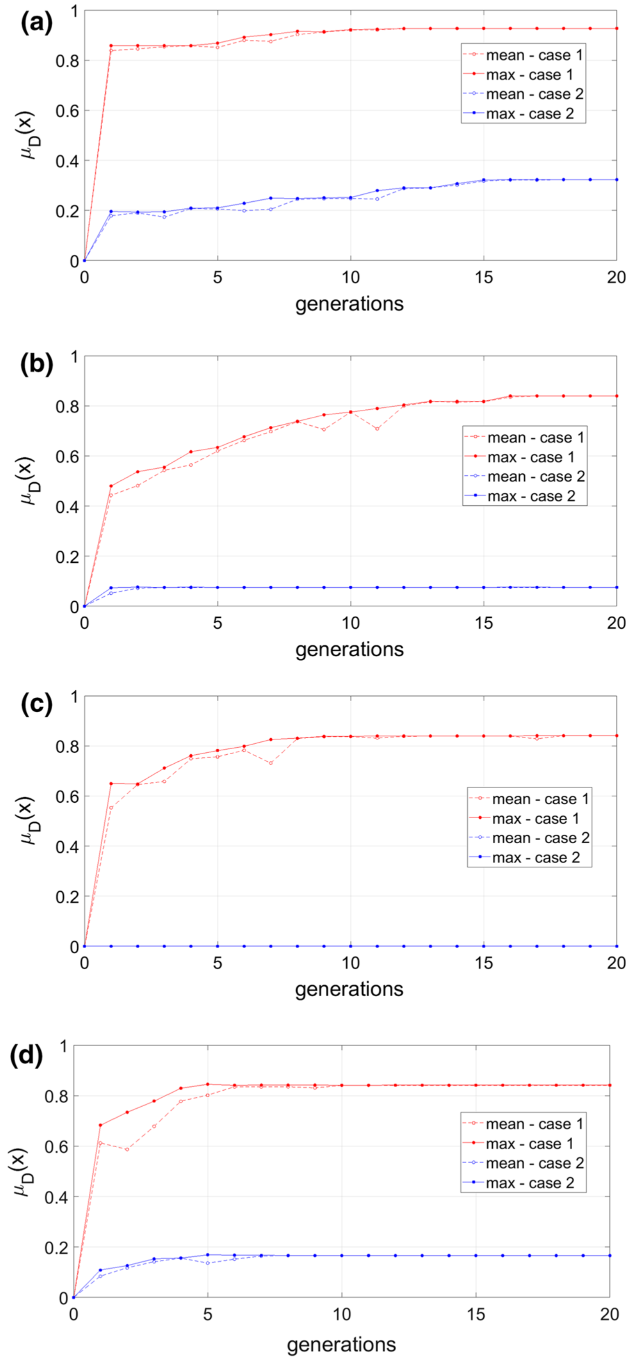

Fig. 9 Results of the fuzzy optimization process, using the following t-norms for objective and fuzzy constraints aggregation: minimum (**a**), product (**b**), Łukasiewicz (**c**), Hamacher product (**d**)

and constraints. So, the application of Łukasiewicz t-norm makes impossible to evaluate solutions generated by genetic algorithm.

The use of the product t-norm and the Hamacher product t-norm enables to obtain the results in the space between expectation of fulfillment in a full degree of maximum number of fuzzy objectives and constraints (Łukasiewicz t-norm) and the maximization of minimum fulfillment of fuzzy objectives and constraints (minimum t-norm).

In case of the reduced requirements concerning grinding process efficiency, there are no significant technological differences in the values of grinding parameters obtained for different t-norms. The highest efficiency was achieved with the use of the minimum t–norm (for $v_w = 4.83$ mm/s, $Q_p = 281$ pcs/h). An increase in the grinding process efficiency requirements significantly differentiates the optimization results. This is due to the inability of complete fulfillment of the requirements imposed on the process. The best grinding process parameters were obtained when minimum t-norm for aggregation requirements imposed on the grinding process was used. The obtained highest rotary table speed $v_w = 5.59$ mm/s, for which the grinding process efficiency $Q_p = 319$ pcs/h.

6 Summary and conclusions

The stochastic nature of grinding process, caused by the multiplicity of factors affecting its results, induces the use of fuzzy logic methods in the decision-making process for the selection of grinding parameters.

The use of classic methods to define objectives and constraints results in the fact that the machining parameters obtained as a result of optimization are often located at the border of acceptable areas. The variability of the grinding process, resulting from the changes of grinding wheel active surface condition, causes the process to pass from the set of admissible parameters into the space of parameters that do not provide the required quality of the process.

Furthermore, the imposition of many restrictions on the machining process, in particular sequential processing, can lead to a situation in which there is no parameter space that meets all the objectives and constraints imposed on the process. (The problem is infeasible.)

In such cases, the application of the fuzzy set theory allows to determine the area of acceptable changes in the value of objectives and constraints imposed on the machining process. It leads to the possibility of assessing the degree of deviation of the optimization results from the expected values.

The article presents the fuzzy optimization of the process of sequential grinding of small ceramic components. The objectives and fuzzy constraints described in the process output parameters space were defined. These parameters were related to grinding efficiency, accuracy of the dimension and shape and quality of workpieces surface. The experimental study allowed to develop the models enabling the transfer of fuzzy constraints definition from the process output parameters to the input parameters space (i.e., process

Table 6 Summary of membership function values for particular fuzzy objective and constraints

	t-norm	Value of membership function of fuzzy objectives and constraints				
	$\mu\Delta h$	$\mu\Delta p$	μF_{n1}	μF_{n2}	μF_{n3}	μQp
Case 1						
Minimum	0.94	1.00	0.93	0.93	1.00	0.93
Product	1.00	1.00	1.00	0.99	1.00	0.85
Łukasiewicz	1.00	1.00	1.00	1.00	1.00	0.84
Hamacher	1.00	1.00	0.99	1.00	1.00	0.85
Case 2						
Minimum	0.38	0.62	0.36	0.36	1.00	0.36
Product	1.00	1.00	0.66	0.67	1.00	0.17
Łukasiewicz	No evaluation possibility, $\mu_D(x) = 0, \quad \forall x \in X$					
Hamacher	0.9	1.00	0.51	0.51	1.00	0.25

Table 7 The results of fuzzy optimization

	t-norm	Process parameters x		
	v_w (mm/s)	a_1 (μm)	a_2 (μm)	a_3 (μm)
Case 1				
Minimum	4.92	151.16	132.40	16.44
Product	4.84	150.62	132.74	16.64
Łukasiewicz	4.84	150.93	132.35	16.73
Hamacher	4.85	151.36	132.00	16.64
Case 2				
Minimum	5.59	153.79	131.68	14.53
Product	5.17	155.28	134.01	10.71
Łukasiewicz	No evaluation possibility, $\mu_D(x) = 0, \quad \forall x \in X$			
Hamacher	5.34	155.75	134.64	9.62

setting parameters: allowance a_i and grinder rotary table speed v_w). The results of the analyses allow to conclude:

- The application of fuzzy logic to define the fuzzy objectives and constraints allows to consider the degree of fulfillment of the contradictory objectives and constraints imposed on the grinding process during making decisions of the selection of grinding parameters.
- The development of models concerning the effect of grinding parameters on the selected output parameters of the grinding process allows to determine the aggregated function of the fuzzy decision in the space of the decision parameters (process input parameters).
- The results of the fuzzy optimization are significantly dependent on the t-norm applied to the aggregation of constraints and the objectives of the grinding, and the greater the area of grinding parameters in which the aggregated value of the fuzzy decision takes values different from 0 or 1, the greater the differentiation of the decisions made.
- In the case of sequential grinding of small ceramic elements, the use of minimum t-norm for an aggregation of grinding objective and constraints allows to achieve the highest process efficiency.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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