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On the idea of 'democratisation', 'modern mathematics' and mathematics teaching in France

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Abstract After the two World Wars, political powers in France recognised a need to democratise education. Successive governments had to cope with population growth and attempted to expand recruitment of the elite because the country lacked an adequate number of engineers and scientists. Many teachers and intellectuals grappled with the question: sociologists, philosophers, psychologists, educators, mathematicians, linguists, and more. In the reform of science education, modern mathematics occupy special place, on account of the very important theoretical contribution of the Bourbaki group, and because mathematics is used in all other sciences. This article examines the different forms in which democratization was instituted from 1920 to 1972, and how mathematics was involved.

Keywords Education · Democratisation · Selection · Bourbaki · Modern mathematics · Mathematics teaching

1 The idea of democratisation: 'modern' mathematics and its teaching

Between the end of World War I and 1975, for the first time the idea of 'democratisation' has taken shape in France, by means of the unification of the educational system and the opening of scholastic instruction to the greatest possible number of students. Here we undertake to answer several questions: What were the vital forces and who were the leaders behind this transformation? What role was played by the economic imperatives of capitalism, the social sciences and mathematics (in how it is done, in how it was formulated by the Bourbaki mathematicians, in its transmission through teachers and their professional association and in each of its social representations subject to socio-economic and political pressure?). What forms has the idea of democratisation assumed over the course of time and to what extent is 'modern' mathematics indirectly involved?

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2 The educational system at the time of the first Bourbakis

André Weil entered the École Normale Supérieure in 1922; Henri Cartan followed him the next year. They were brilliant young men who represented the fruit of an elitist, chauvinistic education received at the hands of their family and secondary schools. The two found themselves, a few years later, at the University of Strasbourg, charged with teaching the course in differential and integral calculus, and they took advantage of the opportunity to investigate the best possible general formulation of Stokes' theorem. Out of this experience was born, in 1934, the Bourbaki group (see Appendix: Who was Nicolas Bourbaki?) (Fig. 1), whose initial objective (soon forgotten) was, in a certain sense, pedagogical, given that they determined to write collectively, with the assistance of other mathematician friends, a basic course of analysis for the university to replace Goursat's textbook—a classic at that time—which they believed to be inadequate [22].

What was the situation of teaching at that time in France? In the course of the nineteenth century multiple measures had been taken to guarantee the equality of access and extend elementary education in order to give all men the education necessary to perform the duties of

Fig. 1 The Bourbaki group in 1938. From left: Simone Weil, Charles Pisot, André Weil, Jean Dieudonné, Claude Chabauty, Charles Ehresmann, Jeab Delsarte. Image: Public domain

citizenship, and give women sufficient means to maintain a household and see to the education of their children. Excluded instead was any attempt to distribute social standing: each social class had its school, with a specific scholastic *cursus* and a teaching adapted to it. Two quite distinct orders of teaching co-existed—primary and secondary—with separate administrations, different institutions and distinct corps of teachers. Until the 1930s, secondary schools had to be paid for. The passage from primary education to secondary education, which occurred quite rarely, was not defined in terms of either age or of program.

Given this state of affairs, for young men born at the end of World War I, there were three possible scholastic *cursus* [19]. For 80 % of them, obligatory education in a primary school ended in manual labour. For 14 %—the best of those who attended primary school—it was possible to extend primary education in order to obtain a supplementary qualification through upper-level primary schools and complementary courses, or through professional schools (the practical schools of commerce and industry); this extension of primary education served to train teachers, elite workers and those who had a supervisory role in the production work. The remaining 6 % attended secondary school, from the lower grades that provided education primary all the way to the *baccalauréat*, or diploma. Those who completed this course formed the small elite occupying positions of responsibility in politics, economics and culture. (In 1936, only 2.7 % of an age group obtained the *baccalauréat*.)

The secondary education for young women, created in 1880 by the legislation of Camille Sée, was strictly separated from that of young men, and its objective was not to prepare young women to exercise a profession, but rather to train them for the tradition role of motherhood. In fact, secondary education for young women was integrated into the overall program for education only in 1924, when the uniformity of programs permitted young women to study Latin and enter university.

3 The first cracks in a rigid educational system: the ‘fair selection’

3.1 The *Compagnons de l’Université nouvelle*

At the end of World War I, in reaction to the loss of almost an entire generation of young men during the war, a group of officials, teachers and engineers met to inject new life in France to the idea of a new model of teaching aimed at all, without distinguishing social classes. This led to the development of a program, principles and proposals that were collected in a two-volume publication entitled *L’Université nouvelle*, with the two volumes published respectively in November 1918 and July 1919.[1]

The declarations of the group were animated by the idea of social fraternisation. The premise of unifying the educational system responded to an ideal of social justice in a framework that was utilitarian and functional. There was no reason not to use young talent that had previously been systematically excluded by some professions solely because they came from less-privileged classes. Instead, it was in the best interests of the nation to broaden the social recruitment of the elite. Without questioning further the economic foundations of a system seen as non-egalitarian, they proposed a model of a school based on meritocracy.

4 The Langevin-Wallon plan

At the time of the liberation, the topic of the democratisation of teaching was taken up once more as an essential objective by the National Council of the Resistance, and in November 1944 the provisional government named a Ministerial Commission to study the reform of teaching, led by two great intellectuals, Paul Langevin (1872–1946), a well-known physicist and member of the French Communist Party, and Henri Wallon (1879–1962), a

[1] For more regarding this, see the article by Jean-Yves Seguy [18].

philosopher and psychologist, also a member of the French Communist Party.

The commission, combining the concern for meritocracy with attention to egalitarianism, did not release its findings until June 1947. This plan provided a thorough reflection on education and became a reference and a source of inspiration for a good many of the decision makers who came after (technical education, 'new classes', educational psychology, etc.).

The reformist discussion was based on two principles: the principle of justice, so that everyone could find a place in society according to abilities rather than fortune, and a principle of culture and development to raise the cultural level of the nation as a whole.

For the first time there was a discussion of the relationship between kind of teaching adopted and the public to be reached; for the first time there was a consideration about what kind of orientation would allow a classification of workers based on both individual skills and social needs [17].

Specifically, the Commission recommended a single school program lasting for a longer period (18 years), in which levels of education operating in parallel would be replaced by a succession of levels. However, later governments did not understand the importance of a comprehensive reform of the school, the Cold War drove the communists ministers out of the government, and the plan was not implemented.

5 At the origins of the new mathematics: the axiomatic method and structures

The Bourbaki group was formed during the 1930s, as we have seen, at a time when the role of science was exalted and almost synonymous with the progress of human development. Mathematics and the physical sciences were present everywhere in day-to-day objects and techniques. The Bourbakis were not interested in secondary education, although some of them, such as Jean Dieudonné and Pierre Samuel, occasionally participated in certain meetings or commissions on the teaching of mathematics. Still, they and others published articles defending a modernised vision of mathematics. Convinced followers—André Revuz and Lucienne Félix, for example—supported the cause of 'modern mathematics' and sought to spread the spirit into textbooks. This was a popularization in which the meaning of the original ideas would eventually drown.

For mathematicians, this raised the need for an internal ordering of results accumulated in many different fields, on specific concepts with specific methods: geometry, arithmetic, algebra, functions, and more. The Bourbaki enterprise consisted in shining a single light on this apparent diversity and producing an exhibition of mathematics since its inception in the form of successive books; the method is axiomatic and proceeds mostly from general to specific. Set theory was to support the entire edifice.

A general idea is given by an article entitled 'L'architecture des Mathématiques' signed Nicolas Bourbaki [3, 4]. The author demonstrates how the 'axiomatic method' provides the profound intelligibility of mathematics and confers on it the unity 'of an organism at the height of its development'. He takes care to distinguish the logical formalism that is one aspect ('To lay down the rules of this language, to set up its vocabulary and to clarify its syntax' [4, p. 223]). The method is based on the notion of 'structure', which he never defines but of which he does give examples. The concept designated 'mathematical structure' applies to elements whose nature is not specified; these elements satisfy one or more independent relations which then become the 'axioms' of the structure envisioned. To construct the axiomatic theory of a given structure is to deduce the logical consequences of the axioms posited, which are all the general theorems relating to structures of that type. Whence the economy of thought that allows the axiomatic method; this makes it possible, for example, to embrace within the same point of view—that of the structure of a group—the real numbers with addition, the integers modulo a prime number with multiplication, movement in Euclidean space with composition.

Three structures, the 'parent structures' [6], lie at the heart of the architecture of mathematics: algebra, topology and order. The others derived from these by multiplication or combination. Alongside the organic metaphor, the paper uses an industrial metaphor: 'The "structures" are tools for the mathematician … One could say that the axiomatic method is nothing but the "Taylor system" for mathematics' [4, p. 227]. But the mathematician is distinguished by his use of a special intuition that regarding the behaviour of mathematical entities, which have become familiar to him after long association: the structural similarities between a known domain and one to be explored will enable him, by transfer illuminate 'with a new light the mathematical landscape in which he is moving about' [4, p. 227]. The intuition always present in the genesis of discoveries 'possesses the powerful tools furnished by the theory of the great types of structures; in a single view, it sweeps over immense domains, now unified by the axiomatic method, but which were formerly in a completely chaotic state' [4, p. 228].

6 The mathematisation of the humanities: a new scientism

Structuralism was the intellectual mainstream at the time, and was particularly strong in 1950–1960. Without being directly related, the ideas of Bourbaki were in line with this movement, which, beginning with the linguistics use of language in the early twentieth century, had spread to all the humanities, to the point that in the discourse that followed, language served as the structural paradigm and permitted analogies that were not impervious to misinterpretation.

There was in effect an explosion of the mathematised social sciences in the years following World War II, in parallel with the growing visibility of new mathematics and enthusiasm for structuralism. Beyond the numerical and statistical processing, mathematics opened a field considered qualitative, extending its ability to work in the humanities. The explicit goal was for modern mathematics to do the same work in the humanities as classical mathematics had done in the natural sciences. Mathematics offers, in fact, a collection of abstract shapes available for any use, because they are without any significance, with a simple and universal language, which is the greatest guarantor of scientificity.

The start to this movement was given by Ferdinand de Saussure (1857–1913), who radically transformed linguistics by introducing the idea that language is a system of signs whose terms are purely differential, defined not by their content but in their relations with the other terms of the system. The consideration of objects not for what they are but for the ways in which they are interconnected, can be transferred from language to various aspects of society; the anthropologist Claude Lévi-Strauss (1908–2009) did not fail to make use of it. Deemed largely responsible for the influence of Saussure's linguistic structuralism, and the introduction of structuralist thought in France, Lévi-Strauss is also related to the ideas of Bourbaki. A refugee in New York during the World War II, he exchanged letters particularly frequently with the Russian-born linguist Roman Jakobson and mathematician André Weil.

Using an approach similar to that of Saussure, he says he wants to exceed the level of classification of multiple perceptual social facts [10]. For the work of theorisation comprised of the production of assumptions and models, he relies on the help of mathematicians. A complex problem of kinship treated mathematically by Weil [21] confirmed a hypothesis that had been advanced earlier, legitimating the theoretical approach. Mathematics is only a tool, but the use of algebra to treat kinship opened new doors, and beyond the factual importance of the event, confirmed mathematics as the universal language of science.

7 The teaching of science: an international problem

7.1 The teaching of mathematics

The question that arose was how to develop science education in line with the technical and technological progress of science, and be able to quickly overcome the shortage of engineers and skilled workers. The community of mathematicians reflected on this at the same time it discussed the new architecture of mathematics initiated by Bourbaki. International commissions were formed to consider this.

The International Commission on Mathematical Instruction (ICMI) had been founded on the occasion of the International Congress of Mathematicians in Rome in 1908, but was only slightly active during the two wars. Refounded in 1952, it launched a vast survey via a questionnaire sent to mathematicians in member countries, regarding the 'Role of the mathematician and mathematics in modern times'. Djuro Kurepa (1907–1993) of the University of Zagreb presented a report on it to the 1954 congress in Amsterdam.

His report and the communications of the different national sub-commissions provided the basis for discussion. All extolled the extraordinary effectiveness of the discipline; never had mathematics been as vital, active and fruitful in the art of engineering and technical research in all its forms. It occupied a place at the crossroads of human activities and participated in the development of the scientific world (touching as well the arts, psychology, etc.). It provided models, methods of measurement and analysis, but also found in other fields material for new impulses. All denounced the problems with education, poor adaptation of programs of secondary school to the needs of future producers or future users of mathematics.

The International Commission for the Study and Improvement of Mathematics Teaching (CIEAEM), created independently in 1950, operated in another manner. It is multidisciplinary (mathematics, pedagogy, psychology, philosophy), coordinates international research teams, organises meetings and seminars on specific topics that are places of exchange between researchers and teachers. Seven meetings were organized between 1950 in London and 1955 in Bellano. A first publication entitled *L'enseignement des mathématiques* [12] shed light on questions of the day. This is a collection of articles by six founding members of the Committee: the Swiss psychologist Jean Piaget (mathematical structures and operating structures of intelligence); the mathematician and teacher of Egyptian origin Caleb Cattegno (mathematics pedagogy); the French mathematicians Jean-Dieudonne (abstraction in mathematics and the development of algebra), Gustave Choquet (on the teaching of elementary geometry), and André Lichnerowicz (introduction of the spirit of modern algebra in elementary algebra and geometry), and the logician and philosopher of science Dutch Evert W. Beth (reflection on the organization and method of mathematics teaching).

7.2 The economic management of education

However, these reflections within the intellectual world did not affect the organisation of education. Following the post-war reconstruction, industrial expansion had taken over and industrial upgrading was the order of the day. The dominant economies had an immediate need for skilled labour and management; the development of education was seen as the foundation on which capitalism could support its ascent. International organisations were created, which while not intervening directly in the educational policies of member countries, guided government and public opinion, normative incentives and educational discourse, nourished scientific arguments [20] based on economic theories, and grew to organise teaching in an industrial manner.

UNESCO (United Nations Educational, Scientific and Cultural Organization), founded in 1945, launched the 'race to education' in 1957, recommending that governments dedicate 5 % of their gross national product to education.

The OEEC (Organisation for European Economic Cooperation) was founded in 1948 to administer the Marshall Plan. It became OECD (Organization for Economic Cooperation and Development) in 1961, and expanded its work globally to promote policies to higher economic growth, employment and global trade. This was the group that gave the kick-off to decisive reforms. In effect, an event caused panic in the Western world: in 1957, during the Cold War, the Soviet Union successfully launched its first satellite, Sputnik. The West, seriously worried about the technology gap, sought to remedy this by reforming the teaching of science and technology. In 1959 the OEEC organised a ten-day symposium in Royaumont, with the objective of launching a reform of contents and methods of teaching mathematics in secondary schools. French mathematicians took part and defended the ideas of Bourbaki. Choquet and Dieudonné's apostrophe "Down with Euclid!" became famous. Following this conference, the general principles were specified in Dubrovnik (1960) and Bologna (1961).[2] Then came the time for action, particularly marked in Belgium by the experimental teaching of Papy: *Un programme moderne de mathématique pour l'enseignement secondaire* (A modern program of mathematics for secondary teaching) was published in 1961 in Paris under the name *Mathématiques nouvelles* (new mathematics), with emphasis on set theory and structures.

The growing need and demand for education was evident throughout the world in the form of a massive increase in enrolment, a trend of continued in-service studies, the increased use of means of extra schooling and a constant increase of the share of national resources devoted to education.

8 The explosion of education and the reform of 'modern' mathematics in France

8.1 Quantitative democratisation

In the years 1950–1960, in France as elsewhere in Europe, an increase in the number of students at all levels was linked to population growth and increasing social demand.[3] Greater prosperity and changes in working conditions encouraged parents to extend their children's education, and formal instruction became the main driver of upward social mobility. On the other hand, the shortage of engineers and scientists trained in research threatened to hinder the country's economic development; the problem to be solved was that the school system did not produce enough science graduates. The policy of the Fifth Republic was therefore aimed at democratising the basis on which managers were recruited and strengthen education. This led to the extension of compulsory schooling and a continuation of the process of transforming post-primary education into a course of 7 years organized in successive levels. Moreover, for this to work, it was necessary to be selective (but without daring to ask clearly), thus diversifying the channels of study and orientation, while avoiding too many students being led to long-term studies with no real gratification as a result, which could produce social unrest.[4]

8.2 The democratisation of success

Society's view of the education of its children was also evolving. The increasing number of students, and the development of experimental psychology raised questions about the reason for scholastic backwardness and failure, and led to the study of school dropouts, attributing this less to the lack of individual talent and more to social determinism [14]. The focus shifted to individual trajectories. In particular, the influence of family background and intellectual stimulation it provided was studied for its effect on the construction of operative intelligence and language skills. An awareness was created that selection on academic merit perpetuated the same social inequalities that education was aimed at abolishing, and that the problem had to be addressed at the base. The battle against educational failure is the essence of the democratisation of

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[2] On the teaching of mathematics in schools, see also the UNESCO guidelines (for example, those of Budapest, August-September 1962).

[3] See [9]. For a historic overview, see [15].

[4] Note that this is completely relative, since 1970 only 20 % of an age group has passed the diploma examinations.

success (long before the slogan 'success for all' became popular in the 1980s).

Even outside the circles of specialists, these years saw a broader questioning of teaching itself. In this context of transformations in many areas, the relatively unchanged content of education appeared obsolete, behind the times in relation to renewals and changing perspectives in science, particularly in language and mathematics. Teaching methods seemed archaic, undercut by studies in psychology and pedagogy. There was a general consensus for the idea of modernising content and creating a new pedagogy that relied more on the motivations of students, their work, their life. This led to the creation of two committees on education: the first, in 1963, for the teaching of French in elementary school, headed by the inspector general Marcel Rouchette; the second, in 1966, for teaching mathematics at the secondary level, chaired by the mathematician André Lichnerowicz. The renovations in physics and the natural sciences would be less radical.

For mathematics, the approach to the work was determined by the years of reflection and discussion that preceded it: mathematics is everywhere; it is are a key to physical and social world; modern mathematics is alive, and provides a simple language that can be used in all applications; it provides an economy of thought; it must be taught to all. The achievement of this objective raised the questions of how to define the content to be taught, and how to reorganise pedagogy. An additional theoretical argument helped overcome reluctance: mathematical structures are in harmony with the development of structures of intelligence; the work of Jean Piaget, promoter of genetic psychology, was central.

8.3 The Bourbaki parent structures in psychopedagogy

During the 1952 conference of CIEAEM 1952 (in La Rochette, near Melun), whose theme was 'Mathematics and mental structures', Jean Piaget responded to a presentation by Jean Dieudonné. He drew a parallel between Bourbaki parent structures and the establishment of operational structures of intelligence in children, and demonstrated the way in which the three basic Bourbaki structures correspond to basic structures of intelligence, of which the first constitute the formalised extension and not naturally direct expression [13]. This justifies the introduction of modern mathematics education. As in psychology, it is assumed that operations are derived from actions that, in being internalised, are coordinated in structures; the pedagogy called active learning is the right method for a progressive acquisition of operational structures.

8.4 Teachers: the protagonists of the reform

Long before the ministerial committee was created at the end of 1966, the Association des Professeurs de Mathématiques de l'Enseignement Public (APMEP, the professional association of mathematics teachers in public education) was engaged in reform under the leadership of its president, Gilbert Walusinski. The APMEP, in contact with the community of university research and education on the one hand, and with the secondary school teachers on the other hand, played a crucial role in transmitting to teachers the new spirit of mathematics, in the spread of the axiomatic of elementary mathematics, in the organization of a discussions on programs, in a direct involvement in committees set up by the government, and ultimately in the development of textbooks by many of its members.

L'APMEP [14] was not concerned with the problem of democratisation.[5] Its members were interested in reforming mathematics education in the whole curriculum 'from kindergarten to university', for all children; but implicitly it was secondary education and the scientific sections of the *baccalauréat* which were given priority, the rest followed by default. The central theme was that modern mathematics is useful and accessible to all, with arguments derived from Bourbaki formulated by skilled mathematicians and re-elaborated by teachers (often professors in Parisian schools).

The APMEP was also concerned with training teachers. In 1960 André Revuz offered a genuine course, in the form of lectures given over a period of 3 years for an hour and a half every 2 weeks. This resulted in the publication of three volumes of APM courses: volume 1, groups, rings, solids; volume 2, vector spaces; volume 3, elements of topology. These courses were taught in parallel with television programs, 'Les chantiers mathématiques' (Mathematics construction site), aimed at teachers. These were APMEP's first steps in in-service teacher training, without the interference of politics. The presentation of these courses (without reference to any program) never set the so-called modern mathematics against classical mathematics, but wanted to show that the new angle had several advantages, as stated in the introduction to the first volume: consistency, clarity of fundamental ideas, arranged in order by theories, highlighted by the profound reason of for results. The tables of contents are simplified transfers of those contained in the corresponding books of the *Elements* of Bourbaki (set theory, algebra, etc.), but the intent is different: the elements are not presented to construct an edifice, but rather to give teachers the means to reflect on their own knowledge from a structuralist perspective. The

[5] For more about this history, see [2].

introduction of new objects is given in the form of problems and linked to many examples.

All levels of the hierarchy were mobilised: André Huisman, Inspector of the Academy of Paris, would give a practical and detailed variation of these courses, more aimed at teaching, with many exercises.

The relationship of the APMEP with the ministerial committee was much closer than the association had foreseen, and beginning in 1966 a commission for 'reform and research' was created within it that worked in parallel; this leaned towards both curriculum development (kindergarten, primary, secondary) and a plan for teacher training (initial and ongoing) that integrated educational research. Thus, the first report of the Lichnerowicz Commission, published in March 1967, presented the association's ideas on and proposals for teacher training: the creation of University Institutes and Institutes for Research Education organised by disciplines [16]. The report provided a comprehensive overview of the role and organization of research institutes for mathematical education.

In January 1968, la Charte du Chambéry (Chambéry Charter) [1] reprised the set of arguments in favour of reform, proposals for it, and outlooks: a reform of mathematics teaching 'from kindergarten to the universities':

In summary: the reform of mathematics teaching that we recommend is based:

- on the guiding principles that animate contemporary mathematical life;
- on the psycho-pedagogical studies that have highlighted the importance of active methods and the need for a very gradual access to the most abstract concepts;
- on the essential role that mathematics plays in social organisation and the production of goods and services [1, §1.3].

The stages of the reform—transitional organisation of teaching, the setting up of in-service training of teachers, and the creation of research institutes for mathematics—were called for. At the end of 1968, three such institutes were created in Paris, Strasbourg and Lyon. Others would follow at a rate of one or two a year.

8.5 The 1968 turning point: first acceleration, then the brakes

The movement to reform secondary education was not really affected by the events of May 1968 that upset the academic world [5], but the arrival of Edgar Faure as Minister of National Education accelerated it. Higher education depends on secondary education, from a quantitative and qualitative point of view, and the minister, in charge of the reform of the university, was concerned with the entire education program. His political speeches of the time, given on behalf of the government,[6] carry an echo of the goal of democratising the whole of teaching, from nursery school to university ('school for all') and the necessity of adapting educational culture to the demands of society (implying a society steeped in techno-sciences, where the keys to understanding the modern world are found in scientific disciplines).

This conservative endorsed the criticism of curricula and examination procedures that favour children who possessed the greatest quality and facility of means of expression, and notions of general culture acquired in their families. He argued that the qualitative route to democratisation lay in strengthening science teaching at the expense of classical humanities, and that Latin—studied to be neither used nor spoken, but emblematic sign of distinction of a higher social class—should be abolished. In October 1968, he established for the sixth year,[7] a common course of study without Latin, but with three 'languages': French (mother tongue), modern mathematics ('simple and precise language' available for all sciences) and a foreign language.

The new sixth-year program became effective in September 1969, that of the fifth-year took effect the following year; but as work was to go forward on the fourth- and third-year programs, dissension broke out in the Lichnerowicz Commission, which was dissolved in 1973, with the resignation of its chairman. This quickly put the brakes on reform. By 1972, with the introduction of the new fourth-year program, the very spirit of modern mathematics had come under attack. Criticism was no longer limited to a few detractors, and began to spread among scientists, the public and the press. From all sides came a condemnation of excessive abstraction, cumbersome new programs and dogmatism. Within the APMEP itself, there arose voices against reform as it has been implemented. Henry Bareil (president of the association and promoter of reform) called for 'a pause'. In 1974, Pierre Samuel, one of the pioneers of the reform, published an appeal for a *deténte*, and Dieudonné himself wondered if modern mathematics should still be taught [7]. It must be said that textbooks had given considerable publicity to and distorted these new mathematics, offering a caricature of a building built following an internal manner unrelated to reality, in which the handling of vocabulary occupies a considerable place. While

[6] Especially in the debates in the National Assembly of 23 July 1968 and 8 October 1968 and in the speech given to UNESCO on 18 October 1968.

[7] Translator's note: In the French secondary educational system, the *sixiéme* ('sixth year') is for students 11–12 years old (equivalent to US sixth grade, or UK seventh year); the *cinquième* ('fifth year') for students 12–13; the *quatrième* ('fourth year') for students 13-14; the troisième ('third year') for students 14-15. These are roughly equivalent to junior high school. The years that follow, *seconde*, *première* and *terminale*, culminate in the *baccalauréat*.

among teachers the program of the APMEP had taken root in a fund of knowledge already constructed, the situation among young students was quite different, and adaptation consisted above all in the establishment of an abstract language that turned in on itself, introduced by pseudo-concrete activities, sometimes to the limits of absurdity. The effect of a discriminatory selection based upon expressive ability was redoubled, the scientific content was not assimilated, and teachers and students were left behind.

Reform was reorganised, then quickly abandoned. It had faced many obstacles: haste on the part of the Ministry, corporatism, the conservatism of the guardians of the temple of Euclid, 'the mathematical illiteracy of almost the entire population' [6], the ignorance on the part of the mathematicians and psychologists (the promoters of reform) regarding the actual conditions of secondary teaching.

In fact, the policy adopted ended up opening to the entire population an education that remained elitist, rather than constructing a genuine mass education.

9 In conclusion: learning to be

In the 1970s, no solution was found to the problem of how to organise a unified mass education which would produce just enough elite to run the country without resorting to an explicit selection that would incite young people to protest. The question has become more difficult because of rising unemployment, the race for diplomas and their consequent inflation that diminishes their value, highlighting the fact that longer and longer years of study is not always a guarantee of finding a social position that compensates the effort at qualification. The ambition to make the school an instrument of equal opportunity has failed, the watchword of mass schooling has led to an impasse. It was at this point that the report carried out on behalf of UNESCO in 1972 by Faure, *Apprendre à être* [8] (Learning to be) signalled a turnaround in policy.

It reprises the criticism of teaching as 'old-fashioned and outdated', 'overly theorised and memorised' non-egalitarian in its operation, in which the teacher-student relationship is 'in the nature of a relationship of dominated to dominant', etc. It reaffirms that democratisation cannot be reduced to assuring equal access but should also aim at equal success, which is far from being realised. To break the deadlock, the syncretic speech conserves this objective by changing the meaning.

It is based on the representation of the scientific-technological modernity at the time as 'the mobility of knowledge and renewal of innovations' to divert the teaching of distribution of knowledge acquired to 'learning the methods of acquisition (learning to learn)'.

Unemployment of too large a number of graduates poses a problem, because it works to disconnect the degree from employment, and the organization of flexibility will make it possible 'to reconcile the democratization of education with economic rationality', 'to optimize mobility of labour and encourage the ongoing desire to learn and train'.

The watchword became 'lifelong learning', thus bypassing the problem of success as a result: as the educational process becomes continuous, the notions of success and failure will change.

The individual becomes the master and the author of his own cultural progress and great value is placed on self-learning. With regard to the teacher-student dyad of the school system, it is necessary 'to assume that the learner is at the centre of the educational process' and it is recommended that the terms of teacher training be profoundly modified, in order to train teachers to be essentially educators more than specialists in the transmission of programmed knowledge'.

At a moment when a long-meditated comprehensive reform, structured and generous in its objectives, had collided with a human reality that its protagonists had neglected, this report (too briefly sketched here), retrieved the libertarian ideas of the time while expressing a submission to the needs of the economy, and is a forerunner of the educational project that was deployed starting in the 1980s. It put an end to the ways in which the democratisation of the education system had been conceived in the previous five decades.

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Appendix: Who was Nicolas Bourbaki?

The Bourbaki group, as such, is far removed from the so-called movement of 'modern mathematics'. Some of its members fought for the renovation of the university, which would take place in France around 1958, but the group was not interested in elementary and secondary education, nor in the efforts to democratise the latter. The imposition of an abstract axiomatic style in secondary education was more the result of zealous followers outside the group, and an intellectual context impregnated structuralism.

Instead, Bourbaki profoundly changed the face of mathematics: in its encyclopaedic enterprise to reconstruct and rewrite all of classical and modern mathematics, it relied on a method—the axiomatic—involving a hierarchy of abstract structures.

The collective pseudonym 'Nicolas Bourbaki' initially referred to a group of a dozen young men, from the École Normale Supérieure in Rue d'Ulm, which hosted and supported their association.

The five principal founding members were Henri Cartan, André Weil, Claude Chevalley, Jean Delsarte and Jean Dieudonné, who were soon joined by René de Possel, Charles Ehresman and Szolem Mandelbrojt. The preliminary meeting took place in summer 1935 in Besse-en-Chandesse, a charming little village in the Auvergne; this was the starting point of a custom of regularly holding pastoral meetings in quiet, attractive places to work. There was no explicit hierarchy in the group, which was in effect a secret society [11].

To carry out the encyclopaedic undertaking of the *Eléments de Mathématique* (Elements of Mathematics), they invented a method of work, comprised of permanent construction/deconstruction, based on a collective text subject to mutual, uncompromising criticism, resulting in an anonymous publication. The *Fascicule de résultats de théorie des ensembles*, which appeared in 1939, was the first publication. The war dispersed them, but other books followed after the war, and the first Bourbaki seminar, which would become an institution in the mathematical community, took place in December 1948.

The period from 1950 to 1970 was glorious. The group became the matrix for the training of a whole generation of mathematicians. It was led by strong personalities, brilliant mathematicians and a covey of Fields medallists: Laurent Schwartz (1950), Jean-Pierre Serre (1954), Alexander Grothendieck (1966). Never exceeding a dozen members at a time, the group was renewed regularly (the rule was to leave after 50 years of age). Its composition varied constantly throughout its history and is never very clear.[8] Members were co-opted after a successful immersion in violent discussions, insults, and being questioned at the meetings.

A passion for mathematics in action did not prohibit humour and schoolboy high jinks. The group cultivated its legends, which helped build a collective myth.

[8] In addition to those already mentioned, other members of the Bourbaki group included: Alain Connes, Jacques Dixmier, Adrien Douady, Jean-Louis Koszul, Charles Pisot and Pierre Samuel. There were also several mathematicians who were not French: the Swiss Armand Borel, the Americans Samuel Eilenberg and Serge Lang. On the other hand, several great French mathematicians such as René Thom, André Lichnerowicz, Marcel Berger and Jean Leray preferred to remain outside.

A long decline began after 1975. Living mathematics is being done elsewhere in the world, in connection with new problems arising from other disciplines or other human activities, as well as new opportunities from the computer.

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