Lett Mat Int (2014) 2:25–34
DOI 10.1007/s40329-014-0048-y

When an academician becomes professor: the case of Joseph-Louis Lagrange

Bruno Belhoste

Published online: 21 May 2014

Abstract With the French Revolution, the role of the mathematician gradually evolved from protégés of princes to official professor. Lagrange provides a remarkable illustration of this fundamental change. Until 1792, this famous but discreet scientist had a European career, first in Turin and then to Berlin and finally Paris, ignoring national boundaries and rivalries between powers but always subject to the system of enlightened despotism. His life was that of a court mathematician, working in the narrow confines of academic institutions. The French Revolution completely changed his career and relaunched his work. He actively participated in the creation of the metric system and taught analysis in new educational institutions established by the Convention: the École Normale and the École Polytechnique. This is where he explained his theory of analytical functions, where differential and integral calculus were reduced to the study of the expansion of functions.

Keywords Lagrange · Mathematics education · Mathematics teaching · French Revolution · Scientific academies · Paris Academy of Sciences · École Normale · École Polytechnique

1 Introduction

It is often underlined that the period around 1800 marked a turning point in the history of mathematics, at least from the political, social and cultural points of view. It was in effect beginning from this date that the figure of the mathematician was transformed little by little from a protégé of a prince to an official professor. The change itself is associated with the progressive professionalisation of the activities of mathematics in the course of the nineteenth century. The majority of the great mathematicians, and more generally those who left a mark in the field of science back in the second half of the seventeenth and the eighteenth centuries, had never performed teaching work, or if they had, it played a minor role in their research work. Let us take the case of Newton, who was Professor of Mathematics at the University of Cambridge. Except for his lessons in optics at the beginning of the 1670s, nothing remains of the activities of his chair. Newton then stopped teaching altogether after the publication of the *Principia*. In contrast, in the course of the nineteenth century, the activity of teaching became essential for the professional mathematician: Cauchy, Liouville and Hermite taught at the École Polytechnique. In Germany, there was Humboldtian university model, where research was closely associated with teaching, which took over little by little. The mathematicians were thus able to expound and freely develop their research in courses and seminars.

This change, which would reach its full development in the second half of the nineteenth century, began in France with the Revolution, which led to a profound reorganisation of the scientific world and its institutions. The ancient academies and universities disappeared, making room for establishments of a new kind. The École Normale, and above all the École Polytechnique, offered positions to the best mathematicians. Lagrange, who represented the archetype of the scientist-academician of the eighteenth century, relaunched his mathematical career within this new framework. The momentum of this appears even more spectacular if we consider that he had become totally unproductive before his arrival in Paris in 1788. Thanks to

B. Belhoste (✉)
Université Paris 1 Panthéon Sorbonne, Centre Malher, 9 rue Malher, 75004 Paris, France
e-mail: bruno.belhoste@univ-paris1.fr

teaching, he took up his full-time activity anew, composing, starting from his lectures, a series of treatises that would exert a profound influence on the new generation of mathematicians of the nineteenth century.

2 A European scientist

Lagrange was born in 1736 in Turin, where his father, of a family of French origin, was treasurer of the Artillery. His mother came from a well-to-do family of Cambiano, in Piedmont. It was at the University of Turin that he discovered mathematics, for which he immediately had a strong predilection. He devoted himself entirely to it, earning the title of *maestro d'arte* in 1752, avidly reading the great authors Newton, Euler and d'Alembert, instead of studying law as his parents wished. It is to this period that his first mathematical work dates, regarding the problem of maxima and minima, as well as the beginning of his correspondence with Euler. In September 1755, fortified by his initial success, he obtained, with the help of his father, the position of *professore supplente* (substitute) of mathematics in the Royal School of Artillery. He was 19 years old.[1] He would hold this position for 11 years, until his departure for Berlin.

Even though Lagrange began his mathematical career as a teacher, this fact mustn't mislead us. His position at the Royal School of Artillery gave him the economic independence that he needed, since his family was undergoing serious financial hardships, but offered him neither prospects nor advantages regarding his projects for research. To be sure, Lagrange wrote, in the years following the appointment, two works for teaching: the first, today lost, on mechanics, and the second, of which a manuscript copy survives, on differential and integral calculus. But it was in another context that he carried out the main part of his mathematical activity in those years.

He took part, in 1757, in the founding of the "Società Privata Torinese", the original nucleus of the Turin Academy of Sciences. It was in the memoirs of this emerging scientific society that he would publish his first mathematical work. Lagrange was able at the same time to insert himself into the intellectual society of Europe. He was named an associate member of the Berlin Academy in 1756, and maintained for years a regular correspondence not only with Euler, but also with Daniel Bernoulli and d'Alembert. In autumn 1763, he left to spend more than 6 months in Paris, where he formed friendships with the French scientists. This was a decisive experience in his decision to leave Turin permanently, where for some time he had felt too isolated and unsupported. Thanks to d'Alembert, he obtained the position of director of the Class of Mathematics of the Berlin Academy, a position that Euler left vacant when he moved to St. Petersburg: In August 1766, he left Turin forever, renouncing at the same time his position at the Royal School of Artillery.

His arrival in Berlin marked the beginning of a new phase in his life, certainly the most productive one. At this point he had no other activities than those of an academician who lived entirely for his research, without any obligation to teach. His mathematical work, of great variety, developed according to the more or less secret influence exerted on him by his contemporaries. He worked with the regularity of a metronome, in the manner of Euler, and for 20 years published on all manner of subjects. Pursuing a project that he had begun with during his years in Turin, he collected and refined the material for a great treatise on mechanics, his *Méccanica analitique*, which he would complete only in 1786. This masterpiece also marked the end of his period in Berlin.

Nothing, in effect, held him any longer in the Prussian capital. Lagrange had lost, with the death of Frederick II, a respectful protector of his genius. The new king, Frederick William II had nothing to do with science, and was mistrustful of foreigners. The French authorities took advantage of the situation to lure him to Paris, where his *Mécanique analytique* was in the course of publication. In May 1787, Lagrange entered into the service of the French king after more than 20 years spent in Berlin, and the Paris Academy of Sciences welcomed him as a member. He was by then considered the greatest geometer in Europe but, tired and depressed, he seemed to have lost his enthusiasm for mathematics. "*J'ai presque pris congé de la géométrie en quittant Berlin*" (I have just quit geometry and will soon leave Berlin), he wrote to the secretary of the Turin Academy, Tommaso Valperga Caluso. But even if the years that followed would be, at least apparently, scientifically sterile, they were marked by the extraordinary events that would give a new direction to his life and relaunch his mathematical career in an unexpected way. The French Revolution began.

Before looking at the Parisian phase, the last of Lagrange's life, let us take a look back at his 30 years of activity, which go from his first mathematical work in 1755, to his departure from Berlin, in 1786. Through his European career, in three capitals, the mathematician from Turin epitomised the model of the stateless cosmopolitan scientist that seemed to characterise the century of

[1] It is possible that he had taught previously at this school without a formal title. On his deathbed, Lagrange himself said that he had been a professor in the School of Artillery in Torino at the age of 15 or 16 years, adding, "*Tous mes élèves avaient quelques années de plus que moi. Je me suis attaché trois ou quatre de ces jeunes gens et en ai fait mes amis*" (All of my students were some years older than me. I attached myself to three or four of these young men and they became my friends). From the Bibliothèque de l'Institut de France, ms. 2041.

Enlightenment. He thought and acted as a European citizen, ignoring the borders and rivalries between powers. His favoured language was French, even though, despite his French name, his mother tongue was Italian. He corresponded, generally in French, with all the great mathematicians of Europe. Even though he maintained an epistolary contact with his country of origin, he privileged the connections with the academic elite of Paris, at that time the centre of gravity of the "Republic of letters" across the continent.

Another striking trait of Lagrange's European career is his submission to the system of enlightened despotism that prevailed in the continent at the time. In Turin, his activity had been confined to the circle of those who served the House of Savoy, with a very particular kind of tie to the Artillery, to which his father, as treasurer, was already associated. The private society of Turin, to whose creation he contributed, was itself tied to the Artillery, its principal founder, Count Saluzzo, having been a member of it, as was Foncenex, a student of Lagrange, who would later direct the Sardinian navy. In Berlin, where he kept to the role of director of the Class of Mathematics and lived a retiring life, far from court, Lagrange's only principle was respect for the authority of the king who had welcomed him. Such discretion contrasted with the attitude of his French friends, d'Alembert and Condorcet, who were, during the same period, the leaders of the philosophical faction and sought the support of enlightened opinion.

3 Revolutionary involvement

Lagrange had arrived in Paris with concern for his tranquillity. If he had left Berlin, it was because he felt a foreigner and threatened [9]. Now, just arrived, he witnessed the huge problems that shook the monarchy and its capital. In spring 1789, he found himself at risk of being executed, then taken by chance in a sedition. He seriously considered returning to Berlin, but did nothing. He remained in Paris and his life was completely transformed. Very quickly, in effect, the Academy of Sciences and the scientific world were directly affected by the political events. One of the Academy's members, the astronomer Jean Sylvain Bailly, became president of the new National Assembly and when he stepped down from the post, at the beginning of July 1789, the Academy sent a delegation to congratulate him on the way in which he had performed it. Lagrange was part of the delegation. As a foreigner, he had been kept somewhat apart, which suited his cautious nature. He had not the least interest and sympathy with what he had seen. A man of the Enlightenment and enemy of the Jesuits, how could he not echo the sentiments of the great principles of liberty and equality everywhere proclaimed? The idea of leaving continued to haunt him until the summer of 1791, but too many things kept him by then in Paris.

Since the seventeenth century, not to say since the Renaissance, the intellectual elite of Europe had been freed from the control of the church to find itself under that of princes and monarchs. The academic system that existed in Paris had spread through the continent, closely tying the man of science to the interests of the secular powers. Further, when one became a member of the Paris Academy of Sciences, one became *ipso facto* a scientist of the King. Lagrange himself had long since interiorised the relationship of personal dependence. In Berlin, Frederick II interested himself personally in the affairs of the Academy and the activity of its scientists. In Paris, where the personal ties were much weaker, the scientists of the Academy participated in the functions of the impersonal and bureaucratic machine, without having contact with the royal person. All things considered, there remained nevertheless the idea that service to the State was one with service to the King. Moreover, when Lagrange arrived in Paris, Louis XVI in person oversaw the conditions of his instalment. And when he married, with the daughter of his colleague, the astronomer Le Monnier, in May 1792, the King and Queen signed the marriage contract. On his eve of his death, Lagrange recalled once again with gratitude the benevolence of Marie-Antoinette, who regarded him as German.

In spite of the ties that bound him to the society of the Ancien regime and the royal family, Lagrange was swept up, as were his colleagues, by the irresistible dynamics of the Revolution [11]. The system of protection and privileges that governed the scientific world, like the rest of society, suddenly vanished and the authority of the old scientific institutions were by then greatly contested. The Academy of Sciences had to revise its regulations and the great institutions, such as the King's Garden and the Observatory, were called to reorganise themselves. Lagrange approved of these reorganisations. As he declared in a letter of 24 October 1791 to an Italian friend, in which he for once expressed his inner feelings, "*je ne regretterai pas d'avoir assisté à un spectacle, le plus intéressant pour les philosophes mêmes, celui d'une grande nation qui se crée un nouveau gouvernement, non par la force de armes, mais par celle de la parole et de l'opinion publique*" (I do not regret having assisted in a spectacle, the most interesting for philosophers themselves, that of a great nations that creates a new government, not by force of arms, but by that of words and public opinion) ([7], vol. XIV, pp. 283–284). Far from being content with the role of observer, he played his part in the changes. Within the Academy, he felt close to the elite of the reformers, above all Condorcet, who he had met during his first stay in Paris, and to Lavoisier, who he admired profoundly. He took part assiduously in the work of the Bureau for Arts and Crafts,

where he was in charge of the review of inventions. Above all, he was a very active member of the Commission for Weights and Measures of the Academy, which was in charge of the reforms in metrology decided on by the Constituent Assembly of 1790.

Lagrange's work within this commission truly marked his return to scientific life after several years of being almost inactive. In his interest in questions of metrology, scientific motives were intimately connected to economical and political motives. The institution of a system of uniform measures first of all presented evident advantages for the work of scientists because it simplified the comparison of given experiments and the performance of calculations. Lagrange was particularly aware of these aspects. In facilitating commercial operations, it also represented a substantial economic contribution. Finally—and this was not its least advantage—the natural and universal nature of such a system could contribute to uniting all men, without distinction between nationality, status or race. For all of these reasons, the metrological reform carried out by the Revolution fit perfectly into the program of the Enlightenment. It gave the Paris Academy the legitimacy that it had sought, when it was battling the criticisms that it was a "caste of scientists".

In the past, Lagrange had already given his opinion on the standardisation of the weights and measures, and had pronounced in favour of a system based on the length of a pendulum beating a second. Now, it is on this basis that the Constituent Assembly asked the Academy of Sciences to prepare the metrological reform. Later, the Assembly decided, on the advice of the Academy, to take instead the length of a terrestrial meridian as the basis. Lagrange rallied to this solution happily, which offered the Academy a reason, or rather the pretext, to persevere in its geodesic operations that it had directed since its founding. More generally, he contributed quite actively to all of the work of the Commission for Weights and Measures, of which he signed all the reports. He seemed particularly involved with the scale of division, which was the object of a report of October 1790. He pleaded for the general implementation of the decimal division, which presented the greatest simplicity taking into account the system of numeration in vigour. Later, he would likewise defend the revolutionary calendar, which was also decimal, and the decimal hour.

The participation of Lagrange in the Commission for Weights and Measures brought him closer to his colleagues. He was quite affected by the execution of Bailly, and above all by that of Lavoisier, whose salon at the Arsenal he had frequented. He remained close to the Jacobin scientists, unlike Laplace. He agreed to purge the Commission for Weights and Measures over which he presided of members who were not considered "trustworthy", and he worked under the orders of the Committee for public health in year II. In sum, whatever private reserves he might have had, he was clearly for the whole of this period on the side of the revolutionary government.

In a certain way, it might be thought that if he rallied without a word to the new power, he was only following the rules of conduct that he had imposed on himself in 1778, consisting in, as befits a wise man, "*se conformer strictement aux lois du pays dans lequel on vit, quand même il y en aurait de déraisonnables*" (conforming strictly to the laws of the country in which one lives, even when these appear unreasonable).[2]

But to think that would be to ignore Lagrange's sincere involvement in the ideas of the Revolution. In the letter cited earlier of 24 October 1791, he underlined that his stay in Paris had lost none of its advantages and agreeableness, and that he had even "*acquis un plus grand intérêt par la discussion publique des objets du gouvernement*" (acquired a greater interest in the public discussions of the objects of government) ([7], vol. XIV, p. 284) Even if the violence and disturbances inspired disgust in him, he felt a deep sympathy with the directions taken by the Revolution and did not hesitate to involve himself personally, to the point of seeing his life and scientific career disrupted.

4 Lagrange and public education

If the Revolution was a decisive moment for the mathematical work of Lagrange, it was above all because it led him to teach. As we have seen, Lagrange was not completely without teaching experience in this area, since in his youth, for more than 10 years, he had taught in the School of Artillery [2, 3, 4]. The treatise on analysis, which he had written at this time around 1750, represented an interesting attempt to explain the principles of this discipline and its application to the study of curves. Lagrange showed gifts for clarity and rigour not unlike those of Euler. He based differential calculus on that of finite differences, according to the Newtonian method of first and last ratios, and introduced the notion of integral by geometrical means, starting with the consideration of finite sums.

This first treatise, unpublished, does not seem however to have had any consequences. It was only in 1772 that Lagrange set out his ideas on the foundations of differential and integral calculus, on the occasion of his work on analogies between positive powers and the differentials and the negative powers and integrals. Wanting to present "*quelques notions générales et préliminaires sur la nature des fonctions d'une ou de plusieurs variables, lesquelles pourraient servir d'introduction à une théorie générale des fonctions*" (some general and preliminary notions on the

[2] Letter of 11 July 1778, to a friend in Turin ([7], vol. XIV, p. 274).

nature of functions of one or more variables, that could serve as an introduction to a general theory of functions), he indicates how differential calculus consists in determining the "derived" functions of a given function and how these functions are defined by means of successive coefficients of their series expansion. Here we find the germ of the idea that was at the basis of his theory of analytical functions. However, Lagrange had not yet thought, it appears, of writing a treatise on the topic. It was only during the Revolution that he returned to it, thanks to teaching.

Education was at the heart of the revolutionary project. After the constitution had been drafted, it was then up to the be citizens to put it into action. Ideas on education were thus political, but the interest in its development was rooted in deeper considerations, based on the notion of human perfectibility (both collective and individual) which the philosophers of the 1700s had dwelled upon at length. This perfectibility justified a fundamental optimism: with improved education, men would be freer and happier; their government would be improved; their wealth increased. Reason was the principal instrument of this emancipation and education was supposed to be both rational and moral and in proportion to the capacity of each. All of this required the construction of a coherent educational system, one which would make a clean break from the past and invent new forms and new contents of education.

The Constituent Assembly decided in consequence to prepare a master plan for public education. The project, presented the following year by Talleyrand, was composed in concert with the scientists of the Academy. Lagrange was consulted on that occasion. The Constituent Assembly, having adjourned before addressing the project, the discussion of the questions fell to the Legislative Assembly, which immediately nominated a Committee for Public Education, presided over by the Secretary of the Academy, Condorcet. That Committee in its turn prepared an ambitious plan for public education, in which five successive degrees were distinguished: for the first level (primary) would be organised a teaching that was universal and free, with at least one school for each community of over 400 inhabitants; at the top, crowning all, was a Society of Sciences and Arts, heir to the old royal academies, directing public education and guiding all research work. At all levels mathematics and physical sciences were to occupy a pre-eminent position. In entrusting the education of children to scientists, Condorcet assigned to pedagogical institutions the historic task that was associated with scientific knowledge: that of permitting the human species to improve itself unceasingly. This mission, which he retraced vividly a little later in his famous *Esquisse d'un tableau historique des progrès de l'esprit humain* (Sketch of a Historical Picture of the Progress of the Human Mind), a grandiose and optimistic vision of a man then hunted to the edge of the grave, to be read as a testament to Enlightenment.

Condorcet presented his plan to the Legislative Assembly in April 1792. The representatives, pressed by more urgent tasks – the same day that Condorcet stood before the Assembly, they proposed to the king that he declare war on Austria – once again adjourned the discussion. It was thus the Convention and his Committee of Public Instruction that took up the problem once again at the beginning of 1793. In the meantime, the idea of the Society of Sciences and Arts had been vigorously contested. Some feared that control of education of children would be taken from the nation to be entrusted to the caste of scientists, seen as a kind of new clergy. More profoundly, many rejected the very idea of an education founded on the sciences. Faced with such opposition, the project for the Society of Sciences and Arts was quickly abandoned and the academies, like the corporations of another age, were completely suppressed. The measure, adopted by the Convention in August 1793, marked the definitive abandonment of Condorcet's plan.

In the months that followed, the organisation of public education was thus limited to primary education. After having adopted the radical plan of Lepeletier advocating egalitarian education for all, the members of the Convention enacted in December 1793 a law on primary education that called for compulsory education and the creation of public schools for both boys and girls, in which teachers would be paid and lodged by the Republic. To help these instructors in their teaching, the Convention established a competition for the writing of official works for elementary schools. A jury, designated in July 1794 by the Committee of Public Instruction, was charged with selecting the best ones. Lagrange took part. The manuscripts submitted having been deemed quite inadequate, the Committee of Public Instruction decided in October to entrust the drafting of these elementary textbooks to "eminent men". Called to draft the "elements of calculus and geometry", Lagrange asked that Legendre be named his associate. In the meantime, he was called to be part of a new institution created to quickly train the future professors and teachers: the École normale, or "Normal School of Year III" [5, 6].

For Lagrange this nomination marked the debut of his new career as a teacher. The idea behind the Normal School was to train future teachers of the Republic by putting them in contact with the best minds. Also, the teachers appointed were all famous personalities, scientists and men of letters. The teaching itself was to consist of magisterial lectures and discussions, allowing students to interact with their masters. The program, of encyclopaedic scope, reserved the place of privilege for science. Lagrange, who was in charge of mathematics, asked that

Laplace be assigned as his associate. Monge, on his side, had to teach descriptive geometry.

The school opened its doors on 20 January 1795 in the amphitheatre of the Museum of Natural History. Fifteen hundred students were expected. The lessons in mathematics were the first; Laplace inaugurated the course with a lesson on numeration and the operations of arithmetic. Lagrange was present at the session, but did not begin his own teaching until ten days later, with a kind of improvised lecture on elementary arithmetic. Between the two mathematicians, the division of work was very clear: Laplace gave the lessons the first day of the ten-day week (the *primidi*) and Lagrange completed it with lectures, the sixth day (the *sextidi*). This dual teaching continued more or less regularly for three months, until the definitive closing of the school, on 19 May 1795.

For the opening of the course, on 20 January, the program for the lessons in mathematics proposed the following:

> présenter les plus importantes découvertes que l'on ait faites, en développer les principes, faire remarquer les idées fines et heureuses qui leur ont donné naissance, indiquer la voie la plus directe qui peut y conduire, les meilleures sources où l'on peut en puiser les détails, ce qui reste encore à faire, la marche qu'il faut suivre pour s'élever à de nouvelles découvertes
> (to present the most important discoveries ever made, and develop the principles, to point out the fine and fortunate ideas which given birth to them, to indicate the most direct route that leads to them, the best sources from which one can peruse the details, what remains to be done, the steps to follow to rise to new discoveries).

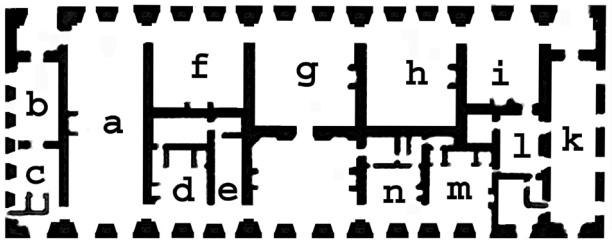
It was intended, in sum, to be at once methodological and encyclopaedic. In was in just such a spirit that the two professors prepared their lessons. Lagrange, in particular, set out to make evident the methods and retrace the history. Beginning with the most elementary notions, he rose quite quickly to considerations of a high level, often borrowing from his own research work. The majority of students, it appears, had some difficulties following him. Joseph Fourier, who was his student, testified that he felt "*assez peu d'accueil*" (not very welcome), but, he added immediately, "*les professeurs le dédommagent*" (the teachers make up for that). Lagrange certainly took pleasure in preparing lessons and without a doubt gave them himself. The fact is that his teaching at the Normal School permanently revived his interest in mathematics.

The professors had to attend the courses of their colleagues. Lagrange was present at the lessons of Laplace and Monge, and certainly profited from that experience. He was also interested in the descriptive geometry that Monge revealed for the first time in public. When Laplace and Monge sparred covertly regarding the respective role of algebra and geometry, he intervened – him, the master of algebra – to underline the role of geometry. He gave as an example of "*une espèce d'application de la géométrie à l'algèbre*" (a kind of application of geometry to algebra) the principle of intermediate values and in illustrating his comments with several figures. The lessons of the professors were to be taken down by stenographers and published immediately. This work was carried out only in part, since only five of Lagrange's lessons were reproduced in the collection of sessions of the school. At least they constituted his first publication since 1788 and they were widely distributed.

5 Professor at the École Polytechnique

Lagrange's teaching at the Normal School was just a taste of things to come. Having reacquired his relish for mathematics, Lagrange also accepted to teach a course at the École centrale des travaux publics (the Central School of Public Works) – the École Polytechnique starting in 1795 – , which was founded by Monge in Paris. This remarkable school, destined to last (in contrast to the ephemeral Normal School of Year III), had a double mission: that of training the engineers which the Republic in war urgently needed, and that, higher, of assuring the progress and dissemination of the sciences [1]. The very particular conditions of its foundation made its hybrid character explicit. On one hand, in effect, the Central School had succeeded the former schools of engineering, which had been suppressed or reorganised. Monge had been largely inspired, for its plan of studies, by the military engineering school at Mézières, where he himself had taught for a number of years. On the other hand, the new school was designed to be placed at the top of the pyramid of instruction provided for by the grand plan which had been awaiting adoption since the beginning of the Revolution. That is why teaching, organized after an encyclopaedic model, was placed under the tutelage of scientists. Monge had performed the feat of reconciling these two missions, professional and academic, closely associating in the new school theoretical and purely scientific teaching with practical and utilitarian applications.

The Central School of Public Works and the Normal School had been instituted at the same time and in parallel. Although designed independently, they shared many common aspects: a very large number of students (400 for the former and 1500 for the latter), a body of teachers recruited from among the best scientists and an extremely ambitious teaching program. The Central School opened its



a. Amphithéâtre
b. Cabinet de l'instituteur
c. Laboratoire
d. Cabinet de physique
e. Atelier
f, g, h. Salles de mathématiques
i, k, l. Cabinets des instituteurs de géométrie descriptive
m. Cabinet de l'instituer d'analyse
n. Cabinet de l'inspecteur temporaire

Fig. 1 Plan of the Hôtel de Lassay where Lagrange gave his lectures at the Central School (the future École Polytechnique)

doors a month ahead of the Normal School, on 21 December 1794, at the other end of Paris, in the Hôtel de Lassay, next to the Palais Bourbon (Fig. 1). The first course was "revolutionary", that is to say, exceptional and accelerated. When the ordinary courses were introduced, on 24 May 1795, the Normal School was just closing its doors. It was only at this date that Lagrange truly began his teaching, that is to say, after having terminated his lessons at the Normal School. In fact, as we shall see, his lessons at the Palais Bourbon were a direct continuance of those that he had given earlier.

That he had also chosen to pursue teaching was a surprise. He had accepted in November 1794 to be appointed professor of analysis at the Central School, on the condition that he never had to give lessons. In effect he contented himself over the winter with taking part in the school's direction, without contributing to the revolutionary courses. He changed his mind in the spring for two reasons, it appears. On one hand, he certainly wanted to contribute to saving the Central School, threatened with the same fate as the Normal School, that is, of being suppressed. He began his lessons at a crucial moment for the new establishment, since Monge had fled to avoid being arrested because of his Jacobin past. On the other side, he found in the Central School the possibility of continuing what he had begun at the Normal School, where he had had to interrupt his lessons before they were finished. This shows that he had rediscovered, with teaching, his taste for mathematics.

The revolutionary courses given over the winter had revealed the very poor level in mathematics of the students of the Central School. He had then to improvise a teaching of initiation, that was extended to the opening of the ordinary courses. Joseph Fourier, spotted while he was a student at the Normal School, was charged with this task. As for Lagrange's lessons in analysis, these were set for *quintidi*, the fifth day of the ten-day week, a day off for students; they were optional and only attended by a small group that was able to understand, who were joined by some external auditors. A short while later, Monge, who had come out of hiding, once again took up teaching analysis applied to geometry, which he had begun in March and which was also reserved for the most advanced students. Thus, for some months there were two high-level courses at the Central School, one by Lagrange and the other by Monge, expounding two parallel and opposite, if not contradictory, conceptions of analysis. While Monge closely associated algebra to geometry, systematically relying on visual intuition, Lagrange developed a purely algebraic point of view, avoiding any recourse to geometric and mechanical representations.

In a little-known lesson of 1797, which served as an unpublished preface to the *Traité des fonctions analytiques* [8], Lagrange very clearly explained approach:

> On reconnaît, après une étude un peu suivie de l'algèbre, que tout le calcul se réduit à de simples transformations, à mettre sous une forme différente un résultat donné d'une autre manière. Or le calcul des fonctions n'est autre chose qu'une transformation fort simple; il est vrai que les résultats qu'on obtient par-là ont une application très heureuse et fort naturelle à la géométrie et à la mécanique, mais ils en sont évidemment tout à fait indépendants. Ce calcul n'a rien qui le distingue de l'algèbre proprement dite.[3]
> (One recognises, after having studied a little following algebra, that all calculations reduce to simple transformations, of putting in a different form a result produced in another manner. Yet the calculation of functions is nothing other than a quite simple transformation; it is true that the results that one obtains here and there are a very fortunate and quite natural application of geometry and of mechanics, but they are evidently altogether independent. That calculation has nothing that distinguishes itself from algebra proper.)

Going back to his teaching at the Normal School, he began with a dozen lessons in arithmetic and elementary algebra, systems of numeration, continued fractions, logarithms and geometric series. Starting in autumn 1795, he approached higher analysis by expounding his theory of functions. This ambition was to avoid recourse to the notions of infinitely small and limits and reduce infinitesimal calculus to simple algebraic operations on functions. More precisely, as in 1772, Lagrange reduced the rules of

[3] This text was published by Luigi Pepe in (1986).

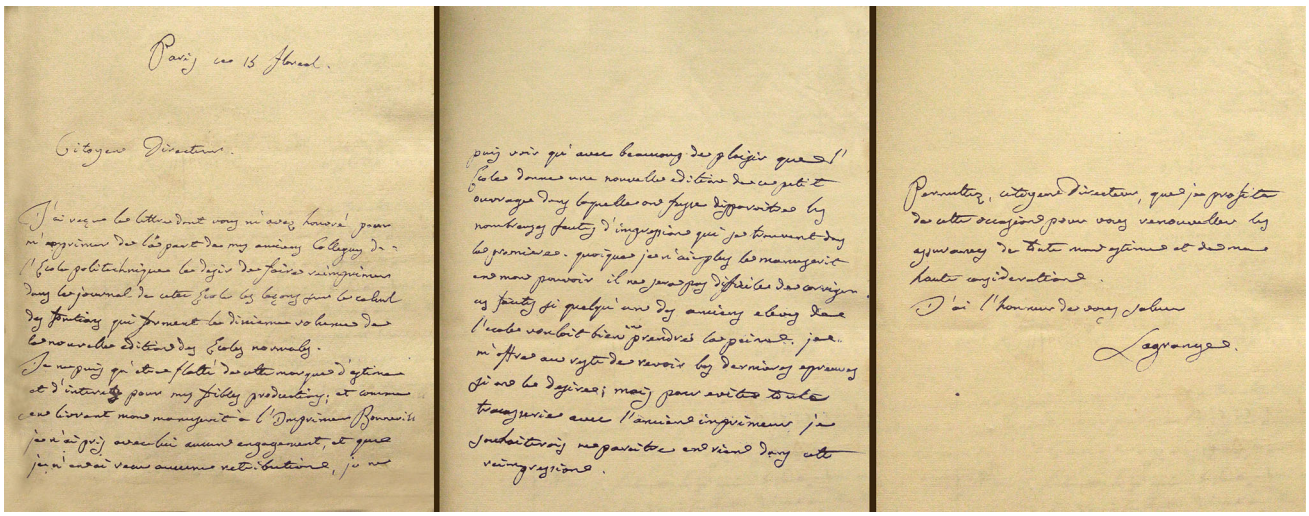


Fig. 2 Letter by Lagrange regarding the new edition of his "Leçons sur le Calcul des fonctions" in *Journal de l'École polytechnique*, 15 floréal XII (*Archives de l'École Polytechnique*, art. VI, §1, sezione b2). Reproduced by permission

differential calculus to the operation of derivation, which went from a function called "primitive" to a function called "derived", and the inverse operation, which went from a derived function to its primitive function. He then obtained the truncated expansion by means of Taylor's formula, which he used to study the contacts of curves and surfaces and easily re-find all the classical results of differential geometry, such as the theory of evolutes or that of developable surfaces.

Lagrange ended the course of mechanics in the course of winter and spring 1797. The *Théorie des fonctions analytiques*, derived from his lessons, appeared in May 1797. Lagrange continued however to teach after the publication of this work. He seems to have taught mechanics the following year, but it is possible that he also presented the subject of his *Traité de la résolution des équations numériques de tous les degrés*, published in 1798. He devoted the years 1798-1799 to a new exposition of the principles of his theory of functions and presented on that occasion a very simple and direct proof of the formulas of Maclaurin and Taylor with the Lagrange remainders. That new exposition of the principles of the theory of functions appeared in 1801 in the *Journal de L'École polytechnique* under the title "Leçons sur le calcul des fonctions" (Fig 2). But by that time, Lagrange had been retired from teaching for almost two years. He effectively handed in his resignation for health reasons on 12 November 1799, three days after the Brumaire coup d'etat, when the École Polytechnique finally had a stable statute.

In spite of their very short duration and marginal place in the plan of studies, the classes given by Monge and Lagrange exerted a large influence, inside and outside the École Polytechnique, and their works drawn from their lessons quickly became classics. Within the École, a research activity, encouraged by Monge and Lagrange, grew up around their teaching. If the project formalised by Lagrange, aimed at reducing infinitesimal calculus to algebra, found few supporters, since it collided with the empiricist conception of mathematics that prevailed in France, the teachers of the École Polytechnique sought to integrate his theory of derived functions and formula for remainders into a more traditional presentation based on the theory of limits or the infinitely small. The young Poisson, who in 1799 had been remarked by Lagrange for his method of binomial expansion, also proposed in 1801 a new proof for developing a function into a Taylor series. Ampére articulated, in 1806, a theory of derived functions obtained from expansions in series. Finally, in a memorable synthesis that is a critical account of Lagrange's contributions, Cauchy was able, after 1816, to found differential and integral calculus on a solid basis, without beginning with the analysis of functions.

In proposing this theory of functions, Lagrange did not aim to impose it on teaching, because his point of view was never that of a mathematician-pedagogue. He was rather, as Amy Dahan [5] has aptly noted with regard to his lessons at the Normal School, a mathematician-philosopher. He hardly believed, in reality, in the utility of professors and textbooks for studying mathematics. Speaking at the end of his life, he underlined that what one learns well, one learns on one's own and that, in his own case, he had studied without a master and discovered the methods by working with applications, contenting himself with sometimes consulting reference works. Finally, he had accepted to teach, first at the Normal School and then at the École Polytechnique, principally to develop the theories that he had at heart for a long time and not to train disciples.

In his treatises, his method of exposition consists in beginning with a few fundamental principles, such as that of virtual velocity in mechanics and that of the analyticity of functions in analysis, and then deducing, by means of transformations and successive developments, a systematic theory. However, he never considered an exposition "by principles" the best one possible to approach a science. His philosophical conceptions were those of sensualism. He thus thought that man learns from sensual experience and that the activity of his understanding must always rest of the lessons of that experience. This is why, far from rejecting geometry, even though he himself avoided use of figures in his treatises, he underlined its importance and its utility. From this point of view, as from others, he may be closer to Monge than to Laplace.

6 The Emperor's mathematician

Like many of his contemporaries, Lagrange rallied willingly to the Napoleonic regime. It corresponded, to be frank, to his ideal of government. Napoleon seemed to embody a synthesis of the enlightened despot, in the manner of Frederick II, and the democratic ideal of the Revolution, although it its most authoritarian version. For Lagrange, the Napoleonic enterprise represented still more. As a cosmopolitan, Italian-born scientist, European by vocation and French by naturalization, he recognized himself in Napoleon's transnational ambition. To be sure, Napoleon was the Emperor of the French, but his project, extending beyond the borders of the former Kingdom of France, included all the nations of Europe. Because it was based on war, domination and often plunder, he was rejected by the people. Lagrange, situated in the heart of the Empire, remained blind to these aspects. On his deathbed, he once more expressed its attachment to the Emperor:

> J'ai été comblé par l'Empereur de fortune et de faveur. Je ne suis pas de ces philosophes qui méprisent l'une et l'autre; chez eux ce prétendu mépris a pour principe la jalousie ou l'orgueil. Ces belles maximes sont bonnes pour les livres, mais elles ne sont pas dans le cœur de l'homme et la philosophie n'est point dans les phrases d'un livre. L'Empereur vient encore de m'envoyer la grand'croix de la réunion, je l'ai reçue avec reconnaissance; cette distinction honore les sciences; et dans le premier ouvrage que je publierai, je prendrai ce nouveau titre. Si mon ouvrage survit, on verra que l'Empereur honorait les savans. Je lui donne les meilleurs ouvrages que je pense, il me donne des rubans et ces rubans sont des distinctions honorables pour moi. Je le répète, la philosophie ne s'apprend pas dans les livres, elle doit être dans le sentiment de soi-même et dans l'indulgence pour les autres; elle ne consiste pas à haïr, à s'isoler, à mépriser, mais à aimer, à servir le gouvernement de son pays, Voilà ma vie, voilà mes principes; tout y est limpide et de facile exécution.

(I have been rewarded by the Emperor with fortune and favour. I am not one of those philosophers who despise one another; among them their alleged contempt has its roots in jealousy or pride. These fine maxims are good for books, but they are not in the heart of man and philosophy is not developed in the phrases of a book. The Emperor wants to send me the Grand Cross of the [Order of the] Reunion, which I have received gratefully; this distinction honours science; and in the next work that I publish I will take this new title. If my work survives, it will be seen that the Emperor honoured the erudite. I give him the best works I can conceive, which bring me ribbons, and those ribbons are honourable distinctions for me. I repeat, philosophy is not learned from books, it must be had in the sentiment of self-knowledge and indulgence for others; it does not consist in hate, in being isolated, in despising, but in love, in serving the government of your country, look at my life, look at my principles; all is clear and easily done.)

These were his last words.[4]

(Translated from the French by Kim Williams)

References

1. Belhoste, B.: La Formation d'une technocratie. L'Ecole polytechnique et ses élèves de la Révolution au Second Empire. Belin, Paris (2003)
2. Borgato, M.T.: ed. Giuseppe Luigi Lagrange: Principi di Analisi sublime. Bollettino di Storia delle Scienze Matematiche **7**(2), 45–200 (1987)
3. Borgato, M.T., Pepe, L.: Lagrange a Torino (1750-1759) e le sue lezioni inedite nelle R. Scuole di Artiglieria. Bollettino di Storia delle Scienze Matematiche **7**(2), 3–44 (1987)
4. Borgato, M.T., Pepe, L.: Lagrange. Appunti per una biografia scientifica. La Rosa editrice, Turin (1990)
5. Dahan, A. La méthode critique du mathématicien philosophe. In: L'Ecole normale de l'an III, leçons de mathématiques. Dhombres, J, (ed.) Dunod, Paris, pp. 171–192 (1992)

[4] From the last conversation with Lagrange, 8 April 1813, written down by Chaptal (Bibliothèque de l'Institut de France, ms 2041). After having thus spoken, he said only to Lacépède, "*Dès que je serai guéri, et ceci sera long, vous nous donnerez à dîner à votre campagne, à Mr Monge, Chaptal et moi et je vous dirai bien d'autres choses*" (As soon as I am well, and it will take a long time, we will have dinner at your farm, with Mr Monge, Chaptal and myself, and I will tell you many other things).

6. Julia, D., ed. L'École normale de l'an III. Une institution révolutionnaire et ses élèves. Introduction historique aux leçons. Éditions Rue d'Ulm, Paris (2014)
7. Lagrange, J.-L.: Oeuvres de Lagrange (14 vols). Joseph Alfred Serret, (ed.) Paris: Gauthier-Villars (1867–1892)
8. Pepe, L.: Tre, "prime edizioni" ed un' introduzione inedita della Théorie des fonctions analytiques di Lagrange. Bollettino di Storia delle Scienze Matematiche **6**, 17–44 (1986)
9. Taton, R.Le: départ de Lagrange de Berlin et son installation à Paris en 1787. Revue d'histoire des sciences **41**, 39–74 (1988)
10. Taton, R. Lagrange et l'Académie royale des sciences (1763-1793). Atti della Accademia delle Scienze di Torino 124, suppl. n. 2 (La "Mécanique analytique" de Lagrange et son héritage), pp. 1–23 (1990)
11. Taton, R. Lagrange et la Révolution française (juillet 1789-novembre 1795). Atti della Accademia delle Scienze di Torino 126, suppl. n. 2 (La "Mécanique analytique" de Lagrange et son héritage), pp. 215–255 (1992)

Bruno Belhoste is a historian of sciences, and specialises in the period of the eighteenth and nineteenth centuries. In particular he works on the history of mathematics and its teaching. He is the author of a biography of Cauchy, a history of the Ècole Polytechnique, and numerous articles on scientific life in France. His most recent book, *Paris savant. Parcours et rencontres au temps des Lumières* (Paris: Armand Colin, 2011) is dedicated to the scientific world of Paris in the second half of the eighteenth century. He currently teaches at the Université Paris 1 Panthéon Sorbonne.