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Special issue in honour of Alexander Shnirelman's 75th birthday

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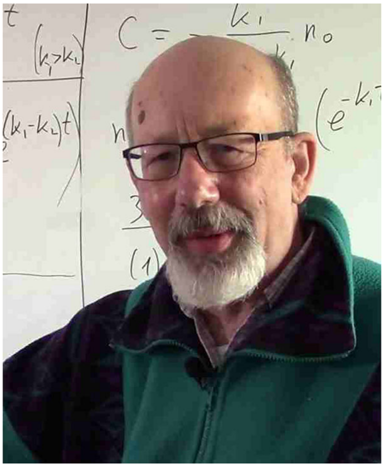

Aesop's fable about the Lioness and the Vixen describes a Fox who rebuked Lioness for not having more than one cub. The Lioness replied, "but that one's a lion." In the world of mathematical ideas, Sasha Shnirelman produced several lions, and a handful of his contributions became classics by now!

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Alexander Shnirelman's short note "Ergodic properties of eigenfunctions" published back in 1974 [1] had a fundamental impact on the area of quantum chaos and essentially initiated the mathematical research in this field. The eigenfunctions, i.e. vibration modes, were studied from the time when a drum was invented, or at least since the works of Laplace and Chladni. Shnirelman's remarkably novel observation was that the ergodicity of the classical (geodesic) flow implies the equidistribution of (almost) all eigenfunctions at high energies, which is a quantum mechanical property. This result was extended and generalized in a myriad directions and had a lasting influence on modern mathematics, from geometric analysis to number theory. In this issue, the reader will find a modern take on Shnirelman's quantum ergodicity theorem as well as the translation of his pioneering paper [2] giving a more detailed exposition of the results presented in the note mentioned above. This paper has originally appeared in Russian in a conference proceedings and has never been published in English previously.

One more cornerstone paper of Sasha's was on ideal hydrodynamics and the geometry of diffeomorphism groups [3]. He proved that the two point geodesic problem for the Euler equation in three dimensions might not have a smooth solution. Furthermore, he discovered that the group of volume-preserving diffeomorphisms in three dimensions has a very peculiar geometry: for instance, it has a finite diameter for diffeomorphisms of a cube.

Another seminal contribution by Shnirelman is to the study of weak solutions of the Euler equations. In 1997 he published a paper [4] on the nonuniqueness of weak solutions which had a long-term effect and eventually led to an impressive progress in the area of nonlinear partial differential equations. In particular, revisiting V. Scheffer's example, Sasha constructed a surprising weak solution of the Euler equations which was compactly supported in space and time and hence did not preserve the energy. Sasha's further contributions to hydrodynamics include, in particular, a development of new techniques to show the existence of weak solutions of the Euler equations with monotonically decreasing energy [5] and a description of two-dimensional fluids via continuous braids.

This volume combining the works of leading specialists in quantum chaos, fluid dynamics, and partial differential equations is a tribute to Alexander Shnirelman's groundbreaking contributions and a way to celebrate his 75th birthday.

Many happy returns, Sasha!

References

1. A. Shnirelman, *Ergodic properties of eigenfunctions*, Uspekhi Mat. Nauk **29** (1974), no. 6 (180), 181–182.
2. A. Shnirelman, *Statistical properties of eigenfunctions*, in: Proceedings of the all–USSR school in differential equations with infinite number of independent variables and in dynamical systems with infinitely many degrees of freedom, Dilijan, Armenia, May 21–June 3, 1973. Armenian Academy of Sciences, Erevan, 1974. *English translation:* Ann. Math. Québec, this issue.
3. A. Shnirelman, *On the geometry of the group of diffeomorphisms and the dynamics of an ideal incompressible fluid*, Mat. Sbornik (N.S.) **128 (170)** (1985), no. 1 (9), 82–109. *English translation:* Mathematics of the USSR-Sbornik, **56** (1987), no. 1, 79–105.
4. A. Shnirelman, *On the nonuniqueness of weak solution of the Euler equation*, Comm. Pure Appl. Math. **50** (1997), no. 12, 1261–1286.
5. A. Shnirelman, *Weak solutions with decreasing energy of incompressible Euler equations*, Comm. Math. Phys. **210** (2000), no. 3, 541–603.