Walter K. Hayman FRS (1926–2020) A Biographical Sketch

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Walter Kurt Hayman was born in Cologne, Germany, on 6th January 1926. He died in London, England, on 1st January 2020 just days before his 94th birthday. He had attended the One Day Function Theory Meeting, an annual meeting that he was instrumental in establishing and at which he was always the principal participant, at De Morgan House the previous September; when he arrived, the entire audience stood up and applauded. Walter's health deteriorated between then and Christmas following a fall, and he passed away peacefully in hospital on New Year's Day 2020. An event had been planned by Eleanor Lingham (Sheffield-Hallam) and Rémi Lodh (Springer) to mark the publication of *Research Problems in Function Theory* by Walter Hayman and Eleanor Lingham [15], the 50th anniversary edition of the original problem collection [8]. The event was arranged to coincide with Walter's birthday on 6th January. Instead, and with great sadness, those present met to remember Walter, joined by his daughter Carolyn.

Walter Hayman was, for decades, the dominant figure in the study of entire and meromorphic functions and associated fields. He was well aware of his responsibilities to the subject and its practitioners, and unfailingly fulfilled them. He became a central

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figure in complex analysis, not only by virtue of his own research but also on account of his energetic participation in conferences and correspondence with mathematicians worldwide. That was one reason why a stream of mathematicians visited Imperial College, many on sabbatical or leave. TC recalls a year (1984/85) spent at Imperial College as a student of Walter, who would spend every Monday morning answering his mathematical correspondence. Walter relied heavily on secretarial support, a necessary but not sufficient requirement being the ability to decipher his legendary handwriting. He valued the Pure Mathematics secretaries greatly and gave each the respect and recognition they deserved [14][p. 78]. Correspondence dealt with, it was time for coffee followed by ‘Hayman’s Seminar’ where the great and the good came to lay their offerings at his feet, followed by lunch and tea. The Monday seminar, with eminent speakers and an audience from across London and beyond, was one of the highlights of the mathematical week at Imperial.

Walter was gifted with great technical ability. His computational power and insight regularly opened a path into areas that were previously believed to be impenetrable, allowing others to follow. He was, first and foremost, a problem solver. Indeed, the vast majority of his research papers could be classified as ‘problem + complete solution’, often with difficult examples constructed to show the sharpness of his theorems. An early representative case is the family of counterexamples he constructed to Wiman’s conjecture on the size of the minimum modulus of an entire function relative to its maximum modulus [3]. The minimum modulus and maximum modulus of an entire function f on the circle of radius r are denoted by $m(r, f)$ and $M(r, f)$, respectively. For $\rho > 0$, let $C(\rho)$ be the best constant for which the inequality

$$\limsup_{r \rightarrow \infty} \frac{\log m(r, f)}{\log M(r, f)} \geq C(\rho)$$

holds for every entire f of order ρ . If $0 < \rho \leq 1$, then $C(\rho) = \cos(\pi\rho)$; this is the celebrated ‘ $\cos(\pi\rho)$ theorem’. For $\rho > 1$, Wiman [17] proved that $C(\rho) = -1$ if it is further assumed that f is zero-free, and conjectured that this restriction can be removed. Indeed, Beurling [1] proved that, for any non-constant entire f and for fixed θ ,

$$\limsup_{r \rightarrow \infty} \frac{\log |f(re^{i\theta})|}{\log M(r, f)} \geq -1,$$

which strongly supported Wiman’s conjecture. However, Walter proved in what Wegert [16] describes as ‘a technical *tour de force*’ that, for arbitrarily large order ρ , the correct order of magnitude is $C(\rho) \sim -\log \rho$. In Walter’s own words, ‘there exist functions of finite order for which $m(r, f) < M(r, f)^{-C}$, $r > r_0$, however large the constant C ’. This and other early work catapulted him to recognition. In 1955, Walter was awarded the Junior Berwick Prize by the London Mathematical Society.

Shortly afterwards, Walter established the Asymptotic Bieberbach Conjecture [4]: if $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ is mean univalent in the unit disc $\mathbb{D} = \{|z| < 1\}$ then

$$\alpha_f = \lim_{n \rightarrow \infty} \frac{|a_n|}{n}$$

Fig. 1 Walter K. Hayman 2017 at the Mathematical Research Institute Oberwolfach. Picture by Jennifer Hinneburg, with kind permission from the picture archive of the Mathematical Research Institute Oberwolfach, Germany.

exists, with $\alpha_f \leq 1$, equality only for (rotations of) the Koebe function. Univalent functions form an important special case of mean univalent functions, and only de Branges' deep proof [2] of the Bieberbach Conjecture, $|a_n|/n \leq 1$, valid only in the univalent case, is comparable to [4]. The results in [4] for mean p -valent functions ($p > 1$) still have no improvement for p -valent functions.

In 1963, Walter showed [6] that there is an absolute constant A such that, if $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ is areally mean univalent in the unit disc $\mathbb{D}$, then $||a_{n+1}| - |a_n|| \leq A$ for all n . This remarkable result on the difference in the size of successive coefficients of areally mean univalent functions improved on previous work by Golusin who had, for univalent functions, the bound $O(n^{1/4} \log n)$, and by Biernacki who had the bound $O(\log n)^{3/2}$. In the Mathematical Review for this article, Nehari writes 'This result which, although known to hold for starlike univalent functions, was hardly suspected to be true in the general case, constitutes a major advance in the theory of univalent functions.'

Another beautiful result, obtained jointly with Jang-Mei Wu, states that the preimage of a line or circle under a conformal mapping from the unit disc to a simply-connected domain has total Euclidean length bounded by an absolute constant [10]. The best bound in the Hayman-Wu Theorem, as it has come to be known, is conjectured to be π^2 .

Fig. 2 Picture taken during Walter's presentation at the Bieberbach conference, March 1985, Purdue University. Picture by David Drasin

Fig. 3 Walter discussing with Christian Pommerenke. Picture taken by David Drasin at Imperial College for Walter's 90th birthday celebration, January 2016

These are but a sample of Walter's striking research. His several monographs have also had significant influence, none more so than *Meromorphic Functions* [7]. Completing a project initiated by his student Paddy Kennedy, *Subharmonic Functions* [9] is a lovely self-contained account of the topic. The volume *Subharmonic Functions II* [11] is a comprehensive account of recent developments in the area. Walter brought out in 1994 [12] an expanded second edition of his Cambridge Tract *Multivalent Functions* from 1958 [5]. *Meromorphic Functions* and *Multivalent Functions* were translated into Russian. His problem collections [8] also did much to popularize the field in the pre-internet era when even international contact could be restricted. They included established problems, as well as submissions by international mathematicians, and many of his own invention. It is perhaps noteworthy that his last published word on mathematics is a problem collection [15], written with Eleanor Lingham,

Fig. 4 Walter making music with Gerald Schmieder. Picture taken by David Drasin at the music room at Oberwolfach, February 17, 1999

that sets out his future agenda for complex analysis (see also the *Mathematical Review* of this collection written by Christopher Bishop).

Walter was a vigorous traveller and lecturer, unstinting in his efforts when it came to mathematics. He was once heard to say—as a fact, not as a boast—that he read Titchmarsh at the rate of three pages an hour. Walter’s remarkable memory for mathematics complemented his work ethic: Kersti Haliste once commented that if you needed to know the statement of a theorem from what was, to most others, a long forgotten talk, Walter could quote chapter and verse. He was eager to talk to and encourage all mathematicians with whom he came into contact. Gender, race, creed, nationality, politics, never entered the equation in his dealings with people. He would be equally patient and encouraging towards people at every level, treating everyone equally in terms of mathematics. He was invited to lecture all over the world, and would take the opportunity in his lectures and discussions to promote work done by others, especially junior mathematicians.

His sense of duty towards his subject and those working in function theory never failed. The British Mathematical Colloquium (the BMC)¹ is the largest UK pure mathematics conference, held annually at various venues, usually around Easter. The conference was especially popular with complex analysts in the 1980’s. Over the three days of the conference, as well as invited ‘morning speakers’ and ‘plenary speakers’², the afternoons were set aside for ‘special sessions’—populated in the main by PhD students giving short talks on their work. Walter attended every one, sitting at the front as overly enthusiastic students competed to get through the greatest number of slides in 20 minutes. At the end of each talk, Walter would patiently ask a question or make a comment which invariably cut to the heart of the matter and could set the direction for the next year of that student’s work. The students themselves might not have understood the value of Walter’s insights, but their supervisors certainly did.

Walter grew up in an academic family in Cologne—his father was Professor of Roman Law at Cologne and a classicist; his mother, Ruth Hensel, was the eldest

¹ <https://mathshistory.st-andrews.ac.uk/BMC/>

² Walter himself was twice an invited plenary speaker (1952, 1956) and twice an invited morning speaker (1970, 1980)

daughter of Kurt Hensel of p -adic numbers fame, and the great granddaughter of Fanny Mendelssohn, sister of Felix. In the 1930's, as Walter writes, 'the storm clouds were gathering'. In September 1938, aged only 12, Walter moved to Britain to attend Duffus House and then Gordonstoun, a private school that had been founded by a cousin of his mother. His parents escaped to England in April 1939, while his married sister Ilse remained in Rome. After Gordonstoun, Walter studied at Cambridge from October 1943, progressing to research under the supervision of Dame Mary Cartwright, FRS. Walter then held lectureships at Newcastle and Exeter. In 1947, he married Margaret Crann whom he had met at a Quaker meeting in Cambridge. Margaret was a mathematics educator, and they cofounded the British Mathematical Olympiad in 1966. In October 1947, the young couple moved to Exeter. From the beginning, Walter considered training future mathematicians to be an essential responsibility. His first doctoral student, P.B. Kennedy, came to work with Walter from University College Cork, defending his thesis in 1954. His second student, P.D. Barry, was also from Cork and succeeded Kennedy as professor there.

In 1956, Walter was both elected to the Royal Society and appointed Professor of Pure Mathematics at Imperial College. His career at Imperial stretched over 30 years, during which time the Pure Mathematics Section grew from a single professor to one of the strongest in the country. Walter took recruitment seriously, and was proud of Imperial's prestige throughout the international mathematical community.

Walter retired from Imperial in 1985 and moved with Margaret to York, Margaret's home county. They divided their time between the city of York, where the university had offered Walter a half time Professorship in Pure Mathematics, and the small village of Goathland, near Whitby. Margaret's health had been slowly deteriorating and she passed away in 1994. Walter subsequently returned to London and to Imperial where he was appointed Senior Research Fellow. He married his former student, Waficka Al-Katifi, in 1995. Waficka passed away in 2001 having been in declining health for many years.

All this and more is recounted by Walter in his autobiography *My Life and Functions* [14] which is a candid and personal account of his life. In the preface he thanks Tony O'Farrell 'who has edited and published this work when other friends told me it was not publishable'. *My Life and Functions* expands on his article 'Background and Early Life' [13], which appeared in a volume of *CMFT* [Volume 8 (2008), Numbers 1 & 2] dedicated to Walter on the occasion of his 80th birthday. The final chapter of his autobiography is entitled 'Marie'. After five years of widowhood, following a serendipitous meeting with Marie Jennings MBE, he proposed marriage on his second visit saying that he was getting on and didn't have time to waste. Walter writes [14, P. 121] 'A wedding of two old people has been described as a triumph of hope over experience. In our case the hope has been amply fulfilled.' Marie passed away on 27th April 2015.

Walter Hayman will be remembered not only for his mathematics but also as a warm, kind and generous man. He is greatly missed by his many friends and colleagues and the many people he helped over a lifetime.

Honours Prizes

- Junior Berwick Prize (1955).
- Senior Berwick Prize (1964).
- De Morgan Medal (1995).

Societies

- The Royal Society (FRS) (1956)
- Finnish Academy of Science and Letters, “Foreign Member” (1978)
- Bavarian Academy (1982)
- Accademia dei Lincei, “Foreign Member” (1985)

Honorary Doctorates

- University of Exeter (1981)
- University of Birmingham (1985)
- Uppsala University, Sweden (1992)
- Giessen University, Germany (1992)
- National University of Ireland (1997)

References

1. Beurling, A.: Some theorems on boundedness of analytic functions. *Duke Math. J.* **16**, 355–359 (1949)
2. de Branges, L.: A proof of the Bieberbach conjecture. *Acta Math.* **154**(1–2), 137–152 (1985). <https://doi.org/10.1007/BF02392821>
3. Hayman, W.K.: The minimum modulus of large integral functions. *Proc. Lond. Math. Soc.* **2**(3), 469–512 (1952). <https://doi.org/10.1112/plms/s3-2.1.469>
4. Hayman, W.K.: The asymptotic behaviour of p -valent functions. *Proc. Lond. Math. Soc.* **5**(3), 257–284 (1955). <https://doi.org/10.1112/plms/s3-5.3.257>
5. Hayman, W.K.: *Multivalent Functions*, Cambridge Tracts in Mathematics and Mathematical Physics, No. 48. Cambridge University Press, Cambridge (1958)
6. Hayman, W.K.: On successive coefficients of univalent functions. *J. Lond. Math. Soc.* **38**, 228–243 (1963). <https://doi.org/10.1112/jlms/s1-38.1.228>
7. Hayman, W.K.: *Meromorphic Functions*, Oxford Mathematical Monographs. Clarendon Press, Oxford (1964)
8. Hayman, W.K.: *Research Problems in Function Theory*. The Athlone Press University of London, London (1967)
9. Hayman, W.K., Kennedy, P.B.: *Subharmonic Functions*. Vol. I, London Mathematical Society Monographs, No. 9. Academic Press [Harcourt Brace Jovanovich, Publishers], London-New York (1976)
10. Hayman, W.K., Wu, J.M.G.: Level sets of univalent functions. *Comment. Math. Helv.* **56**(3), 366–403 (1981). <https://doi.org/10.1007/BF02566219>
11. Hayman, W.K.: *Subharmonic Functions*. Vol. 2, London Mathematical Society Monographs, vol. 20. Academic Press Inc. [Harcourt Brace Jovanovich, Publishers], London (1989)
12. Hayman, W.K.: *Multivalent Functions*, 2nd ed., Cambridge Tracts in Mathematics, vol. 110. Cambridge University Press, Cambridge (1994)
13. Hayman, W.K.: My background and early life. *Comput. Methods Funct. Theory* **8**(1–2), xi–xxvi (2008)
14. Hayman, W.K.: *My Life and Functions*. Logic Press, Kildare (2014)
15. Hayman, W.K., Lingham, E.F.: *Research Problems in Function Theory*, Problem Books in Mathematics. Springer, Cham (2019) (Fiftieth anniversary edition of Ref. [8])
16. Wegert, Elias: About the cover: the minimum modulus problem for entire functions. *Comput. Methods Funct. Theory* **21**(1), 1–5 (2021). <https://doi.org/10.1007/s40315-021-00366-9>

17. Wiman, A.: Über den Zusammenhang zwischen dem Maximalbetrage einer analytischen Funktion und dem grössten Betrage bei gegebenem Argumente der Funktion. *Acta Math.* **41**(1), 1–28 (1916). <https://doi.org/10.1007/BF02422938>. (German)

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