Walter Kurt Hayman: 1926–2020

David Drasin¹

Published online: 1 June 2020

© Springer-Verlag GmbH Germany, part of Springer Nature 2020

After a long and distinguished career, a dominant force in complex analysis passed away on January 1, 2020, a few days short of his 94th birthday. Walter Hayman's research contributions already established his reputation by age 30, but his famous Monday morning seminar at Imperial College and his decades of international contacts, initiatives and research advocacy were a stable presence for well over 60 years. Several aspects of his personal history have been documented for broader audiences [2,8].

Walter was born in Cologne to an academic family, and shared his middle name with his grandfather Kurt Hensel, today remembered for introducing p -adic numbers. Although his family considered itself Protestant, their Jewish ancestry and the ascendent Hitlerian racial laws made imperative his leaving for the UK in 1938, his parents were fortunate to later find refuge there also.

Communicated by Doron Lubinsky.

✉ David Drasin
drasin@purdue.edu

¹ Department of Mathematics, Purdue University, 150 N. University Street, West Lafayette, IN 47907-2067, USA

In his autobiography [8] Walter indicates that he was attracted to mathematics as a child. He was fortunate to begin his research at Cambridge, home to Littlewood and his supervisor, Mary Cartwright, where complex analysis was of major interest.

His research style was apparent from the start: incorporating fundamental research tools as springboard for his own methods and questions concerning analytic functions. The first results yielded concrete examples of how restrictions on the range of an analytic/meromorphic function are reflected in limiting the function's growth. The model is Schottky's famous theorem: if f is analytic in $\mathbb{D} = \{|z| < 1\}$ and omits (normalization) 0 and 1, then

$$\log |f(z)| \leq C(|f(0)|) \frac{1 + |z|}{1 - |z|},$$

and Walter augmented this to show how omitting a sequence of values further modifies the right side; some problems he raised are still open, see [10, Ch. 5].

The welding of his widely-recognized research power to producing formal collections of open problems was Walter's special contribution. This was manifested in two ways. In the first place, his books were convenient introductions to several research areas, the most prominent being Multivalent Functions [3], Meromorphic Functions [4] and Subharmonic Functions (two volumes, the first with Kennedy) [6,7]) almost all of which remain in print, some also in translation. Some results appear only in these volumes, and generations of analysts (including this writer) began independent research relying on them. The style was always direct and non-pompous, leading to the key results with a minimum of formalism. As an example a reader of [4] masters the fundamentals of Nevanlinna's theory of value distribution in less than 50 pages, while Nevanlinna's earlier classic book [12] needs over 250 pages to attain that summit.

In addition to these formal texts, the two collections [9] and its revised version [10] develop a catalogue of open problems covering most of complex analysis. Each chapter begins with a short survey of a specific area, with background discussion and bibliographical references. This was initiated not only in pre-internet days, but at a period where professional communication could be very restricted, and at times impossible. As a result, libraries and offices held onto yellowed copies of [9] for many decades. Walter thus became the natural node of correspondence with analysts everywhere, and his initiatives, suggestions and letters inspired and encouraged generations of mathematicians. The announcement of Walter's death by Imperial College [11] calls attention to his mentoring contributions, even at the end of his career. This helped develop an international research community whose vitality and scope is reflected in the orientation of this journal. Of course, Walter was on the CMFT editorial board from the first issue.

That the genealogy product credits Walter with 20 students thus greatly understates his full contribution. Many other analysts were trained by other members of the Imperial College Function Theory group, or spent sabbaticals or post-docs at I. C. In addition, Walter had an influence with pre-university UK students by joining his wife Margaret to have Britain participate in the International Mathematical Olympiad.

MathSciNet credits Walter with 227 publications; (some of these are useful surveys, in recent years the density of obituaries increased). A detailed account of the research

is not appropriate here. However, we note that his breakthroughs concerning minimum modulus and univalent functions (some presented in [3]) led him to both Junior and Senior Berwick Prizes, and he was elected a Fellow of the Royal Society at age 30. He also gave invited addresses to the ICM twice.

An account of some of the results and consequences of his most cited paper [5] may offer a glimpse of this picture. We have mentioned Schottky's theorem, but this was linked to Picard's original breakthrough that if f is meromorphic and omits three values (say 0, 1, ∞) it must be constant; here is the genesis of the Nevanlinna theory (recall [4]). Miranda had modified Picard's theorem by replacing $f \neq 1$ to $f' \neq 1$ but in [5], Hayman showed that Miranda's theorem remains valid for *meromorphic functions* with no additional restrictions: if $f \neq 0$, $f' \neq 1$, then f is constant even if f is meromorphic. This led to many fascinating and deep questions to reveal additional secrets incorporated in the derivative of a function, and [5] when united with earlier work of Pólya form the prelude to [4, Ch. 3]. A famous conjecture introduced there was that if f is meromorphic with $f \neq 0$, $f^{(m)} \neq 0$ for some $m \geq 2$, then $f(z) = e^{az+b}$ or $f(z) = (az + b)^{-k}$, $k \geq 1$. Walter discovered that a claimed partial solution was inadequate, and he settled the case $m = 2$ for entire functions in [5]. The chapter brought attention to Pólya's work on related questions, culminating in results of Frank [1] and others.

Colleagues who were able to share his love of music and literature carry additional warm memories. Walter became interested in the Quaker religion at Cambridge, and met Margaret there; he remained dedicated to their creed and spirit for the rest of his life. This included support for human rights and welcoming refugees.

References

1. Frank, G.: Eine Vermutung von Hayman über Nullstellen meromorpher Funktionen. *Math. Z.* **149**, 29–36 (1976)
2. Hayman, S.: Mendelssohn, the Nazis and me, video. <https://www.imdb.com/title/tt1557715>
3. Hayman, W.K.: *Multivalent Functions*. Cambridge University Press, Cambridge (1958)
4. Hayman, W.K.: *Meromorphic Functions*. Oxford University Press, Oxford (1964)
5. Hayman, W.K.: Picard values of meromorphic functions and their derivatives. *Ann. Math.* **2**, 70 (1959)
6. Hayman, W.K., Kennedy, P.B.: *Subharmonic Functions*, vol. 1. Academic Press, London (1976)
7. Hayman, W.K.: *Subharmonic Functions*, vol. 2. Academic Press, London (1989)
8. Hayman, W.K.: *My Life and Functions*. Logic Press, Kildare (2014)
9. Hayman, W.K.: *Research Problems in Function Theory*. Althlone Press, London (1967)
10. Hayman, W.K., Lingham, E.F.: *Research Problems in Function Theory* Fiftieth anniversary edition of [9]. Springer, New York (2019)
11. <https://www.imperial.ac.uk/news/194794/tributes-paid-professor-walter-hayman-launch/>
12. Nevanlinna, R.: *Analytic Functions*. Springer, New York (1970)

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.