Why I think ‘*Mathematics for the Citizen*’ is an unfortunate slogan

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Abstract The author criticizes the slogan ‘Mathematics for the Citizen, showing how in Italy teaching mathematics is not really aimed towards its applications, which can be used in everyday life but how, instead, everyday cases in real life are used as an introduction to the study of particular mathematical situations. In this paper we examine the dangers which derive from oversimplifying complex issues such as the evaluation of cognitive learning processes using standard mathematical methods, which has already met with criticism in the world of economics.

Keywords Mathematics for the citizen · Slogans · Mathematics teaching · Evaluation

In 2012 I took part in a conference on teacher training, and during my talk I stated, “‘Mathematics for the citizen’ is, in my opinion, an unfortunate slogan’.¹ I was then asked to explain or expound on this beyond the passing mention made in a complex context, and I was given 5–6 pages of space to do so. This is decidedly too little to address such a pertinent topic in the detail it deserved, but it is sufficient to give an idea of what I wanted to say. The form, example and tones used would be inappropriate in a broader treatment, but I hope that they will be suggestive of the right context for summing up and communicate my doubts.

1 Mixed up objectives

In my talk I spoke of ‘unfortunate slogan’, with the intention of focusing attention on the word ‘slogan’ as it is usually understood. *Wikipedia* defines it like this:

A slogan is a memorable motto or phrase used in a political, commercial, religious, and other context as a repetitive expression of an idea or purpose. ...Slogans vary from the written and the visual to the chanted and the vulgar. Their simple rhetorical nature usually leaves little room for detail and a chanted slogan may serve more as social expression of unified purpose than as communication to an intended audience.

In our case, slogan has two connotations: it expresses both an idea—the necessity of teaching mathematics while privileging its practical aspects for the benefit of a citizen who must deal with a society that is by now quite complex, and which asked him to be able to manage percentages, pie graphs and statistics—as well as an aspiration to make learning mathematics more pleasurable and natural, finally grounding the subject in reality.

Going to see what has been proposed by the Americans, the masters in this field, I found James–Stein, professor of mathematics at California State University, who has this to say in a recent book:

How much math do you need to be a productive citizen, to enrich your life and the groups of which you are a part? Amazingly enough, sixth grade

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¹ The conference dedicated to the training of mathematics teachers was entitled “La formazione degli insegnanti di Matematica” and took place on 8 June 2012 at the Università di Roma “Tor Vergata”; see [Catastini 3].

arithmetic will take you an awfully long way if you just use it right, and you can go further with only a few extra tools that are easy to pick up. You don't need algebra, geometry, trigonometry or calculus.

... I have yet to find a good explanation for why the math establishment insists on stuffing algebra down everyone's throat, starting in about seventh grade. After all, who really needs algebra? Certainly anyone planning a career in the sciences or engineering does, and it's useful in the investment arena, but that's about it. Algebra is mandatory on the high school exit exam of many states, despite overwhelming evidence that outside of the people who really need algebra (the groups mentioned previously), almost nobody needs algebra or ever uses it once they put down their pencils at the SAT [8].

The whole book then grounds mathematics (although in fact, I would almost say only arithmetic) in real life, *training* the citizen to use the subject in the most productive way, in solving concrete daily problems. I use the verb 'training' to underline what is effectively the transfer of arithmetic notions to the solution of practical problems that are of greater or lesser degrees of complexity. To give a clearer idea, here are just a few of the many problems dealt with: Are service contracts for appliances a rip-off? What are the probabilities of winning at roulette? Is going to college worth it? Who should you invite to the senior prom? Why are women fickle and men dependable? How did statistics prevent cholera in London in the nineteenth century? Are you more likely to meet a man who is more than six feet tall or one who is more than a 100 years old? How do you estimate the cost of a disaster?

'Probability' and 'prediction' are the terms used most often in the solution to these problems. The solutions are then found by only using knowledge of the appropriate context, the one in which the problems themselves arise, and a great part of the work of explaining them consists in the description of these indispensable contexts. In the first problem mentioned, for example, he explains how all electrical appliances, large and small, behave the same way in terms of the possibility that they will break, and he presents a survey, a table that describes in terms of percentage the number of products that require repairs within a given number of years, making us see how among these computers are at the top, with 43 % of repairs, while cathode-ray tube televisions are way below that, with only 6 %. He suggests calculating the cost of extending the guarantee and the average cost or replacement: if the cost of extending the guarantee is higher than the replacement cost, then that contract is a rip-off.

This is truly doing what the slogan 'Mathematics for the citizen' suggests: *presenting* sensible and up-to-date

Fig. 1 $x + y < b$

Fig. 2 What is the probability of hitting the target at a point closer the centre than to the circumference?

problems of everyday life, *explaining and giving information*, sometimes even quite complex, about the real-life contexts where they are found, *teaching* the citizen the techniques he needs to solve them, reducing mathematics to a few necessary things you need to know to add figures, read tables, calculate percentages, compare results. This is what Stein offers, in keeping with his premises.

Wanting to compare this type of citizen's problem with those proposed by the Union of Italian Mathematicians, I chose the breadstick problem² proposed as part of the 'Progetto 2003' for a fourth-year high school class: 'A breadstick of length b is broken into three random parts. What is the probability that the three pieces will form a triangle?' Here is the beginning of the treatment of that problem:

The students are invited to formulate conjectures (perhaps by trying themselves to break a breadstick into three parts). From the discussion it comes out that in order for the three pieces of breadstick to form a triangle they must have a certain length!. The need of setting conditions for length leads to the translation of the problem into mathematical language: the breadstick is considered like a line segment that must be divided into parts so that it can form a triangle. Represent this with segment of length b and let x and y be two of its parts (Fig. 1).

It is easy to understand that $x + y < b$ is the translation into formal language of the condition that the segment can be divided into three parts. It is known that each of the three parts, in order to form a triangle, must have a length that is less than the sum of the two other lengths, which

² See "Grissini e triangoli" in [1]. http://www2.dm.unito.it/pagine_personali/arzarello/matematica2003/acd/iibiennio.pdf (accessed 5 December 2013).

translated into formal terms is: $x < b/2$; $y < b/2$; $x + y > b/2$. The transition from the breadstick to the segment is not trivial! It requires a process of abstraction that, in this context, is performed by using the skills related to the calculation of probability. To arrive at this, it is helpful if the students have already become accustomed to calculating probability in the case of infinite space of events as ratios of areas, by giving them examples like this:

‘We want to hit a round target. What is the probability of hitting the target at a point closer the centre than to the circumference?’ (Fig. 2).

Here it must be observed that the segment, in contrast to the breadstick, is uniform with respect to its points, and thus it can be conjectured that all points are equiprobable (that is, the segment can be ‘broken’ at every point); then it is possible to apply the classic definition of probability as a ratio between favourable and unfavourable possible cases.

Going forward in this way in modelling real situations, the solution is reached: with p indicating the probability being sought, we have $p = 1/4$.

The context in which this problem is discussed is (to my relief) that of probability and of geometry; the activities proposed are: inferring, conjecturing, proving. The reality to be faced by the citizen in his everyday life is constituted by a real breadstick, to be broken at will into three pieces that form the ‘sides’ of a triangle, and from the question he might ask himself: what is the probability that this will happen? I think that it appears evident that, in this case, in contrast to the ‘mathematics of the American citizen’, the breadstick is only a pretence to conceal a mathematical problem that is interesting in itself,³ and solvable by calculating the ratios between two areas defined by two systems of inequality in x and y . To solve this question it is necessary to have studied topics of theoretical mathematics that range from geometry to algebra and probability, and to know how to manoeuvre within them, as required by our national mathematics programs. My predilection for the formative aspect of mathematics means that I am more in favour of maintaining these arguments than of studying arithmetic alone for the utilitarian purposes proposed by Stein. So then, I ask myself, *why do we need this slogan?*

To go into the question more deeply, reading the aims cited in the introduction to the project entitled ‘Matematica 2003’ [1], I find:

³ We can underline the fact that a mathematical model, in order to be applied to reality and permit realistic predictions, uses a series of simplifying hypotheses that are necessary to pass to a model that is very far removed from reality, extremely complex, given that the breadstick is not uniform, is almost cylindrical, and has a probability of breaking that is different at each of its points. That is, we can underline how the ‘objective’ value of the results of a mathematical prediction become ‘subjective’ if account is taken of how they are related to the simplifying choices introduced by the solver.

Mathematical education must contribute, along with all the other disciplines, to the cultural education of the citizen, in such a way as to permit him to participate in social life with awareness and critical capability. The skills the citizen should have, and to the acquisition of which mathematics contributes, are, for example: to adequately express information, to intuit and imagine, to solve and pose problems, to design and construct models of real situations, to make choices in conditions of uncertainty. Knowledge of the scientific languages, including first of all that of mathematics, reveals itself to be increasingly essential for the acquisition of a capacity for correct judgment. In particular, mathematics teaching must gradually accustom the student, beginning with fields of experience that are rich for him, to the use of mathematical language and reasoning, as an instrument for interpreting reality, and must not consist solely in a fund of abstract notions [1].

‘Accustom the student, beginning with fields of experience that are rich for him, to the use of mathematical language and reasoning, as an instrument for interpreting reality’: this is one of the new objectives, but it isn’t easy to identify in it meanings that are different from those given by Prof. Stein. Not even the questions that appear on the school exit examination help to understand; rather, they heighten the ambiguity. Here, for example, is the ‘saucepan question’ that appeared on the 2008 exit examination for scientific high schools: ‘*Among the saucepans shaped like cylinders with the same surface S (that of the sides plus the bottom), which is the one with the greatest volume?*’ From the premises given above, which call for knowledge of mathematical language and reasoning, we would expect a process in which the student is supposed to know that the saucepan isn’t to be considered like a cooking vessel, but rather treated geometrically as what it is: an object that is solid (since the sides have a thickness), concave (it can ideally be obtained subtracting a smaller cylinder from one that is coaxial and larger), with its own volume and capacity (the volume of the smaller cylinder). Knowledge of scientific languages and the habit of reasoning would then lead to calculating the maximum volume of the concave solids like saucepans (also because otherwise it would have involved calculating the capacity), but what isn’t completely clear is why the problem asks in reality the maximum volume of all the cylinders, solid and convex, that have the same surface, equal to ‘that of the side plus the bottom’ of the saucepan. Which skills are supposed to be measured by this problem? Perhaps something like ‘making choices in conditions of uncertainty and interpreting the thought of those who posed the question?’

2 Mathematics and complexity, mathematics and politics

Reading in detail the projects presented by the Union of Italian Mathematicians, I believe I can say that one of the aspects that most emerges is the exemplification and the exhortation to the use of laboratory kinds of teaching in mathematics, such as that proposed by Emma Castelnuovo, which were successively experimented in the *Piano Nazionale Lauree Scientifiche* (PNLS, the National Plan for Scientific Degrees), promoted by the Ministry for Education and Research in 2004 and still in vigour. There is, however, a substantial difference between the two proposals: the PNLS general views laboratories and the topics addressed in them as extracurricular, opportunities to address the mathematical modelling of reality by means of problems that are often drawn from the history of the discipline and not necessarily from the program to be followed in the classroom, while the proposal of the Union of Italian Mathematicians concerns mathematics programs and mathematics teaching during classroom hours, and suggests significant modifications in the topics traditionally dealt with at school. It can thus be considered opportune, for example, to give greater emphasis and space to probability and statistics, and less to trigonometry, the method of proof, or synthetic geometry. The Ministry's *Indicazioni Nazionali* ('National guidelines', see [MIUR 7]), on the other hand, in line with these suggestions, no longer contain lists of the mathematical topics deemed fundamental and required for a particular course of study—the scientific high school, for example—but is concerned 'only' with certain aspects of education. This is how the 2010 document begins:

The National Guidelines for the specific objectives of learning for high schools represent the disciplinary declensions of the student's educative, cultural and professional Profile at the conclusion of the high school programme. The Profile and the Guidelines thus constitute the framework upon which the scholastic institutions design their own Plan for teaching, the teachers construct their own didactic itineraries, and the students are placed in condition to achieve the objectives for learning and maturing the skills proper to high school education and its articulation [MIUR 7].

The move from 'topics for learning' to 'objectives for learning' that has occurred in these years has led various schools to inflate their programs, often without seeing a satisfactory return in terms of the students' skills. In a recent private colloquium, the current president of the Union of Italian Mathematicians, Ciro Ciliberto, complained to me about the excessive accumulation of contents in secondary school programs, and of the increasingly numerous and serious gaps in first-year university students,

who show a profound lack of techniques of calculating, reasoning and proving, and in the basic knowledge indispensable for dealing with their chosen courses. By now, these considerations are heard more and more often in Italian universities, which have also been shaken by the discussions of the Italian National Agency of Evaluation of the University System and Research (Agenzia nazionale di valutazione del sistema universitario e della ricerca, AN-VUR). How did it come to this?

The by now constant presence of systems of standardised evaluation on the educational scene force us take a second look at the slogan 'Mathematics for the Citizen'. Now what it really appears to want to say is 'Mathematics for the evaluation of the European citizen'. In effect, what has determined the choices of the Union of Italian Mathematicians is found explicitly in the Lisbon European Council of 2,000, and all the Councils that have followed up to the present, which have asked the countries of the European Union to adjust their educational systems to international principles dictated by the need to evaluate and compare national results by means of evaluations that are statistical and standardised. The need to evaluate and compare complex processes has introduced as indispensable, within the subject and its teaching, classificatory categories and terms that, in contrast to scientific terms, are highly ambiguous (for example, 'competence', 'skills', 'mobilisation of knowledge', etc.), and are therefore not optimal as assumptions for a statistical kind of measurement. The ambition to increase the students' performance and make it measurable through international methods of mathematical modelling has led the Union of Italian Mathematicians to formulate the classificatory identification of the core topics—*number; space and shapes; ratios; information and predictions*—and three transversal nuclei centred on the processes of the students—*inferring and conjecturing; measuring; solving and posing problems*. The proposed nuclei were then adopted by the Italian National Institute for the Evaluation of the Educational System (Istituto nazionale per la valutazione del sistema educativo di istruzione e di formazione, INVALSI) as its own frame of reference for the evaluation examinations for mathematical learning.

You see, in my opinion it is still too early to think about being able to catalogue and evaluate phenomena of an intellectual nature as complex and delicate as the personal processes of learning, which are only in part quantitative, by means of arbitrary frameworks or nuclei whose validity for measuring is not yet confirmed, and indeed, over which, in certain cases, there hangs a heavy cloud of doubt not only within the psycho-pedagogical community (see, for example, [4]) but also by recent findings in neuroscience. This topic is extremely complex and cannot be discussed in such a short space as this. Only to give an idea, let us bear in mind, for example, that the connections between the two

cerebral hemispheres, which are besides all else essential for reasoning and making decisions in situations that are not routine, are not complete in the human brain until after sexual maturity has been reached, and thus can vary greatly in time from individual to individual. Our brain is so powerful in part because it continues to grow and develop throughout adolescence. Hence reasoning has different limits and colours for each twelve-year-old. Some particular performances are not only tied to the teacher's ability to teach and the student's willingness to learn, but also to the particular stage of that student's personal growth and neural maturity. The problems assigned in the classroom, or in the exit examinations at the end of the third year of middle school, still sensibly evaluate the student's level of learning by adopting forms that respect these complex biological characteristics but, paradoxically, this is exactly why they do not serve the purposes of a linear standardisation of the type used by the INVALSI. This difficult relationship between the complexity of a system and its verification by means of standardised systems of evaluation has been described by Alain Berthoz⁴ in a recent work on the complexity of systems, including living systems:

In the face of the complexity (of the world, of society) we are seeing a proliferation of methods for simplifying it. Such methods, aimed at avoiding collective and individual madness due to our brains' impossibility to elaborate the immense quantity of information necessary for living, acting and understanding, display an apparent simplicity expressed by means of abstruse mathematical theories that mask the inability of their authors to grasp reality. These mathematical models, tied to the private interests that they conceal, regularly provoke dramatic situations, as shown by the recent financial crisis and the failure of banking systems. We can give another example: to facilitate decisions there is a tendency to reduce man to a series of logical processes and to model him by means of a series of logical-mathematical theories that simplify the reality of life. But, in spite of efforts aimed at finding efficacious solutions, the 'simple heuristics that make us smart',⁵ we must necessarily recognise that man today is like a Theseus lost in a labyrinth, without Ariadne's thread to help him find the way [Berthoz [2]: VIII; my trans.].

With regard to the widespread methods used for the statistical description and prediction of economical-social phenomena of the kind Berthoz is talking about, my own opinion is echoed in the words of Vincenzo Zeno-Zencovich, rector of the Università degli Studi Internazionali di

Roma–UNINT, who, in a recent workshop on the role and responsibility of mathematical models in the present economic crisis, underlined the importance of the criteria implicit in the collection of the data to be interpolated:

Those who wish to look at the questions from a holistic perspective can draw from the crisis in the relationships between economics and mathematics, highlighted by the failure of some predictive models, cues that suggest, certainly not the abandonment of a dialogue that has lasted for more than 200 years, but rather a reconsideration of the function and utility of that relationship. The jurist draws the attention of the economists ... to the *raison d'être* of their science, that is, the description of economic-social phenomena. Only if this is as correct as possible—if, that is, it uses appropriate instruments to gather data—it can be useful for grasping the constants and accidentals in the behaviour of the community in question. It doesn't make much sense then to blame mathematics or econometrics when the starting datum is erroneous or imprecise. This leads us to take care to avoid dogmatic approaches (the facts must conform to the theories, and if they don't, it must be the facts that are 'wrong') ... This solicits a further caveat: while economical science must base itself on data that is faithfully gathered, it is possible to intuit how these vary remarkably not only from one continent to another (e.g., the infantile pretense of applying American models to Europe) but also within a single national market [10]: XII; my trans.].

He further adds:

That which the jurist asks of the mathematician is the capacity to evolve his own very refined language to express and describe phenomena that are only in part quantitative and appertain to intellectual and sensorial experiences. There is a long way to go but the studies—fascinating and ever deeper—in the field of neurosciences shows that it is possible to give form to something that just a short time before had appeared unknowable or simply irrelevant [10]: XII; my trans.].

I don't really know if we are like so many Theseuses without a lifeline—as Berthoz said—but we are certainly in a moment that is extremely delicate for the fate of our culture and our scientific research. The state of agitation is perceived not only in schools, where the INVALSI led to rebellions on the part of teachers, but also in the universities. Just recently there was the episode about the release of the classifications of the ANVUR and much nervousness on the part of research centres.⁶ We have already had unfortunate experiences, both in our Italian history and in European history of the twentieth century, in which the

⁴ Alain Berthoz is a professor of the physiology of perception and action at the Collège de France and a member of the Académie des sciences in Paris. Since 1989 he has directed the Laboratoire de physiologie de la perception et de l'action (CNRS, Collège de France).

⁵ For the economic sector, see (Gigerenzer et al.[5]).

⁶ A competent, non-specialist explanation of what happened can be found at: <http://www.roars.it/online/archimede-pitagorico-ci-spiega-perche-le-classifiche-vqr-non-hanno-senso/>.

requirements and the choices of national necessities had repressive consequences for mathematical culture and for scientific communities. But it is precisely the complex, fundamental nature of the questions at stake that must, as Zeno-Zencovich put it, ‘suggest certainly not the abandonment of a dialogue that has lasted for more than 200 years, but rather a reconsideration of the function and utility of that relationship’, excluding the acritical acceptance of proposals whose scientific validity, in the sense that the Rector intended, is doubtful to say the least. As Angelo Guerraggio and Pietro Nastasi warn in their book *L'Italia degli scienziati*, the consensus of scientists is fundamental:

[The role of science in the development of society] is a public good that cannot be left to improvisation, but to the contrary, one whose growth should be encouraged. Our history tells of national governments that were disposed to spend, and invest, but wanted to have a say about the lines of research. They think that the efficiency of the scientific community must be in some way verified and inserted into a broader logic. What remains open is the question of the intensity and modality of this attention on the part of the government ... It is unthinkable that basic research be separated and absolutely distinct from research which is more applied. Nor is it conceivable to punish it, even in the case where the intention is to aim at applications. It is precisely the applications that demand a greater involvement in fundamental research. To construct the necessary network of channels of collaboration and communication between government, politics, society and science takes time and skill, a widespread scientific education, a development of productive strengths, and the search for a consensus among scientists [6]; my trans.].

Schools and education have always been involved in this turbine of interests. We can only hope that our mathematicians, our scientists, called today to contribute to Europe's economic plan with measures and guidelines for education, are capable of imposing the time and methods necessary to identify the right instruments that will lead to proposals that are well-supported scientifically and adequate to the complexity of the topics.⁷ Not least because Italian mathematicians *still* possess the excellent capacities and mentality needed to do it!

Translated from the Italian by Kim Williams.

⁷ For more on the term ‘evaluation’, I suggest the recent article by Giorgio Tassinari [9], full professor of economical statistics at the University of Bologna.

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