



# A new spherical aggregation function with the concept of spherical fuzzy difference for spherical fuzzy EDAS and its application to industrial robot selection

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## Abstract

In this article, a new fully fuzzy approach is developed for the evaluation based on distance from average solution (EDAS) for multi-criteria decision-making (MCDM) using spherical fuzzy sets (SFSs). The proposed approach avoids the current limitations and drawbacks of distance-based methods in general and the EDAS method in particular using spherical fuzzy information namely, early defuzzification, the flaws of distance measures, and the undefined spherical fuzzy subtraction and division operations. First, the approach employs the score function only in the final step for ranking. Second, the concept of the spherical fuzzy difference is introduced to make up for the subtraction operation which is the backbone of EDAS and as a substitute for distance measures. The spherical fuzzy difference is utilized to indicate any increase or decrease in the membership degree, the non-membership degree, and the hesitancy degree in the performance of an alternative for a criterion than that of its peer in the average solution. Then, the weighted spherical differences are calculated. The total weighted spherical differences from the average solution of each alternative for the assessment criteria are aggregated in the appraisal score. Due to a flaw in the extant aggregation operators, their results might be misleading. Therefore, an aggregation function is introduced that guarantees a balanced and fair aggregation. The appraisal scores are defuzzified, and the alternative with the highest appraisal score is the best. Two practical examples in MCDM are solved and a comparative study is presented to demonstrate and validate the algorithm.

**Keywords** Spherical fuzzy sets · Aggregation functions · The EDAS method · Multi-criteria decision-making · Supplier selection · Industrial robot selection

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## 1 Introduction

Multi-criteria decision-making (MCDM) encompasses plenty of methods that help decision-makers in the assessment and the selection of the best candidate from a set of alternatives based on many diverse, even conflicting, criteria. A new era in MCDM began when Zadeh (1965) introduced the notion of fuzzy sets. The fuzzy set theory proved to be powerful in handling various MCDM problems to overcome uncertainty, imprecise data, and ambiguous information. Fuzzy MCDM has been extensively used in a wide range of real-life applications, e.g., evaluation of mergers and acquisitions (Zhang et al. 2016), assessment of alternative vehicle technologies' performance (Onat et al. 2016), green supplier chain management (Ali et al. 2021), evaluation of site selection indicators for locating vehicle shredding facilities (Deveci et al. 2022c), and the evaluation of safe e-scooter strategies (Deveci et al. 2022a). Over the past decades, several researchers have proposed many extensions and generalizations of fuzzy sets. Zadeh (1975) introduced a more elaborate form of fuzzy sets, type-2 fuzzy sets. However, due to the computational complications of type-2 fuzzy sets, several simple variants were proposed, e.g., interval-valued and interval type-2 fuzzy sets. Atanassov (1986) introduced intuitionistic fuzzy sets (IFSs) to better depict the alternatives' information under incomplete and uncertain data. Nevertheless, IFSs may fail in some cases to fulfill the condition on the sum of the membership degree and the non-membership degree to be at most one. Several extensions of IFSs were introduced, their main goal is to handle uncertainty and clarify the indeterminacy with a high level of reliability (Ilbahar et al. 2018).

Smarandache (1998) introduced neutrosophic fuzzy sets (NFSs) as an extension of IFSs, defining the degree of indeterminacy as an independent parameter of the set. Therefore, real-life problems can be represented more accurately. The sum of three parameters of the set can be between 0 and 3, with each parameter varying between 0 and 1, independently. Yager (2014) introduced the Pythagorean fuzzy sets (PFSs). The condition on the sum of the membership degree and the non-membership degree is modified to take the sum of the squares instead. This gives more room for decision-makers to express their preference. Yager (2017) also introduced the q-rung orthopair fuzzy sets as a generalization of IFSs and PFSs. They are characterized by a pair of degrees, a membership degree ( $\mu$ ) and a non-membership degree ( $\nu$ ), satisfying the relation  $\mu^q + \nu^q \leq 1$ , for  $q \geq 1$ .

Intuitionistic and Pythagorean fuzzy sets or q-rung orthopair sets can handle the hesitancy of the decision-makers in a limited way because they do not consider an independent hesitancy degree. Regarding neutrosophic sets, independent hesitancy is considered, but different complications arise from the condition on the sum of the three parameters, making it difficult to utilize in scientific and engineering applications. Spherical fuzzy sets (SFSs) were developed to handle these issues, and their operations are proposed (Gül 2021). SFSs are new members of the fuzzy sets' family. They were introduced to fulfill the aspiration of researchers and practitioners to express human perception and cognition effectively and efficiently. The novel concept of SFSs has a major advantage; the decision-makers are free to express their hesitance independent of the membership degree and non-membership degree. Hence, higher independence opportunities are given to the decision-makers. In addition, the geometric spherical surface gives the decision-makers a broader preference domain and makes experts more free to assign the values of parameters of the set. Similar to PFSs, the

sum of the squares of the membership degree, the non-membership degree, and the hesitancy degree is considered. Yet, for SFSs the sum is less than or equal to one.

The development of novel MCDM methods and the improvement of the extant techniques or the integration of these techniques to build efficient and robust structures that can handle complex decisions arising from recent emerging technologies receive enormous interest among researchers. For example, Gorcun et al. (2021) proposed an integrated fuzzy approach consisting of the fuzzy Step-Wise Weight Assessment Ratio Analysis (F-SWARA) and the combinative Distance-based assessment (F-CODAS) techniques to select the best tanker vehicle option. Iordache et al. (2022) proposed an improved interval rough Measurement Alternatives and Ranking according to the COMpromise Solution (MARCOS) methodology for hydrogen gas grid development via natural gas network conversion. Deveci et al. (2022b) improved the Weighted Aggregated Sum Product Assessment (WASPAS) approach to evaluate and rank the prioritization of climate change mitigation strategies using triangular fuzzy numbers. Yu and Xu (2022) established the advantage matrix and designed new effective and reliable algorithms.

The evaluation based on distance from average solution (EDAS) method is one of the recently proposed MCDM techniques that simplify the traditional decision-making process (Liang 2020). The EDAS method has a significant role in solving MCDM problems especially when more conflicting criteria exist. It is steady and progressive when various weights of the criteria are allocated. In addition, it has the potential to produce more accurate results with fewer calculations (Batool et al. 2021). It also makes the compromise solution method more stable (Gül 2021).

The EDAS method is one of the distance-based multi-criteria decision-making methods similar to the technique of order preference by similarity to the ideal solution (TOPSIS) and multi-criteria optimization and compromise solution (VIKOR). TOPSIS and VIKOR consider the measure of the distances between each alternative and the positive and the negative ideal solutions as a decision criterion. Then, a positive and a negative ideal solution must be determined. EDAS can eliminate this additional step since it measures the distance between an alternative and the average solution. Therefore, decision-makers do not have to obtain positive and negative ideal solutions; they just need to compute the average performance scores of each attribute. For this, the EDAS method simplifies the calculation of distances and determines the final decision rapidly (Karaşan and Kahraman 2018). EDAS also outperforms TOPSIS and VIKOR in terms of time complexity due to the elimination of non-prospective candidates. Besides, the method is favorable when there is information about the preferred average value of the attributes' evaluation (Ilieva et al. 2018).

In the spherical fuzzy environment, distance-based methods have two main deficiencies that can lead to incorrect results or hinder their implementation. First, as SFSs are recently introduced, the newly developed score functions might produce different ranks since they have not been extensively studied (Kutlu Gündoğdu and Kahraman 2020). As a result, the positive and negative ideal solutions might change according to the score function employed. Second, the distance between the positive and negative ideal solutions might not be the largest. Therefore, the EDAS method is superior in this environment since it has the merit of using the average solution not the positive and the negative ideal solutions. In addition, it is always better to develop fully fuzzy approaches to utilize the score functions in the final step only for ranking with the aid of the accuracy function, if required, to lessen their effect.

A major concern in distance measures in a spherical fuzzy environment is handling the three parameters of a SFS in the same manner despite the different influences of each parameter. For example, an increase in the membership degree has the same effect as an increase

in the non-membership degree, although the former is an advantage and the latter is a disadvantage. Herein, it is important to find a substitute for distance measures that takes the different influences of each parameter of a SFS into consideration when comparing the distance between SFSs.

In this article, EDAS is extended to handle MCDM problems with SFSs. Since SFSs are non-negative and the subtraction operation is not defined for SFSs, the main challenge is to perform the algorithm under these conditions. Instead of using subtraction, the spherical fuzzy differences are utilized to indicate any increase or decrease in the membership degree, the non-membership degree, and the hesitancy degree in the performance of an alternative for a criterion than that of its peer in the average solution. The weighted differences are calculated. Then, the total weighted differences are aggregated in the appraisal score of each alternative. Due to a drawback in the extant weighted averaging operators, they might not be able to represent the actual situation. Therefore, an aggregation function is introduced which guarantees a balanced aggregation and depicts the actual situation fairly. The appraisal scores are defuzzified, and the alternatives are ranked in descending order. The alternative with the highest appraisal score is the best candidate. Two practical examples are solved with the proposed SF-EDAS to demonstrate the algorithm. The results are compared mainly with the results of some extant MCDM methods to validate the algorithm.

In the nutshells, the main contributions of the article are:

- (1) Due to the deficiencies of distance-based methods in the spherical fuzzy environment, a spherical fully fuzzy EDAS algorithm is developed that surpasses these deficiencies and encompasses all the advantages of SFSs over the other fuzzy set types.
- (2) It is known that a proposed MCDM method is robust if they obtained results are invariant under different distance metrics (Ilieva et al. 2018). Hence, the concept of spherical fuzzy difference is introduced as a substitute for distance measures and to account for negative distances due to the absence of the subtraction operation.
- (3) Due to certain flaw in the extant aggregation operators, a new aggregation function is proposed that ensures balanced aggregation, increasing the accuracy of the results.

The article is organized as follows. The basic operations on SFSs, a literature review on the EDAS method, and the ordinary EDAS algorithm are given in Sect. 2. The concept of spherical fuzzy difference, the proposed aggregation function, and the SF-EDAS are developed in Sect. 3. In Sect. 4, two practical examples are solved to demonstrate the method, and a comparative analysis is presented. Finally, the conclusion is given in Sect. 5.

## 2 Preliminaries

In this section, we briefly review some basic concepts about the SFSs.

### 2.1 Spherical fuzzy sets

A SFS is expressed by

$$\tilde{A}_S = \left\{ \left\langle u, \left( \mu_{\tilde{A}_S}(u), \nu_{\tilde{A}_S}(u), \pi_{\tilde{A}_S}(u) \right) \mid u \in U \right\rangle \right\},$$

where  $\mu_{\tilde{A}_S} : U \rightarrow [0, 1]$  is the membership degree,  $\nu_{\tilde{A}_S} : U \rightarrow [0, 1]$  is the non-membership degree,  $\pi_{\tilde{A}_S} : U \rightarrow [0, 1]$  is the hesitancy degree, and  $0 \leq \mu_{\tilde{A}_S}^2(u) + \nu_{\tilde{A}_S}^2(u) + \pi_{\tilde{A}_S}^2(u) \leq 1, \forall u \in U$ .

**Definition 2.1.1** (Gündoğdu and Kahraman 2019). Let  $\tilde{A}_s = (\mu_{\tilde{A}_s}, \nu_{\tilde{A}_s}, \pi_{\tilde{A}_s})$  and  $\tilde{B}_s = (\mu_{\tilde{B}_s}, \nu_{\tilde{B}_s}, \pi_{\tilde{B}_s})$  be two SFSs. The operational laws of SFSs are given as follows.

Addition

$$\tilde{A}_s \oplus \tilde{B}_s = \left\{ \left( \mu_{\tilde{A}_s}^2 + \mu_{\tilde{B}_s}^2 - \mu_{\tilde{A}_s}^2 \mu_{\tilde{B}_s}^2 \right)^{1/2}, \nu_{\tilde{A}_s} \nu_{\tilde{B}_s}, \left( (1 - \mu_{\tilde{A}_s}^2) \pi_{\tilde{A}_s}^2 + (1 - \mu_{\tilde{B}_s}^2) \pi_{\tilde{B}_s}^2 - \pi_{\tilde{A}_s}^2 \pi_{\tilde{B}_s}^2 \right)^{1/2} \right\}, \tag{1}$$

and

$$\tilde{A}_s \oplus \tilde{B}_s = \tilde{B}_s \oplus \tilde{A}_s.$$

Multiplication by a scalar; For real  $\lambda > 0$

$$\lambda \odot \tilde{A}_s = \left\{ \left( 1 - (1 - \mu_{\tilde{A}_s}^2)^\lambda \right)^{1/2}, \nu_{\tilde{A}_s}^\lambda, \left[ (1 - \mu_{\tilde{A}_s}^2)^\lambda - (1 - \mu_{\tilde{A}_s}^2 - \pi_{\tilde{A}_s}^2)^\lambda \right]^{1/2} \right\} \tag{2}$$

and.

$$\lambda \odot (\tilde{A}_s \oplus \tilde{B}_s) = \lambda \odot \tilde{A}_s \oplus \lambda \odot \tilde{B}_s.$$

Multiplication

$$\tilde{A}_s \otimes \tilde{B}_s = \left\{ \mu_{\tilde{A}_s} \mu_{\tilde{B}_s}, \left( \nu_{\tilde{A}_s}^2 + \nu_{\tilde{B}_s}^2 - \nu_{\tilde{A}_s}^2 \nu_{\tilde{B}_s}^2 \right)^{1/2}, \left( (1 - \nu_{\tilde{A}_s}^2) \pi_{\tilde{A}_s}^2 + (1 - \nu_{\tilde{B}_s}^2) \pi_{\tilde{B}_s}^2 - \pi_{\tilde{A}_s}^2 \pi_{\tilde{B}_s}^2 \right)^{1/2} \right\}. \tag{3}$$

**Definition 2.1.2** (Gündoğdu and Kahraman 2019). The score and accuracy functions for ranking SFSs are given by:

$$\text{Score}(\tilde{A}_s) = (\mu_{\tilde{A}_s} - \pi_{\tilde{A}_s})^2 - (\nu_{\tilde{A}_s} - \pi_{\tilde{A}_s})^2. \tag{4}$$

$$\text{Accuracy}(\tilde{A}_s) = \mu_{\tilde{A}_s}^2 + \nu_{\tilde{A}_s}^2 + \pi_{\tilde{A}_s}^2. \tag{5}$$

An order relation between two SFSs  $\tilde{A}_s$  and  $\tilde{B}_s$  is stated as

$$\tilde{A}_s < \tilde{B}_s \text{ if } \text{Score}(\tilde{A}_s) < \text{Score}(\tilde{B}_s),$$

$$\text{or } \text{Score}(\tilde{A}_s) = \text{Score}(\tilde{B}_s) \text{ and } \text{Accuracy}(\tilde{A}_s) < \text{Accuracy}(\tilde{B}_s).$$

**Definition 2.1.3** (Ashraf et al. 2019). The conjugate of a SFS is given by:

$$\tilde{A}_s^c = (\nu_{\tilde{A}_s}, \mu_{\tilde{A}_s}, \pi_{\tilde{A}_s}). \tag{6}$$

**Definition 2.1.4** (Gündoğdu and Kahraman 2019). The aggregation operators namely, the spherical weighted arithmetic mean (SWAM) and the spherical weighted geometric mean operators (SWGGM) are given by:

$$\begin{aligned} \text{SWAM}_w(\tilde{A}_{S_1}, \tilde{A}_{S_2}, \dots, \tilde{A}_{S_n}) &= w_1 \tilde{A}_{S_1} \oplus w_2 \tilde{A}_{S_2} \oplus \dots \oplus w_n \tilde{A}_{S_n} \\ &= \left\{ \left[ 1 - \prod_{i=1}^n (1 - \mu_{\tilde{A}_{S_i}}^2)^{w_i} \right]^{1/2}, \prod_{i=1}^n \nu_{\tilde{A}_{S_i}}^{w_i}, \left[ \prod_{i=1}^n (1 - \mu_{\tilde{A}_{S_i}}^2)^{w_i} - \prod_{i=1}^n (1 - \mu_{\tilde{A}_{S_i}}^2 - \pi_{\tilde{A}_{S_i}}^2)^{w_i} \right]^{1/2} \right\}, \end{aligned} \tag{7}$$

where  $w_i \in [0, 1]; \sum_{i=1}^n w_i = 1$ .

$$\begin{aligned} \text{SWGGM}_w(\tilde{A}_{S_1}, \tilde{A}_{S_2}, \dots, \tilde{A}_{S_n}) &= \tilde{A}_{S_1}^{w_1} \otimes \tilde{A}_{S_2}^{w_2} \otimes \dots \otimes \tilde{A}_{S_n}^{w_n} \\ &= \left\{ \begin{aligned} &\prod_{i=1}^n \mu_{\tilde{A}_{S_i}}^{w_i}, \left[ 1 - \prod_{i=1}^n \left( 1 - \nu_{\tilde{A}_{S_i}}^2 \right)^{w_i} \right]^{1/2}, \\ &\left[ \prod_{i=1}^n \left( 1 - \nu_{\tilde{A}_{S_i}}^2 \right)^{w_i} - \prod_{i=1}^n \left( 1 - \nu_{\tilde{A}_{S_i}}^2 - \pi_{\tilde{A}_{S_i}}^2 \right)^{w_i} \right]^{1/2} \end{aligned} \right\}, \end{aligned} \tag{8}$$

where  $w_i \in [0, 1]; \sum_{i=1}^n w_i = 1$ .

The Euclidean distance between two SFSs is given by (Kutlu Gündoğdu and Kahraman 2019)

$$\text{dis}_E(\tilde{A}_s, \tilde{B}_s) = \sqrt{(\mu_{\tilde{A}_s} - \mu_{\tilde{B}_s})^2 + (\nu_{\tilde{A}_s} - \nu_{\tilde{B}_s})^2 + (\pi_{\tilde{A}_s} - \pi_{\tilde{B}_s})^2}. \tag{9}$$

The Xu and Zhang’s distance is given by:

$$\text{dis}_{XZ}(\tilde{A}_s, \tilde{B}_s) = \frac{1}{2} \left| (\mu_{\tilde{A}_s} - \mu_{\tilde{B}_s})^2 + (\nu_{\tilde{A}_s} - \nu_{\tilde{B}_s})^2 + (\pi_{\tilde{A}_s} - \pi_{\tilde{B}_s})^2 \right|. \tag{10}$$

The spherical distance is given by:

$$\text{dis}_{Sp}(\tilde{A}_s, \tilde{B}_s) = \frac{2}{\pi} \left( \arccos(\mu_{\tilde{A}_s} \mu_{\tilde{B}_s} + \nu_{\tilde{A}_s} \nu_{\tilde{B}_s} + \pi_{\tilde{A}_s} \pi_{\tilde{B}_s}) \right). \tag{11}$$

## 2.2 The EDAS method

### 2.2.1 A literature review

The evaluation based on distance from average solution (EDAS) method is a recent MCDM method. Ghorabae et al. (2015) introduced EDAS as an effective method for managing and controlling a large number of inventory items or stock-keeping units (SKUs), i.e., inventory classification with crisp data. The evaluation process in the EDAS method is based on the distances of the alternatives from the average scores of the attributes for the total appraisal of the alternatives. Two types of distances are defined, the positive distance from the average (PDA), and the negative distance from the average (NDA) for the assessment of the alternatives. The alternative with the highest PDA total score and lowest NDA total score is considered the best. Thus, the EDAS model owns the merit of taking the average solutions only into account concerning the intangibility of decision-makers and the uncertainty of the decision-making environment to obtain more accurate and effective aggregation results (Zhang et al. 2019a). Compared to other decision-making and classification methods, the EDAS method has high efficiency and needs fewer computations (He et al. 2019). It is more accurate and effective for considering the conflicting attributes (Zhang et al. 2019b).

The EDAS method was extended to handle MCDM problems with trapezoidal type-1 (Ghorabae et al. 2016) and trapezoidal interval type-2 fuzzy sets (Ghorabae et al. 2017b). Ghorabae et al. (2017a) proposed a new integrated model based on IT2FSSs and the EDAS method for supplier evaluation and order allocation with environmental consideration. Keshavarz Ghorabae et al. (2017) proposed a stochastic EDAS method to handle problems in which the performance values of alternatives on each criterion follow the normal distribution, and used the proposed method to evaluate the performance of bank branches.

Kahraman et al. (2017) proposed an intuitionistic fuzzy EDAS for the management of solid wastes. Peng and Liu (2017) proposed three algorithms to solve single-valued neutrosophic soft decision-making problems based on EDAS, similarity measure, and level soft set. Ecer (2018) proposed a novel integrated model composed of the fuzzy Analytical Hierarchy Process (AHP) and EDAS to select proper third-party logistics providers, in which Fuzzy AHP is used for calculating priority weights of each criterion and EDAS is employed to achieve the final ranking. Feng et al. (2018) proposed an EDAS method for the extended hesitant fuzzy linguistic environment. Ilieva (2018) proposed two variants of EDAS for interval type-2 fuzzy sets to improve the computations. In addition, Ilieva et al. (2018) proposed a new variant of the EDAS method for trapezoidal type-1 fuzzy sets which reduces the number of needed calculations without affecting the quality of the solution. Karaşan and Kahraman (2018) introduced an interval-valued neutrosophic version of EDAS with the advantage of considering experts' truthiness, falsity, and indeterminacy simultaneously. The proposed method was applied to the prioritization of The United Nations' national sustainable development goals. Keshavarz-Ghorabae et al. (2018) proposed a fuzzy dynamic MCGDM approach based on the EDAS method for subcontractor evaluation.

He et al. (2019) proposed a novel integrated model using information entropy and EDAS under probabilistic uncertain linguistic sets, in which information entropy is used for deriving priority weights of each attribute and EDAS is employed to obtain the final ranking of a green supplier. Li et al. (2019) extended the traditional EDAS method to solve the MAGDM problem in the picture fuzzy environment. Wang et al. (2019) presented a 2-tuple linguistic neutrosophic EDAS model and employed this model for the safety assessment of construction projects. Mi and Liao (2019) utilized the Best Worst Method and the EDAS method with hesitant fuzzy information for choosing commercial endowment insurance products. Zhang et al. (2019a) designed an EDAS model with picture 2-tuple linguistic numbers for green supplier selection. Zhang et al. (2019b) also constructed a picture fuzzy EDAS model for green supplier selection.

Li et al. (2020) extended the EDAS method for multi-attribute group decision-making (MAGDM) under  $q$ -rung orthopair fuzzy sets. Liang (2020) extended the EDAS method to intuitionistic fuzzy environments to solve some MAGDM issues and utilized the method for evaluating green building energy-saving design projects. Yanmaz et al. (2020) extended the EDAS method using interval-valued Pythagorean fuzzy numbers for MCDM problems and applied the extended method to the car selection problem. Batool et al. (2021) established a multi-attribute decision-making approach based on the EDAS method under Pythagorean probabilistic hesitant fuzzy information to select an ideal drug from certain drugs with efficacy values for coronavirus disease. They also developed an algorithm to address the uncertainty in the selection of drugs in emergency decision-making issues concerning clinical analysis. Krishankumar et al. (2021) proposed an integrated framework composed of a novel attitudinal evidence-based Bayesian approach for criteria weight estimation, variance approach to determine experts' weights, and EDAS approach to prioritize zero-carbon measures. Li et al. (2021) proposed a new EDAS method using PFSs based on the prospect theory. They applied the proposed approach to select a suitable project for building a high-grade highway. Finally, Gül (2021) modified EDAS for the spherical fuzzy environment with Entropy-based objective attribute weighting to avoid the undesired potential effects of subjective weighting. In his study, Gül (2021) had to consider the score function value of the average solution rather than the average solution itself since there is no proposition for division operation of SFS and the development of division operation on SFS requires complex mathematics. This situation can be considered a limitation of the study.

From the previous studies, we conclude that the EDAS as an efficient, robust, and accurate method but it has not been well studied using spherical fuzzy information. Hence, it is required to propose new versions of the EDAS algorithm that can dodge the current limitations and drawbacks of the method namely the undefined spherical fuzzy subtraction and division operations, early defuzzification, and the flaws of distance measures.

### 2.2.2 The conventional EDAS algorithm

For a MCDM problem, let  $A = \{a_1, a_2, \dots, a_n\}$  be the set of alternatives,  $C = \{c_1, c_2, \dots, c_m\}$  the set of criteria, and  $D = \{D_1, D_2, \dots, D_k\}$  the set of decision-makers. The EDAS method can be summarized in the following steps (Ghorabae et al., 2015).

**Step 1:** Determine the most relevant criteria that describe the alternatives.

**Step 2:** Construct the decision matrix and the weighting matrix of the criteria according to the evaluation of the decision-makers.

$$X = [x_{ij}] = \begin{matrix} & c_1 & c_2 & \dots & c_m \\ \begin{matrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{matrix} & \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1m} \\ x_{21} & x_{22} & \dots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \dots & x_{nm} \end{bmatrix} \end{matrix} \text{ and } W = [w_1 \ w_2 \ \dots \ w_m],$$

where  $x_{ij}$  is the performance of the  $i$ th alternative for the  $j$ th criterion, and  $w_j$  is the weight of the  $j$ th criterion.

**Step 3:** Determine the average solution for each criterion.

$$av_j = \frac{\sum_{i=1}^n x_{ij}}{n}. \tag{12}$$

**Step 4:** Compute the matrices of the PDA and the NDA.

$$PDA = [PDA_{ij}], \quad NDA = [NDA_{ij}],$$

$$\text{where } PDA_{ij} = \begin{cases} \frac{\max(0, (x_{ij} - av_j))}{av_j} & \text{if } j \in \text{benefit criteria} \\ \frac{\max(0, (av_j - x_{ij}))}{av_j} & \text{if } j \in \text{cost criteria,} \end{cases} \tag{13}$$

$$\text{and } NDA_{ij} = \begin{cases} \frac{\max(0, (av_j - x_{ij}))}{av_j} & \text{if } j \in \text{benefit criteria} \\ \frac{\max(0, (x_{ij} - av_j))}{av_j} & \text{if } j \in \text{cost criteria.} \end{cases} \tag{14}$$

**Step 5:** Compute the weighted sum of the PDA and the NDA for each alternative.

$$SP_i = \sum_{j=1}^m w_j PDA_{ij}, \tag{15}$$

$$SN_i = \sum_{j=1}^m w_j NDA_{ij}. \tag{16}$$

**Step 6:** Normalize the weighted sums for all the alternatives.

$$NSP_i = \frac{SP_i}{\max_i(SP_i)}, \tag{17}$$



$$NSN_i = 1 - \frac{SN_i}{\max_i(SN_i)}. \tag{18}$$

**Step 7:** Compute the appraisal score for all the alternatives.

$$AS_i = \frac{1}{2}(NSP_i + NSN_i), \text{ where } 0 \leq AS_i \leq 1. \tag{19}$$

**Step 8:** Rank the alternatives in decreasing order according to the appraisal score. The alternative with the highest appraisal score is the best candidate.

### 3 The proposed SF-EDAS algorithm

In this section, a new method named as SF-EDAS is proposed. In this method, the performance of the alternatives and the weights of the criteria are represented in the form of SFSs. Since the subtraction operation is not defined for SFSs, the algorithm will count on the spherical fuzzy difference instead. First, the formation of the spherical fuzzy difference to express the difference in the performance of an alternative from the average solution is explained. Second, the proposed spherical weighted averaging operator is developed. Third, the steps of the algorithm are demonstrated. In the case of cost criteria, the conjugate given by Eq. (6) is used. The first two steps, the selection of the criteria and the formation of the decision and weight matrices, are standard in most MCDM methods. Hence, the illustration of the algorithm will start after these three steps.

#### 3.1 The formation of spherical fuzzy difference

Let  $\tilde{x}_{ij} = (\mu_{ij}, \nu_{ij}, \pi_{ij})$  be the performance of the  $i$ th alternative for the  $j$ th criterion. A membership degree  $\mu_{ij}$  denotes the extent an alternative fulfills a criterion. A non-membership degree  $\nu_{ij}$  denotes the extent an alternative does not fulfill a criterion. A hesitancy degree  $\pi_{ij}$  denotes the reliability of the information represented by an alternative (Szmidt and Kacprzyk 2004).

The distance between the two performances  $\tilde{x}_{ij}$  and the average solution  $\tilde{a}v_j = (\mu_j, \nu_j, \pi_j)$  is calculated using the Hamming distance as follows.

$$d_H(\tilde{x}_{ij}, \tilde{a}v_j) = |\mu_{ij} - \mu_j| + |\nu_{ij} - \nu_j| + |\pi_{ij} - \pi_j|. \tag{20}$$

Then, the performance  $\tilde{x}_{ij}$  is compared with the average solution  $\tilde{a}v_j$  per degree to form the spherical fuzzy difference  $\tilde{S}_{ij}^\Delta = (\mu_\Delta, \nu_\Delta, \pi_\Delta)$ . An increment in  $\mu$  indicates an increase in the agreement on an alternative. An increment in  $\nu$  indicates an increase in the disagreement on an alternative. While an increment in  $\pi$  indicates an increase in the hesitation margin and thus less reliability. Alternatively, a decrement in  $\mu$  indicates an increase in the disagreement on an alternative. A decrement in  $\nu$  indicates an increase in the agreement on an alternative. While a decrement in  $\pi$  indicates a decrease in the hesitation margin and thus more reliability, hence an increase in the agreement on the alternative.

First, compare the membership degrees. If  $\mu_{ij} > \mu_j$ , in this case, the increase in the membership degree of  $\tilde{x}_{ij}$  than its peer in the average  $\tilde{a}v_j$  is considered an advantage. Therefore, we set the membership degree

$$\mu_{\Delta\mu^+} = \frac{|\mu_{ij} - \mu_j|}{d_H}, \tag{21}$$

which represents the percentage of agreement on the alternative according to its increment from the average.

On the contrary, if  $\mu_{ij} < \mu_j$ , the decrease in the membership degree of  $\tilde{x}_{ij}$  than its peer in the average  $\tilde{a}\tilde{v}_j$  is considered as a disadvantage. Therefore, we set the non-membership degree.

$$v_{\Delta\mu^-} = \frac{|\mu_{ij} - \mu_j|}{d_H}, \tag{22}$$

which represents the percentage of rejection of the alternative due to its decrement from the average.

Second, compare the non-membership degrees. If  $v_{ij} > v_j$ , in this case, the increase in the non-membership degree of  $\tilde{x}_{ij}$  than its peer in the average  $\tilde{a}\tilde{v}_j$  is considered as a disadvantage. Therefore, we set the non-membership degree

$$v_{\Delta v^+} = \frac{|v_{ij} - v_j|}{d_H}, \tag{23}$$

which represents the percentage of rejection of the alternative according to its increment from the average.

On the other hand, if  $v_{ij} < v_j$ , the decrease in the non-membership degree of  $\tilde{x}_{ij}$  than its peer in the average  $\tilde{a}\tilde{v}_j$  is considered as an advantage. Therefore, we set the membership degree.

$$\mu_{\Delta v^-} = \frac{|v_{ij} - v_j|}{d_H}, \tag{24}$$

which represents the percentage of agreement on the alternative according to its decrement from the average.

Third, compare the hesitancy degrees. If  $\pi_{ij} > \pi_j$ , in this case, the increase in the hesitancy degree of  $\tilde{x}_{ij}$  than its peer in the average  $\tilde{a}\tilde{v}_j$  is considered as a disadvantage. Therefore, we set the hesitancy degree

$$\pi_{\Delta\pi^+} = \frac{|\pi_{ij} - \pi_j|}{d_H}, \tag{25}$$

which represents the percentage of reliability according to its increment from the average. Else if  $\pi_{ij} < \pi_j$ , the decrease in the hesitancy degree of  $\tilde{x}_{ij}$  than its peer in the average  $\tilde{a}\tilde{v}_j$  is considered as an advantage. Therefore, we set the membership degree.

$$\mu_{\Delta\pi^-} = \frac{|\pi_{ij} - \pi_j|}{d_H}, \tag{26}$$

which represents the percentage of agreement on the alternative according to its decrement from the average.

Finally, compute the spherical fuzzy difference  $\tilde{S}_{ij}^\Delta = (\mu_\Delta, v_\Delta, \pi_\Delta)$ , where

$$\mu_\Delta = \mu_{\Delta\mu^+} + \mu_{\Delta v^-} + \mu_{\Delta\pi^-}, v_\Delta = v_{\Delta\mu^-} + v_{\Delta v^+} \text{ and } \pi_\Delta = \pi_{\Delta\pi^+} \tag{27}$$

Consequently, we can form the spherical fuzzy difference matrix  $\tilde{S} = [\tilde{S}_{ij}^\Delta]$ , i.e., the matrix of spherical fuzzy differences. The matrix  $\tilde{S}$  accentuates the increase or decrease of the membership degree, the non-membership degree, and the hesitancy degree of the performance of each alternative from their peer in the average solution for the whole criteria. Figure 1 illustrates the computation of the spherical fuzzy difference between two SFSSs.

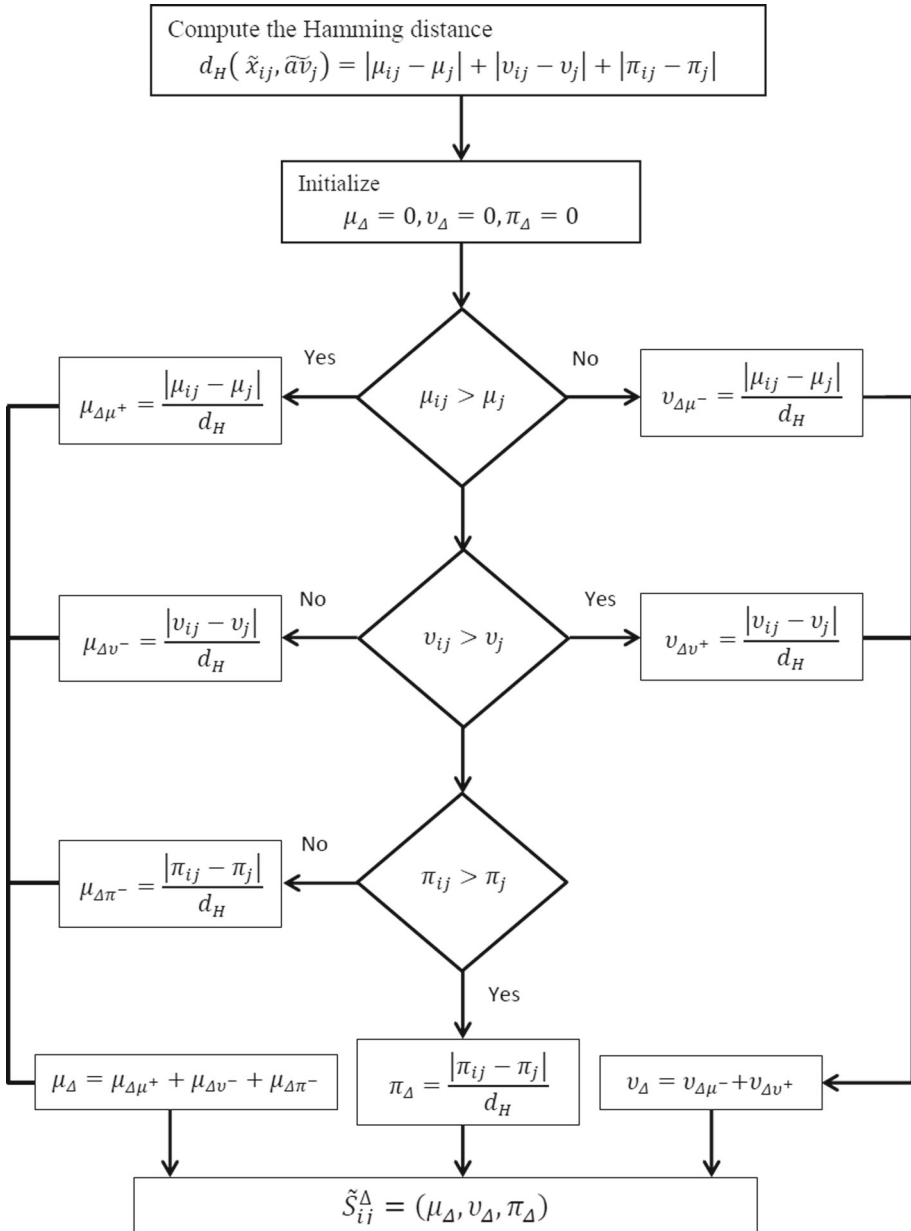


Fig. 1 Computation of the proposed spherical fuzzy difference

### 3.2 The proposed spherical aggregation function

The extant aggregation operators have a flaw when employing a SFS with either a membership degree “ $\mu = 0$ ” or a non-membership degree “ $\nu = 0$ ” e.g.,  $(1, 0, 0)$  and  $(1, 0, 0)$ . It is clear from the definition of the SWAM operator (7) that the resulting non-membership degree

is the product of all the non-membership degrees,  $\prod_{i=1}^n v_{\tilde{A}_{S_i}}^{w_i}$ . Therefore, whenever any SFS has a zero non-membership degree, this zero will dominate and the resulting SFS will have a zero non-membership degree regardless of the values of the other non-membership degree. For example, consider the SFSs (1, 0, 0), (0.2, 0.8, 0.2), and (0.1, 0.9, 0.1). The result of aggregation using (7) is always (1, 0, 0), despite the second and third non-membership degrees being relatively high. Similarly, from the definition of the SWGM operator (8), the resulting membership degree is the product of all the membership degrees,  $\prod_{i=1}^n \mu_{\tilde{A}_{S_i}}^{w_i}$ . Hence, whenever any SFS has a zero membership degree, this zero will dominate and the resulting SFS will have a zero membership degree. For example, consider the SFSs (0, 1, 0), (0.8, 0.2, 0.2), and (0.9, 0.1, 0.1). The result of aggregation using (8) is (0, 1, 0), although the second and third membership degrees are relatively high.

Thus, it is required to develop a balanced aggregation function that is not affected by a zero either in the membership or the non-membership degree and reflects the actual average of the aggregated SFSs. The proposed aggregation function is based on Yager’s definition of aggregation functions and does not rely on the operational laws of SFSs (1) and (2). According to Yager (2014) an aggregation function should satisfy the following three conditions.

**Definition 3.2.1** (Yager 2014) A function  $\text{Agg} : [0, 1]^n \rightarrow [0, 1]$  is an aggregation function if

$$i) \text{Agg}(0, \dots, 0) = 0,$$

$$ii) \text{Agg}(1, \dots, 1) = 1,$$

$$iii) \text{Agg}(a_1, \dots, a_n) \geq \text{Agg}(b_1, \dots, b_n) \text{ if } a_i \geq b_i \text{ for all } i.$$

**Definition 3.2.2** For the SFSs  $\{\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n\}$  with weights  $(w_1, w_2, \dots, w_n); w_i \in [0, 1]$  and  $\sum_{i=1}^n w_i = 1$ , the average value obtained using the spherical aggregation (SFAF) is given by:

$$\text{SFAGg}(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) = \left( \sqrt[n]{\sum_{i=1}^n w_i \mu_{\tilde{A}_i}^2}, \sqrt[n]{\sum_{i=1}^n w_i v_{\tilde{A}_i}^2}, \sqrt[n]{\sum_{i=1}^n w_i \pi_{\tilde{A}_i}^2} \right). \quad (28)$$

**Theorem** For SFSs, the SFAg operator defined in Eq. (28) satisfies the properties given in Definition 3.2.1 and the resulting number is again a SFS.

**Proof** Applying the SFAg for SFSs  $\{(1, 0, 0), \dots, (1, 0, 0)\}$

$$\begin{aligned} \text{SFAGg}\{(1, 0, 0), \dots, (1, 0, 0)\} &= \left( \sqrt[n]{\sum_{i=1}^n w_i \mu_{\tilde{A}_i}^2}, \sqrt[n]{\sum_{i=1}^n w_i v_{\tilde{A}_i}^2}, \sqrt[n]{\sum_{i=1}^n w_i \pi_{\tilde{A}_i}^2} \right) \\ &= \left( \sum_{i=1}^n w_i, \sum_{i=1}^n 0, \sum_{i=1}^n 0 \right) = (1, 0, 0) \end{aligned}$$

Similarly, the aggregation of the SFSs  $\{(0, 1, 0), \dots, (0, 1, 0)\}$  is (0, 1, 0) and the aggregation of the SFSs  $\{(0, 0, 1), \dots, (0, 0, 1)\}$  is (0, 0, 1).

Further, by applying the SFagg for the SFSs  $\{\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n\}$ , we get

$$\text{SFagg}\{\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n\} = \left( \sqrt{\sum_{i=1}^n w_i \mu_{\tilde{A}_i}^2}, \sqrt{\sum_{i=1}^n w_i \nu_{\tilde{A}_i}^2}, \sqrt{\sum_{i=1}^n w_i \pi_{\tilde{A}_i}^2} \right),$$

and SFagg for the SFSs  $\{\tilde{B}_1, \tilde{B}_2, \dots, \tilde{B}_n\}$ , we get

$$\text{SFagg}\{\tilde{B}_1, \tilde{B}_2, \dots, \tilde{B}_n\} = \left( \sqrt{\sum_{i=1}^n w_i \mu_{\tilde{B}_i}^2}, \sqrt{\sum_{i=1}^n w_i \nu_{\tilde{B}_i}^2}, \sqrt{\sum_{i=1}^n w_i \pi_{\tilde{B}_i}^2} \right).$$

Suppose  $\mu_{\tilde{A}_i} \geq \mu_{\tilde{B}_i}, \nu_{\tilde{A}_i} \geq \nu_{\tilde{B}_i}$ , and  $\pi_{\tilde{A}_i} \geq \pi_{\tilde{B}_i} \forall i$ , then it is clear from definition that

$$\sqrt{\sum_{i=1}^n w_i \mu_{\tilde{A}_i}^2} \geq \sqrt{\sum_{i=1}^n w_i \mu_{\tilde{B}_i}^2}, \sqrt{\sum_{i=1}^n w_i \nu_{\tilde{A}_i}^2} \geq \sqrt{\sum_{i=1}^n w_i \nu_{\tilde{B}_i}^2} \text{ and } \sqrt{\sum_{i=1}^n w_i \pi_{\tilde{A}_i}^2} \geq \sqrt{\sum_{i=1}^n w_i \pi_{\tilde{B}_i}^2}.$$

Finally, it is required to prove that the result is a SFS.

$$\begin{aligned} & \left( \sqrt{\sum_{i=1}^n w_i \mu_{\tilde{A}_i}^2} \right)^2 + \left( \sqrt{\sum_{i=1}^n w_i \nu_{\tilde{A}_i}^2} \right)^2 + \left( \sqrt{\sum_{i=1}^n w_i \pi_{\tilde{A}_i}^2} \right)^2 \\ &= \sum_{i=1}^n w_i \mu_{\tilde{A}_i}^2 + \sum_{i=1}^n w_i \nu_{\tilde{A}_i}^2 + \sum_{i=1}^n w_i \pi_{\tilde{A}_i}^2 = \sum_{i=1}^n w_i (\mu_{\tilde{A}_i}^2 + \nu_{\tilde{A}_i}^2 + \pi_{\tilde{A}_i}^2) \leq \sum_{i=1}^n w_i = 1. \end{aligned}$$

This completes the proof.

To demonstrate the working of these stated operators, we illustrate them with the help of two examples.

**Example** Suppose the performance of two alternatives for three different criteria whose weights are  $\{0.2, 0.3, 0.5\}$  is given by the following decision matrix.

$$\tilde{D} = \begin{matrix} & c_1 & c_2 & c_3 \\ \begin{matrix} a_1 \\ a_2 \end{matrix} & \begin{bmatrix} (1, 0, 0) \\ (0.9, 0.1, 0.1) \end{bmatrix} & \begin{bmatrix} (0.2, 0.8, 0.2) \\ (0.8, 0.2, 0.2) \end{bmatrix} & \begin{bmatrix} (0.1, 0.9, 0.1) \\ (0.9, 0.1, 0.1) \end{bmatrix} \end{matrix}.$$

It is clear from the decision matrix that the ratings of “ $a_2$ ” are far better than “ $a_1$ ” for the second and the third criteria that have higher weights. Meanwhile, the rating of “ $a_1$ ” is a bit better than that of “ $a_2$ ” for the first criterion only that has the lowest weight. Therefore, “ $a_2$ ” is better than “ $a_1$ ” intuitively.

Applying the SWAM aggregation operator (7) and the score function (4), we get  $\text{SWAM}(a_1) = (1, 0, 0)$  with  $\text{Sc}(a_1) = 1$ , and  $\text{SWAM}(a_2) = (0.8774, 0.1231, 0.1274)$  with score  $\text{Sc}(a_2) = 0.5625$ . Then, we have  $a_1 > a_2$  although it is the worst by intuition. When using the SWAM aggregation operator, a single criterion with the rating  $(1, 0, 0)$  will dominate and eliminate the effect of all the other assessment criteria regardless of its weight. Hence, the alternative having a rating of  $(1, 0, 0)$  for a criterion is selected regardless of its ratings for the other criteria leading to an incorrect ranking. However, by using the proposed SFagg (28),  $\text{SFagg}(a_1) = (0.4658, 0.7727, 0.1304)$  with  $\text{Sc}(a_1) = -0.3001$ , and  $\text{SFagg}(a_2) = (0.8712, 0.1378, 0.1378)$  with score  $\text{Sc}(a_2) = 0.5379$ . Here, the aggregation is balanced and the ranking  $a_2 > a_1$  is logic.

**Example** Suppose the performance of two alternatives for three different criteria whose weights are {0.2, 0.3, 0.5} is given by the following decision matrix.

$$\tilde{D} = \begin{matrix} & c_1 & c_2 & c_3 \\ a_1 & (0, 1, 0) & (0.8, 0.2, 0.2) & (0.9, 0.1, 0.1) \\ a_2 & (0.1, 0.9, 0.1) & (0.2, 0.8, 0.2) & (0.1, 0.9, 0.1) \end{matrix}$$

The ratings of “ $a_1$ ” exceed that of “ $a_2$ ” for the second and third criteria with higher weights. On the other hand, the rating of “ $a_2$ ” is a bit better than that of “ $a_1$ ” for the first criterion with the lowest weight. Accordingly, “ $a_1$ ” is better than “ $a_2$ ” intuitively. Applying the SWGM aggregation operator (8) and the score function (4), we get  $SWGM(a_1) = (0, 1, 0)$  with  $Sc(a_1) = -1$  and  $SWGM(a_2) = (0.1382, 0.8688, 0.1389)$  with  $Sc(a_2) = -0.5328$ . Consequently,  $a_2 > a_1$  this is not true by intuition. When using the SWGM aggregation operator, a single criterion with the rating (0, 1, 0) will dominate and eliminate the effect of the other evaluation criteria. Hence, the alternative having a rating of (0, 1, 0) for a criterion is selected regardless of its ratings for the other criteria leading to an incorrect ranking. On the other hand, by using the proposed SFAGg (28),  $SFAGg(a_1) = (0.7727, 0.4658, 0.1304)$  with  $Sc(a_1) = 0.3001$ , and  $SFAGg(a_2) = (0.1378, 0.8712, 0.1378)$  with score  $Sc(a_2) = -0.5379$ . Hence, the aggregation is also balanced and the ranking  $a_1 > a_2$  is logic.

### 3.3 The steps of the proposed SF-EDAS algorithm

In this section, we describe the steps of the proposed algorithm to solve the decision-making problems as follows.

**Step 1:** Determine the most relevant criteria that describe the alternatives.

**Step 2:** Construct the spherical fuzzy decision matrix ( $\tilde{X}$ ) and the weighting matrix ( $\tilde{W}$ ) of the criteria according to the evaluation of the decision-makers.

$$\tilde{X} = [\tilde{x}_{ij}] = \begin{matrix} & c_1 & c_2 & \dots & c_m \\ a_1 & \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1m} \\ a_2 & \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_n & \tilde{x}_{n1} & \tilde{x}_{n2} & \dots & \tilde{x}_{nm} \end{matrix} \text{ and } \tilde{W} = [\tilde{w}_j] = [\tilde{w}_1 \tilde{w}_2 \dots \tilde{w}_m],$$

where  $\tilde{x}_{ij}$  (the performance of the  $i$ th alternative for the  $j$ th criteria) and  $\tilde{w}_j$  (the weight of the  $j$ th criteria) are SFSs given by  $\tilde{x}_{ij} = (\mu_{ij}, \nu_{ij}, \pi_{ij})$ , and  $\tilde{w}_j = (\mu_j, \nu_j, \pi_j)$ .

**Step 3:** Determine the average solution for each criterion.

The performances of the alternatives for the  $j$ th criterion are aggregated using the spherical weighted arithmetic mean (7) to compute the average solution.

$$\tilde{a}v_j = SWAM_w(\tilde{x}_{1j}, \tilde{x}_{2j}, \dots, \tilde{x}_{nj}) = \left( \mu_j^{av}, \nu_j^{av}, \pi_j^{av} \right), \text{ where } w_i = \frac{1}{n}. \tag{29}$$

**Step 4:** Construct the spherical fuzzy difference matrix.

The spherical fuzzy difference ( $\tilde{S}_{ij}^\Delta$ ) between the performance of an alternative for a criterion ( $\tilde{x}_{ij}$ ) and the average solution ( $\tilde{a}v_j$ ) is calculated as shown by the flowchart in

Fig. 1. Then, the matrix of spherical fuzzy differences  $\tilde{S} = [\tilde{S}_{ij}^\Delta]$  is constructed.

$$\tilde{S} = \begin{bmatrix} \tilde{S}_{11}^\Delta & \tilde{S}_{12}^\Delta & \dots & \tilde{S}_{1m}^\Delta \\ \tilde{S}_{21}^\Delta & \tilde{S}_{22}^\Delta & & \tilde{S}_{2m}^\Delta \\ \vdots & \ddots & \ddots & \vdots \\ \tilde{S}_{n1}^\Delta & \tilde{S}_{n2}^\Delta & \dots & \tilde{S}_{nm}^\Delta \end{bmatrix} \tag{30}$$

**Step 5:** Construct the weighted spherical fuzzy difference matrix  $\tilde{S}_w$ .

The weighted difference matrix is computed by multiplying each spherical fuzzy difference  $(\tilde{S}_{ij}^\Delta)$  by the weight of the criterion ( $\tilde{w}_j$ ) using Eq. (3).

$$\tilde{S}_w = [\tilde{S}_{ij}], \tag{31}$$

where  $\tilde{S}_{ij} = \tilde{w}_j \otimes \tilde{S}_{ij}^\Delta$ .

It is noted that if the weights of the criteria are crisp values, then the operational law (2) is used instead.

**Step 6:** Compute the appraisal scores of the alternatives.

The appraisal scores of the alternatives are calculated by aggregating the weighted differences of each alternative for the whole criteria, i.e., aggregating the elements of each row in the weighted difference matrix. This is accomplished by applying the proposed aggregation operator SFAgg (30) to ensure balanced aggregation, hence correct ranking.

$$\begin{aligned} \widetilde{ASc} &= [\widetilde{asc}_i], \quad \widetilde{asc}_i = (\mu_{as_i}, \nu_{as_i}, \pi_{as_i}), \\ \widetilde{asc}_i &= \text{SFAgg}_w \left\{ \tilde{S}_{i1}, \tilde{S}_{i2}, \dots, \tilde{S}_{im} \right\}, \quad w_j = \frac{1}{m}. \end{aligned} \tag{32}$$

**Step 7:** Calculate the score of each alternative, and the accuracy if required (in case of equal scores).

The appraisal scores are defuzzified using the score function (4), and if two alternatives have the same score, the accuracy function (5) is also calculated.

$$\begin{aligned} \text{Score}(\widetilde{asc}_i) &= (\mu_{\widetilde{as}_i} - \pi_{\widetilde{as}_i})^2 - (\nu_{\widetilde{as}_i} - \pi_{\widetilde{as}_i})^2, \\ \text{Accuracy}(\widetilde{asc}_i) &= \mu_{\widetilde{as}_i}^2 + \nu_{\widetilde{as}_i}^2 + \pi_{\widetilde{as}_i}^2. \end{aligned}$$

**Step 8:** Rank the alternatives: Finally, the alternatives are ranked in descending order. The alternative with the highest score is the best.

The flowchart of the proposed SF-EDAS approach is summarized in Fig. 2.

### 4 Applications

In this section, two examples are solved to demonstrate the proposed method and compare the results with the results of other methods. The first example is due to Gündoğdu and Kahraman (2019) in supplier selection. The second example is industrial robot selection (Kutlu Gundogdu and Kahraman 2019).

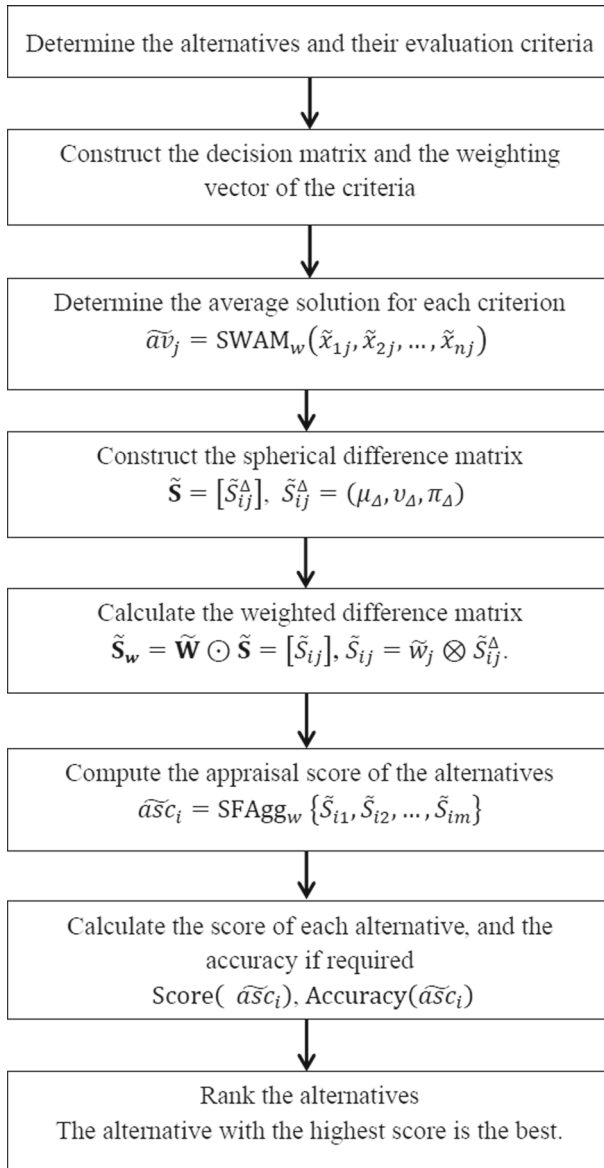


Fig. 2 The proposed SF-EDAS algorithm

### 4.1 A supplier selection problem

It is required to determine the best air condition supplier from four suppliers  $A = \{a_1, a_2, a_3, a_4\}$ . Four criteria  $C = \{c_1, c_2, c_3, c_4\}$  are chosen for assessment: price, quality, delivery and performance, respectively. Only the price is a cost criterion, the rest are benefit criteria. Three decision-makers  $D = \{D_1, D_2, D_3\}$  participate in the evaluation process. The solution steps are illustrated using the decision matrix and the weights obtained



by the SWAM operator directly. The details of the example can be found in Gündoğdu and Kahraman (2019). The solution steps are as follows.

**Step 1:** Determine the most relevant criteria that describe the alternatives.

Price ( $c_1$ ), quality ( $c_2$ ), delivery ( $c_3$ ), and performance ( $c_4$ ).

**Step 2:** Construct the decision matrix and the weighting matrix of the criteria according to the evaluation of the decision-makers.

$$\tilde{X} = \begin{matrix} & c_1 & c_2 & c_3 & c_4 \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{matrix} & \begin{bmatrix} (0.80,0.20,0.21) & (0.70,0.30,0.30) & (0.82, 0.19,0.23) & (0.81,0.20,0.23) \\ (0.77,0.24,0.22) & (0.74,0.28,0.32) & (0.74,0.27,0.27) & (0.45,0.55,0.46) \\ (0.57,0.44,0.32) & (0.53,0.49,0.37) & (0.74,0.27,0.28) & (0.52,0.52,0.31) \\ (0.48,0.56,0.29) & (0.62,0.39,0.39) & (0.65,0.37,0.25) & (0.62,0.39,0.34) \end{bmatrix} \end{matrix},$$

and

$$\tilde{W} = [ (0.57, 0.44, 0.32) (0.82, 0.19, 0.23) (0.80, 0.20, 0.21) (0.81, 0.20, 0.23) ].$$

**Step 3:** Determine the average solution for each criterion using (29).

$$\tilde{a}v_j = \text{SWAM}_w(\tilde{x}_{1j}, \tilde{x}_{2j}, \dots, \tilde{x}_{nj}) = (\mu_j^{av}, \nu_j^{av}, \pi_j^{av}), \text{ where } w_i = \frac{1}{4}.$$

$$\tilde{a}v_j = [ (0.69, 0.33, 0.26) (0.66, 0.36, 0.035) (0.75, 0.27, 0.26) (0.64, 0.39, 0.33) ].$$

**Step 4:** Construct the spherical fuzzy difference matrix (30).

$$\tilde{S} = \begin{matrix} & c_1 & c_2 & c_3 & c_4 \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{matrix} & \begin{bmatrix} [1 \ 0 \ 0] & [1 \ 0 \ 0] & [1 \ 0 \ 0] & [1 \ 0 \ 0] \\ [1 \ 0 \ 0] & [1 \ 0 \ 0] & [0 \ 0.44 \ 0.56] & [0 \ 0.73 \ 0.27] \\ [0 \ 0.78 \ 0.22] & [0 \ 0.91 \ 0.09] & [0 \ 0.29 \ 0.71] & [0.08 \ 0.92 \ 0] \\ [0 \ 0.93 \ 0.07] & [0 \ 0.62 \ 0.38] & [0.04 \ 0.96 \ 0] & [0 \ 0.74 \ 0.26] \end{bmatrix} \end{matrix}.$$

**Step 5:** Construct the weighted spherical fuzzy difference matrix (31).

$$\tilde{S}_w = \begin{matrix} & c_1 & c_2 & c_3 & c_4 \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{matrix} & \begin{bmatrix} (0.57,0.44,0.32) & (0.82,0.19,0.23) & (0.80,0.20,0.21) & (0.81,0.2,0.23) \\ (0.57,0.44,0.32) & (0.82,0.19,0.23) & (0.00,0.47,0.57) & (0.00,0.75,0.30) \\ (0.00,0.83,0.27) & (0.00,0.92,0.12) & (0.00,0.34,0.71) & (0.07,0.92,0.09) \\ (0.00,0.94,0.13) & (0.00,0.64,0.41) & (0.04,0.96,0.06) & (0.00,0.75,0.29) \end{bmatrix} \end{matrix}.$$

**Step 6:** Compute the appraisal score of the alternatives using (32).

$$\tilde{ASc} = [a\tilde{s}c_i], a\tilde{s}c_i = (\mu_{as_i}, \nu_{as_i}, \pi_{as_i}),$$

$$\tilde{a}\tilde{s}c_i = \text{SFAgg}_w \{ \tilde{s}_{i1}, \tilde{s}_{i2}, \dots, \tilde{s}_{im} \}, w_j = \frac{1}{4}.$$

$$\tilde{ASc} = [ (0.76, 0.28, 0.25) (0.50, 0.50, 0.38) (0.03, 0.80, 0.39) (0.02, 0.83, 0.26) ].$$

**Step 7:** Calculate the score of each alternative using the score function (4).

$$AS = [ 0.26 \ 0.00 \ -0.04 \ -0.27 ].$$

**Step 8:** Rank the alternatives in descending order. The alternative with the highest score is the best candidate. According to the obtained scores, the best alternative is  $a_1$ , and the overall ranking is

$$a_1 > a_2 > a_3 > a_4.$$

#### 4.1.1 Comparative analysis of the supplier selection problem

This subsection presents a comparative analysis between the proposed SF-EDAS and both the SF-TOPSIS (Gündoğdu and Kahraman 2019) and the SF-VIKOR (Kutlu Gündoğdu and Kahraman 2019) to show its applicability and validity. The aforementioned supplier selection problem is solved using these techniques being the most successful distance-based methods from which SF-EDAS stems.

Throughout this section, the score function (4) is utilized. Three distance measures will be employed namely, the Euclidean distance (9), Xu and Zhang’s distance (10), and the Spherical distance (11). First, the SF-TOPSIS is applied. The weighted decision matrix is given in Table 1 (Gündoğdu and Kahraman 2019).

The score function values of the weighted decision matrix with the positive (PIS) and negative ideal solutions (NIS) are given in Table 2.

When employing the three distance measures (9), (10), and (11), it turned out that the only distance measure that can be utilized is the Euclidean distance (9).

Regarding Xu and Zhang distance (10), the PIS for the fourth criterion " $c_4$ " is " $a_1$ " with a weighted rating of (0.46, 0.48, 0.36). On the other hand, the NIS is " $a_3$ " with a weighted rating of (0.42, 0.55, 0.35). The distance between the PIS and the NIS equals  $dis_{XZ}(a^+, a^-) = dis_{XZ}(a_1, a_4) = 0.2506$ . Meanwhile, the distance between the PIS " $a_1$ " and the second

**Table 1** Weighted decision matrix for the supplier selection problem

	$c_1$	$c_2$	$c_3$	$c_4$
$a_1$	(0.46, 0.48, 0.36)	(0.57, 0.35, 0.36)	(0.66, 0.27, 0.30)	(0.65, 0.28, 0.32)
$a_2$	(0.44, 0.50, 0.37)	(0.60, 0.33, 0.37)	(0.59, 0.33, 0.34)	(0.37, 0.57, 0.48)
$a_3$	(0.33, 0.60, 0.40)	(0.43, 0.52, 0.41)	(0.59, 0.33, 0.34)	(0.42, 0.55, 0.35)
$a_4$	(0.27, 0.67, 0.36)	(0.51, 0.43, 0.43)	(0.53, 0.42, 0.31)	(0.50, 0.43, 0.39)

**Table 2** Score function values of the weighted decision matrix

	$c_1$	$c_2$	$c_3$	$c_4$
$a_1$	- 0.0041	0.0454	0.1248	0.1157
$a_2$	- 0.0105	0.0510	0.0696	0.0037
$a_3$	- 0.0346	- 0.0115	0.0657	- 0.0327
$a_4$	- 0.0876	0.0064	0.0335	0.0113
PIS ( $a^+$ )	$a_1$	$a_2$	$a_1$	$a_1$
NIS ( $a^-$ )	$a_4$	$a_3$	$a_4$	$a_3$

alternative " $a_2$ " equals  $dis_{XZ}(a_1, a_2) = 0.3301$  Therefore, the distance between the PIS and the NIS is not the largest. Here, SF-TOPSIS might not give a correct result.

As for the spherical distance (11), the distance between a SFS and itself is not necessarily equal to zero. From the definition of the Spherical distance (11), this distance is equal to zero only if  $\mu^2 + \nu^2 + \pi^2 = 1$ . This condition is not always achieved for a SFS as it suffices that  $\mu^2 + \nu^2 + \pi^2 \leq 1$ . Moreover, the distance between a SFS and itself might be the largest. For the first criterion " $c_1$ ", while the distance between the PIS and the NIS equals  $dis_{Sp}(a^+, a^-) = dis_{Sp}(a_1, a_4) = 0.6125$ , the distance between the PIS " $a_1$ " and itself equals  $dis_{Sp}(a_1, a_1) = 0.6192$ . Hence, in this example, the spherical distance (11) is not applicable. Finally, when employing the Euclidean distance (9), the distance between the PIS and the NIS is the largest for the whole criteria. Then, the Euclidean distance can be applied and guarantees a correct result. The obtained rank is  $a_1 > a_2 > a_4 > a_3$ .

Second, the SF-VIKOR is applied. The score function values for the decision matrix and the positive and negative ideal solutions are given in Table 3.

When employing the three distance measures in the SF-VIKOR, the distance between the PIS and the NIS is the largest for the whole criteria. Consequently, the results are correct and reliable for comparison. Concerning the Spherical fuzzy distance, despite the distance between an ideal solution and itself is not zero, it is the smallest relative to its distance from other alternatives. Different values of  $\gamma$ , the weight of "the maximum group utility" strategy, are also examined.

The results of comparison are summarized in Table 4. The results of the intuitionistic fuzzy TOPSIS (IF-TOPSIS) is due to (Gündoğdu and Kahraman 2019).

**Table 3** Score function values of the weighted decision matrix for the supplier selection problem

	$c_1$	$c_2$	$c_3$	$c_4$
$a_1$	0.3480	0.1600	0.3465	0.3355
$a_2$	0.3021	0.1748	0.2209	- 0.0080
$a_3$	0.0481	0.0112	0.2115	0
$a_4$	- 0.0368	0.0529	0.1456	0.0759
PIS ( $a^+$ )	$a_1$	$a_2$	$a_1$	$a_1$
NIS ( $a^-$ )	$a_4$	$a_3$	$a_4$	$a_2$

**Table 4** Ranking using SF-EDAS, SF-TOPSIS, SF-VIKOR, and IF-TOPSIS for the supplier selection problem

Method	Ranking
Proposed SF-EDAS	$a_1 > a_2 > a_3 > a_4$
SF-TOPSIS (Euclidean distance)	$a_1 > a_2 > a_4 > a_3$
SF-VIKOR (Euclidean distance for $\gamma = 0$ )	$a_1 > a_2 > a_3 > a_4$
SF-VIKOR (Euclidean distance for $\gamma = 0.1$ to 1)	$a_1 > a_2 > a_4 > a_3$
SF-VIKOR (Xu and Zhang distance for $\gamma = 0$ and 0.1)	$a_1 > a_2 > a_3 > a_4$
SF-VIKOR (Xu and Zhang distance for $\gamma = 0.2$ to 1)	$a_1 > a_2 > a_4 > a_3$
SF-VIKOR (Spherical distance $\gamma = 0$ to 1)	$a_1 > a_2 > a_4 > a_3$
IF-TOPSIS	$a_1 > a_2 > a_4 > a_3$

From Table 4, it is clear that the best alternative obtained using the SF-EDAS is " $a_1$ " which is also the best alternative obtained by the methods used for comparison. The overall ranking is  $a_1 > a_2 > a_3 > a_4$  which coincides with the overall ranking of the SF-VIKOR using the Euclidean distance for  $\gamma = 0$ , SF-VIKOR using Xu and Zhang distance for  $\gamma = 0$  and 0.1. The third and fourth alternatives exchange positions from one method to another.

### 4.2 An industrial robot selection problem

In today’s technologically advanced society, most industries focus on improving automated-driven systems to increase productivity and lower production costs. In this environment, technical decisions are essential, such as robot selection. Robots are computer-programmed automated material handling devices that perform various tasks, e.g. welding, spray painting, part assembly, loading, and unloading. They increase system flexibility and production, improve material flow efficiency, improve facility utilization, and reduce lead time and labor costs. Due to the wide range of industrial robots with different performances and various technical characteristics, managers sometimes face difficulties in selecting the proper robot to accomplish the required task. Improper industrial robot selection not only reduces productivity but also harms the organization’s reputation. Hence, proper robot selection is crucial to increasing the production rate with the highest precision and accuracy (Goswami et al. 2021).

Consider the problem of choosing one of the most used robots, 6-axis robots, Scara robots, Dual-arm robots, redundant robots, and Cartesian robots. Four criteria are used for the assessment process, efficiency, suitability, automation, and ergonomics. Three experts, with different levels of experience, are involved in the decision-making process. The judgments of the experts for the performance of the robots for the evaluation criteria and the weights of the criteria are aggregated using the SWAM operator (7). The aggregated decision matrix and the aggregated weights of the criteria are given in Table 5. For the details of the problem, the reader is referred to Kutlu Gundogdu and Kahraman (2019).

Since Steps 1 and 2 are summarized in Table 5, Step 3 is presented directly.

**Step 3:** Determine the average solution for each criterion using (29) with  $w_i = \frac{1}{3}$ .

$$\tilde{a}_j = \left[ (0.64, 0.37, 0.35) \ (0.70, 0.31, 0.32) \ (0.67, 0.34, 0.30) \ (0.56, 0.45, 0.41) \right].$$

**Step 4:** Construct the spherical fuzzy difference matrix (30).

**Table 5** The aggregated decision matrix and the aggregated weights of the criteria

	Efficiency ( $c_1$ )	Suitability ( $c_2$ )	Automation ( $c_3$ )	Ergonomics ( $c_4$ )
6-axis ( $a_1$ )	(0.78, 0.23, 0.27)	(0.63, 0.37, 0.37)	(0.77, 0.23, 0.23)	(0.55, 0.46, 0.40)
Scara ( $a_2$ )	(0.40, 0.60, 0.40)	(0.71, 0.30, 0.32)	(0.65, 0.35, 0.36)	(0.58, 0.43, 0.44)
Dual-arm ( $a_3$ )	(0.56, 0.45, 0.41)	(0.80, 0.20, 0.20)	(0.77, 0.23, 0.23)	(0.65, 0.35, 0.36)
Redundant ( $a_4$ )	(0.64, 0.36, 0.36)	(0.66, 0.34, 0.34)	(0.53, 0.50, 0.31)	(0.40, 0.61, 0.41)
Cartesian ( $a_5$ )	(0.70, 0.30, 0.30)	(0.62, 0.38, 0.39)	(0.53, 0.49, 0.38)	(0.57, 0.43, 0.43)
Criteria weights	(0.78, 0.23, 0.27)	(0.53, 0.50, 0.31)	(0.77, 0.23, 0.23)	(0.66, 0.34, 0.34)

$$\tilde{S} = \begin{matrix} & c_1 & c_2 & c_3 & c_4 \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{matrix} & \begin{bmatrix} [1 & 0 & 0] \\ [0 & 0.9 & 0.1] \\ [0 & 0.72 & 0.28] \\ [0.28 & 0.18 & 0.54] \\ [1 & 0 & 0] \end{bmatrix} & \begin{bmatrix} [0 & 0.72 & 0.28] \\ [1 & 0 & 0] \\ [1 & 0 & 0] \\ [0 & 0.78 & 0.22] \\ [0 & 0.68 & 0.32] \end{bmatrix} & \begin{bmatrix} [1 & 0 & 0] \\ [0 & 0.35 & 0.65] \\ [1 & 0 & 0] \\ [0 & 0.97 & 0.03] \\ [0 & 0.79 & 0.21] \end{bmatrix} & \begin{bmatrix} [0.3 & 0.7 & 0] \\ [0.56 & 0 & 0.45] \\ [1 & 0 & 0] \\ [0 & 1 & 0] \\ [0.58 & 0 & 0.2] \end{bmatrix} \end{matrix}.$$

**Step 5:** Construct the weighted spherical fuzzy difference matrix (31).

$$\tilde{S}_w = \begin{matrix} & c_1 & c_2 & c_3 & c_4 \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{matrix} & \begin{bmatrix} (0.78,0.23,0.27) \\ (0,0.9,0.15) \\ (0,0.74,0.32) \\ (0.22,0.29,0.57) \\ (0.78,0.23,0.27) \end{bmatrix} & \begin{bmatrix} (0,0.8,0.31) \\ (0.53,0.5,0.31) \\ (0.53,0.5,0.31) \\ (0,0.84,0.26) \\ (0,0.77,0.34) \end{bmatrix} & \begin{bmatrix} (0.77,0.23,0.23) \\ (0,0.41,0.65) \\ (0.77,0.23,0.23) \\ (0,0.97,0.06) \\ (0,0.8,0.25) \end{bmatrix} & \begin{bmatrix} (0.2,0.74,0.24) \\ (0.37,0.34,0.52) \\ (0.66,0.34,0.34) \\ (0,1,0) \\ (0.38,0.34,0.5) \end{bmatrix} \end{matrix}.$$

**Step 6:** Compute the appraisal score of the alternatives using (32) with  $w_j = \frac{1}{4}$ .

$$\widetilde{ASc} = \left[ (0.5, 0.51, 0.24) (0.29, 0.52, 0.4) (0.51, 0.44, 0.27) (0.1, 0.74, 0.28) (0.39, 0.53, 0.32) \right].$$

**Step 7:** Calculate the score of each alternative using the score function (4).

$$AS = \left[ -0.0054 \quad -7.3517 \times 10^{-4} \quad 0.0297 \quad -0.1733 \quad -0.04 \right].$$

**Step 8:** Rank the alternatives in descending order. The alternative with the highest score is the best candidate. According to the obtained scores, the best alternative is "a<sub>3</sub>", and the overall ranking is

$$a_3 > a_2 > a_1 > a_5 > a_4.$$

### 4.2.1 Comparative analysis of the robot selection problem

First, apply the SF-TOPSIS. The weighted decision matrix is given in Table 6. The score function values of the weighted decision matrix with the positive and negative ideal solutions are given in Table 7. The problem is solved using the three distance measures (9), (10), and (11). The Euclidean distance (9) is the only distance measure that guarantees that the distance between the PIS and the NIS is the largest.

**Table 6** Weighted decision matrix for the robot selection problem

	c <sub>1</sub>	c <sub>2</sub>	c <sub>3</sub>	c <sub>4</sub>
a <sub>1</sub>	(0.61, 0.32, 0.36)	(0.33, 0.59, 0.42)	(0.59, 0.32, 0.31)	(0.36, 0.55, 0.46)
a <sub>2</sub>	(0.31, 0.63, 0.43)	(0.38, 0.56, 0.39)	(0.50, 0.41, 0.40)	(0.38, 0.53, 0.49)
a <sub>3</sub>	(0.44, 0.49, 0.45)	(0.42, 0.53, 0.34)	(0.59, 0.32, 0.31)	(0.43, 0.47, 0.45)
a <sub>4</sub>	(0.50, 0.42, 0.42)	(0.35, 0.58, 0.40)	(0.41, 0.54, 0.35)	(0.26, 0.67, 0.45)
a <sub>5</sub>	(0.55, 0.37, 0.38)	(0.33, 0.60, 0.43)	(0.41, 0.53, 0.41)	(0.38, 0.53, 0.49)

**Table 7** Score function values of the weighted decision matrix for the robot selection problem

	$c_1$	$c_2$	$c_3$	$c_4$
$a_1$	0.0577	- 0.0253	0.0788	0.0023
$a_2$	- 0.0239	- 0.0288	0.0095	0.0109
$a_3$	- 0.0015	- 0.0279	0.0788	0
$a_4$	0.0062	- 0.0296	- 0.0309	- 0.0130
$a_5$	0.0272	- 0.0202	- 0.0139	0.0103
PIS ( $a^+$ )	$a_1$	$a_5$	$a_1, a_3$	$a_2$
NIS ( $a^-$ )	$a_2$	$a_4$	$a_4$	$a_4$

When employing Xu and Zhang’s distance (10), the weighted rating of the PIS and the NIS for the second criterion “ $c_2$ ” is (0.33, 0.60, 0.43) and (0.35, 0.58, 0.40), respectively. Yet,  $dis_{XZ}(a_5, a_4) = 0.0285 < dis_{XZ}(a_5, a_3) = 0.1067$ . Then, it is inconvenient to be utilized. As for the spherical distance (11), the previously mentioned flaws exist. Hence, it cannot be utilized. The distance between an alternative and itself is larger than the its distance from another alternative,  $dis_{Sp}(a_2, a_2) = 0.5337 > dis_{Sp}(a_2, a_4) = 0.5284$ . We also have for the second criterion the distance between the PIS and the NIS is not the largest,  $dis_{Sp}(a_5, a_3) = 0.5957 > dis_{Sp}(a_5, a_4) = 0.5746$ .

Second, the SF-VIKOR is applied. The distance between the ideal solutions is the largest when the Euclidean and Xu and Zhang’s distance are utilized. The Spherical distance still cannot be applied. The results of the SF-WASPAS and the IF-TOPSIS are due to Kutlu Gundogdu and Kahraman (2019). The results of the comparison are summarized in Table 8.

From Table 8, the best alternative obtained by the proposed SF-EDAS is the alternative “ $a_3$ ” which is ranked second by the other methods. The difference in ranking of the SF-EDAS from the SF-TOPSIS, the SF-VIKOR, and the SF-WASPAS can be attributed to the utilization of score functions in the early steps of these methods. In the SF-TOPSIS and the SF-VIKOR the score function is used to find the PIS and the NIS. When processing the SF-WASPAS, the criteria weights are defuzzified. The normalized criteria weights after

**Table 8** Ranking using SF-EDAS, SF-TOPSIS and SF-VIKOR for the robot selection problem

Method	Ranking
Proposed SF-EDAS	$a_3 > a_2 > a_1 > a_5 > a_4$
SF-TOPSIS (Euclidean distance)	$a_1 > a_3 > a_5 > a_4 > a_2$
SF-VIKOR (Euclidean distance for $\gamma = 0$ to 0.1)	$a_1 > a_3 > a_2 > a_4 > a_5$
SF-VIKOR (Euclidean distance for $\gamma = 0.2$ to 0.4)	$a_1 > a_3 > a_2 > a_5 > a_4$
SF-VIKOR (Euclidean distance for $\gamma = 0.5$ to 1)	$a_1 > a_3 > a_5 > a_2 > a_4$
SF-VIKOR (Xu and Zhang distance for $\gamma = 0$ to 0.3)	$a_1 > a_3 > a_2 > a_4 > a_5$
SF-VIKOR (Xu and Zhang distance $\gamma = 0.4$ to 0.5)	$a_1 > a_3 > a_2 > a_5 > a_4$
SF-VIKOR (Xu and Zhang distance $\gamma = 0.6$ to 1)	$a_1 > a_3 > a_5 > a_2 > a_4$
SF-WASPAS	$a_1 > a_3 > a_5 > a_4 > a_2$
IF-TOPSIS	$a_1 > a_3 > a_2 > a_5 > a_4$

defuzzification are  $\{0.26681, 0.00001, 0.49129, 0.24189\}$ . Since the weight of the second criterion is approximately zero, when multiplying the ratings of the alternatives for the second criteria by its weight in the weighted sum model, the five alternatives have the same value  $(0, 1, 0)$ , which is the additive neutral. Hence, the effect of the second criterion is eliminated despite the third alternative having the best performance for this criterion. Consequently, it is preferable to use fully fuzzy approaches. Regarding the IF-TOPSIS, the difference is normal due to the alteration of the preference domain with an independent hesitation degree.

## 5 Conclusion

In this study, a new fully fuzzy approach is proposed to develop EDAS for MCDM problems using SFSs. In the spherical fuzzy environment, the implementation of distance-based methods faces some deficiencies. First, the reference points might change according to the score function employed. Second, the distance between the reference points is not necessarily the largest. The proposed SF-EDAS approach averts these deficiencies. Then, the contributions of the study are.

- A spherical fully fuzzy EDAS algorithm is developed that avoids the current limitations and drawbacks of distance-based methods in general and the SF-EDAS, in particular, using spherical fuzzy information namely, early defuzzification, the flaws of distance measures, and the undefined spherical fuzzy subtraction and division operations.
- The concept of spherical fuzzy difference is introduced to make up for the undefined subtraction operation and as an alternative to distance measures to count for the increase or decrease in the various degrees of the alternatives' performance for the criteria than that of their peers in the average solutions.
- The extant aggregation operators have a flaw when handling a zero membership or non-membership degree, the aggregation result might be unfair. Therefore, an aggregation function is introduced which guarantees a balanced aggregation and reflects the actual values fairly.

Two practical examples were solved and a comparative study is presented for each example. The data of the two examples are SFSs. Each example exemplifies one of the flaws encountered in the implementation of distance-based methods in a spherical fuzzy environment. In examples, the SF-TOPSIS and the SF-VIKOR were employed for comparison. Three distance measures were examined in their implementation namely the Euclidean distance, Xu and Zhang distance, and the spherical fuzzy distance.

In the first example, the results of the proposed SF-EDAS are compared with the results of the SF-TOPSIS, the SF-VIKOR, and the IF-TOPSIS. In the implementation of the SF-TOPSIS, only the Euclidean distance maintained the distance between the PIS and the NIS as the largest. Meanwhile, the other two distance measures failed. As for the SF-VIKOR, the Euclidean distance and Xu and Zhang's distance were valid. The Spherical fuzzy distance is not valid except on the surface of the unit sphere. Therefore, it is not advisable to be utilized, in general. The results revealed that the alternatives in the first and second places obtained by the proposed SF-EDAS are identical to those obtained by these methods. The alternatives in the third and fourth places exchange ranks from one method to another.

In the second example, the results of the proposed SF-EDAS are compared with the results of the SF-TOPSIS, the SF-VIKOR, the SF-WASPAS, and the IF-TOPSIS. Similar to the first example, not all distance measures were convenient in the implementation of the SF-TOPSIS and the SF-VIKOR. The Euclidean distance was valid for both methods; Xu

and Zhang distance was valid for the SF-VIKOR. In the SF-WASPAS, the weights of the alternatives were defuzzified. As a result, the weight of one of the criteria was approximately zero and its effect was eliminated although the third alternative has the best performance for this criterion. In consequence, the third alternative which is ranked first by the proposed SF-EDAS recedes to second place in the SF-WASPAS. The rest of the ranking list changes from one method to another.

The main limitation of the proposed SF-EDAS method is the lack of objective evaluations. The method handles only subjective evaluations in the spherical fuzzy environment. Thus, future research should develop a SF-EDAS method for both subjective and objective evaluations.

In addition, future research should investigate the employment of the concept of the spherical fuzzy in other distance-based methods, e.g., TOPSIS and VIKOR as a substitute for distance measures to avoid the flaw that might occur, i.e., the distance between the positive and negative ideal solutions is not the largest between the alternatives.

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## References

- Ali Z, Mahmood T, Ullah K, Khan Q (2021) Einstein geometric aggregation operators using a novel complex interval-valued pythagorean fuzzy setting with application in green supplier chain management. *Rep Mech Eng* 2:105–134. <https://doi.org/10.31181/rme2001020105t>
- Ashraf S, Abdullah S, Mahmood T et al (2019) Spherical fuzzy sets and their applications in multi-attribute decision making problems. *J Intell Fuzzy Syst* 36:2829–2844
- Atanassov KT (1986) Intuitionistic fuzzy sets. *Fuzzy Sets Syst* 20:87–96
- Batool B, Abosuliman SS, Abdullah S, Ashraf S (2021) EDAS method for decision support modeling under the Pythagorean probabilistic hesitant fuzzy aggregation information. *J Ambient Intell Humaniz Comput*. <https://doi.org/10.1007/s12652-021-03181-1>
- Deveci M, Gokasar I, Pamucar D et al (2022a) Safe E-scooter operation alternative prioritization using a q-rung orthopair Fuzzy Einstein based WASPAS approach. *J Clean Prod* 347:131239. <https://doi.org/10.1016/J.JCLEPRO.2022.131239>
- Deveci M, Pamucar D, Gokasar I et al (2022b) Fuzzy Einstein WASPAS approach for the economic and societal dynamics of the climate change mitigation strategies in urban mobility planning. *Struct Chang Econ Dyn* 61:1–17. <https://doi.org/10.1016/J.STRUECO.2022.01.009>
- Deveci M, Simic V, Karagoz S, Antucheviciene J (2022c) An interval type-2 fuzzy sets based Delphi approach to evaluate site selection indicators of sustainable vehicle shredding facilities. *Appl Soft Comput* 118:108465. <https://doi.org/10.1016/J.ASOC.2022.108465>
- Ecer F (2018) Third-party logistics (3PLs) provider selection via fuzzy AHP and EDAS integrated model. *Technol Econ Dev Econ* 24:615–634. <https://doi.org/10.3846/20294913.2016.1213207>
- Feng X, Wei C, Liu Q (2018) EDAS method for extended hesitant fuzzy linguistic multi-criteria decision making. *Int J Fuzzy Syst* 20:2470–2483. <https://doi.org/10.1007/s40815-018-0504-5>
- Ghorabae MK, Zavadskas EK, Olfat L, Turskis Z (2015) Multi-criteria inventory classification using a new method of evaluation based on distance from average solution (EDAS). *Informatica* 26:435–451. <https://doi.org/10.15388/Informatica.2015.57>
- Ghorabae M, Zavadskas E, Amiri M, Turskis Z (2016) Extended EDAS method for fuzzy multi-criteria decision-making: an application to supplier selection. *Int J Comput Commun*. <https://doi.org/10.15837/ijccc.2016.3.2557>
- Ghorabae M, Amiri M, Zavadskas EK et al (2017a) A new multi-criteria model based on interval type-2 fuzzy sets and EDAS method for supplier evaluation and order allocation with environmental considerations. *Comput Ind Eng* 112:156–174. <https://doi.org/10.1016/j.cie.2017.08.017>
- Ghorabae MK, Amiri M, Zavadskas EK, Turskis Z (2017b) Multi-criteria group decision-making using an extended EDAS method with interval type-2 fuzzy sets. *E + M Ekon Manag* 20:48–68. <https://doi.org/10.15240/tul/001/2017-1-004>



- Gorcun OF, Senthil S, Küçükönder H (2021) Evaluation of tanker vehicle selection using a novel hybrid fuzzy MCDM technique. *Decis Mak Appl Manag Eng* 4:140–162. <https://doi.org/10.31181/dmame210402140g>
- Goswami SS, Behera DK, Afzal A et al (2021) Analysis of a robot selection problem using two newly developed hybrid MCDM models of TOPSIS-ARAS and COPRAS-ARAS. *Symmetry (basel)*. <https://doi.org/10.3390/sym13081331>
- Gül S (2021) Spherical fuzzy version of EDAS and an application. *Int J Adv Eng Pure Sci*. <https://doi.org/10.7240/jeps.783060>
- Gündoğdu FK, Kahraman C (2019) Spherical fuzzy sets and spherical fuzzy TOPSIS method. *J Intell Fuzzy Syst* 36:337–352. <https://doi.org/10.3233/JIFS-181401>
- He Y, Lei F, Wei G et al (2019) Edas method for multiple attribute group decision making with probabilistic uncertain linguistic information and its application to green supplier selection. *Int J Comput Intell Syst* 12:1361–1370. <https://doi.org/10.2991/ijcis.d.191028.001>
- Ilbahar E, Karaşan A, Cebi S, Kahraman C (2018) A novel approach to risk assessment for occupational health and safety using Pythagorean fuzzy AHP & fuzzy inference system. *Saf Sci* 103:124–136. <https://doi.org/10.1016/j.ssci.2017.10.025>
- Ilieva G (2018) Group decision analysis algorithms with EDAS for interval fuzzy sets. *Cybern Inf Technol* 18:51–64. <https://doi.org/10.2478/cait-2018-0027>
- Ilieva G, Yankova T, Klisarova-Belcheva S (2018) Decision analysis with classic and fuzzy EDAS modifications. *Comput Appl Math* 37:5650–5680. <https://doi.org/10.1007/s40314-018-0652-0>
- Iordache M, Pamucar D, Deveci M et al (2022) Prioritizing the alternatives of the natural gas grid conversion to hydrogen using a hybrid interval rough based Dombi MARCOS model. *Int J Hydrogen Energy* 47:10665–10688. <https://doi.org/10.1016/J.IJHYDENE.2022.01.130>
- Kahraman C, Keshavarz Ghorabae M, Zavadskas EK et al (2017) Intuitionistic fuzzy EDAS method: an application to solid waste disposal site selection. *J Environ Eng Landsc Manag* 25:1–12. <https://doi.org/10.3846/16486897.2017.1281139>
- Karaşan A, Kahraman C (2018) A novel interval-valued neutrosophic EDAS method: prioritization of the United Nations national sustainable development goals. *Soft Comput* 22:4891–4906. <https://doi.org/10.1007/s00500-018-3088-y>
- Keshavarz Ghorabae M, Amiri M, Zavadskas EK et al (2017) Stochastic EDAS method for multi-criteria decision-making with normally distributed data. *J Intell Fuzzy Syst* 33:1627–1638. <https://doi.org/10.3233/JIFS-17184>
- Keshavarz-Ghorabae M, Amiri M, Zavadskas EK et al (2018) A dynamic fuzzy approach based on the EDAS method for multi-criteria subcontractor evaluation. *Information*. <https://doi.org/10.3390/info9030068>
- Krishankumar R, Pamucar D, Deveci M, Ravichandran KS (2021) Prioritization of zero-carbon measures for sustainable urban mobility using integrated double hierarchy decision framework and EDAS approach. *Sci Total Environ* 797:149068. <https://doi.org/10.1016/J.SCITOTENV.2021.149068>
- Kutlu Gundogdu F, Kahraman C (2019) Extension of WASPAS with spherical fuzzy sets. *Informatica* 30:269–292. <https://doi.org/10.15388/Informatica.2019.206>
- Kutlu Gündoğdu F, Kahraman C (2019) A novel VIKOR method using spherical fuzzy sets and its application to warehouse site selection. *J Intell Fuzzy Syst* 37:1197–1211. <https://doi.org/10.3233/JIFS-182651>
- Kutlu Gündoğdu F, Kahraman C (2020) A novel spherical fuzzy analytic hierarchy process and its renewable energy application. *Soft Comput* 24:4607–4621. <https://doi.org/10.1007/s00500-019-04222-w>
- Li X, Ju Y, Ju D et al (2019) Multi-attribute group decision making method based on EDAS under picture fuzzy environment. *IEEE Access* 7:141179–141192. <https://doi.org/10.1109/ACCESS.2019.2943348>
- Li Z, Wei G, Wang R et al (2020) EDAS method for multiple attribute group decision making under q-rung orthopair fuzzy environment. *Technol Econ Dev Econ* 26:86–102. <https://doi.org/10.3846/tede.2019.11333>
- Li P, Liu J, Wei C, Liu J (2021) A new EDAS method based on prospect theory for Pythagorean fuzzy set and its application in selecting investment projects for highway. *Kybernetes*. <https://doi.org/10.1108/K-01-2021-0066>
- Liang Y (2020) An EDAS method for multiple attribute group decision-making under intuitionistic fuzzy environment and its application for evaluating green building energy-saving design projects. *Symmetry (basel)*. <https://doi.org/10.3390/SYM12030484>
- Mi X, Liao H (2019) An integrated approach to multiple criteria decision making based on the average solution and normalized weights of criteria deduced by the hesitant fuzzy best worst method. *Comput Ind Eng* 133:83–94. <https://doi.org/10.1016/J.CIE.2019.05.004>
- Onat NC, Gumus S, Kucukvar M, Tatari O (2016) Application of the TOPSIS and intuitionistic fuzzy set approaches for ranking the life cycle sustainability performance of alternative vehicle technologies. *Sustain Prod Consum* 6:12–25. <https://doi.org/10.1016/j.spc.2015.12.003>

- Peng X, Liu C (2017) Algorithms for neutrosophic soft decision making based on EDAS, new similarity measure and level soft set. *J Intell Fuzzy Syst* 32:955–968. <https://doi.org/10.3233/JIFS-161548>
- Smarandache F (1998) Neutrosophy. Neutrosophic Probability, Set, and Logic. ProQuest Information and Learning. Ann Arbor, Michigan, USA
- Szmidt E, Kacprzyk J (2004) LNAI 3070: a similarity measure for intuitionistic fuzzy sets and its application in supporting medical diagnostic reasoning. Springer, Berlin
- Wang P, Wang J, Wei G (2019) EDAS method for multiple criteria group decision making under 2-tuple linguistic neutrosophic environment. *J Intell Fuzzy Syst* 37:1597–1608. <https://doi.org/10.3233/JIFS-179223>
- Yager RR (2014) Pythagorean membership grades in multicriteria decision making. *IEEE Trans Fuzzy Syst* 22:958–965. <https://doi.org/10.1109/TFUZZ.2013.2278989>
- Yager RR (2017) Generalized orthopair fuzzy sets. *IEEE Trans Fuzzy Syst* 25:1222–1230. <https://doi.org/10.1109/TFUZZ.2016.2604005>
- Yanmaz O, Turgut Y, Can EN, Kahraman C (2020) Interval-valued pythagorean fuzzy EDAS method: an application to car selection problem. *J Intell Fuzzy Syst* 38:4061–4077. <https://doi.org/10.3233/JIFS-182667>
- Yu B, Xu Z (2022) Advantage matrix: two novel multi-attribute decision-making methods and their applications. *Artif Intell Rev*. <https://doi.org/10.1007/s10462-021-10126-9>
- Zadeh LA (1965) Fuzzy sets. *Inf. Control* 8:338–353
- Zadeh LA (1975) The concept of a linguistic variable and its applications to approximate reasoning. *Inf Sci (NY)* 8:199–249
- Zhang C, Li D, Ren R (2016) Pythagorean fuzzy multigranulation rough set over two universes and its applications in merger and acquisition. *Int J Intell Syst* 31:921–943. <https://doi.org/10.1002/int.21811>
- Zhang S, Gao H, Wei G et al (2019a) Evaluation based on distance from average solution method for multiple criteria group decision making under picture 2-tuple linguistic environment. *Mathematics*. <https://doi.org/10.3390/math7030243>
- Zhang S, Wei G, Gao H et al (2019b) EDAS method for multiple criteria group decision making with picture fuzzy information and its application to green suppliers selections. *Technol Econ Dev Econ* 25:1123–1138. <https://doi.org/10.3846/tede.2019.10714>

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