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Aggregation operators and VIKOR method based on complex q-rung orthopair uncertain linguistic informations and their applications in multi-attribute decision making

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Abstract

Complex q-rung orthopair uncertain linguistic set (CQROULS) is a combination of complex q-rung orthopair fuzzy set (CQROFS) and uncertain linguistic variable set (ULVS) as a proficient technique to express uncertain and awkward information in real decision theory. CQROULS contains uncertain linguistic variables, truth and falsity grades, which gives extensive freedom to decision makers for taking a decision as compared to CQROFS and their special cases. In this article, a new concept of fuzzy set, called CQROULS using CQROFS and ULVS is explored, and this can examine the qualitative assessment of decision makers and gives them extensive freedom in reflecting their belief about allowable truth grades. Based on the established operational laws and comparison methods for CQROULSs, the notions of complex q-rung orthopair uncertain linguistic weighted-averaging aggregation operator and complex q-rung orthopair uncertain linguistic weighted geometric aggregation operator are explored. Some special cases and the desirable properties of the explored operators are also established and studied. Additionally, the notion of VIseKriterijumska Optimizacija I KOmpromisno Resenje (VIKOR) method based on CQROULSs is explored, with the help of a numerical example, it is verified and also its comparative study is established. Moreover, based on the above analysis, we establish a method to solve the multi-attribute group decision making problems, in which the evaluation information is shown as CQROULNs. Finally, we solve some numerical examples using some decision making steps and explain the verity and proficiency of the explored operators by comparing with other methods, the advantages and graphical interpretation of the explored work are also discussed.

Keywords Complex q-rung orthopair uncertain linguistic sets · Aggregation operators · Weighted aggregation operators · Multi-attribute decision making

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1 Introduction

Multi-attribute decision making (MADM) is a proficient technique to solve various problems in our real-decision environment. MADM is used for the purpose of examining to rank the family of alternatives and to examine the best one. But due to the increase in day by day complexity and difficulty in the environment of the decision-making process, the decision-maker cannot face much longer these kind of problems, which are in the form of numerical values. To solve these kind of problems, the theory of fuzzy set was explored by Zadeh (1965), as characterized by the grade of truth belonging to the unit interval. Atanassove's (1999) modified the theory of FS to explore the idea of intuitionistic FS (IFS), contains the grade of truth and the grade of falsity with the condition that is the sum of both is restricted to the unit interval. Various scholars have studied and utilized it in the environment of different areas (Krishankumar et al. 2020; Seker 2020; Zhang et al. 2020a; Demircioğlu and Ulukan 2020). But there are various problems where the sum of the truth and falsity grades is exceeded form unit interval. For example, when a student is qualified for the Ph.D. test, the group of teachers who set in the interview and indicates their grade whose in the form of yes is 0.6, and in the form of no is 0.7. It is clear that the sum of both is exceeded form unit interval. To address such types of problems, Yager (2013) presented the Pythagorean FS (PFS) with the condition that is the sum of the squares of the both is not exceeded form unit interval. The PFS has utilized in various areas. For example, the divergence measure for PFS was elaborated by Zhou et al. (2020). Song et al. (2020) presented the Pythagorean fuzzy analytic hierarchy process. The Chebyshev distance measures for PFS was explored by Chen (2020). Riaz et al. (2020) examined the TOPSIS method by using the Pythagorean m-polar fuzzy soft sets. Sarkar and Biswas (2020) established the entropy measure, linear programing and modified technique for ideal solution by using the PFSs. But these were still a problem, when a group of teaches indicates their opinion in the form of yes is 0.9 and in the form of no is 0.8, then probably the sum of the squares of the both is exceeded form the unit interval. For addressing such types of difficulties, Yager (2016) again explored the q-rung orthopair FS (QROFS) with a condition that is the sum of q-powers of the truth and falsity grades is not exceeded form unit interval. QROFS is extensive proficient technique to resolve real-decision activities. QROFS have received extensive attention form a scholars and various researchers have applied it to in various areas (Li et al. 2020; Liu and Wang 2020; Tang et al. 2020; Qin et al. 2020; Liu and Huang 2020).

As for the above existing studies, it has been analyzed that they have investigated the MADM problems under the FS, IFS, PFS, QROFS or its generalizations, which are only able to deal with the uncertainty and vagueness that exists in preferences given by the decision makers. None of these models are able to represent the partial ignorance of the data and its fluctuations at a given phase of time. However, in complex data sets, uncertainty and vagueness in the data occur concurrently with changes to the phase (periodicity) of the data. To handle these, Ramot et al. (2002) presented complex FS (CFS), which is characterized by the grade of truth in the form of complex-valued, whose real and imaginary parts are belonging to unit interval. Allah and Salleh (2012) modified the theory of CFS is to explore the idea of complex IFS (CIFS), contains the grade of truth and the grade of falsity in the form of polar coordinates with a condition that is the sum of the real part (also for the imaginary part) of the both is restricted to the unit interval. Various scholars have studied and utilized it in the environment of different areas (Garg and Rani 2020a, b; Ngan et al. 2020). But there are various problems where the sum of the real part (also for imaginary part) of the truth and the real part (also for imaginary part) of the falsity grades is exceeded form unit interval. For example, when a decision maker gives the grades for yes is $0.7e^{i2\pi(0.7)}$ and for no is $0.6e^{i2\pi(0.6)}$. It is clear that the sum of the real part (also for imaginary part) of the both is exceeded form unit interval. To address such types of problems, Ullah et al. (2019a) presented the complex PFS (CPFS) with a condition that is the sum of the squares of the real part (also for imaginary part) of the both is not exceeded form unit interval. The CPFS have utilized in various areas (Akram and Naz 2019). But these was still a problem, when a decision maker indicate their opinion in the form of yes is $0.9e^{i2\pi(0.9)}$ and in the form of no is $0.8e^{i2\pi(0.8)}$, then probably the sum of the squares of the real part (also for imaginary part) of the both is exceeded form the unit interval. For addressing such types of difficulties, Liu et al. (2019a, b) explored the complex QROFS (CQROFS) with a condition that is the sum of q-powers of the real part (also for imaginary part) of the truth and the real part (also for imaginary part) of the falsity grades is not exceeded form unit interval. CQROFS is extensive proficient technique to resolve real-decision activities.

It is actually complicated for a decision maker to give directly the quantitative assessment data, in various real decision problems. For addressing such kinds of complications, the theory of linguistic variable (LV) was explored by Zadeh (1974) as an efficient technique to address with complicated and awkward information. Further, various scholars have modified the theory of LV is to explore uncertain linguistic variable (Xu 2004). The theory of intuitionistic fuzzy uncertain aggregation operators was explored by Liu and Jin (2012). Liu et al. (2014) established the intuitionistic uncertain linguistic Bonferroni mean operators. Liu and Liu (2017) presented the intuitionistic uncertain linguistic partitioned Bonferroni mean operators. Lu and Wei (2017) explored the Pythagorean uncertain linguistic aggregation mean operators. Liu et al. (2017) examined the Pythagorean uncertain linguistic partitioned Bonferroni mean operators. The q-rung orthopair fuzzy uncertain linguistic aggregation operators was explored by Liu et al. (2019b).

From the above analysis it is clear that, various researchers have utilized the aggregation operators in the environment of IFS, PFS, and QROFS (Wang et al. 2012, 2019a, b; Xing et al. 2019a, b; Ullah et al. 2018a, 2020; Ghorabaee et al. 2017; Shen and Wang 2018; Jana et al. 2020a, b; Liu et al. 2020b; Wang and Zhang 2012; Zhang et al. 2020b, c, d, e; Zhan et al. 2020a, b; Jiang et al. 2020) to evaluate the ambiguities which occurred in the problem of MADM. But there is still a problem when a decision-maker provides the information in the form of groups and say to find the best one, it is very difficult to find the relation between them especially when it is in the form of two-dimensional information in a single set. For instance, when a decision maker gives $0.8e^{i2\pi(0.7)}$ for complex-valued truth grade, $0.7e^{i2\pi(0.8)}$ for complex-valued falsity grade, and $\left[\dot{S}_2, \dot{S}_3\right]$ for uncertain linguistic term, then the existing notions like IFS, PFS, QROFS, CIFS, CPFS, CQROFS, and their extensions. For handling such kinds of problems, the aims of this manuscript are summarized as follows:

1. To present the new CQROULS and their special properties.
2. The aggregation operators called averaging and geometric aggregation operators based on CQROULSs are explored and also studied with their important properties.
3. Moreover, based on the above analysis, we establish a method to solve the multi-attribute group decision making problems, in which the evaluation information is shown as CQROULNs.
4. To explore VIKOR method based on novel CQROULNs and compare with some other methods and to examine the reliability and effectiveness of the explored methods.

5. Finally, we solve some numerical examples using some decision making steps and explain the verity and proficiency of the explored operators by comparing with other methods; the advantages and graphical interpretation of the explored work are also discussed.

The purpose of this article is summarized in the following ways: In Sect. 2, we review the CQROFSs, and the notion of uncertain linguistic variable set (ULVS) and their basic properties. In Sect. 3, the novel approach of CQROULS is explored, which is the combination of CQROFS and ULVS is a proficient technique to express uncertain and awkward information in real decision theory. CQROULS contains uncertain linguistic variable, truth and falsity grades, which gives extensive freedom to a decision makers for taking a decision is compared to CQROFS and their special cases. In this article, a new concept of fuzzy set is called CQROULS using CQROFS and ULVS is explored, and this can examine the qualitative assessment of decision makers and gives them extensive freedom in reflecting their belief about allowable truth grades. In Sect. 4, based on the established operational laws and comparison methods for CQROULSs, the notions of complex q-rung orthopair uncertain linguistic weighted averaging aggregation operator and complex q-rung orthopair uncertain linguistic weighted geometric aggregation operator are explored. Some special cases and the desirable properties of the explored operators are also established and studied. Additionally, the VIKOR method based on CQROULSs are also explored and verified it with the help of numerical example. In Sect. 5, based on the above analysis, we establish a method to solve the multi-attribute group decision making problems, in which the evaluation information is shown as CQROULNs. Finally, we solve some numerical examples using some decision making steps and explain the verity and proficiency of the explored operators by comparing with other methods; the advantages and graphical interpretation of the explored work are also discussed.

2 Preliminaries

For better understanding the established work in the next section, we concisely review some useful notions of CQROFS (Liu et al. 2019a, b; Zadeh 1974) and their operational laws. The notion of linguistic term set and uncertain linguistic term set are also discussed. Further, the symbols X_U, μ, and η are represented by the universal grade of truth, and the grade of falsity. Where q^{SC}, $\delta^{SC} \geq 1$.

Definition 1 (Liu et al. 2019a, 2020a; Zadeh 1974) A CQROFS is of the form:

$$\mathcal{Q}^{CQ} = \left\{ \left(\mu_{\mathcal{Q}^{CQ}}(x), \eta_{\mathcal{Q}^{CQ}}(x) \right) : x \in X_U \right\} \tag{1}$$

where $\mu_{\mathcal{Q}^{CQ}} = \mu_{\mathcal{Q}^{RP}} e^{i2\pi W_{\mu_{\mathcal{Q}^{IP}}}}$ and $\eta_{\mathcal{Q}^{CQ}} = \eta_{\mathcal{Q}^{RP}} e^{i2\pi W_{\eta_{\mathcal{Q}^{IP}}}}$, with a conditions: $0 \leq \mu^{q^{SC}}_{\mathcal{Q}^{RP}}(x) + \eta^{q^{SC}}_{\mathcal{Q}^{IP}}(x) \leq 1$, $0 \leq W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP}}}(x) + W^{q^{SC}}_{\eta_{\mathcal{Q}^{IP}}}(x) \leq 1$. Moreover, $\zeta_{\mathcal{Q}^{CQ}}(x) = \zeta_{\mathcal{Q}^{RP}} e^{i2\pi W_{\zeta_{\mathcal{Q}^{IP}}}} = \left(1 - \left(\mu^{q^{SC}}_{\mathcal{Q}^{RP}}(x) + \eta^{q^{SC}}_{\mathcal{Q}^{IP}}(x) \right)^{\frac{1}{q^{SC}}} \right) e^{i2\pi \left(1 - \left(W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP}}}(x) + W^{q^{SC}}_{\eta_{\mathcal{Q}^{IP}}}(x) \right)^{\frac{1}{q^{SC}}} \right)}$ is called refusal grade, the complex q-rung orthopair fuzzy number (CQROFN) is represented by $\mathcal{Q}^{CQ} = \left(\mu_{\mathcal{Q}^{CQ}}(x), \eta_{\mathcal{Q}^{CQ}}(x) \right) = \left(\mu_{\mathcal{Q}^{RP}}(x) e^{i2\pi W_{\mu_{\mathcal{Q}^{IP}}}(x)}, \eta_{\mathcal{Q}^{RP}}(x) e^{i2\pi W_{\eta_{\mathcal{Q}^{IP}}}(x)} \right)$.

Definition 2 (Liu et al. 2019a, 2020a; Zadeh 1974) For any two CQROFNs $\mathcal{Q}^{CQ-1} = \left(\mu_{\mathcal{Q}^{RP-1}}(x) e^{i2\pi W_{\mu_{\mathcal{Q}^{IP-1}}}(x)}, \eta_{\mathcal{Q}^{RP-1}}(x) e^{i2\pi W_{\eta_{\mathcal{Q}^{IP-1}}}(x)} \right)$ and $\mathcal{Q}^{CQ-2} = \left(\mu_{\mathcal{Q}^{RP-2}}(x) e^{i2\pi W_{\mu_{\mathcal{Q}^{IP-2}}}(x)}, \eta_{\mathcal{Q}^{RP-2}}(x) e^{i2\pi W_{\eta_{\mathcal{Q}^{IP-2}}}(x)} \right)$, then

1.

$$\mathcal{Q}^{CQ-1} \oplus_{CQ} \mathcal{Q}^{CQ-2} = \left(\begin{array}{c} \left(\begin{array}{c} \mu^{q^{SC}}_{\mathcal{Q}^{RP-1}}(x) + \mu^{q^{SC}}_{\mathcal{Q}^{RP-2}}(x) - \\ \mu^{q^{SC}}_{\mathcal{Q}^{RP-1}}(x) \mu^{q^{SC}}_{\mathcal{Q}^{RP-2}}(x) \end{array} \right)^{\frac{1}{q^{SC}}} e^{i2\pi \left(\begin{array}{c} W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-1}}}(x) + W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-2}}}(x) - \\ W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-1}}}(x) W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-2}}}(x) \end{array} \right)^{\frac{1}{q^{SC}}}}, \\ \left(\eta_{\mathcal{Q}^{RP-1}}(x) \eta_{\mathcal{Q}^{RP-2}}(x) \right) e^{i2\pi \left(W_{\eta_{\mathcal{Q}^{IP-1}}}(x) W_{\eta_{\mathcal{Q}^{IP-2}}}(x) \right)} \end{array} \right);$$

2.

$$\mathcal{Q}^{CQ-1} \otimes_{CQ} \mathcal{Q}^{CQ-2} = \left(\begin{array}{c} \left(\mu_{\mathcal{Q}^{RP-1}}(x) \mu_{\mathcal{Q}^{RP-2}}(x) \right) e^{i2\pi \left(W_{\mu_{\mathcal{Q}^{IP-1}}}(x) W_{\mu_{\mathcal{Q}^{IP-2}}}(x) \right)}, \\ \left(\begin{array}{c} \eta^{q^{SC}}_{\mathcal{Q}^{RP-1}}(x) + \eta^{q^{SC}}_{\mathcal{Q}^{RP-2}}(x) - \\ \eta^{q^{SC}}_{\mathcal{Q}^{RP-1}}(x) \eta^{q^{SC}}_{\mathcal{Q}^{RP-2}}(x) \end{array} \right)^{\frac{1}{q^{SC}}} e^{i2\pi \left(\begin{array}{c} W^{q^{SC}}_{\eta_{\mathcal{Q}^{IP-1}}}(x) + W^{q^{SC}}_{\eta_{\mathcal{Q}^{IP-2}}}(x) - \\ W^{q^{SC}}_{\eta_{\mathcal{Q}^{IP-1}}}(x) W^{q^{SC}}_{\eta_{\mathcal{Q}^{IP-2}}}(x) \end{array} \right)^{\frac{1}{q^{SC}}}} \end{array} \right);$$

3.

$$\mathcal{Q}^{CQ-1^{\delta^{SC}}} = \left(\begin{array}{c} \mu^{\delta^{SC}}_{\mathcal{Q}^{CQ-1}}(x) e^{i2\pi W^{\delta^{SC}}_{\mu_{\mathcal{Q}^{CQ-1}}}(x)}, \\ \left(1 - \left(1 - \eta^{q^{SC}}_{\mathcal{Q}^{CQ-1}}(x)(x) \right)^{\delta^{SC}} \right)^{\frac{1}{q^{SC}}} e^{i2\pi \left(1 - \left(1 - W^{q^{SC}}_{\eta_{\mathcal{Q}^{CQ-1}}}(x) \right)^{\delta^{SC}} \right)^{\frac{1}{q^{SC}}}} \end{array} \right);$$

4.

$$\delta^{SC} \mathcal{Q}^{CQ-1} = \left(\begin{array}{c} \left(1 - \left(1 - \mu^{q^{SC}}_{\mathcal{Q}^{CQ-1}}(x) \right)^{\delta^{SC}} \right)^{\frac{1}{q^{SC}}} e^{i2\pi \left(1 - \left(1 - W^{q^{SC}}_{\mu_{\mathcal{Q}^{CQ-1}}}(x) \right)^{\delta^{SC}} \right)^{\frac{1}{q^{SC}}}}, \\ \eta^{\delta^{SC}}_{\mathcal{Q}^{CQ-1}}(x) e^{i2\pi W^{\delta^{SC}}_{\eta_{\mathcal{Q}^{CQ-1}}}(x)} \end{array} \right).$$

Definition 3 (Liu et al. 2019a, 2020a; Zadeh 1974) For any two CQROFNs $\mathcal{Q}^{CQ-1} = \left(\mu_{\mathcal{Q}^{RP-1}}(x) e^{i2\pi W_{\mu_{\mathcal{Q}^{IP-1}}}(x)}, \eta_{\mathcal{Q}^{RP-1}}(x) e^{i2\pi W_{\eta_{\mathcal{Q}^{IP-1}}}(x)} \right)$ and $\mathcal{Q}^{CQ-2} = \left(\mu_{\mathcal{Q}^{RP-2}}(x) e^{i2\pi W_{\mu_{\mathcal{Q}^{IP-2}}}(x)}, \eta_{\mathcal{Q}^{RP-2}}(x) e^{i2\pi W_{\eta_{\mathcal{Q}^{IP-2}}}(x)} \right)$, the score and accuracy function is given by:

$$\underset{\cdot}{S}\left(\mathcal{Q}^{CQ-1} \right) = \frac{\left(\mu^{qSC}_{\mathcal{Q}_{RPTL-1}}(x) + W_{\mu_{\mathcal{Q}_{IPTL-1}}}(x) - \eta^{qSC}_{\mathcal{Q}_{RPTL-1}}(x) - W_{\eta_{\mathcal{Q}_{IPTL-1}}}(x) \right)}{2} \tag{2}$$

$$\check{H}\left(\mathcal{Q}^{CQ-1} \right) = \frac{\left(\mu^{qSC}_{\mathcal{Q}_{RPTL-1}}(x) + W_{\mu_{\mathcal{Q}_{IPTL-1}}}(x) + \eta^{qSC}_{\mathcal{Q}_{RPTL-1}}(x) + W_{\eta_{\mathcal{Q}_{IPTL-1}}}(x) \right)}{2} \tag{3}$$

Based on the above two notions, the compassion between two CQROFNs is given by:

1. If $\underset{\cdot}{S}\left(\mathcal{Q}^{CQ-1}\right) > \underset{\cdot}{S}\left(\mathcal{Q}^{CQ-2}\right)$, the $\mathcal{Q}^{CQ-1} > \mathcal{Q}^{CQ-2}$;
2. If $\underset{\cdot}{S}\left(\mathcal{Q}^{CQ-1}\right) = \underset{\cdot}{S}\left(\mathcal{Q}^{CQ-2}\right)$, then:
   1. If $\check{H}\left(\mathcal{Q}^{CQ-1}\right) > \check{H}\left(\mathcal{Q}^{CQ-2}\right)$, the $\mathcal{Q}^{CQ-1} > \mathcal{Q}^{CQ-2}$;
   2. If $\check{H}\left(\mathcal{Q}^{CQ-1}\right) = \check{H}\left(\mathcal{Q}^{CQ-2}\right)$, the $\mathcal{Q}^{CQ-1} = \mathcal{Q}^{CQ-2}$.

Definition 4 (Xu 2004) For a linguistic term set $\dot{S} = \left\{ \dot{S}_j / j = 0, 1, 2, \ldots, z-1 \right\}$ with odd cardinality, where, z is the cardinality of $\dot{S}$, and $\dot{S}_j$ is a linguistic variable. A possible linguistic term set is given by:

$\dot{S} = \{\dot{S}_0, \dot{S}_1, \dot{S}_2, \dot{S}_3, \dot{S}_4, \dot{S}_5, \dot{S}_6\} = \{very\ poor, poor, slightly\ poor, fair, slightly\ good, good, very\ good\}$ by using the values of $z = 7$, then $z - 1 = 6$. The linguistic terms are expressed by Pythagorean fuzzy sets for five and seven terms. Further, $\overline{\dot{S}} = \{\dot{S}_\theta : \theta \in R^+\}$ is called continuous linguistic term sets if the following conditions are holds true:

1. The ordered set: $\dot{S}_\theta < \dot{S}_\varphi$ iff $\theta < \varphi$;
2. The negation operator: $Neg(\dot{S}_\theta) = \dot{S}_\varphi$ such that $\varphi = 2z - \theta$;
3. if $\theta \le \varphi$, then $\max(\dot{S}_\theta, \dot{S}_\varphi) = \dot{S}_\varphi$ and $\min(\dot{S}_\theta, \dot{S}_\varphi) = \dot{S}_\theta$.

Definition 5 (Liu and Jin 2012) For an uncertain linguistic variable $\dot{S} = [\dot{S}_\theta, \dot{S}_\varphi]$,$\dot{S}_\theta, \dot{S}_\varphi \in \dot{S}$ is the upper and lower limits of $\dot{S}$ with $0 < \theta \le \varphi$. For any two ULVs $\dot{S}_1 = [\dot{S}_{\theta_1}, \dot{S}_{\varphi_1}]$ and $\dot{S}_2 = [\dot{S}_{\theta_2}, \dot{S}_{\varphi_2}]$, $\delta^{SC} \ge 0$, then

$$\dot{S}_1 \oplus \dot{S}_2 = [\dot{S}_{\theta_1}, \dot{S}_{\varphi_1}] \oplus [\dot{S}_{\theta_2}, \dot{S}_{\varphi_2}] = \left[\dot{S}_{\theta_1+\theta_2-\frac{\theta_1\theta_2}{z}}, \dot{S}_{\varphi_1+\varphi_2-\frac{\varphi_1\varphi_2}{z}}\right];$$

$$\dot{S}_1 \otimes \dot{S}_2 = [\dot{S}_{\theta_1}, \dot{S}_{\varphi_1}] \otimes [\dot{S}_{\theta_2}, \dot{S}_{\varphi_2}] = \left[\dot{S}_{\frac{\theta_1\times\theta_2}{z}}, \dot{S}_{\frac{\varphi_1\times\varphi_2}{z}}\right];$$

$$\delta^{SC}\dot{S}_1 = \delta^{SC}[\dot{S}_{\theta_1}, \dot{S}_{\varphi_1}] = \left[\dot{S}_{z-z\left(1-\frac{\theta_1}{z}\right)^{\delta^{SC}}}, \dot{S}_{z-z\left(1-\frac{\varphi_1}{z}\right)^{\delta^{SC}}}\right];$$

$$\dot{S}_1^{\delta^{SC}} = [\dot{S}_{\theta_1}, \dot{S}_{\varphi_1}]^{\delta^{SC}} = \left[\dot{S}_{z\left(1-\frac{\theta_1}{z}\right)^{\delta^{SC}}}, \dot{S}_{z\left(1-\frac{\varphi_1}{z}\right)^{\delta^{SC}}}\right].$$

3 Complex q-rung orthopair uncertain linguistic variables

To improve the quality of the proposed work, in this study, we present the novel approach of CQROULS and their fundamental operational laws. Basically, the CQROULS is a mixture of CQROFS and ULS to cope with unpredictable and unreliable information in our day to day life. Based on the existing notion which is discussed in Sect. 2, the explored approaches are follow as:

Definition 6 A CQROULS is given by

$$\mathcal{Q}_{CQUL} = \left\{x, \left([\dot{S}_{\theta(x)}, \dot{S}_{\varphi(x)}], \left(\mu^{\mathcal{Q}_{CQUL}}(x), \eta^{\mathcal{Q}_{CQUL}}(x)\right)\right) : x \in R\right\} \tag{4}$$

where $\mu_{QCQ} = \mu_{QRP} e^{i2\pi W_{\mu_{QIP}}}$ and $\eta_{QCQ} = \eta_{QRP} e^{i2\pi W_{\eta_{QIP}}}$, with a conditions: $0 \le \mu_{QRP}^{q^{SC}}(x) + \eta_{QIP}^{q^{SC}}(x) \le 1$, $0 \le W_{\mu_{QIP}}^{q^{SC}}(x) + W_{\eta_{QIP}}^{q^{SC}}(x) \le 1$ with a ULV $[\dot{S}_{\theta(x)}, \dot{S}_{\varphi(x)}]$. Moreover, $\zeta_{QCQ}(x) = \zeta_{QRP} e^{i2\pi W_{\zeta_{QIP}}} = \left(1 - \left(\mu_{QRP}^{q^{SC}}(x) + \eta_{QIP}^{q^{SC}}(x)\right)^{\frac{1}{q^{SC}}}\right) e^{i2\pi\left(1-\left(W_{\mu_{QIP}}^{q^{SC}}(x)+W_{\eta_{QIP}}^{q^{SC}}(x)\right)^{\frac{1}{q^{SC}}}\right)}$ is called refusal grade, the complex q-rung orthopair uncertain linguistic number (CQROFN) or complex q-rung orthopair uncertain linguistic variable (CQROULV) is represented by $\mathcal{Q}^{CQUL} = \left([\dot{S}_{\theta(x)}, \dot{S}_{\varphi(x)}], \left(\mu^{\mathcal{Q}_{CQUL}}(x), \eta^{\mathcal{Q}_{CQUL}}(x)\right)\right) = \left([\dot{S}_{\theta(x)}, \dot{S}_{\varphi(x)}], \begin{pmatrix} \mu_{QRP}(x) e^{i2\pi W_{\mu_{QIP}}(x)}, \\ \eta_{QRP}(x) e^{i2\pi W_{\eta_{QIP}}(x)} \end{pmatrix}\right)$.

Definition 7 For any two CQROULNs $\mathcal{Q}^{CQUL-1} = \left(\left[\dot{S}_{\theta_1(x)}, \dot{S}_{\varphi_1(x)}\right], \begin{pmatrix} \mu_{\mathcal{Q}^{RP-1}}(x) e^{i2\pi W_{\mu_{\mathcal{Q}^{IP-1}}}(x)}, \\ \eta_{\mathcal{Q}^{RP-1}}(x) e^{i2\pi W_{\eta_{\mathcal{Q}^{IP-1}}}(x)} \end{pmatrix}\right)$ and $\mathcal{Q}^{CQUL-2} = \left(\left[\dot{S}_{\theta_2(x)}, \dot{S}_{\varphi_2(x)}\right], \begin{pmatrix} \mu_{\mathcal{Q}^{RP-2}}(x) e^{i2\pi W_{\mu_{\mathcal{Q}^{IP-2}}}(x)}, \\ \eta_{\mathcal{Q}^{RP-2}}(x) e^{i2\pi W_{\eta_{\mathcal{Q}^{IP-2}}}(x)} \end{pmatrix}\right)$, then

1.

$$\mathcal{Q}^{CQUL-1} \oplus_{CQUL} \mathcal{Q}^{CQUL-2} = \begin{pmatrix} \left[\dot{S}_{\theta_1(x)+\theta_2(x)-\frac{\theta_1(x)\theta_2(x)}{z}}, \dot{S}_{\varphi_1(x)+\varphi_2(x)-\frac{\varphi_1(x)\varphi_2(x)}{z}}\right], \\ \begin{pmatrix} \begin{pmatrix} \mu^{q^{SC}}_{\mathcal{Q}^{RP-1}}(x) + \mu^{q^{SC}}_{\mathcal{Q}^{RP-2}}(x) - \\ \mu^{q^{SC}}_{\mathcal{Q}^{RP-1}}(x)\mu^{q^{SC}}_{\mathcal{Q}^{RP-2}}(x) \end{pmatrix}^{\frac{1}{q^{SC}}} e^{i2\pi \begin{pmatrix} W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-1}}}(x) + W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-2}}}(x) - \\ W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-1}}}(x) W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-2}}}(x) \end{pmatrix}^{\frac{1}{q^{SC}}}}, \\ \left(\eta_{\mathcal{Q}^{RP-1}}(x)\eta_{\mathcal{Q}^{RP-2}}(x)\right) e^{i2\pi\left(W_{\eta_{\mathcal{Q}^{IP-1}}}(x) W_{\eta_{\mathcal{Q}^{IP-2}}}(x)\right)} \end{pmatrix} \end{pmatrix};$$

2.

$$\mathcal{Q}^{CQUL-1} \otimes_{CQUL} \mathcal{Q}^{CQUL-2} = \begin{pmatrix} \left[\dot{S}_{\frac{\theta_1(x)\theta_2(x)}{z}}, \dot{S}_{\frac{\varphi_1(x)\varphi_2(x)}{z}}\right], \\ \begin{pmatrix} \left(\mu_{\mathcal{Q}^{RP-1}}(x)\mu_{\mathcal{Q}^{RP-2}}(x)\right) e^{i2\pi\left(W_{\mu_{\mathcal{Q}^{IP-1}}}(x) W_{\mu_{\mathcal{Q}^{IP-2}}}(x)\right)}, \\ \begin{pmatrix} \eta^{q^{SC}}_{\mathcal{Q}^{RP-1}}(x) + \eta^{q^{SC}}_{\mathcal{Q}^{RP-2}}(x) - \\ \eta^{q^{SC}}_{\mathcal{Q}^{RP-1}}(x)\eta^{q^{SC}}_{\mathcal{Q}^{RP-2}}(x) \end{pmatrix}^{\frac{1}{q^{SC}}} e^{i2\pi \begin{pmatrix} W^{q^{SC}}_{\eta_{\mathcal{Q}^{IP-1}}}(x) + W^{q^{SC}}_{\eta_{\mathcal{Q}^{IP-2}}}(x) - \\ W^{q^{SC}}_{\eta_{\mathcal{Q}^{IP-1}}}(x) W^{q^{SC}}_{\eta_{\mathcal{Q}^{IP-2}}}(x) \end{pmatrix}^{\frac{1}{q^{SC}}}} \end{pmatrix} \end{pmatrix};$$

3.

$$\mathcal{Q}^{CQUL-1^{\delta^{SC}}} = \begin{pmatrix} \left[\dot{S}_{z\left(\frac{\theta_1(x)}{z}\right)^{\delta^{SC}}}, \dot{S}_{z\left(\frac{\varphi_1(x)}{z}\right)^{\delta^{SC}}}\right], \\ \begin{pmatrix} \mu^{\delta^{SC}}_{\mathcal{Q}^{CQ-1}}(x) e^{i2\pi W^{\delta^{SC}}_{\mu_{\mathcal{Q}^{CQ-1}}}(x)}, \\ \left(1-\left(1-\eta^{q^{SC}}_{\mathcal{Q}^{CQ-1}}(x)(x)\right)^{\delta^{SC}}\right)^{\frac{1}{q^{SC}}} e^{i2\pi\left(1-\left(1-W^{q^{SC}}_{\eta_{\mathcal{Q}^{CQ-1}}}(x)\right)^{\delta^{SC}}\right)^{\frac{1}{q^{SC}}}} \end{pmatrix} \end{pmatrix};$$

4.

$$\delta^{SC}\mathcal{Q}^{CQUL-1} = \begin{pmatrix} \left[\dot{S}_{z-z\left(1-\frac{\theta_1(x)}{z}\right)^{\delta^{SC}}}, \dot{S}_{z-z\left(1-\frac{\varphi_1(x)}{z}\right)^{\delta^{SC}}}\right], \\ \begin{pmatrix} \left(1-\left(1-\mu^{q^{SC}}_{\mathcal{Q}^{CQ-1}}(x)\right)^{\delta^{SC}}\right)^{\frac{1}{q^{SC}}} e^{i2\pi\left(1-\left(1-W^{q^{SC}}_{\mu_{\mathcal{Q}^{CQ-1}}}(x)\right)^{\delta^{SC}}\right)^{\frac{1}{q^{SC}}}}, \\ \eta^{\delta^{SC}}_{\mathcal{Q}^{CQ-1}}(x) e^{i2\pi W^{\delta^{SC}}_{\eta_{\mathcal{Q}^{CQ-1}}}(x)} \end{pmatrix} \end{pmatrix}.$$

Definition 8 For any two CQROULNs $\mathcal{Q}^{CQUL-1} = \left(\left[\dot{S}_{\theta_1(x)}, \dot{S}_{\varphi_1(x)}\right], \begin{pmatrix} \mu_{\mathcal{Q}^{RP-1}}(x) e^{i2\pi W_{\mu_{\mathcal{Q}^{IP-1}}}(x)}, \\ \eta_{\mathcal{Q}^{RP-1}}(x) e^{i2\pi W_{\eta_{\mathcal{Q}^{IP-1}}}(x)} \end{pmatrix}\right)$ and $\mathcal{Q}^{CQUL-2} = \left(\left[\dot{S}_{\theta_2(x)}, \dot{S}_{\varphi_2(x)}\right], \begin{pmatrix} \mu_{\mathcal{Q}^{RP-2}}(x) e^{i2\pi W_{\mu_{\mathcal{Q}^{IP-2}}}(x)}, \\ \eta_{\mathcal{Q}^{RP-2}}(x) e^{i2\pi W_{\eta_{\mathcal{Q}^{IP-2}}}(x)} \end{pmatrix}\right)$, the expectation and accuracy function is given by:

$$\dot{S}\left(\mathcal{Q}^{CQUL-1}\right) = \dot{S}_{\frac{(\theta_1(x)+\varphi_1(x))\times\left(\mu^{q^{SC}}_{\mathcal{Q}_{RPTL-1}}(x)+W_{\mu_{\mathcal{Q}_{IPTL-1}}}(x)-\eta^{q^{SC}}_{\mathcal{Q}_{RPTL-1}}(x)-W_{\eta_{\mathcal{Q}_{IPTL-1}}}(x)\right)}{4}} \quad (5)$$

$$\check{H}\left(\mathcal{Q}^{CQUL-1}\right) = \dot{S}_{\frac{(\theta_1(x)+\varphi_1(x))\times\left(\mu^{q^{SC}}_{\mathcal{Q}_{RPTL-1}}(x)+W_{\mu_{\mathcal{Q}_{IPTL-1}}}(x)+\eta^{q^{SC}}_{\mathcal{Q}_{RPTL-1}}(x)+W_{\eta_{\mathcal{Q}_{IPTL-1}}}(x)\right)}{4}} \quad (6)$$

Based on the above two notions, the compassion between two CQROULNs is given by:
1. If $\underset{\cdot}{S}(\mathcal{Q}^{CQUL-1}) > \underset{\cdot}{S}(\mathcal{Q}^{CQUL-2})$, the $\mathcal{Q}^{CQUL-1} > \mathcal{Q}^{CQUL-2}$;
2. If $\underset{\cdot}{S}(\mathcal{Q}^{CQUL-1}) = \underset{\cdot}{S}(\mathcal{Q}^{CQUL-2})$, then:
1. If $\check{H}(\mathcal{Q}^{CQUL-1}) > \check{H}(\mathcal{Q}^{CQUL-2})$, the $\mathcal{Q}^{CQUL-1} > \mathcal{Q}^{CQUL-2}$;
2. If $\check{H}(\mathcal{Q}^{CQUL-1}) = \check{H}(\mathcal{Q}^{CQUL-2})$, the $\mathcal{Q}^{CQUL-1} = \mathcal{Q}^{CQUL-2}$.

Theorem 1 *For any two CQROULNs* $\mathcal{Q}^{CQUL-1} = \left(\left[\dot{S}_{\theta_1(x)}, \dot{S}_{\varphi_1(x)}\right], \begin{pmatrix} \mu_{\mathcal{Q}^{RP-1}}(x)e^{i2\pi W_{\mu_{\mathcal{Q}^{IP-1}}}(x)}, \\ \eta_{\mathcal{Q}^{RP-1}}(x)e^{i2\pi W_{\eta_{\mathcal{Q}^{IP-1}}}(x)} \end{pmatrix}\right)$ *and* $\mathcal{Q}^{CQUL-2} = \left(\left[\dot{S}_{\theta_2(x)}, \dot{S}_{\varphi_2(x)}\right], \begin{pmatrix} \mu_{\mathcal{Q}^{RP-2}}(x)e^{i2\pi W_{\mu_{\mathcal{Q}^{IP-2}}}(x)}, \\ \eta_{\mathcal{Q}^{RP-2}}(x)e^{i2\pi W_{\eta_{\mathcal{Q}^{IP-2}}}(x)} \end{pmatrix}\right)$ *with scalers* δ^{SC-1}, $\delta^{SC-2} \geq 0$, *then*

$$\mathcal{Q}^{CQUL-1} \oplus_{CQUL} \mathcal{Q}^{CQUL-2} = \mathcal{Q}^{CQUL-2} \oplus_{CQUL} \mathcal{Q}^{CQUL-1};$$
$$\mathcal{Q}^{CQUL-1} \otimes_{CQUL} \mathcal{Q}^{CQUL-2} = \mathcal{Q}^{CQUL-2} \otimes_{CQUL} \mathcal{Q}^{CQUL-1};$$
$$\delta^{SC-1}\left(\mathcal{Q}^{CQUL-1} \oplus_{CQUL} \mathcal{Q}^{CQUL-2}\right) = \delta^{SC-1}\mathcal{Q}^{CQUL-2} \oplus_{CQUL} \delta^{SC-1}\mathcal{Q}^{CQUL-1};$$
$$\delta^{SC-1}\mathcal{Q}^{CQUL-1} \oplus_{CQUL} \delta^{SC-2}\mathcal{Q}^{CQUL-1} = \left(\delta^{SC-1} + \delta^{SC-2}\right)\mathcal{Q}^{CQUL-1};$$
$$\mathcal{Q}^{CQUL-1^{\delta^{SC-1}}} \otimes_{CQUL} \mathcal{Q}^{CQUL-2^{\delta^{SC-1}}} = \left(\mathcal{Q}^{CQUL-1} \otimes_{CQUL} \mathcal{Q}^{CQUL-2}\right)^{\delta^{SC-1}};$$
$$\mathcal{Q}^{CQUL-1^{\delta^{SC-1}}} \otimes_{CQUL} \mathcal{Q}^{CQUL-1^{\delta^{SC-2}}} = \mathcal{Q}^{CQUL-1^{\left(\delta^{SC-1}+\delta^{SC-1}\right)}}.$$

Proof Straightforward.

4 Aggregation operators for CQROULSs

To improve the quality of the proposed work, in this study, we present the aggregation operators using the CQROULS and also study their special cases. Basically, we explored the averaging and geometric aggregation and with their weight vector for CQROULS. Based on the established notions which are discussed in Sect. 3, the explored operators are follow as:

Definition 9 For a collection CQROULNs $\mathcal{Q}^{CQUL-j} = \left(\left[\dot{S}_{\theta_j(x)}, \dot{S}_{\varphi_j(x)}\right], \begin{pmatrix} \mu_{\mathcal{Q}^{RP-j}}(x)e^{i2\pi W_{\mu_{\mathcal{Q}^{IP-j}}}(x)}, \\ \eta_{\mathcal{Q}^{RP-j}}(x)e^{i2\pi W_{\eta_{\mathcal{Q}^{IP-j}}}(x)} \end{pmatrix}\right)$, $j = 1, 2, \ldots, n$, the CQROULWA operator is given by:

$$CQROULWA\left(\mathcal{Q}^{CQUL-1}, \mathcal{Q}^{CQUL-2}, \ldots, \mathcal{Q}^{CQUL-n}\right) = \sum_{j=1}^{n} \omega^{w-j}\mathcal{Q}^{CQUL-j} \tag{7}$$

where $\omega^{w} = \left(\omega^{w-1}, \omega^{w-2}, \ldots, \omega^{w-n}\right)^{T}$ denotes the weight vectors with a condition $\sum_{j=1}^{n} \omega^{w-j} = 1$.

Theorem 2 *Suppose a collection CQROULNs* $\mathcal{Q}^{CQUL-j} = \left(\left[\dot{S}_{\theta_j(x)}, \dot{S}_{\varphi_j(x)}\right], \begin{pmatrix} \mu_{\mathcal{Q}^{RP-j}}(x)e^{i2\pi W_{\mu_{\mathcal{Q}^{IP-j}}}(x)}, \\ \eta_{\mathcal{Q}^{RP-j}}(x)e^{i2\pi W_{\eta_{\mathcal{Q}^{IP-j}}}(x)} \end{pmatrix}\right)$, $j = 1, 2, \ldots, n$, *the aggregated value of the* Eq. (7) *is again a CQROULN, we have*

$$CQROULWA\left(\mathcal{Q}^{CQUL-1}, \mathcal{Q}^{CQUL-2}, \ldots, \mathcal{Q}^{CQUL-n}\right) = \sum_{j=1}^{n} \omega^{w-j}\mathcal{Q}^{CQUL-j}$$

$$= \left(\begin{array}{c} \left[\dot{S}_{z - z \prod_{j=1}^{n} \left(1 - \frac{\theta_j(x)}{z}\right)^{\omega^{w-j}}}, \dot{S}_{z - z \prod_{j=1}^{n} \left(1 - \frac{\varphi_j(x)}{z}\right)^{\omega^{w-j}}} \right], \\ \left(\begin{array}{c} \left(1 - \prod_{j=1}^{n} \left(1 - \mu^{q^{SC}}_{\mathcal{Q}^{RP-j}}(x)\right)^{\omega^{w-j}}\right)^{\frac{1}{q^{SC}}} e^{i2\pi \left(1 - \prod_{j=1}^{n} \left(1 - W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-j}}}(x)\right)^{\omega^{w-j}}\right)^{\frac{1}{q^{SC}}}}, \\ \prod_{j=1}^{n} \eta^{\omega^{w-j}}_{\mathcal{Q}^{RP-j}}(x) e^{i2\pi \prod_{j=1}^{n} W^{\omega^{w-j}}_{\eta_{\mathcal{Q}^{IP-j}}}(x)} \end{array} \right) \end{array} \right) \tag{8}$$

Proof By using the method of induction, we prove Eq. (8); the steps of the induction is as follows:

1. For choosing the value of parameter $n = 1$, then Eq. (8) is hold true.
2. For choosing the value of parameter $n = 2$ and using def. (7), we have

$$\omega^{w-1} \mathcal{Q}^{CQUL-1} = \left(\begin{array}{c} \left[\dot{S}_{z - z \left(1 - \frac{\theta_1(x)}{z}\right)^{\omega^{w-1}}}, \dot{S}_{z - z \left(1 - \frac{\varphi_1(x)}{z}\right)^{\omega^{w-1}}} \right], \\ \left(\begin{array}{c} \left(1 - \left(1 - \mu^{q^{SC}}_{\mathcal{Q}^{RP-1}}(x)\right)^{\omega^{w-1}}\right)^{\frac{1}{q^{SC}}} e^{i2\pi \left(1 - \left(1 - W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-1}}}(x)\right)^{\omega^{w-1}}\right)^{\frac{1}{q^{SC}}}}, \\ \eta^{\omega^{w-1}}_{\mathcal{Q}^{RP-1}}(x) e^{i2\pi W^{\omega^{w-1}}_{\eta_{\mathcal{Q}^{IP-1}}}(x)} \end{array} \right) \end{array} \right)$$

$$\omega^{w-2} \mathcal{Q}^{CQUL-2} = \left(\begin{array}{c} \left[\dot{S}_{z - z \left(1 - \frac{\theta_2(x)}{z}\right)^{\omega^{w-2}}}, \dot{S}_{z - z \left(1 - \frac{\varphi_2(x)}{z}\right)^{\omega^{w-2}}} \right], \\ \left(\begin{array}{c} \left(1 - \left(1 - \mu^{q^{SC}}_{\mathcal{Q}^{RP-2}}(x)\right)^{\omega^{w-2}}\right)^{\frac{1}{q^{SC}}} e^{i2\pi \left(1 - \left(1 - W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-2}}}(x)\right)^{\omega^{w-2}}\right)^{\frac{1}{q^{SC}}}}, \\ \eta^{\omega^{w-2}}_{\mathcal{Q}^{RP-2}}(x) e^{i2\pi W^{\omega^{w-2}}_{\eta_{\mathcal{Q}^{IP-2}}}(x)} \end{array} \right) \end{array} \right)$$

then

$$\omega^{w-1} \mathcal{Q}^{CQUL-1} \oplus_{CQUL} \omega^{w-2} \mathcal{Q}^{CQUL-2}$$

$$= \left(\begin{array}{c} \left[\begin{array}{l} \dot{S}_{z - z \left(1 - \frac{\theta_1(x)}{z}\right)^{\omega^{w-1}} + z - z \left(1 - \frac{\theta_2(x)}{z}\right)^{\omega^{w-2}} - \frac{\left(z - z \left(1 - \frac{\theta_1(x)}{z}\right)^{\omega^{w-1}} \times z - z \left(1 - \frac{\theta_2(x)}{z}\right)^{\omega^{w-2}}\right)}{z}}, \\ \dot{S}_{z - z \left(1 - \frac{\varphi_1(x)}{z}\right)^{\omega^{w-1}} + z - z \left(1 - \frac{\varphi_2(x)}{z}\right)^{\omega^{w-2}} - \frac{\left(z - z \left(1 - \frac{\varphi_1(x)}{z}\right)^{\omega^{w-1}} \times z - z \left(1 - \frac{\varphi_2(x)}{z}\right)^{\omega^{w-2}}\right)}{z}} \end{array} \right], \\ \left(\begin{array}{c} \left(\begin{array}{c} \left(1 - \left(1 - \mu^{q^{SC}}_{\mathcal{Q}^{RP-1}}(x)\right)^{\omega^{w-1}}\right)^{\frac{q^{SC}}{q^{SC}}} + \left(1 - \left(1 - \mu^{q^{SC}}_{\mathcal{Q}^{RP-2}}(x)\right)^{\omega^{w-2}}\right)^{\frac{q^{SC}}{q^{SC}}} - \\ \left(1 - \left(1 - \mu^{q^{SC}}_{\mathcal{Q}^{RP-1}}(x)\right)^{\omega^{w-1}}\right)^{\frac{q^{SC}}{q^{SC}}} \times \left(1 - \left(1 - \mu^{q^{SC}}_{\mathcal{Q}^{RP-2}}(x)\right)^{\omega^{w-2}}\right)^{\frac{q^{SC}}{q^{SC}}} \end{array} \right) \\ e^{i2\pi \left(\begin{array}{c} \left(1 - \left(1 - W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-1}}}(x)\right)^{\omega^{w-1}}\right)^{\frac{q^{SC}}{q^{SC}}} + \left(1 - \left(1 - W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-2}}}(x)\right)^{\omega^{w-2}}\right)^{\frac{q^{SC}}{q^{SC}}} - \\ \left(1 - \left(1 - W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-1}}}(x)\right)^{\omega^{w-1}}\right)^{\frac{q^{SC}}{q^{SC}}} \times \left(1 - \left(1 - W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-2}}}(x)\right)^{\omega^{w-2}}\right)^{\frac{q^{SC}}{q^{SC}}} \end{array} \right)}, \\ \eta^{\omega^{w-1}}_{\mathcal{Q}^{RP-1}}(x) \eta^{\omega^{w-2}}_{\mathcal{Q}^{RP-2}}(x) e^{i2\pi \left(W^{\omega^{w-1}}_{\eta_{\mathcal{Q}^{IP-1}}}(x) W^{\omega^{w-2}}_{\eta_{\mathcal{Q}^{IP-2}}}(x) \right)} \end{array} \right) \end{array} \right)$$

$$= \left(\begin{bmatrix} \dot{S}_{z-z\left(1-\frac{\theta_1(x)}{z}\right)^{\omega^{w-1}}+z-z\left(1-\frac{\theta_2(x)}{z}\right)^{\omega^{w-2}}-z\left(1-\left(1-\frac{\theta_1(x)}{z}\right)^{\omega^{w-1}}-\left(1-\frac{\theta_2(x)}{z}\right)^{\omega^{w-2}}+\left(1-\frac{\theta_1(x)}{z}\right)^{\omega^{w-1}}\left(1-\frac{\theta_2(x)}{z}\right)^{\omega^{w-2}}\right)}, \\ \dot{S}_{z-z\left(1-\frac{\varphi_1(x)}{z}\right)^{\omega^{w-1}}+z-z\left(1-\frac{\varphi_2(x)}{z}\right)^{\omega^{w-2}}-z\left(1-\left(1-\frac{\varphi_1(x)}{z}\right)^{\omega^{w-1}}-\left(1-\frac{\varphi_2(x)}{z}\right)^{\omega^{w-2}}+\left(1-\frac{\varphi_1(x)}{z}\right)^{\omega^{w-1}}\left(1-\frac{\varphi_2(x)}{z}\right)^{\omega^{w-2}}\right)} \end{bmatrix}, \left(\begin{array}{c} \left(\begin{array}{c} 1-\left(1-\mu^{q^{SC}}_{\mathcal{Q}^{RP-1}}(x)\right)^{\omega^{w-1}}+1-\left(1-\mu^{q^{SC}}_{\mathcal{Q}^{RP-2}}(x)\right)^{\omega^{w-2}}- \\ \left(1-\left(1-\mu^{q^{SC}}_{\mathcal{Q}^{RP-1}}(x)\right)^{\omega^{w-1}}\right)\left(1-\left(1-\mu^{q^{SC}}_{\mathcal{Q}^{RP-2}}(x)\right)^{\omega^{w-2}}\right) \end{array} \right)^{\frac{1}{q^{SC}}} \\ e^{i2\pi\left(\begin{array}{c} 1-\left(1-W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-1}}}(x)\right)^{\omega^{w-1}}+1-\left(1-W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-2}}}(x)\right)^{\omega^{w-2}}- \\ \left(1-\left(1-W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-1}}}(x)\right)^{\omega^{w-1}}\right)\times\left(1-\left(1-W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-2}}}(x)\right)^{\omega^{w-2}}\right) \end{array} \right)^{\frac{1}{q^{SC}}}}, \\ \eta^{\omega^{w-1}}_{\mathcal{Q}^{RP-1}}(x)\,\eta^{\omega^{w-2}}_{\mathcal{Q}^{RP-2}}(x)\,e^{i2\pi\left(W^{\omega^{w-1}}_{\eta_{\mathcal{Q}^{IP-1}}}(x)\,W^{\omega^{w-2}}_{\eta_{\mathcal{Q}^{IP-2}}}(x)\right)} \end{array} \right) \right)$$

$$= \left(\begin{array}{c} \begin{bmatrix} \dot{S}_{z-z\left(1-\frac{\theta_1(x)}{z}\right)^{\omega^{w-1}}\left(1-\frac{\theta_2(x)}{z}\right)^{\omega^{w-2}}}, \\ \dot{S}_{z-z\left(1-\frac{\varphi_1(x)}{z}\right)^{\omega^{w-1}}\left(1-\frac{\varphi_2(x)}{z}\right)^{\omega^{w-2}}} \end{bmatrix}, \\ \left(\begin{array}{c} \left(1-\left(1-\mu^{q^{SC}}_{\mathcal{Q}^{RP-1}}(x)\right)^{\omega^{w-1}}\left(1-\mu^{q^{SC}}_{\mathcal{Q}^{RP-2}}(x)\right)^{\omega^{w-2}}\right)^{\frac{1}{q^{SC}}} \\ e^{i2\pi\left(1-\left(1-W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-1}}}(x)\right)^{\omega^{w-1}}\left(1-W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-2}}}(x)\right)^{\omega^{w-2}}\right)^{\frac{1}{q^{SC}}}}, \\ \eta^{\omega^{w-1}}_{\mathcal{Q}^{RP-1}}(x)\,\eta^{\omega^{w-2}}_{\mathcal{Q}^{RP-2}}(x)\,e^{i2\pi\left(W^{\omega^{w-1}}_{\eta_{\mathcal{Q}^{IP-1}}}(x)\,W^{\omega^{w-2}}_{\eta_{\mathcal{Q}^{IP-2}}}(x)\right)} \end{array} \right) \end{array} \right)$$

$$= \left(\begin{array}{c} \left[\dot{S}_{z-z\prod_{j=1}^{2}\left(1-\frac{\theta_j(x)}{z}\right)^{\omega^{w-j}}}, \dot{S}_{z-z\prod_{j=1}^{2}\left(1-\frac{\varphi_j(x)}{z}\right)^{\omega^{w-j}}} \right], \\ \left(\begin{array}{c} \left(1-\prod_{j=1}^{2}\left(1-\mu^{q^{SC}}_{\mathcal{Q}^{RP-j}}(x)\right)^{\omega^{w-j}}\right)^{\frac{1}{q^{SC}}} e^{i2\pi\left(1-\prod_{j=1}^{2}\left(1-W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-j}}}(x)\right)^{\omega^{w-j}}\right)^{\frac{1}{q^{SC}}}}, \\ \prod_{j=1}^{2}\eta^{\omega^{w-j}}_{\mathcal{Q}^{RP-j}}(x)\,e^{i2\pi\prod_{j=1}^{2}W^{\omega^{w-j}}_{\eta_{\mathcal{Q}^{IP-j}}}(x)} \end{array} \right) \end{array} \right)$$

The Eq. (8) is truly holds for $n = 2$. Further we check for $n = k$

$$CQROULWA\left(\mathcal{Q}^{CQUL-1}, \mathcal{Q}^{CQUL-2}, \ldots, \mathcal{Q}^{CQUL-k}\right)$$

$$= \left(\begin{array}{c} \left[\dot{S}_{z-z\prod_{j=1}^{k}\left(1-\frac{\theta_j(x)}{z}\right)^{\omega^{w-j}}}, \dot{S}_{z-z\prod_{j=1}^{k}\left(1-\frac{\varphi_j(x)}{z}\right)^{\omega^{w-j}}} \right], \\ \left(\begin{array}{c} \left(1-\prod_{j=1}^{k}\left(1-\mu^{q^{SC}}_{\mathcal{Q}^{RP-j}}(x)\right)^{\omega^{w-j}}\right)^{\frac{1}{q^{SC}}} e^{i2\pi\left(1-\prod_{j=1}^{k}\left(1-W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-j}}}(x)\right)^{\omega^{w-j}}\right)^{\frac{1}{q^{SC}}}}, \\ \prod_{j=1}^{k}\eta^{\omega^{w-j}}_{\mathcal{Q}^{RP-j}}(x)\,e^{i2\pi\prod_{j=1}^{k}W^{\omega^{w-j}}_{\eta_{\mathcal{Q}^{IP-j}}}(x)} \end{array} \right) \end{array} \right)$$

Moreover, we have to check for $n = k + 1$, then

$$CQROULWA\left(\mathcal{Q}^{CQUL-1}, \mathcal{Q}^{CQUL-2}, \ldots, \mathcal{Q}^{CQUL-k+1}\right)$$

$$= CQROULWA\left(\mathcal{Q}^{CQUL-1}, \mathcal{Q}^{CQUL-2}, \ldots, \mathcal{Q}^{CQUL-k}\right) \oplus_{CQUL} \omega^{w-k+1}\mathcal{Q}^{CQUL-k+1}$$

$$= \left(\begin{array}{c} \left[\dot{S}_{z - z\prod_{j=1}^{k}\left(1-\frac{\theta_j(x)}{z}\right)^{\omega^{w-j}}}, \dot{S}_{z - z\prod_{j=1}^{k}\left(1-\frac{\varphi_j(x)}{z}\right)^{\omega^{w-j}}} \right], \\ \left(\begin{array}{c} \left(1-\prod_{j=1}^{k}\left(1-\mu^{q^{SC}}_{Q^{RP-j}}(x)\right)^{\omega^{w-j}}\right)^{\frac{1}{q^{SC}}} e^{i2\pi\left(1-\prod_{j=1}^{k}\left(1-W^{q^{SC}}_{\mu_{Q^{IP-j}}}(x)\right)^{\omega^{w-j}}\right)^{\frac{1}{q^{SC}}}}, \\ \prod_{j=1}^{k}\eta^{\omega^{w-j}}_{Q^{RP-j}}(x) e^{i2\pi\prod_{j=1}^{k}W^{\omega^{w-j}}_{\eta_{Q^{IP-j}}}(x)} \end{array}\right) \end{array}\right) \oplus_{CQUL} \left(\begin{array}{c} \left[\dot{S}_{z - z\left(1-\frac{\theta_{k+1}(x)}{z}\right)^{\omega^{w-k+1}}}, \dot{S}_{z - z\left(1-\frac{\varphi_{k+1}(x)}{z}\right)^{\omega^{w-k+1}}} \right], \\ \left(\begin{array}{c} \left(1-\left(1-\mu^{q^{SC}}_{Q^{RP-k+1}}(x)\right)^{\omega^{w-k+1}}\right)^{\frac{1}{q^{SC}}} e^{i2\pi\left(1-\left(1-W^{q^{SC}}_{\mu_{Q^{IP-k+1}}}(x)\right)^{\omega^{w-k+1}}\right)^{\frac{1}{q^{SC}}}}, \\ \eta^{\omega^{w-k+1}}_{Q^{RP-k+1}}(x) e^{i2\pi W^{\omega^{w-k+1}}_{\eta_{Q^{IP-k+1}}}(x)} \end{array}\right) \end{array}\right)$$

$$= \left(\begin{array}{c} \left[\begin{array}{c} \dot{S}_{z-z\prod_{j=1}^{k}\left(1-\frac{\theta_j(x)}{z}\right)^{\omega^{w-j}} + z-z\left(1-\frac{\theta_{k+1}(x)}{z}\right)^{\omega^{w-k+1}} - \frac{\left(z-z\prod_{j=1}^{k}\left(1-\frac{\theta_j(x)}{z}\right)^{\omega^{w-j}} \times z-z\left(1-\frac{\theta_{k+1}(x)}{z}\right)^{\omega^{w-k+1}}\right)}{z}}, \\ \dot{S}_{z-z\prod_{j=1}^{k}\left(1-\frac{\varphi_j(x)}{z}\right)^{\omega^{w-j}} + z-z\left(1-\frac{\varphi_{k+1}(x)}{z}\right)^{\omega^{w-k+1}} - \frac{\left(z-z\prod_{j=1}^{k}\left(1-\frac{\varphi_j(x)}{z}\right)^{\omega^{w-j}} \times z-z\left(1-\frac{\varphi_{k+1}(x)}{z}\right)^{\omega^{w-k+1}}\right)}{z}} \end{array} \right], \\ \left(\begin{array}{c} \left(\begin{array}{c} \left(1-\prod_{j=1}^{k}\left(1-\mu^{q^{SC}}_{Q^{RP-j}}(x)\right)^{\omega^{w-j}}\right)^{\frac{q^{SC}}{q^{SC}}} + \left(1-\left(1-\mu^{q^{SC}}_{Q^{RP-k+1}}(x)\right)^{\omega^{w-k+1}}\right)^{\frac{q^{SC}}{q^{SC}}} - \\ \left(1-\prod_{j=1}^{k}\left(1-\mu^{q^{SC}}_{Q^{RP-j}}(x)\right)^{\omega^{w-j}}\right)^{\frac{q^{SC}}{q^{SC}}} \times \left(1-\left(1-\mu^{q^{SC}}_{Q^{RP-k+1}}(x)\right)^{\omega^{w-k+1}}\right)^{\frac{q^{SC}}{q^{SC}}} \end{array} \right) \\ e^{i2\pi \left(\begin{array}{c} \left(1-\prod_{j=1}^{k}\left(1-W^{q^{SC}}_{\mu_{Q^{IP-j}}}(x)\right)^{\omega^{w-j}}\right)^{\frac{q^{SC}}{q^{SC}}} + \left(1-\left(1-W^{q^{SC}}_{\mu_{Q^{IP-k+1}}}(x)\right)^{\omega^{w-k+1}}\right)^{\frac{q^{SC}}{q^{SC}}} - \\ \left(1-\prod_{j=1}^{k}\left(1-W^{q^{SC}}_{\mu_{Q^{IP-j}}}(x)\right)^{\omega^{w-j}}\right)^{\frac{q^{SC}}{q^{SC}}} \times \left(1-\left(1-W^{q^{SC}}_{\mu_{Q^{IP-k+1}}}(x)\right)^{\omega^{w-k+1}}\right)^{\frac{q^{SC}}{q^{SC}}} \end{array} \right)}, \\ \prod_{j=1}^{k}\eta^{\omega^{w-j}}_{Q^{RP-j}}(x)\eta^{\omega^{w-k+1}}_{Q^{RP-k+1}}(x) e^{i2\pi\left(\prod_{j=1}^{k}W^{\omega^{w-j}}_{\eta_{Q^{IP-j}}}(x) W^{\omega^{w-k+1}}_{\eta_{Q^{IP-k+1}}}(x)\right)} \end{array} \right) \end{array} \right)$$

$$= \left(\begin{array}{c} \left[\begin{array}{c} \dot{S}_{z-z\prod_{j=1}^{k}\left(1-\frac{\theta_j(x)}{z}\right)^{\omega^{w-j}} + z-z\left(1-\frac{\theta_{k+1}(x)}{z}\right)^{\omega^{w-k+1}} - z\left(\begin{array}{c} 1-\prod_{j=1}^{k}\left(1-\frac{\theta_j(x)}{z}\right)^{\omega^{w-j}} - \left(1-\frac{\theta_{k+1}(x)}{z}\right)^{\omega^{w-k+1}} + \\ \prod_{j=1}^{k}\left(1-\frac{\theta_j(x)}{z}\right)^{\omega^{w-j}}\left(1-\frac{\theta_{k+1}(x)}{z}\right)^{\omega^{w-k+1}} \end{array} \right)}, \\ \dot{S}_{z-z\prod_{j=1}^{k}\left(1-\frac{\varphi_j(x)}{z}\right)^{\omega^{w-j}} + z-z\left(1-\frac{\varphi_{k+1}(x)}{z}\right)^{\omega^{w-k+1}} - z\left(\begin{array}{c} 1-\prod_{j=1}^{k}\left(1-\frac{\varphi_j(x)}{z}\right)^{\omega^{w-j}} - \left(1-\frac{\varphi_{k+1}(x)}{z}\right)^{\omega^{w-k+1}} + \\ \prod_{j=1}^{k}\left(1-\frac{\varphi_j(x)}{z}\right)^{\omega^{w-j}}\left(1-\frac{\varphi_{k+1}(x)}{z}\right)^{\omega^{w-k+1}} \end{array} \right)} \end{array} \right], \\ \left(\begin{array}{c} \left(\begin{array}{c} 1-\prod_{j=1}^{k}\left(1-\mu^{q^{SC}}_{Q^{RP-j}}(x)\right)^{\omega^{w-j}} + 1-\left(1-\mu^{q^{SC}}_{Q^{RP-k+1}}(x)\right)^{\omega^{w-k+1}} - \\ \left(1-\prod_{j=1}^{k}\left(1-\mu^{q^{SC}}_{Q^{RP-j}}(x)\right)^{\omega^{w-j}}\right) \times \left(1-\left(1-\mu^{q^{SC}}_{Q^{RP-k+1}}(x)\right)^{\omega^{w-k+1}}\right) \end{array} \right) \\ e^{i2\pi \left(\begin{array}{c} 1-\prod_{j=1}^{k}\left(1-W^{q^{SC}}_{\mu_{Q^{IP-j}}}(x)\right)^{\omega^{w-j}} + 1-\left(1-W^{q^{SC}}_{\mu_{Q^{IP-k+1}}}(x)\right)^{\omega^{w-k+1}} - \\ \left(1-\prod_{j=1}^{k}\left(1-W^{q^{SC}}_{\mu_{Q^{IP-j}}}(x)\right)^{\omega^{w-j}}\right) \times \left(1-\left(1-W^{q^{SC}}_{\mu_{Q^{IP-k+1}}}(x)\right)^{\omega^{w-k+1}}\right) \end{array} \right)}, \\ \prod_{j=1}^{k}\eta^{\omega^{w-j}}_{Q^{RP-j}}(x)\eta^{\omega^{w-k+1}}_{Q^{RP-k+1}}(x) e^{i2\pi\left(\prod_{j=1}^{k}W^{\omega^{w-j}}_{\eta_{Q^{IP-j}}}(x) W^{\omega^{w-k+1}}_{\eta_{Q^{IP-k+1}}}(x)\right)} \end{array} \right) \end{array} \right)$$

$$= \left(\begin{array}{c} \left[\begin{array}{l} \dot{S}_{z - z\prod_{j=1}^{k}\left(1-\frac{\theta_j(x)}{z}\right)^{\omega^{w-j}}\left(1-\frac{\theta_{k+1}(x)}{z}\right)^{\omega^{w-k+1}}}, \\ \dot{S}_{z - z\prod_{j=1}^{k}\left(1-\frac{\varphi_j(x)}{z}\right)^{\omega^{w-j}}\left(1-\frac{\varphi_{k+1}(x)}{z}\right)^{\omega^{w-k+1}}} \end{array} \right], \\ \left(\begin{array}{c} \left(1-\prod_{j=1}^{k}\left(1-\mu^{q^{SC}}_{\mathcal{Q}^{RP-j}}(x)\right)^{\omega^{w-j}}\left(1-\mu^{q^{SC}}_{\mathcal{Q}^{RP-k+1}}(x)\right)^{\omega^{w-k+1}}\right)^{\frac{1}{q^{SC}}} \\ e^{i2\pi\left(1-\prod_{j=1}^{k}\left(1-W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-j}}}(x)\right)^{\omega^{w-j}}\left(1-W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-k+1}}}(x)\right)^{\omega^{w-k+1}}\right)^{\frac{1}{q^{SC}}}}, \\ \prod_{j=1}^{k+1}\eta^{\omega^{w-j}}_{\mathcal{Q}^{RP-j}}(x)e^{i2\pi\prod_{j=1}^{k+1}W^{\omega^{w-j}}_{\eta_{\mathcal{Q}^{IP-j}}}(x)} \end{array} \right) \end{array} \right)$$

$$= \left(\begin{array}{c} \left[\dot{S}_{z - z\prod_{j=1}^{k+1}\left(1-\frac{\theta_j(x)}{z}\right)^{\omega^{w-j}}}, \dot{S}_{z - z\prod_{j=1}^{k+1}\left(1-\frac{\varphi_j(x)}{z}\right)^{\omega^{w-j}}} \right], \\ \left(\begin{array}{c} \left(1-\prod_{j=1}^{k+1}\left(1-\mu^{q^{SC}}_{\mathcal{Q}^{RP-j}}(x)\right)^{\omega^{w-j}}\right)^{\frac{1}{q^{SC}}} e^{i2\pi\left(1-\prod_{j=1}^{k+1}\left(1-W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-j}}}(x)\right)^{\omega^{w-j}}\right)^{\frac{1}{q^{SC}}}}, \\ \prod_{j=1}^{k+1}\eta^{\omega^{w-j}}_{\mathcal{Q}^{RP-j}}(x)e^{i2\pi\prod_{j=1}^{k+1}W^{\omega^{w-j}}_{\eta_{\mathcal{Q}^{IP-j}}}(x)} \end{array} \right) \end{array} \right)$$

The result has been proved. Further, we evaluate some properties for CQROULNs like idempotency, monotonicity and boundedness.

Theorem 3 *Suppose a collection* CQROULNs $\mathcal{Q}^{CQUL-j} = \left(\left[\dot{S}_{\theta_j(x)}, \dot{S}_{\varphi_j(x)}\right], \left(\begin{array}{c}\mu_{\mathcal{Q}^{RP-j}}(x)e^{i2\pi W_{\mu_{\mathcal{Q}^{IP-j}}}(x)}, \\ \eta_{\mathcal{Q}^{RP-j}}(x)e^{i2\pi W_{\eta_{\mathcal{Q}^{IP-j}}}(x)}\end{array}\right)\right)$, $j = 1, 2, \ldots, n$ *and* $\mathcal{Q}^{CQUL} = \left(\left[\dot{S}_{\theta(x)}, \dot{S}_{\varphi(x)}\right], \left(\begin{array}{c}\mu_{\mathcal{Q}^{RP}}(x)e^{i2\pi W_{\mu_{\mathcal{Q}^{IP}}}(x)}, \\ \eta_{\mathcal{Q}^{RP}}(x)e^{i2\pi W_{\eta_{\mathcal{Q}^{IP}}}(x)}\end{array}\right)\right)$, *if* $\mathcal{Q}^{CQUL-j} = \mathcal{Q}^{CQUL}$, *then*

$$CQROULWA\left(\mathcal{Q}^{CQUL-1}, \mathcal{Q}^{CQUL-2}, \ldots, \mathcal{Q}^{CQUL-n}\right) = \mathcal{Q}^{CQUL}.$$

Proof Suppose $\mathcal{Q}^{CQUL-j} = \mathcal{Q}^{CQUL}$, then by using Eq. (8), we have

$$CQROULWA\left(\mathcal{Q}^{CQUL-1}, \mathcal{Q}^{CQUL-2}, \ldots, \mathcal{Q}^{CQUL-n}\right)$$

$$= \left(\begin{array}{c} \left[\dot{S}_{z - z\prod_{j=1}^{n}\left(1-\frac{\theta_j(x)}{z}\right)^{\omega^{w-j}}}, \dot{S}_{z - z\prod_{j=1}^{n}\left(1-\frac{\varphi_j(x)}{z}\right)^{\omega^{w-j}}} \right], \\ \left(\begin{array}{c} \left(1-\prod_{j=1}^{n}\left(1-\mu^{q^{SC}}_{\mathcal{Q}^{RP-j}}(x)\right)^{\omega^{w-j}}\right)^{\frac{1}{q^{SC}}} e^{i2\pi\left(1-\prod_{j=1}^{n}\left(1-W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP-j}}}(x)\right)^{\omega^{w-j}}\right)^{\frac{1}{q^{SC}}}}, \\ \prod_{j=1}^{n}\eta^{\omega^{w-j}}_{\mathcal{Q}^{RP-j}}(x)e^{i2\pi\prod_{j=1}^{n}W^{\omega^{w-j}}_{\eta_{\mathcal{Q}^{IP-j}}}(x)} \end{array} \right) \end{array} \right)$$

$$= \left(\begin{array}{c} \left[\dot{S}_{z - z\prod_{j=1}^{n}\left(1-\frac{\theta(x)}{z}\right)^{\omega^{w}}}, \dot{S}_{z - z\prod_{j=1}^{n}\left(1-\frac{\varphi(x)}{z}\right)^{\omega^{w}}} \right], \\ \left(\begin{array}{c} \left(1-\prod_{j=1}^{n}\left(1-\mu^{q^{SC}}_{\mathcal{Q}^{RP}}(x)\right)^{\omega^{w}}\right)^{\frac{1}{q^{SC}}} e^{i2\pi\left(1-\prod_{j=1}^{n}\left(1-W^{q^{SC}}_{\mu_{\mathcal{Q}^{IP}}}(x)\right)^{\omega^{w}}\right)^{\frac{1}{q^{SC}}}}, \\ \prod_{j=1}^{n}\eta^{\omega^{w}}_{\mathcal{Q}^{RP}}(x)e^{i2\pi\prod_{j=1}^{n}W^{\omega^{w}}_{\eta_{\mathcal{Q}^{IP}}}(x)} \end{array} \right) \end{array} \right)$$

$$= \left(\left[\dot{S}_{\theta(x)}, \dot{S}_{\varphi(x)} \right], \left(\begin{matrix} \mu_{Q^{RP}}(x) e^{i2\pi W_{\mu_{Q^{IP}}}(x)}, \\ \eta_{Q^{RP}}(x) e^{i2\pi W_{\eta_{Q^{IP}}}(x)} \end{matrix} \right) \right) = Q^{CQUL}$$

The result has been proved.

Theorem 4 *Suppose a collection* CQROULNs $Q^{CQUL-j} = \left(\left[\dot{S}_{\theta_j(x)}, \dot{S}_{\varphi_j(x)} \right], \left(\begin{matrix} \mu_{Q^{RP-j}}(x) e^{i2\pi W_{\mu_{Q^{IP-j}}}(x)}, \\ \eta_{Q^{RP-j}}(x) e^{i2\pi W_{\eta_{Q^{IP-j}}}(x)} \end{matrix} \right) \right)$ *and* $Q^{CQUL-*j} = \left(\left[\dot{S}_{\theta_{*j}(x)}, \dot{S}_{\varphi_{*j}(x)} \right], \left(\begin{matrix} \mu_{Q^{RP-*j}}(x) e^{i2\pi W_{\mu_{Q^{IP-*j}}}(x)}, \\ \eta_{Q^{RP-*j}}(x) e^{i2\pi W_{\eta_{Q^{IP-*j}}}(x)} \end{matrix} \right) \right)$, $j = 1, 2, \ldots, n$, *if* $\dot{S}_{\theta_j(x)} \leq \dot{S}_{\theta_{*j}(x)}$, $\dot{S}_{\varphi_j(x)} \leq \dot{S}_{\varphi_{*j}(x)}$, $\mu_{Q^{RP-j}}(x) \leq \mu_{Q^{RP-*j}}(x)$, $W_{\mu_{Q^{IP-j}}}(x) \leq W_{\mu_{Q^{IP-*j}}}(x)$ *and* $\eta_{Q^{RP-j}}(x) \geq \eta_{Q^{RP-*j}}(x)$, $W_{\eta_{Q^{IP-j}}}(x) \geq W_{\eta_{Q^{IP-*j}}}(x)$, then

$$CQROULWA\left(Q^{CQUL-1}, Q^{CQUL-2}, \ldots, Q^{CQUL-n}\right) \leq CQROULWA\left(Q^{CQUL-*1}, Q^{CQUL-*2}, \ldots, Q^{CQUL-*n}\right)$$

Proof Based on Eq. (8), we know that

$$CQROULWA\left(Q^{CQUL-1}, Q^{CQUL-2}, \ldots, Q^{CQUL-n}\right)$$

$$= \left(\begin{matrix} \left[\dot{S}_{z - z\prod_{j=1}^{n}\left(1-\frac{\theta_j(x)}{z}\right)^{\omega^{w-j}}}, \dot{S}_{z - z\prod_{j=1}^{n}\left(1-\frac{\varphi_j(x)}{z}\right)^{\omega^{w-j}}} \right], \\ \left(\begin{matrix} \left(1 - \prod_{j=1}^{n}\left(1 - \mu_{Q^{RP-j}}^{q^{SC}}(x)\right)^{\omega^{w-j}}\right)^{\frac{1}{q^{SC}}} e^{i2\pi\left(1-\prod_{j=1}^{n}\left(1-W_{\mu_{Q^{IP-j}}}^{q^{SC}}(x)\right)^{\omega^{w-j}}\right)^{\frac{1}{q^{SC}}}}, \\ \prod_{j=1}^{n} \eta_{Q^{RP-j}}^{\omega^{w-j}}(x) e^{i2\pi \prod_{j=1}^{n} W_{\eta_{Q^{IP-j}}}^{\omega^{w-j}}(x)} \end{matrix} \right) \end{matrix} \right)$$

and

$$CQROULWA\left(Q^{CQUL-*1}, Q^{CQUL-*2}, \ldots, Q^{CQUL-*n}\right)$$

$$= \left(\begin{matrix} \left[\dot{S}_{z - z\prod_{j=1}^{n}\left(1-\frac{\theta_{*j}(x)}{z}\right)^{\omega^{w-j}}}, \dot{S}_{z - z\prod_{j=1}^{n}\left(1-\frac{\varphi_{*j}(x)}{z}\right)^{\omega^{w-j}}} \right], \\ \left(\begin{matrix} \left(1 - \prod_{j=1}^{n}\left(1 - \mu_{Q^{RP-*j}}^{q^{SC}}(x)\right)^{\omega^{w-*j}}\right)^{\frac{1}{q^{SC}}} e^{i2\pi\left(1-\prod_{j=1}^{n}\left(1-W_{\mu_{Q^{IP-*j}}}^{q^{SC}}(x)\right)^{\omega^{w-*j}}\right)^{\frac{1}{q^{SC}}}}, \\ \prod_{j=1}^{n} \eta_{Q^{RP-*j}}^{\omega^{w-*j}}(x) e^{i2\pi \prod_{j=1}^{n} W_{\eta_{Q^{IP-*j}}}^{\omega^{w-*j}}(x)} \end{matrix} \right) \end{matrix} \right)$$

First, we have to study the uncertain linguistic parts $\dot{S}_{\theta_j(x)} \leq \dot{S}_{\theta_{*j}(x)}$ and $\dot{S}_{\varphi_j(x)} \leq \dot{S}_{\varphi_{*j}(x)}$, we have

$$\dot{S}_{\frac{\theta_j(x)}{z}} \leq \dot{S}_{\frac{\theta_{*j}(x)}{z}} \Rightarrow 1 - \dot{S}_{\frac{\theta_j(x)}{z}} \leq 1 - \dot{S}_{\frac{\theta_{*j}(x)}{z}} \Rightarrow z\prod_{j=1}^{n}\left(1 - \dot{S}_{\frac{\theta_j(x)}{z}}\right) \geq z\prod_{j=1}^{n}\left(1 - \dot{S}_{\frac{\theta_{*j}(x)}{z}}\right)$$
$$\Rightarrow z - z\prod_{j=1}^{n}\left(1 - \dot{S}_{\frac{\theta_j(x)}{z}}\right) \geq z - z\prod_{j=1}^{n}\left(1 - \dot{S}_{\frac{\theta_{*j}(x)}{z}}\right)$$

Hence $\dot{S}_{\theta_j(x)} \le \dot{S}_{\theta_{*j}(x)}$, similarly, we can prove that $\dot{S}_{\varphi_j(x)} \le \dot{S}_{\varphi_{*j}(x)}$. Next, we discuss the real part of the complex-valued truth grade $\mu_{\mathcal{Q}^{RP-j}}(x) \le \mu_{\mathcal{Q}^{RP-*j}}(x)$, $W_{\mu_{\mathcal{Q}^{IP-j}}}(x) \le W_{\mu_{\mathcal{Q}^{IP-*j}}}(x)$, we have

$$\mu^{q^{SC}}_{\mathcal{Q}^{RP-j}}(x) \le \mu^{q^{SC}}_{\mathcal{Q}^{RP-*j}}(x) \Rightarrow 1 - \mu^{q^{SC}}_{\mathcal{Q}^{RP-j}}(x) \ge 1 - \mu^{q^{SC}}_{\mathcal{Q}^{RP-*j}}(x)$$

$$\Rightarrow \prod_{j=1}^{n}\left(1 - \mu^{q^{SC}}_{\mathcal{Q}^{RP-j}}(x)\right)^{\omega^{w-j}} \ge \prod_{j=1}^{n}\left(1 - \mu^{q^{SC}}_{\mathcal{Q}^{RP-*j}}(x)\right)^{\omega^{w-*j}}$$

$$\Rightarrow 1 - \prod_{j=1}^{n}\left(1 - \mu^{q^{SC}}_{\mathcal{Q}^{RP-j}}(x)\right)^{\omega^{w-j}} \le 1 - \prod_{j=1}^{n}\left(1 - \mu^{q^{SC}}_{\mathcal{Q}^{RP-*j}}(x)\right)^{\omega^{w-*j}}$$

$$\Rightarrow \left(1 - \prod_{j=1}^{n}\left(1 - \mu^{q^{SC}}_{\mathcal{Q}^{RP-j}}(x)\right)^{\omega^{w-j}}\right)^{\frac{1}{q^{SC}}} \le \left(1 - \prod_{j=1}^{n}\left(1 - \mu^{q^{SC}}_{\mathcal{Q}^{RP-*j}}(x)\right)^{\omega^{w-*j}}\right)^{\frac{1}{q^{SC}}}$$

The imaginary part of the complex-valued truth grade is same. Next, we prove the real part of the complex-valued falsity grade $\eta_{\mathcal{Q}^{RP-j}}(x) \ge \eta_{\mathcal{Q}^{RP-*j}}(x)$, $W_{\eta_{\mathcal{Q}^{IP-j}}}(x) \ge W_{\eta_{\mathcal{Q}^{IP-*j}}}(x)$, we have

$$\eta_{\mathcal{Q}^{RP-j}}(x) \ge \eta_{\mathcal{Q}^{RP-*j}}(x) \Rightarrow \prod_{j=1}^{n}\eta^{\omega^{w-j}}_{\mathcal{Q}^{RP-j}}(x) \ge \prod_{j=1}^{n}\eta^{\omega^{w-*j}}_{\mathcal{Q}^{RP-*j}}(x)$$

Hence the expectation values of the $CQROULWA\left(\mathcal{Q}^{CQUL-1}, \mathcal{Q}^{CQUL-2}, \ldots, \mathcal{Q}^{CQUL-n}\right) = A$ and $CQROULWA\left(\mathcal{Q}^{CQUL-*1}, \mathcal{Q}^{CQUL-*2}, \ldots, \mathcal{Q}^{CQUL-*n}\right) = B$ then by using the Def. (8), we have If $\underset{\cdot}{S}\left(\mathcal{Q}^{CQUL-1}\right) > \underset{\cdot}{S}\left(\mathcal{Q}^{CQUL-2}\right)$, the $\mathcal{Q}^{CQUL-1} > \mathcal{Q}^{CQUL-2}$. If $\underset{\cdot}{S}\left(\mathcal{Q}^{CQUL-1}\right) = \underset{\cdot}{S}\left(\mathcal{Q}^{CQUL-2}\right)$, then: If $\check{H}\left(\mathcal{Q}^{CQUL-1}\right) > \check{H}\left(\mathcal{Q}^{CQUL-2}\right)$, the $\mathcal{Q}^{CQUL-1} > \mathcal{Q}^{CQUL-2}$ because $\dot{S}_{\theta_j(x)} \le \dot{S}_{\theta_{*j}(x)}$, $\dot{S}_{\varphi_j(x)} \le \dot{S}_{\varphi_{*j}(x)}$, $\mu_{\mathcal{Q}^{RP-j}}(x) \le \mu_{\mathcal{Q}^{RP-*j}}(x)$, $W_{\mu_{\mathcal{Q}^{IP-j}}}(x) \le W_{\mu_{\mathcal{Q}^{IP-*j}}}(x)$ and $\eta_{\mathcal{Q}^{RP-j}}(x) \ge \eta_{\mathcal{Q}^{RP-*j}}(x)$, $W_{\eta_{\mathcal{Q}^{IP-j}}}(x) \ge W_{\eta_{\mathcal{Q}^{IP-*j}}}(x)$. So by using the above properties, we have get the result, such that

$$CQROULWA\left(\mathcal{Q}^{CQUL-1}, \mathcal{Q}^{CQUL-2}, \ldots, \mathcal{Q}^{CQUL-n}\right) \le CQROULWA\left(\mathcal{Q}^{CQUL-*1}, \mathcal{Q}^{CQUL-*2}, \ldots, \mathcal{Q}^{CQUL-*n}\right)$$

The result has been proved.

Theorem 5 *Suppose a collection CQROULNs* $\mathcal{Q}^{CQUL-j} = \left(\left[\dot{S}_{\theta_j(x)}, \dot{S}_{\varphi_j(x)}\right], \begin{pmatrix} \mu_{\mathcal{Q}^{RP-j}}(x)e^{i2\pi W_{\mu_{\mathcal{Q}^{IP-j}}}(x)}, \\ \eta_{\mathcal{Q}^{RP-j}}(x)e^{i2\pi W_{\eta_{\mathcal{Q}^{IP-j}}}(x)} \end{pmatrix}\right) j = 1, 2, \ldots, n$, *if* $\mathcal{Q}^{-CQUL-j} = \left(\left[\dot{S}_{\theta^-_j(x)}, \dot{S}_{\varphi^-_j(x)}\right], \begin{pmatrix} \mu^-_{\mathcal{Q}^{RP-j}}(x)e^{i2\pi W^-_{\mu_{\mathcal{Q}^{IP-j}}}(x)}, \\ \eta^-_{\mathcal{Q}^{RP-j}}(x)e^{i2\pi W^-_{\eta_{\mathcal{Q}^{IP-j}}}(x)} \end{pmatrix}\right)$ *and* $\mathcal{Q}^{+CQUL-j} = \left(\left[\dot{S}_{\theta^+_j(x)}, \dot{S}_{\varphi^+_j(x)}\right], \begin{pmatrix} \mu^+_{\mathcal{Q}^{RP-j}}(x)e^{i2\pi W^+_{\mu_{\mathcal{Q}^{IP-j}}}(x)}, \\ \eta^+_{\mathcal{Q}^{RP-j}}(x)e^{i2\pi W^+_{\eta_{\mathcal{Q}^{IP-j}}}(x)} \end{pmatrix}\right)$, *then*

$$\mathcal{Q}^{-CQUL-j} \le CQROULWA\left(\mathcal{Q}^{CQUL-1}, \mathcal{Q}^{CQUL-2}, \ldots, \mathcal{Q}^{CQUL-n}\right) \le \mathcal{Q}^{+CQUL-j}$$

Proof It is clear that

$\dot{S}_{min\theta_j(x)} \leq \dot{S}_{\theta_j(x)} \leq \dot{S}_{max\theta_j(x)}$, $\dot{S}_{min\varphi_j(x)} \leq \dot{S}_{\varphi_j(x)} \leq \dot{S}_{max\varphi_j(x)}$, $min\mu_{Q^{RP-j}}(x) \leq \mu_{Q^{RP-j}}(x) \leq max\mu_{Q^{RP-j}}(x)$, $minW_{\mu_{Q^{IP-j}}}(x) \leq W_{\mu_{Q^{IP-j}}}(x) \leq maxW_{\mu_{Q^{IP-j}}}(x)$, $max\eta_{Q^{RP-j}}(x) \geq \eta_{Q^{RP-j}}(x) \geq min\eta_{Q^{RP-j}}(x)$ and $maxW_{\eta_{Q^{IP-j}}}(x) \geq W_{\eta_{Q^{IP-j}}}(x) \geq minW_{\eta_{Q^{IP-j}}}(x)$, then by using the theorem 3 and theorem 4, such that

$$CQROULWA\left(Q^{CQUL-1}, Q^{CQUL-2}, \ldots, Q^{CQUL-n}\right)$$
$$\geq CQROULWA\left(Q^{-CQUL-1}, Q^{-CQUL-2}, \ldots, Q^{-CQUL-n}\right) = Q^{-CQUL-j}$$

$$CQROULWA\left(Q^{CQUL-1}, Q^{CQUL-2}, \ldots, Q^{CQUL-n}\right)$$
$$\leq CQROULWA\left(Q^{+CQUL-1}, Q^{+CQUL-2}, \ldots, Q^{+CQUL-n}\right) = Q^{+CQUL-j}$$

therefore

$$Q^{-CQUL-j} \leq CQROULWA\left(Q^{CQUL-1}, Q^{CQUL-2}, \ldots, Q^{CQUL-n}\right) \leq Q^{+CQUL-j}$$

The result has been proved.

***Remark* 1** If we choose the values of imaginary part as zero in Eq. (8), then Eq. (8) is reduced for q-rung orthopair uncertain linguistic sets. Similarly, if we choose the values of $q^{SC} = 2$ in Eq. (8), then Eq. (8) is reduced for complex Pythagorean uncertain linguistic sets and if we choose the values of $q^{SC} = 1$ in Eq. (8), then Eq. (8) is reduced for complex intuitionistic uncertain linguistic sets.

Definition 10 For a collection CQROULNs $Q^{CQUL-j} = \left(\left[\dot{S}_{\theta_j(x)}, \dot{S}_{\varphi_j(x)}\right], \begin{pmatrix} \mu_{Q^{RP-j}}(x)e^{i2\pi W_{\mu_{Q^{IP-j}}}(x)}, \\ \eta_{Q^{RP-j}}(x)e^{i2\pi W_{\eta_{Q^{IP-j}}}(x)} \end{pmatrix}\right)$, $j = 1, 2, \ldots, n$, the CQROULWG operator is given by:

$$CQROULWG\left(Q^{CQUL-1}, Q^{CQUL-2}, \ldots, Q^{CQUL-n}\right) = \prod_{j=1}^{n}\left(Q^{CQUL-j}\right)^{\omega^{w-j}} \quad (9)$$

where $\omega^w = \left(\omega^{w-1}, \omega^{w-2}, \ldots, \omega^{w-n}\right)^T$ denotes the weight vectors with a condition $\sum_{j=1}^{n}\omega^{w-j} = 1$.

Theorem 6 *Suppose a collection CQROULNs* $Q^{CQUL-j} = \left(\left[\dot{S}_{\theta_j(x)}, \dot{S}_{\varphi_j(x)}\right], \begin{pmatrix} \mu_{Q^{RP-j}}(x)e^{i2\pi W_{\mu_{Q^{IP-j}}}(x)}, \\ \eta_{Q^{RP-j}}(x)e^{i2\pi W_{\eta_{Q^{IP-j}}}(x)} \end{pmatrix}\right)$, $j = 1, 2, \ldots, n$, *the aggregated value of the* Eq. (9) *is again a CQROULN, we have*

$$CQROULWG\left(Q^{CQUL-1}, Q^{CQUL-2}, \ldots, Q^{CQUL-n}\right) = \prod_{j=1}^{n}\left(Q^{CQUL-j}\right)^{\omega^{w-j}}$$

$$= \left(\begin{array}{c} \left[\dot{S}_{z \prod_{j=1}^{n} \left(\frac{\theta_j(x)}{z} \right)^{\omega^{w-j}}}, \dot{S}_{z \prod_{j=1}^{n} \left(\frac{\varphi_j(x)}{z} \right)^{\omega^{w-j}}} \right], \\ \left(\begin{array}{c} \prod_{j=1}^{n} \mu^{\omega^{w-j}}_{\mathcal{Q}^{RP-j}}(x) e^{i2\pi \prod_{j=1}^{n} W^{\omega^{w-j}}_{\mu_{\mathcal{Q}^{IP-j}}}(x)}, \\ \left(1 - \prod_{j=1}^{n} \left(1 - \eta^{q^{SC}}_{\mathcal{Q}^{RP-j}}(x)\right)^{\omega^{w-j}}\right)^{\frac{1}{q^{SC}}} e^{i2\pi \left(1 - \prod_{j=1}^{n} \left(1 - W^{q^{SC}}_{\eta_{\mathcal{Q}^{IP-j}}}(x)\right)^{\omega^{w-j}}\right)^{\frac{1}{q^{SC}}}} \end{array} \right) \end{array} \right) \tag{10}$$

Proof Straightforward. (Similar to Theorem 2).

Further, we evaluate some properties for CQROULNs like idempotency, monotonicity and boundedness.

Theorem 7 *Suppose a collection CQROULNs* $\mathcal{Q}^{CQUL-j} = \left(\left[\dot{S}_{\theta_j(x)}, \dot{S}_{\varphi_j(x)} \right], \left(\begin{array}{c} \mu_{\mathcal{Q}^{RP-j}}(x) e^{i2\pi W_{\mu_{\mathcal{Q}^{IP-j}}}(x)}, \\ \eta_{\mathcal{Q}^{RP-j}}(x) e^{i2\pi W_{\eta_{\mathcal{Q}^{IP-j}}}(x)} \end{array} \right) \right)$, $j = 1, 2, \ldots, n$ *and* $\mathcal{Q}^{CQUL} = \left(\left[\dot{S}_{\theta(x)}, \dot{S}_{\varphi(x)} \right], \left(\begin{array}{c} \mu_{\mathcal{Q}^{RP}}(x) e^{i2\pi W_{\mu_{\mathcal{Q}^{IP}}}(x)}, \\ \eta_{\mathcal{Q}^{RP}}(x) e^{i2\pi W_{\eta_{\mathcal{Q}^{IP}}}(x)} \end{array} \right) \right)$, if $\mathcal{Q}^{CQUL-j} = \mathcal{Q}^{CQUL}$, *then*

$$CQROULWG\left(\mathcal{Q}^{CQUL-1}, \mathcal{Q}^{CQUL-2}, \ldots, \mathcal{Q}^{CQUL-n}\right) = \mathcal{Q}^{CQUL}$$

Proof Straightforward. (Similar to Theorem 3).

Theorem 8 *Suppose a collection CQROULNs* $\mathcal{Q}^{CQUL-j} = \left(\left[\dot{S}_{\theta_j(x)}, \dot{S}_{\varphi_j(x)} \right], \left(\begin{array}{c} \mu_{\mathcal{Q}^{RP-j}}(x) e^{i2\pi W_{\mu_{\mathcal{Q}^{IP-j}}}(x)}, \\ \eta_{\mathcal{Q}^{RP-j}}(x) e^{i2\pi W_{\eta_{\mathcal{Q}^{IP-j}}}(x)} \end{array} \right) \right)$ *and* $\mathcal{Q}^{CQUL-*j} = \left(\left[\dot{S}_{\theta_{*j}(x)}, \dot{S}_{\varphi_{*j}(x)} \right], \left(\begin{array}{c} \mu_{\mathcal{Q}^{RP-*j}}(x) e^{i2\pi W_{\mu_{\mathcal{Q}^{IP-*j}}}(x)}, \\ \eta_{\mathcal{Q}^{RP-*j}}(x) e^{i2\pi W_{\eta_{\mathcal{Q}^{IP-*j}}}(x)} \end{array} \right) \right)$, $j = 1, 2, \ldots, n$, *if* $\dot{S}_{\theta_j(x)} \leq \dot{S}_{\theta_{*j}(x)}$, $\dot{S}_{\varphi_j(x)} \leq \dot{S}_{\varphi_{*j}(x)}$, $\mu_{\mathcal{Q}^{RP-j}}(x) \leq \mu_{\mathcal{Q}^{RP-*j}}(x)$, $W_{\mu_{\mathcal{Q}^{IP-j}}}(x) \leq W_{\mu_{\mathcal{Q}^{IP-*j}}}(x)$ *and* $\eta_{\mathcal{Q}^{RP-j}}(x) \geq \eta_{\mathcal{Q}^{RP-*j}}(x)$, $W_{\eta_{\mathcal{Q}^{IP-j}}}(x) \geq W_{\eta_{\mathcal{Q}^{IP-*j}}}(x)$, *then*

$$CQROULWG\left(\mathcal{Q}^{CQUL-1}, \mathcal{Q}^{CQUL-2}, \ldots, \mathcal{Q}^{CQUL-n}\right) \leq CQROULWG\left(\mathcal{Q}^{CQUL-*1}, \mathcal{Q}^{CQUL-*2}, \ldots, \mathcal{Q}^{CQUL-*n}\right)$$

Proof Straightforward. (Similar to Theorem 4).

Theorem 9 *Suppose a collection CQROULNs* $\mathcal{Q}^{CQUL-j} = \left(\left[\dot{S}_{\theta_j(x)}, \dot{S}_{\varphi_j(x)} \right], \left(\begin{array}{c} \mu_{\mathcal{Q}^{RP-j}}(x) e^{i2\pi W_{\mu_{\mathcal{Q}^{IP-j}}}(x)}, \\ \eta_{\mathcal{Q}^{RP-j}}(x) e^{i2\pi W_{\eta_{\mathcal{Q}^{IP-j}}}(x)} \end{array} \right) \right) j = 1, 2, \ldots, n$, *if* $\mathcal{Q}^{-CQUL-j} = \left(\left[\dot{S}_{\theta_j^-(x)}, \dot{S}_{\varphi_j^-(x)} \right], \left(\begin{array}{c} \mu^-_{\mathcal{Q}^{RP-j}}(x) e^{i2\pi W^-_{\mu_{\mathcal{Q}^{IP-j}}}(x)}, \\ \eta^-_{\mathcal{Q}^{RP-j}}(x) e^{i2\pi W^-_{\eta_{\mathcal{Q}^{IP-j}}}(x)} \end{array} \right) \right)$ *and* $\mathcal{Q}^{+CQUL-j} = \left(\left[\dot{S}_{\theta_j^+(x)}, \dot{S}_{\varphi_j^+(x)} \right], \left(\begin{array}{c} \mu^+_{\mathcal{Q}^{RP-j}}(x) e^{i2\pi W^+_{\mu_{\mathcal{Q}^{IP-j}}}(x)}, \\ \eta^+_{\mathcal{Q}^{RP-j}}(x) e^{i2\pi W^+_{\eta_{\mathcal{Q}^{IP-j}}}(x)} \end{array} \right) \right)$, *then*

$$\mathcal{Q}^{-CQUL-j} \leq CQROULWG\left(\mathcal{Q}^{CQUL-1}, \mathcal{Q}^{CQUL-2}, \ldots, \mathcal{Q}^{CQUL-n}\right) \leq \mathcal{Q}^{+CQUL-j}.$$

Proof Straightforward. (Similar to Theorem 5).

***Remark* 2** If we choose the values of imaginary part is zero in Eq. (10), then the Eq. (10) is reduced for q-rung orthopair uncertain linguistic sets. Similarly, if we choose the values of $q^{SC} = 2$ in Eq. (10), then the Eq. (10) is reduced for complex Pythagorean uncertain linguistic sets and if we choose the values of $q^{SC} = 1$ in Eq. (10), then the Eq. (10) is reduced for complex intuitionistic uncertain linguistic sets.

4.1 VIKOR method for complex q-Rung orthopair uncertain linguistic MADM problems

The VIKOR approach, pioneered for multi-attribute optimization problems, concentrate on ranking the alternatives and considered a compromise solution. The decision making problem, which can be solve by VIKOR, is express as follows.

Considered the m alternatives and n attributes $X_1, X_2, \ldots, X_m$ and $\widetilde{\widehat{A}}_1, \widetilde{\widehat{A}}_2, \ldots, \widetilde{\widehat{A}}_n$ with respect to weight vectors such that $w = (w_1, w_2, \ldots, w_n)^{\underset{\frown}{h}}$, $\sum_{j=1}^{n} w_i = 1$, the compromise ranking by VIKOR methods is started with the form of L_p-metric (He et al. 2019).

$$L_{pi} = \left\{\sum_{j=1}^{n}\left[\left(\frac{R_j^* - R_{ij}}{R_j^* - R_j^-}\right)\right]^b\right\}^{\frac{1}{b}}, 1 \leq b \leq \infty, i = 1, 2, \ldots, m \quad (11)$$

In the VIKOR method, the maximum group utility can be gotten by $minS_i$ and minimum individual regret can be gotten by $minS_i^{'}$, where $S_i = L_{1,i}$, and $S_i^{'} = L_{\infty,i}$.

The steps of the VIKOR method is follow as:

Step 1: Computing the virtual positive ideal x_j^* and the virtual negative ideal x_j^- values under the attributes $\widetilde{\widehat{A}}_j$, we have

$$x_j^* = max_i\left(x_{ij}\right), x_j^- = min_i\left(x_{ij}\right) \quad (12)$$

Step 2: Computing the values of group utility S_i and $S_i^{'}$, we have

$$S_i = \frac{\sum_{j=1}^{n} w_j \| R_j^* - R_{ij} \|}{\| R_j^* - R_j^- \|} \quad (13)$$

$$S_i^{'} = \frac{max_j w_j \| R_j^* - R_{ij} \|}{\| R_j^* - R_j^- \|} \quad (14)$$

Step 3: Computing the values of Q_i, $i = 1, 2, \ldots, m$, we have

$$Q_i = \frac{v(S_i - S^*)}{(S^- - S^*)} + \frac{(1 - v)\left(S_i^{'} - S^{'*}\right)}{\left(S^{'-} - S^{'*}\right)} \quad (15)$$

where $S^* = min_i(S_i)$, $S^- = max_i(S_i)$, $S^{'*} = min_i\left(S_i^{'}\right)$ and $S^{'-} = max_i\left(S_i^{'}\right)$, the symbol v is the balance parameter which can balance the group of utility and individual regret. There are three possibilities:

1. If $v > 0.5$ represents the maximum group utility is more than minimum individual regret.
2. If $v < 0.5$ represents the minimum individual regret is more than maximum group utility.
3. If $v = 0.5$ represents the maximum group utility and minimum individual regret are same importance.

Step 4: Using the values of S, $S^{'}$ and Q and ranking the alternatives, then we will obtain the compromise solution.

Step 5: When we get the compromise solution $X^{(1)}$ in steps 4, then it satisfied the following two conditions.

Condition 1: *Acceptable advantages*: $Q\left(X^{(2)}\right) - Q\left(X^{(1)}\right) \geq \frac{1}{m-1}$, where $Q\left(X^{(2)}\right)$ is the Q value in the second position of all ranking alternatives produced by the value of Q and m number of alternatives.

Condition 2: *Acceptable stability*: Alternative $X^{(1)}$ must also in the first position of all ranking alternatives produced by the values of S or $S^{'}$.

If one of the above condition is not met, then we collected the compromise alternatives and not one compromise solution.

1. If condition 2 is not hold, then we will examine the alternatives $X^{(1)}$ and $X^{(2)}$ should be compromise solution.
2. If condition 2 is not hold, then the maximum M eximane by the formula $Q\left(X^{(M)}\right) - Q\left(X^{(1)}\right) < MQ = \frac{1}{m-1}$, we examined the alternatives $X^{(1)}, X^{(2)}, \ldots, X^{(M)}$ are compromise solution.

Based on the above analysis, we will construct the VIKOR method for CQROULSs.

4.2 VIKOR method for CQROULSs

Let $X = \{X_1, X_2, \ldots, X_m\}$ be a collection of m alternatives, $\tilde{\widehat{A}} = \left\{\tilde{\widehat{A}}_1, \tilde{\widehat{A}}_2, \ldots, \tilde{\widehat{A}}_m\right\}$ be the collection of attributes with respect to weight vectors $w = (w_1, w_2, \ldots, w_n)^{\widehat{h}}$, $\sum_{j=1}^{n} w_i = 1$. The decision matrix for CQROULSs is follow as: $R^{'} = \left(r_{jk}^{L}\right)_{n\times m}$, where the Complex q-rung orthopair uncertain linguistic number is represented by $r_{jk}^{L} = \left(\left[\dot{S}_{\theta_{jk}(x)}^{L}, \dot{S}_{\varphi_{jk}(x)}^{L}\right], \begin{pmatrix} \mu_{QRP-jk}^{L}(x)e^{i2\pi W_{\mu_{QIP-jk}}^{L}(x)}, \\ \eta_{QRP-jk}^{L}(x)e^{i2\pi W_{\eta_{QIP-jk}}^{L}(x)} \end{pmatrix}\right)$, the aim of VIKOR method is follow as:

Step 1: Normalize the decision matrix, there are two types of attribute such as benefits B and cost C types attributes, the normalized can be done by the following formula;

$$R_{DM}^{L} = \left(r_{jk}^{L}\right)_{m\times n} = \left(\left[\dot{S}_{\theta_{jk}(x)}^{L}, \dot{S}_{\varphi_{jk}(x)}^{L}\right], \begin{pmatrix} \mu_{QRP-jk}^{L}(x)e^{i2\pi W_{\mu_{QIP-jk}}^{L}(x)}, \\ \eta_{QRP-jk}^{L}(x)e^{i2\pi W_{\eta_{QIP-jk}}^{L}(x)} \end{pmatrix}\right) \quad (16)$$

$$R_{DM}^{L} = \left(r_{jk}^{L}\right)_{m\times n} = \left(\left[\dot{S}_{\theta_{jk}(x)}^{L}, \dot{S}_{\varphi_{jk}(x)}^{L}\right], \begin{pmatrix} \eta_{QRP-jk}^{L}(x)e^{i2\pi W_{\eta_{QIP-jk}}^{L}(x)}, \\ \mu_{QRP-jk}^{L}(x)e^{i2\pi W_{\mu_{QIP-jk}}^{L}(x)} \end{pmatrix}\right) \quad (17)$$

Step 2: Computing the virtual positive ideal x_k^* and the virtual negative ideal x_k^- values under the attributes $\tilde{\widehat{A}}_j$, we have

$$x_k^* = max_j\left(r_{jk}^{L}\right), x_k^- = min_j\left(r_{jk}^{L}\right) \quad (18)$$

Step 3: Computing the values of group utility S_i and $S_i^{'}$, we have

$$S_i = \frac{\sum_{j=1}^{n} w_j \left(\| R_j^* - R_{ij} \| \right)}{\left(\| R_j^* - R_j^- \| \right)} \tag{19}$$

$$S_i^{'} = \frac{max_j w_j \left(\| R_j^* - R_{ij} \| \right)}{\left(\| R_j^* - R_j^- \| \right)} \tag{20}$$

where $\| R_i, R_j \|$ represents the distance between two CQROULNs, which is defined as:

$$d\left(R_i, R_j\right) = \| R_i - R_j \|$$
$$= \frac{1}{2} \sum_{i=1}^{n} \left(\begin{array}{c} \left| \mu_{QRP-i}^{L}{}^{q^{SC}} - \mu_{QRP-j}^{L}{}^{q^{SC}} \right| + \left| \eta_{QRP-i}^{L}{}^{q^{SC}} - \eta_{QRP-j}^{L}{}^{q^{SC}} \right| \\ + \left| W_{\mu_{QIP-i}}^{L}{}^{q^{SC}} - W_{\mu_{QIP-j}}^{L}{}^{q^{SC}} \right| + \left| W_{\eta_{QIP-i}}^{L}{}^{q^{SC}} - W_{\eta_{QIP-j}}^{L}{}^{q^{SC}} \right| \end{array} \right)$$
$$\times \left(\dot{S}_{\varphi_j}^{L} - \dot{S}_{\theta_i}^{L} \right) \tag{21}$$

Step 4: Computing the values of Q_i, $i = 1, 2, \ldots, m$, we have

$$Q_i = \frac{v(S_i - S^*)}{\left(S^- - S^*\right)} + \frac{(1 - v)\left(S_i^{'} - S^{'*}\right)}{\left(S^{'-} - S^{'*}\right)} \tag{22}$$

where $S^* = min_i(S_i)$, $S^- = max_i(S_i)$, $S^{'*} = min_i\left(S_i^{'}\right)$ and $S^{'-} = max_i\left(S_i^{'}\right)$, the symbol v is the balance parameter which can balance the group of utility and individual regret. There are three possibility:

1. If $v > 0.5$ represents the maximum group utility is more than minimum individual regret.
2. If $v < 0.5$ represents the minimum individual regret is more than maximum group utility.
3. If $v = 0.5$ represents the maximum group utility and minimum individual regret are same importance.

Step 4: Using the values of S, $S^{'}$ and Q and ranking the alternatives, then we will obtain the compromise solution.

Step 5: When we get the compromise solution $X^{(1)}$ in steps 4, then it satisfied the following two conditions.

Condition 1: *Acceptable advantages*: $Q\left(X^{(2)}\right) - Q\left(X^{(1)}\right) \geq \frac{1}{m-1}$, where $Q\left(X^{(2)}\right)$ is the Q value in the second position of all ranking alternatives produced by the value of Q and m number of alternatives.

Condition 2: *Acceptable stability*: Alternative $X^{(1)}$ must also in the first position of all ranking alternatives produced by the values of S or $S^{'}$.

Example 1 We take the method from Ref. (Garg et al. 2020) which is a invest selection problem (Table 1). The investment company want to in with one of the following company is denoted by $X_i (i = 1, 2, 3, 4)$ and measured by four attributes, whose detail is discussed in Table 7.

The attributes in Table 2, is the form of CQROULNs, with weight vectors $w = (0.35, 0.25, 0.3, 0.1)$. based on VIKOR method, we solve the following matrix (Table 3).

Step 1: We normalize the Table 2 using the Eqs. (16) and (17), then the new decision matrix is follow as:

Table 1 Representation of different attributes

Symbols	$\boldsymbol{A}_1$	$\boldsymbol{A}_2$	$\boldsymbol{A}_3$	$\boldsymbol{A}_4$
Representations	Anti-risk ability	Growth ability	Social impact	Environment impact

Step 2: We computing the virtual positive ideal x_j^* and the virtual negative ideal x_j^- values under the attributes $\tilde{\widehat{A}}_j$ using the Eq. (18), then

$$x^* = \left\{ \begin{pmatrix} [\dot{S}_2, \dot{S}_4], \\ (0.7e^{i2\pi(0.8)}, 0.3e^{i2\pi(0.22)}) \end{pmatrix}, \begin{pmatrix} [\dot{S}_3, \dot{S}_4], \\ (0.62e^{i2\pi(0.88)}, 0.24e^{i2\pi(0.13)}) \end{pmatrix}, \begin{pmatrix} [\dot{S}_3, \dot{S}_4], \\ (0.87e^{i2\pi(0.86)}, 0.14e^{i2\pi(0.13)}) \end{pmatrix}, \begin{pmatrix} [\dot{S}_3, \dot{S}_4], \\ (0.75e^{i2\pi(0.9)}, 0.24e^{i2\pi(0.13)}) \end{pmatrix} \right\}$$

$$x^- = \left\{ \begin{pmatrix} [\dot{S}_1, \dot{S}_2], \\ (0.63e^{i2\pi(0.73)}, 0.4e^{i2\pi(0.3)}) \end{pmatrix}, \begin{pmatrix} [\dot{S}_1, \dot{S}_3], \\ (0.55e^{i2\pi(0.70)}, 0.43e^{i2\pi(0.33)}) \end{pmatrix}, \begin{pmatrix} [\dot{S}_1, \dot{S}_2], \\ (0.55e^{i2\pi(0.71)}, 0.42e^{i2\pi(0.34)}) \end{pmatrix}, \begin{pmatrix} [\dot{S}_1, \dot{S}_3], \\ (0.6e^{i2\pi(0.62)}, 0.44e^{i2\pi(0.39)}) \end{pmatrix} \right\}$$

Step 3: We compute the values of group utility S_i and $S_i^{'}$ using the Eqs. (19), (20) and (21), if we ignoring the values of uncertain linguistic terms, then

Step 4: We compute the values of Q_i, $i = 1, 2, \ldots, m$ using the Eq. (22), then (Tables 4, 5)

Step 5: Using the values of S, $S^{'}$ and Q and ranking the alternatives, then we will obtain the compromise solution see Table 5.

Step 6: We obtain the compromise results using the condition 1 and condition 2, such that $Q(X_4) = 0$, and the second position is $Q(X_3) = 0.39$, then $MD = \frac{1}{m-1} = \frac{1}{4-1} = 0.333$, so $Q(X_3) - Q(X_4) = 0.39 > 0.333$ which holds the conditions $Q(X_3) - Q(X_4) \geq \frac{1}{4-1}$, but the alternative X_4 is the best ranked by S and $S^{'}$, which holds the condition 1. By calculating, we get

$$Q(X_3) - Q(X_4) = 0.39 > 0.333$$

$$Q(X_2) - Q(X_4) = 1 > 0.333$$

$$Q(X_1) - Q(X_4) = 0.41 > 0.333$$

So, the condition 1 holds accurately, therefore by condition 1, X_1, X_2, X_3 and X_4 are the compromise solutions. When condition 1 cannot hold, we used condition 2 and solved the problems. The comparison between the proposed methods and existing methods for numerical example (1), are discussed in Table 6.

Based on VIKOR methods for existing and proposed approach, the best alternative is $\tilde{\widehat{A}}_4$.

The methods introduced in this manuscript express a wider range of fuzzy information, and they can ask for the sum of square of real part (Also for imaginary) of membership degree and the sum of square of real part (Also for imaginary) of non-membership degree is

Table 2 Decision matrix for complex intuitionistic uncertain linguistic numbers

Symbols	$\boldsymbol{A}_1$	$\boldsymbol{A}_2$	$\boldsymbol{A}_3$	$\boldsymbol{A}_4$
$\boldsymbol{X}_1$	$\left([\dot{S}_1, \dot{S}_2], \left(0.3e^{i2\pi(0.2)}, 0.7e^{i2\pi(0.7)}\right)\right)$	$\left([\dot{S}_1, \dot{S}_3], \left(0.59e^{i2\pi(0.70)}, 0.41e^{i2\pi(0.31)}\right)\right)$	$\left([\dot{S}_2, \dot{S}_3], \left(0.87e^{i2\pi(0.86)}, 0.14e^{i2\pi(0.13)}\right)\right)$	$\left([\dot{S}_3, \dot{S}_4], \left(0.75e^{i2\pi(0.7)}, 0.34e^{i2\pi(0.33)}\right)\right)$
$\boldsymbol{X}_2$	$\left([\dot{S}_2, \dot{S}_3], \left(0.37e^{i2\pi(0.27)}, 0.63e^{i2\pi(0.73)}\right)\right)$	$\left([\dot{S}_3, \dot{S}_4], \left(0.57e^{i2\pi(0.75)}, 0.43e^{i2\pi(0.33)}\right)\right)$	$\left([\dot{S}_1, \dot{S}_2], \left(0.81e^{i2\pi(0.71)}, 0.14e^{i2\pi(0.33)}\right)\right)$	$\left([\dot{S}_1, \dot{S}_3], \left(0.6e^{i2\pi(0.62)}, 0.44e^{i2\pi(0.39)}\right)\right)$
$\boldsymbol{X}_3$	$\left([\dot{S}_2, \dot{S}_4], \left(0.34e^{i2\pi(0.76)}, 0.67e^{i2\pi(0.77)}\right)\right)$	$\left([\dot{S}_1, \dot{S}_3], \left(0.55e^{i2\pi(0.88)}, 0.24e^{i2\pi(0.13)}\right)\right)$	$\left([\dot{S}_1, \dot{S}_4], \left(0.55e^{i2\pi(0.76)}, 0.42e^{i2\pi(0.34)}\right)\right)$	$\left([\dot{S}_3, \dot{S}_4], \left(0.79e^{i2\pi(0.9)}, 0.24e^{i2\pi(0.13)}\right)\right)$
$\boldsymbol{X}_4$	$\left([\dot{S}_1, \dot{S}_4], \left(0.31e^{i2\pi(0.79)}, 0.7e^{i2\pi(0.78)}\right)\right)$	$\left([\dot{S}_2, \dot{S}_4], \left(0.62e^{i2\pi(0.71)}, 0.4e^{i2\pi(0.3)}\right)\right)$	$\left([\dot{S}_3, \dot{S}_4], \left(0.82e^{i2\pi(0.83)}, 0.24e^{i2\pi(0.23)}\right)\right)$	$\left([\dot{S}_3, \dot{S}_4], \left(0.65e^{i2\pi(0.72)}, 0.4e^{i2\pi(0.34)}\right)\right)$

Table 3 Normalized decision matrix

Symbols	$\boldsymbol{A}_1$	$\boldsymbol{A}_2$	$\boldsymbol{A}_3$	$\boldsymbol{A}_4$
$\boldsymbol{X}_1$	$\left(\begin{array}{c}[\dot{S}_1, \dot{S}_2], \\ \left(0.7e^{i2\pi(0.7)}, 0.3e^{i2\pi(0.2)}\right)\end{array}\right)$	$\left(\begin{array}{c}[\dot{S}_1, \dot{S}_3], \\ \left(0.59e^{i2\pi(0.70)}, 0.41e^{i2\pi(0.31)}\right)\end{array}\right)$	$\left(\begin{array}{c}[\dot{S}_2, \dot{S}_3], \\ \left(0.87e^{i2\pi(0.86)}, 0.14e^{i2\pi(0.13)}\right)\end{array}\right)$	$\left(\begin{array}{c}[\dot{S}_3, \dot{S}_4], \\ \left(0.75e^{i2\pi(0.7)}, 0.34e^{i2\pi(0.33)}\right)\end{array}\right)$
$\boldsymbol{X}_2$	$\left(\begin{array}{c}[\dot{S}_2, \dot{S}_3], \\ \left(0.63e^{i2\pi(0.73)}, 0.37e^{i2\pi(0.27)}\right)\end{array}\right)$	$\left(\begin{array}{c}[\dot{S}_3, \dot{S}_4], \\ \left(0.57e^{i2\pi(0.75)}, 0.43e^{i2\pi(0.33)}\right)\end{array}\right)$	$\left(\begin{array}{c}[\dot{S}_1, \dot{S}_2], \\ \left(0.81e^{i2\pi(0.71)}, 0.14e^{i2\pi(0.33)}\right)\end{array}\right)$	$\left(\begin{array}{c}[\dot{S}_1, \dot{S}_3], \\ \left(0.6e^{i2\pi(0.62)}, 0.44e^{i2\pi(0.39)}\right)\end{array}\right)$
$\boldsymbol{X}_3$	$\left(\begin{array}{c}[\dot{S}_2, \dot{S}_4], \\ \left(0.67e^{i2\pi(0.77)}, 0.34e^{i2\pi(0.76)}\right)\end{array}\right)$	$\left(\begin{array}{c}[\dot{S}_1, \dot{S}_3], \\ \left(0.55e^{i2\pi(0.88)}, 0.24e^{i2\pi(0.13)}\right)\end{array}\right)$	$\left(\begin{array}{c}[\dot{S}_1, \dot{S}_4], \\ \left(0.55e^{i2\pi(0.76)}, 0.42e^{i2\pi(0.34)}\right)\end{array}\right)$	$\left(\begin{array}{c}[\dot{S}_3, \dot{S}_4], \\ \left(0.79e^{i2\pi(0.9)}, 0.24e^{i2\pi(0.13)}\right)\end{array}\right)$
$\boldsymbol{X}_4$	$\left(\begin{array}{c}[\dot{S}_1, \dot{S}_4], \\ \left(0.7e^{i2\pi(0.78)}, 0.31e^{i2\pi(0.79)}\right)\end{array}\right)$	$\left(\begin{array}{c}[\dot{S}_2, \dot{S}_4], \\ \left(0.62e^{i2\pi(0.71)}, 0.4e^{i2\pi(0.3)}\right)\end{array}\right)$	$\left(\begin{array}{c}[\dot{S}_3, \dot{S}_4], \\ \left(0.82e^{i2\pi(0.83)}, 0.24e^{i2\pi(0.23)}\right)\end{array}\right)$	$\left(\begin{array}{c}[\dot{S}_3, \dot{S}_4], \\ \left(0.65e^{i2\pi(0.72)}, 0.4e^{i2\pi(0.34)}\right)\end{array}\right)$

Table 4 Values of the group utility

Symbols	Values	Symbols	Values
S_1	0.22	S_1'	0.16
S_2	0.51	S_2'	0.26
S_3	0.35	S_3'	0.2
S_4	0.17	S_4'	0.14

Table 5 Ranking results of the Table 4

Symbols	X_1	X_2	X_3	X_4	Ranking	Compromise solution
S	0.22	0.51	0.35	0.17	$\widetilde{\widehat{A}}_4 \geq \widetilde{\widehat{A}}_1 \geq \widetilde{\widehat{A}}_3 \geq \widetilde{\widehat{A}}_2$	X_4
S'	0.16	0.26	0.2	0.14	$\widetilde{\widehat{A}}_4 \geq \widetilde{\widehat{A}}_1 \geq \widetilde{\widehat{A}}_3 \geq \widetilde{\widehat{A}}_2$	X_4
Q	0.41	1	0.39	0	$\widetilde{\widehat{A}}_4 \geq \widetilde{\widehat{A}}_3 \geq \widetilde{\widehat{A}}_1 \geq \widetilde{\widehat{A}}_2$	X_4
Compromise solution						X_1, X_2, X_3, X_4

greater than one. Our proposed methods are more general and more effective. Because the VIKOR methods for CIULS and CPULS are all the special case of the VIKOR methods for CQROULS. When parameter $q = 1$ the VIKOR methods for CQROULS reduces the VIKOR methods for CIULS. When parameter $q = 2$ the VIKOR methods for CQROULS reduces the VIKOR methods for CPULS. Besides, our approach is more flexible, and decision makers can choose different values of parameter q according to the different risk attitudes.

According to the comparisons and analysis above, the VIKOR methods based on CQROULS proposed in this paper are better than the existing other methods for aggregating the complex intuitionistic uncertain linguistic information and complex Pythagorean uncertain linguistic information. Therefore, they are more suitable to solve the difficult and complicated problems.

5 MADM based on CQROULSs

The purpose of this communication is to explore the MADM problem by using the averaging and geometric aggregation operators based on CQROULSs, to improve the quality of the explore approach. Based on the above analysis, we consider the family of the alternatives and the family of attributes, whose representations are stated as: $\mathcal{A}_{Al} = \{\mathcal{A}_{Al-1}, \mathcal{A}_{Al-2}, \ldots, \mathcal{A}_{Al-m}\}$, $\mathcal{C}_{AT} = \{\mathcal{C}_{AT-1}, \mathcal{C}_{AT-2}, \ldots, \mathcal{C}_{AT-n}\}$. For these informations, we choose a matrix $R^L_{DM} = \left(r^L_{jk}\right)_{m\times n}$, whose every entities are in the form of CQROF2-TLNs provide by the decision maker $\mathcal{D}_{DE-L}(L = 1, 2, .., t)$ for alternatives $\mathcal{A}_{Al-j}(j = 1, 2, 3, .., m)$ and their attributes $\mathcal{C}_{AT-k}(k = 1, 2, 3, \ldots, n)$, where $r^L_{jk} = \left(\left[\dot{S}^L_{\theta_{jk}(x)}, \dot{S}^L_{\varphi_{jk}(x)}\right], \begin{pmatrix} \mu^L_{QRP-jk}(x)e^{i2\pi W^L_{\mu_{QIP-jk}}(x)}, \\ \eta^L_{QRP-jk}(x)e^{i2\pi W^L_{\eta_{QIP-jk}}(x)} \end{pmatrix}\right)$, $L = 1, 2, 3, .., l$, whose related information is given in Sect. 4. The steps of the MADM problem based on CQROULNs are follow as:

Table 6 Comparison between proposed methods with existing methods

Methods	Values for $\boldsymbol{S}$	Values for $\boldsymbol{S}'$	Ranking for $\boldsymbol{S}$ and $\boldsymbol{S}'$
VIKOR Method for CIULS	$X_1 = 0.29,\ X_2 = 0.47,\ X_3 = 0.28,\ X_4 = 0.14$	$X_1 = 0.16,\ X_2 = 0.24,\ X_3 = 0.16,\ X_4 = 0.13$	$\tilde{\widehat{A}}_4 \geq \tilde{\widehat{A}}_3 \geq \tilde{\widehat{A}}_1 \geq \tilde{\widehat{A}}_2$ $\tilde{\widehat{A}}_4 \geq \tilde{\widehat{A}}_3 \geq \tilde{\widehat{A}}_1 \geq \tilde{\widehat{A}}_2$
VIKOR Method for CPULS	$X_1 = 0.25,\ X_2 = 0.49,\ X_3 = 0.31,\ X_4 = 0.15$	$X_1 = 0.16,\ X_2 = 0.25,\ X_3 = 0.18,\ X_4 = 0.13$	$\tilde{\widehat{A}}_4 \geq \tilde{\widehat{A}}_1 \geq \tilde{\widehat{A}}_3 \geq \tilde{\widehat{A}}_2$ $\tilde{\widehat{A}}_4 \geq \tilde{\widehat{A}}_1 \geq \tilde{\widehat{A}}_3 \geq \tilde{\widehat{A}}_2$
VIKOR Method for CQROULS	$X_1 = 0.22,\ X_2 = 0.51,\ X_3 = 0.35,\ X_4 = 0.17$	$X_1 = 0.16,\ X_2 = 0.26,\ X_3 = 0.2,\ X_4 = 0.14$	$\tilde{\widehat{A}}_4 \geq \tilde{\widehat{A}}_3 \geq \tilde{\widehat{A}}_1 \geq \tilde{\widehat{A}}_2$ $\tilde{\widehat{A}}_4 \geq \tilde{\widehat{A}}_3 \geq \tilde{\widehat{A}}_1 \geq \tilde{\widehat{A}}_2$

$Q_1 = 0.41,\ Q_2 = 1,\ Q_3 = 0.39,\ Q_4 = 0$

Step 1: By using the CQROULNs is to construct the decision matrix $R_{DM}^{L} = \left(r_{jk}^{L}\right)_{m\times n}$, and then normalized it with the help of two methods which are discussed below:

1. When the values of attributes $\mathcal{C}_{AT-k}$, $k = 1, 2, \ldots, m$ in the form of benefit kinds, then

$$R_{DM}^{L} = \left(r_{jk}^{L}\right)_{m\times n} = \left(\left[\dot{S}_{\theta_{jk}(x)}^{L}, \dot{S}_{\varphi_{jk}(x)}^{L}\right], \begin{pmatrix} \mu_{\mathcal{Q}^{RP-jk}}^{L}(x)e^{i2\pi W_{\mu_{\mathcal{Q}^{IP-jk}}}^{L}(x)}, \\ \eta_{\mathcal{Q}^{RP-jk}}^{L}(x)e^{i2\pi W_{\eta_{\mathcal{Q}^{IP-jk}}}^{L}(x)} \end{pmatrix}\right) \tag{23}$$

2. When the values of attributes $\mathcal{C}_{AT-k}$, $k = 1, 2, \ldots, m$ in the form of cost kinds, then

$$R_{DM}^{L} = \left(r_{jk}^{L}\right)_{m\times n} = \left(\left[\dot{S}_{\theta_{jk}(x)}^{L}, \dot{S}_{\varphi_{jk}(x)}^{L}\right], \begin{pmatrix} \eta_{\mathcal{Q}^{RP-jk}}^{L}(x)e^{i2\pi W_{\eta_{\mathcal{Q}^{IP-jk}}}^{L}(x)}, \\ \mu_{\mathcal{Q}^{RP-jk}}^{L}(x)e^{i2\pi W_{\mu_{\mathcal{Q}^{IP-jk}}}^{L}(x)} \end{pmatrix}\right) \tag{24}$$

Step 2: To integrate the decision matrix, by using the CQROULWA operator or CQROULWG operator, which is explored below:

$$CQROULWA\left(\mathcal{Q}_{1}^{CQUL-jk}, \mathcal{Q}_{2}^{CQUL-jk}, \ldots, \mathcal{Q}_{l}^{CQUL-jk}\right)$$
$$= \left(\begin{array}{c} \left[\dot{S}_{z - z\prod_{L=1}^{l}\left(1-\frac{\theta_{jk}^{L}(x)}{z}\right)^{\omega^{w-L}}}, \dot{S}_{z - z\prod_{L=1}^{l}\left(1-\frac{\varphi_{jk}^{L}(x)}{z}\right)^{\omega^{w-L}}}\right], \\ \left(\begin{array}{c} \left(1-\prod_{L=1}^{l}\left(1-\mu_{\mathcal{Q}^{RP-jk}}^{q^{SC}\,L}(x)\right)^{\omega^{w-L}}\right)^{\frac{1}{q^{SC}}} e^{i2\pi\left(1-\prod_{L=1}^{l}\left(1-W_{\mu_{\mathcal{Q}^{IP-jk}}}^{q^{SC}\,L}(x)\right)^{\omega^{w-L}}\right)^{\frac{1}{q^{SC}}}}, \\ \prod_{L=1}^{l}\eta_{\mathcal{Q}^{RP-jk}}^{\omega^{w-L}\,L}(x)e^{i2\pi\prod_{L=1}^{l}W_{\eta_{\mathcal{Q}^{IP-jk}}}^{\omega^{w-L}\,L}(x)} \end{array}\right) \end{array}\right) \tag{25}$$

or

$$CQROULWG\left(\mathcal{Q}_{1}^{CQUL-jk}, \mathcal{Q}_{2}^{CQUL-jk}, \ldots, \mathcal{Q}_{l}^{CQUL-jk}\right)$$
$$= \left(\begin{array}{c} \left[\dot{S}_{z\prod_{L=1}^{l}\left(\frac{\theta_{jk}^{L}(x)}{z}\right)^{\omega^{w-L}}}, \dot{S}_{z\prod_{L=1}^{l}\left(\frac{\varphi_{jk}^{L}(x)}{z}\right)^{\omega^{w-L}}}\right], \\ \left(\begin{array}{c} \prod_{L=1}^{l}\mu_{\mathcal{Q}^{RP-jk}}^{\omega^{w-L}\,L}(x)e^{i2\pi\prod_{L=1}^{l}W_{\mu_{\mathcal{Q}^{IP-jk}}}^{\omega^{w-L}\,L}(x)}, \\ \left(1-\prod_{L=1}^{l}\left(1-\eta_{\mathcal{Q}^{RP-jk}}^{q^{SC}\,L}(x)\right)^{\omega^{w-L}}\right)^{\frac{1}{q^{SC}}} e^{i2\pi\left(1-\prod_{L=1}^{l}\left(1-W_{\eta_{\mathcal{Q}^{IP-jk}}}^{q^{SC}\,L}(x)\right)^{\omega^{w-L}}\right)^{\frac{1}{q^{SC}}}} \end{array}\right) \end{array}\right) \tag{26}$$

Step 3: With the aggregated values in step 2, we examine the expectation values by using the Eq. (5).

Step 4: The expectation values, which we obtained in the step 3, rank to all the alternatives and find the best one.

Step 5: The end.

Example 2 To examine the people of which city is more effected form Coronavirus disease (COVID-19) in the duration of lockdown. To resolve the issues of security, which city is more securable form COVID-19 and which is in the dangerous zone and it may be passable the effected people form COVID-19 is increases day by day in the duration of lockdown. For this mission the Pakistan telecom authority (PTA) gives the responsibility of the following four cities which is possible to effect the people of that areas form COVID-19, whose representation is follow as:

$\mathcal{A}_{Al-1}$: Islamabad Areas;

$\mathcal{A}_{Al-2}$: Rawalpindi Areas;

$\mathcal{A}_{Al-3}$: Karachi Areas;

$\mathcal{A}_{Al-4}$: Gilgit Areas.

The information of the effected people form COVID-19 of four cities is collected by the four attributes, whose representation is follow as:

$\mathcal{C}_{AT-1}$: Food distribution during at lockdown;

$\mathcal{C}_{AT-2}$: Water Supply during at lockdown;

$\mathcal{C}_{AT-3}$: Electric Supply during at lockdown;

$\mathcal{C}_{AT-4}$: Money distribution during at lockdown;

For evaluating these types of problems, we consider the weight vector, whose information is of the form $\omega^{w} = (0.4, 0.3, 0.1, 0.2)^{T}$, with a condition $\sum_{j=1}^{n} \omega^{w-j} = 1$. Additionally, the linguistic terms set is stated by $\dot{S} = \left\{\dot{S}_0, \dot{S}_1, \dot{S}_2, \dot{S}_3, \dot{S}_4, \dot{S}_5\right\}$. The examining information is considered in the form of CQROULVs, whose expression is follows: $r_{jk}^{L} = \left(\left[\dot{S}_{\theta_{jk}(x)}^{L}, \dot{S}_{\varphi_{jk}(x)}^{L}\right], \left(\begin{array}{c}\mu_{Q^{RP-jk}}^{L}(x)e^{i2\pi W_{\mu_{Q^{IP-jk}}}^{L}(x)}, \\ \eta_{Q^{RP-jk}}^{L}(x)e^{i2\pi W_{\eta_{Q^{IP-jk}}}^{L}(x)}\end{array}\right)\right)$, $L = 1, 2, 3, .., l$. The complex q-rung orthopair uncertain linguistic decision information is available in Table 7.

The steps of the MADM problem based on CQROULNs are follow as:

Step 1: By using the CQROULNs is to construct the decision matrix $R_{DM}^{L} = \left(r_{jk}^{L}\right)_{m\times n}$, whose information are available in Table 1. Further, we normalized the decision matrix with the help of two methods which are discussed in Eqs. (23) and (24), the information of the normalized decision matrix is discussed in Table 8, which is follow as:

Step 2: To integrate the decision matrix, by using the Eqs. (25) and (26) based on the CQROULWA operator or CQROULWG operator, the aggregated values are discussed below:

Step 3: The aggregated values in step 2, we examine the expectation values by using the Eq. (5), which is follow as:

$\underset{\cdot}{S}(\mathcal{A}_{Al-1}) = \dot{S}_{0.5252}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-2}) = \dot{S}_{0.5565}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-3}) = \dot{S}_{0.4868}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-4}) = \dot{S}_{0.5298}$, (for weighted averaging)

$\underset{\cdot}{S}(\mathcal{A}_{Al-1}) = \dot{S}_{0.3866}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-2}) = \dot{S}_{0.3698}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-3}) = \dot{S}_{0.3484}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-4}) = \dot{S}_{0.3835}$, (for weighted geometric)

Step 4: The expectation values, which we are obtains in the step 3, rank to all the alternatives and we find the best one, which is follow as:

$\mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-4} \geq \mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-3}$, (for weighted averaging)

$\mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-4} \geq \mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-3}$, (for weighted geometric).

Form the above discussion, we obtain the result the city Islamabad and Rawalpindi are more effected form corona various diseases 2019, which is $\mathcal{A}_{Al-2}$ and $\mathcal{A}_{Al-1}$, by using weighted averaging and weighted geometric aggregation operators, which is Islamabad and Rawalpindi areas. It is required for government of Pakistan to supply the necessities' of the people in the duration of lockdown and strictly say to the people of the effected city stay at home to save our life (Table 9).

Step 5: The end.

Table 7 Decision matrix, whose information is in the form of complex intuitionistic uncertain linguistic numbers

Symbols	$\mathcal{C}_{\boldsymbol{AT}-1}$	$\mathcal{C}_{\boldsymbol{AT}-2}$	$\mathcal{C}_{\boldsymbol{AT}-3}$	$\mathcal{C}_{\boldsymbol{AT}-4}$
$\mathcal{A}_{\boldsymbol{Al}-1}$	$\left([\dot{S}_3, \dot{S}_5], \left(0.1e^{i2\pi(0.2)}, 0.4e^{i2\pi(0.5)}\right)\right)$	$\left([\dot{S}_3, \dot{S}_4], \left(0.21e^{i2\pi(0.32)}, 0.54e^{i2\pi(0.5)}\right)\right)$	$\left([\dot{S}_2, \dot{S}_3], \left(0.5e^{i2\pi(0.6)}, 0.4e^{i2\pi(0.3)}\right)\right)$	$\left([\dot{S}_3, \dot{S}_4], \left(0.6e^{i2\pi(0.7)}, 0.3e^{i2\pi(0.2)}\right)\right)$
$\mathcal{A}_{\boldsymbol{Al}-2}$	$\left([\dot{S}_4, \dot{S}_5], \left(0.12e^{i2\pi(0.22)}, 0.42e^{i2\pi(0.52)}\right)\right)$	$\left([\dot{S}_4, \dot{S}_5], \left(0.22e^{i2\pi(0.42)}, 0.52e^{i2\pi(0.42)}\right)\right)$	$\left([\dot{S}_1, \dot{S}_3], \left(0.52e^{i2\pi(0.62)}, 0.41e^{i2\pi(0.31)}\right)\right)$	$\left([\dot{S}_3, \dot{S}_4], \left(0.61e^{i2\pi(0.71)}, 0.31e^{i2\pi(0.21)}\right)\right)$
$\mathcal{A}_{\boldsymbol{Al}-3}$	$\left([\dot{S}_1, \dot{S}_4], \left(0.15e^{i2\pi(0.25)}, 0.45e^{i2\pi(0.55)}\right)\right)$	$\left([\dot{S}_3, \dot{S}_4], \left(0.35e^{i2\pi(0.25)}, 0.35e^{i2\pi(0.55)}\right)\right)$	$\left([\dot{S}_2, \dot{S}_5], \left(0.53e^{i2\pi(0.63)}, 0.42e^{i2\pi(0.32)}\right)\right)$	$\left([\dot{S}_2, \dot{S}_5], \left(0.62e^{i2\pi(0.72)}, 0.32e^{i2\pi(0.22)}\right)\right)$
$\mathcal{A}_{\boldsymbol{Al}-4}$	$\left([\dot{S}_1, \dot{S}_4], \left(0.19e^{i2\pi(0.29)}, 0.49e^{i2\pi(0.59)}\right)\right)$	$\left([\dot{S}_1, \dot{S}_4], \left(0.29e^{i2\pi(0.39)}, 0.59e^{i2\pi(0.49)}\right)\right)$	$\left([\dot{S}_2, \dot{S}_4], \left(0.54e^{i2\pi(0.64)}, 0.44e^{i2\pi(0.34)}\right)\right)$	$\left([\dot{S}_3, \dot{S}_5], \left(0.63e^{i2\pi(0.73)}, 0.33e^{i2\pi(0.23)}\right)\right)$

Table 8 Normalized decision matrix, whose information is also in the form of complex intuitionistic uncertain linguistic numbers

Symbols	$\mathcal{C}_{\boldsymbol{AT}-1}$	$\mathcal{C}_{\boldsymbol{AT}-2}$	$\mathcal{C}_{\boldsymbol{AT}-3}$	$\mathcal{C}_{\boldsymbol{AT}-4}$
$\mathcal{A}_{\boldsymbol{AI}-1}$	$\left(\begin{matrix}[\dot{S}_3, \dot{S}_5],\\ \left(\begin{matrix}0.4e^{i2\pi(0.5)},\\ 0.1e^{i2\pi(0.2)}\end{matrix}\right)\end{matrix}\right)$	$\left(\begin{matrix}[\dot{S}_3, \dot{S}_4],\\ \left(\begin{matrix}0.54e^{i2\pi(0.5)},\\ 0.21e^{i2\pi(0.32)}\end{matrix}\right)\end{matrix}\right)$	$\left(\begin{matrix}[\dot{S}_2, \dot{S}_3],\\ \left(\begin{matrix}0.5e^{i2\pi(0.6)},\\ 0.4e^{i2\pi(0.3)}\end{matrix}\right)\end{matrix}\right)$	$\left(\begin{matrix}[\dot{S}_3, \dot{S}_4],\\ \left(\begin{matrix}0.6e^{i2\pi(0.7)},\\ 0.3e^{i2\pi(0.2)}\end{matrix}\right)\end{matrix}\right)$
$\mathcal{A}_{\boldsymbol{AI}-2}$	$\left(\begin{matrix}[\dot{S}_4, \dot{S}_5],\\ \left(\begin{matrix}0.42e^{i2\pi(0.52)},\\ 0.12e^{i2\pi(0.22)}\end{matrix}\right)\end{matrix}\right)$	$\left(\begin{matrix}[\dot{S}_4, \dot{S}_5],\\ \left(\begin{matrix}0.52e^{i2\pi(0.42)},\\ 0.22e^{i2\pi(0.42)}\end{matrix}\right)\end{matrix}\right)$	$\left(\begin{matrix}[\dot{S}_1, \dot{S}_3],\\ \left(\begin{matrix}0.52e^{i2\pi(0.62)},\\ 0.41e^{i2\pi(0.31)}\end{matrix}\right)\end{matrix}\right)$	$\left(\begin{matrix}[\dot{S}_3, \dot{S}_4],\\ \left(\begin{matrix}0.61e^{i2\pi(0.71)},\\ 0.31e^{i2\pi(0.21)}\end{matrix}\right)\end{matrix}\right)$
$\mathcal{A}_{\boldsymbol{AI}-3}$	$\left(\begin{matrix}[\dot{S}_1, \dot{S}_4],\\ \left(\begin{matrix}0.45e^{i2\pi(0.55)},\\ 0.15e^{i2\pi(0.25)}\end{matrix}\right)\end{matrix}\right)$	$\left(\begin{matrix}[\dot{S}_3, \dot{S}_4],\\ \left(\begin{matrix}0.35e^{i2\pi(0.55)},\\ 0.35e^{i2\pi(0.25)}\end{matrix}\right)\end{matrix}\right)$	$\left(\begin{matrix}[\dot{S}_2, \dot{S}_5],\\ \left(\begin{matrix}0.53e^{i2\pi(0.63)},\\ 0.42e^{i2\pi(0.32)}\end{matrix}\right)\end{matrix}\right)$	$\left(\begin{matrix}[\dot{S}_2, \dot{S}_5],\\ \left(\begin{matrix}0.62e^{i2\pi(0.72)},\\ 0.32e^{i2\pi(0.22)}\end{matrix}\right)\end{matrix}\right)$
$\mathcal{A}_{\boldsymbol{AI}-4}$	$\left(\begin{matrix}[\dot{S}_1, \dot{S}_4],\\ \left(\begin{matrix}0.49e^{i2\pi(0.59)},\\ 0.19e^{i2\pi(0.29)}\end{matrix}\right)\end{matrix}\right)$	$\left(\begin{matrix}[\dot{S}_1, \dot{S}_4],\\ \left(\begin{matrix}0.59e^{i2\pi(0.49)},\\ 0.29e^{i2\pi(0.39)}\end{matrix}\right)\end{matrix}\right)$	$\left(\begin{matrix}[\dot{S}_2, \dot{S}_4],\\ \left(\begin{matrix}0.54e^{i2\pi(0.64)},\\ 0.44e^{i2\pi(0.34)}\end{matrix}\right)\end{matrix}\right)$	$\left(\begin{matrix}[\dot{S}_3, \dot{S}_5],\\ \left(\begin{matrix}0.63e^{i2\pi(0.73)},\\ 0.33e^{i2\pi(0.23)}\end{matrix}\right)\end{matrix}\right)$

Table 9 By using the Eqs. (25, 26), we get the values of weighted averaging and weighted geometric based on explored ideas

Symbol	CQROULWA operator	Symbol	CQROULWG operator
$\mathcal{A}_{Al-1}$	$\left(\begin{array}{c}[\dot{S}_{2.83}, \dot{S}_5], \\ \left(\begin{array}{c}0.49e^{i2\pi(0.55)}, \\ 0.18e^{i2\pi(0.24)}\end{array}\right)\end{array}\right)$	$\mathcal{A}_{Al-1}$	$\left(\begin{array}{c}[\dot{S}_{2.77}, \dot{S}_{4.13}], \\ \left(\begin{array}{c}0.48e^{i2\pi(0.54)}, \\ 0.27e^{i2\pi(0.27)}\end{array}\right)\end{array}\right)$
$\mathcal{A}_{Al-2}$	$\left(\begin{array}{c}[\dot{S}_{3.59}, \dot{S}_5], \\ \left(\begin{array}{c}0.50e^{i2\pi(0.55)}, \\ 0.20e^{i2\pi(0.28)}\end{array}\right)\end{array}\right)$	$\mathcal{A}_{Al-2}$	$\left(\begin{array}{c}[\dot{S}_{2.95}, \dot{S}_{4.41}], \\ \left(\begin{array}{c}0.49e^{i2\pi(0.52)}, \\ 0.28e^{i2\pi(0.32)}\end{array}\right)\end{array}\right)$
$\mathcal{A}_{Al-3}$	$\left(\begin{array}{c}[\dot{S}_{2.20}, \dot{S}_5], \\ \left(\begin{array}{c}0.47e^{i2\pi(0.59)}, \\ 0.26e^{i2\pi(0.26)}\end{array}\right)\end{array}\right)$	$\mathcal{A}_{Al-3}$	$\left(\begin{array}{c}[\dot{S}_{1.72}, \dot{S}_{4.28}], \\ \left(\begin{array}{c}0.45e^{i2\pi(0.58)}, \\ 0.32e^{i2\pi(0.26)}\end{array}\right)\end{array}\right)$
$\mathcal{A}_{Al-4}$	$\left(\begin{array}{c}[\dot{S}_{1.48}, \dot{S}_5], \\ \left(\begin{array}{c}0.55e^{i2\pi(0.60)}, \\ 0.27e^{i2\pi(0.32)}\end{array}\right)\end{array}\right)$	$\mathcal{A}_{Al-4}$	$\left(\begin{array}{c}[\dot{S}_{1.28}, \dot{S}_{4.09}], \\ \left(\begin{array}{c}0.54e^{i2\pi(0.58)}, \\ 0.31e^{i2\pi(0.33)}\end{array}\right)\end{array}\right)$

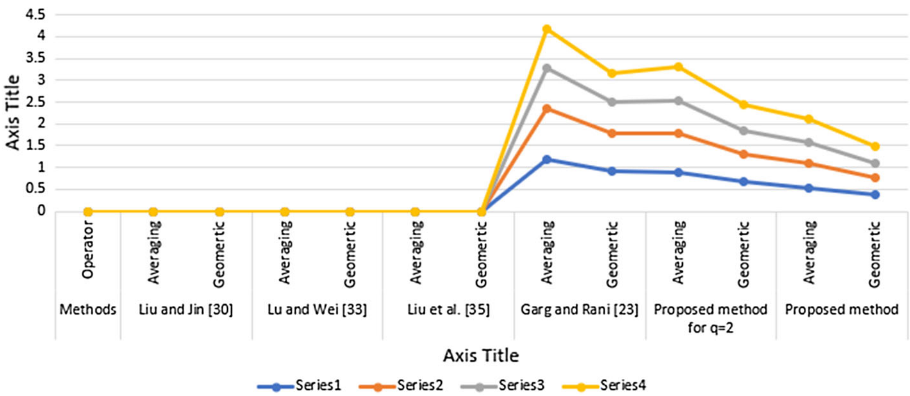

Fig. 1 Graphical representation using the information of Table 10

The comparison of the elaborated approach in this manuscript are examined with the help of some existing operators are discussed in Table 10, whose information is discussed in Table 9, which is stated below.

Form the above discussion, we obtain the result the cities which is more effected form corona various diseases 2019 are $\mathcal{A}_{Al-2}$ and $\mathcal{A}_{Al-1}$, by using weighted averaging and weighted geometric aggregation operators, which is Islamabad and Rawalpindi areas. It is required for government of Pakistan to supply the necessities' of the people in the duration of lockdown and strictly say to the people of the effected city stay at home to save our life. The graphical interpretation of the information, which is discussed in Table 10, are understand with the help of Fig. 1.

5.1 Advantages and comparative analysis with graphical representations

The CQROULS is a mixture of CQROFS and ULVS is a proficient technique to express uncertain and awkward information in real decision theory is explored. The advantage of the

Table 10 Comparison between explored and existing operators

Methods	Operator	Score values	Ranking
Liu and Jin (2012)	Averaging	$CannotbeClassified$	–
	Geometric	$CannotbeClassified$	–
Lu and Wei (2017)	Averaging	$CannotbeClassified$	–
	Geometric	$CannotbeClassified$	–
Liu et al. (2019b)	Averaging	$CannotbeClassified$	–
	Geometric	$CannotbeClassified$	–
Garg and Rani (2020b)	Averaging	$\dot{S}(\mathcal{A}_{Al-1}) = \dot{S}_{1.1725}$, $\dot{S}(\mathcal{A}_{Al-2}) = \dot{S}_{1.1675}$, $\dot{S}(\mathcal{A}_{Al-3}) = \dot{S}_{0.9325}$, $\dot{S}(\mathcal{A}_{Al-4}) = \dot{S}_{0.8875}$	$\mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-3} \geq \mathcal{A}_{Al-4}$
	Geometric	$\dot{S}(\mathcal{A}_{Al-1}) = \dot{S}_{0.9175}$, $\dot{S}(\mathcal{A}_{Al-2}) = \dot{S}_{0.86}$, $\dot{S}(\mathcal{A}_{Al-3}) = \dot{S}_{0.71}$, $\dot{S}(\mathcal{A}_{Al-4}) = \dot{S}_{0.6775}$	$\mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-3} \geq \mathcal{A}_{Al-4}$
Proposed work for $q = 2$ (Complex Pythagorean uncertain linguistic information)	Averaging	$\dot{S}(\mathcal{A}_{Al-1}) = \dot{S}_{0.8725}$, $\dot{S}(\mathcal{A}_{Al-2}) = \dot{S}_{0.9025}$, $\dot{S}(\mathcal{A}_{Al-3}) = \dot{S}_{0.755}$, $\dot{S}(\mathcal{A}_{Al-4}) = \dot{S}_{0.7725}$	$\mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-4} \geq \mathcal{A}_{Al-3}$
	Geometric	$\dot{S}(\mathcal{A}_{Al-1}) = \dot{S}_{0.665}$, $\dot{S}(\mathcal{A}_{Al-2}) = \dot{S}_{0.6325}$, $\dot{S}(\mathcal{A}_{Al-3}) = \dot{S}_{0.5575}$, $\dot{S}(\mathcal{A}_{Al-4}) = \dot{S}_{0.5775}$	$\mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-3} \geq \mathcal{A}_{Al-4}$
Proposed work for $q = 3$ (Complex q-rung orthopair uncertain linguistic information)	Averaging	$\dot{S}(\mathcal{A}_{Al-1}) = \dot{S}_{0.5233}$, $\dot{S}(\mathcal{A}_{Al-2}) = \dot{S}_{0.5575}$, $\dot{S}(\mathcal{A}_{Al-3}) = \dot{S}_{0.4875}$, $\dot{S}(\mathcal{A}_{Al-4}) = \dot{S}_{0.5298}$	$\mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-4} \geq \mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-3}$
	Geometric	$\dot{S}(\mathcal{A}_{Al-1}) = \dot{S}_{0.3866}$, $\dot{S}(\mathcal{A}_{Al-2}) = \dot{S}_{0.3698}$, $\dot{S}(\mathcal{A}_{Al-3}) = \dot{S}_{0.3484}$, $\dot{S}(\mathcal{A}_{Al-4}) = \dot{S}_{0.3835}$	$\mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-4} \geq \mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-3}$

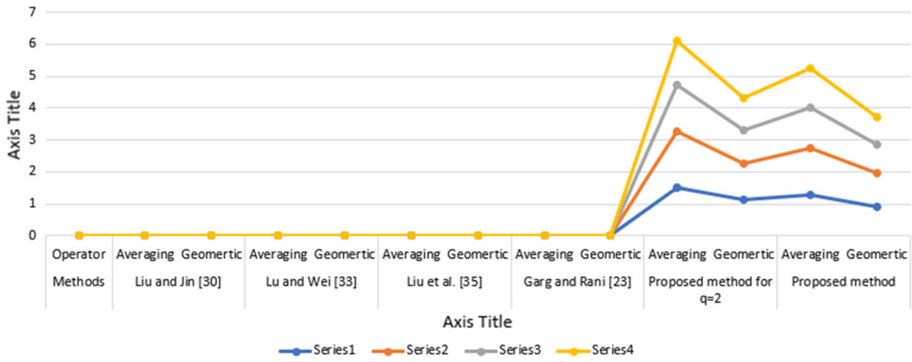

Fig. 2 Graphical representation using the information of Table 12

CQRULS is that it contains the uncertain linguistic variable, truth and falsity grades with a conditions that is the sum of q-power of the real parts (also for imaginary parts) of the truth and falsity grades are not exceeded from unit interval. Further, to explore the proficiency and validity of the established operators based on the novel CQROULVs, we choose some existing operators based on intuitionistic uncertain linguistic variables (Liu and Jin 2012), Pythagorean uncertain linguistic variables (Lu and Wei 2017), q-rung orthopair uncertain linguistic variables (Liu et al. 2019b), complex intuitionistic uncertain linguistic variables (Special case of the explored operators), complex Pythagorean uncertain linguistic variables (Special case of the explored operators), and complex q-rung orthopair uncertain linguistic variables. Further, we choose the complex Pythagorean uncertain linguistic information, which is discussed in Table 11, and solve it by using some existing methods (Liu and Jin 2012; Lu and Wei 2017; Liu et al. 2019b).

The aggregated values of the normalized decision matrix, whose information is given in Table 11, are discussed in Table 6. The comparison of the elaborated approach in this manuscript are examined with the help of some existing operators are discussed in Table 12, whose information is discussed in Table 11, which is stated below.

Form the above discussion, we obtain the result the cities which is more effected form corona various diseases 2019 are $\mathcal{A}_{Al-2}$, by using weighted averaging and weighted geometric aggregation operators, which is Rawalpindi areas. It is required for government of Pakistan to supply the necessities' of the people in the duration of lockdown and strictly say to the people of the effected city stay at home to save our life. The graphical interpretation of the information, which is discussed in Table 12, are understand with the help of Fig. 2.

Further, we choose the complex q-rung orthopair uncertain linguistic information, which is discussed in Table 13, and solve it by using some existing methods (Liu and Jin 2012; Lu and Wei 2017; Liu et al. 2019b).

The aggregated values of the normalized decision matrix for $q^{SC} = 7$, whose information is given in Table 13, are discussed in Table 14. The comparison of the elaborated approach in this manuscript are examined with the help of some existing operators are discussed in Table 14, whose information is discussed in Table 13, which is stated below

Form the above discussion, we obtain the result the cities which is more effected form corona various diseases 2019 are $\mathcal{A}_{Al-1}$, by using weighted averaging and weighted geometric aggregation operators, which is Islamabad areas. It is required for government of Pakistan to supply the necessities' of the people in the duration of lockdown and strictly say to the people of the effected city stay at home to save our life. The graphical interpretation of the

Table 11 Decision matrix, whose information is in the form of complex Pythagorean uncertain linguistic numbers

Symbols	$\mathcal{C}_{\boldsymbol{AT}-1}$	$\mathcal{C}_{\boldsymbol{AT}-2}$	$\mathcal{C}_{\boldsymbol{AT}-3}$	$\mathcal{C}_{\boldsymbol{AT}-4}$
$\mathcal{A}_{\boldsymbol{Al}-1}$	$\left([\dot{S}_3, \dot{S}_5], \left(0.8e^{i2\pi(0.7)}, 0.23e^{i2\pi(0.6)}\right)\right)$	$\left([\dot{S}_3, \dot{S}_4], \left(0.64e^{i2\pi(0.7)}, 0.41e^{i2\pi(0.32)}\right)\right)$	$\left([\dot{S}_2, \dot{S}_3], \left(0.6e^{i2\pi(0.8)}, 0.4e^{i2\pi(0.2)}\right)\right)$	$\left([\dot{S}_3, \dot{S}_4], \left(0.6e^{i2\pi(0.7)}, 0.5e^{i2\pi(0.3)}\right)\right)$
$\mathcal{A}_{\boldsymbol{Al}-2}$	$\left([\dot{S}_4, \dot{S}_5], \left(0.81e^{i2\pi(0.71)}, 0.22e^{i2\pi(0.59)}\right)\right)$	$\left([\dot{S}_4, \dot{S}_5], \left(0.65e^{i2\pi(0.71)}, 0.42e^{i2\pi(0.33)}\right)\right)$	$\left([\dot{S}_1, \dot{S}_3], \left(0.61e^{i2\pi(0.81)}, 0.41e^{i2\pi(0.21)}\right)\right)$	$\left([\dot{S}_3, \dot{S}_4], \left(0.61e^{i2\pi(0.71)}, 0.51e^{i2\pi(0.31)}\right)\right)$
$\mathcal{A}_{\boldsymbol{Al}-3}$	$\left([\dot{S}_1, \dot{S}_4], \left(0.82e^{i2\pi(0.72)}, 0.21e^{i2\pi(0.58)}\right)\right)$	$\left([\dot{S}_3, \dot{S}_4], \left(0.66e^{i2\pi(0.72)}, 0.43e^{i2\pi(0.34)}\right)\right)$	$\left([\dot{S}_2, \dot{S}_5], \left(0.62e^{i2\pi(0.82)}, 0.42e^{i2\pi(0.22)}\right)\right)$	$\left([\dot{S}_2, \dot{S}_5], \left(0.62e^{i2\pi(0.72)}, 0.52e^{i2\pi(0.32)}\right)\right)$
$\mathcal{A}_{\boldsymbol{Al}-4}$	$\left([\dot{S}_1, \dot{S}_4], \left(0.83e^{i2\pi(0.73)}, 0.20e^{i2\pi(0.57)}\right)\right)$	$\left([\dot{S}_1, \dot{S}_4], \left(0.67e^{i2\pi(0.74)}, 0.44e^{i2\pi(0.35)}\right)\right)$	$\left([\dot{S}_2, \dot{S}_4], \left(0.63e^{i2\pi(0.83)}, 0.43e^{i2\pi(0.23)}\right)\right)$	$\left([\dot{S}_3, \dot{S}_5], \left(0.63e^{i2\pi(0.73)}, 0.43e^{i2\pi(0.33)}\right)\right)$

Table 12 Comparison between explored and existing operators

Methods	Operator	Score values	Ranking
Liu and Jin (2012)	Averaging	$CannotbeClassified$	–
	Geometric	$CannotbeClassified$	–
Lu and Wei (2017)	Averaging	$CannotbeClassified$	–
	Geometric	$CannotbeClassified$	–
Liu et al. (2019b)	Averaging	$CannotbeClassified$	–
	Geometric	$CannotbeClassified$	–
Garg and Rani (2020b)	Averaging	$CannotbeClassified$	–
	Geometric	$CannotbeClassified$	–
Proposed work for $q = 2$ (Complex Pythagorean uncertain linguistic information)	Averaging	$\dot{S}(\mathcal{A}_{Al-1}) = \dot{s}_{1.5280}$, $\dot{S}(\mathcal{A}_{Al-2}) = \dot{s}_{1.7315}$, $\dot{S}(\mathcal{A}_{Al-3}) = \dot{s}_{1.4680}$, $\dot{S}(\mathcal{A}_{Al-4}) = \dot{s}_{1.3996}$	$\mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-4} \geq \mathcal{A}_{Al-3}$
	Geometric	$\dot{S}(\mathcal{A}_{Al-1}) = \dot{s}_{1.1248}$, $\dot{S}(\mathcal{A}_{Al-2}) = \dot{s}_{1.2513}$, $\dot{S}(\mathcal{A}_{Al-3}) = \dot{s}_{1.0602}$, $\dot{S}(\mathcal{A}_{Al-4}) = \dot{s}_{0.9877}$	$\mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-4} \geq \mathcal{A}_{Al-3}$
Proposed work for $q = 3$ (Complex q-rung orthopair uncertain linguistic information)	Averaging	$\dot{S}(\mathcal{A}_{Al-1}) = \dot{s}_{1.2852}$, $\dot{S}(\mathcal{A}_{Al-2}) = \dot{s}_{1.4740}$, $\dot{S}(\mathcal{A}_{Al-3}) = \dot{s}_{1.2630}$, $\dot{S}(\mathcal{A}_{Al-4}) = \dot{s}_{1.2181}$	$\mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-4} \geq \mathcal{A}_{Al-3}$
	Geometric	$\dot{S}(\mathcal{A}_{Al-1}) = \dot{s}_{0.9269}$, $\dot{S}(\mathcal{A}_{Al-2}) = \dot{s}_{1.046}$, $\dot{S}(\mathcal{A}_{Al-3}) = \dot{s}_{0.8982}$, $\dot{S}(\mathcal{A}_{Al-4}) = \dot{s}_{0.8481}$	$\mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-4} \geq \mathcal{A}_{Al-3}$

Table 13 Decision matrix, whose information is in the form of complex q-rung orthopair uncertain linguistic numbers

Symbols	$\mathcal{C}_{AT-1}$	$\mathcal{C}_{AT-2}$	$\mathcal{C}_{AT-3}$	$\mathcal{C}_{AT-4}$
$\mathcal{A}_{Al-1}$	$\left(\left[\dot{S}_3, \dot{S}_5\right], \left(0.8e^{i2\pi(0.9)}, 0.79e^{i2\pi(0.6)}\right)\right)$	$\left(\left[\dot{S}_3, \dot{S}_4\right], \left(0.9e^{i2\pi(0.9)}, 0.41e^{i2\pi(0.32)}\right)\right)$	$\left(\left[\dot{S}_2, \dot{S}_3\right], \left(0.96e^{i2\pi(0.8)}, 0.84e^{i2\pi(0.82)}\right)\right)$	$\left(\left[\dot{S}_3, \dot{S}_4\right], \left(0.89e^{i2\pi(0.7)}, 0.5e^{i2\pi(0.3)}\right)\right)$
$\mathcal{A}_{Al-2}$	$\left(\left[\dot{S}_4, \dot{S}_5\right], \left(0.81e^{i2\pi(0.71)}, 0.8e^{i2\pi(0.77)}\right)\right)$	$\left(\left[\dot{S}_4, \dot{S}_5\right], \left(0.92e^{i2\pi(0.91)}, 0.42e^{i2\pi(0.33)}\right)\right)$	$\left(\left[\dot{S}_1, \dot{S}_3\right], \left(0.91e^{i2\pi(0.81)}, 0.41e^{i2\pi(0.21)}\right)\right)$	$\left(\left[\dot{S}_3, \dot{S}_4\right], \left(0.91e^{i2\pi(0.71)}, 0.51e^{i2\pi(0.31)}\right)\right)$
$\mathcal{A}_{Al-3}$	$\left(\left[\dot{S}_1, \dot{S}_4\right], \left(0.82e^{i2\pi(0.72)}, 0.81e^{i2\pi(0.68)}\right)\right)$	$\left(\left[\dot{S}_3, \dot{S}_4\right], \left(0.96e^{i2\pi(0.82)}, 0.43e^{i2\pi(0.34)}\right)\right)$	$\left(\left[\dot{S}_2, \dot{S}_5\right], \left(0.62e^{i2\pi(0.82)}, 0.92e^{i2\pi(0.22)}\right)\right)$	$\left(\left[\dot{S}_2, \dot{S}_5\right], \left(0.62e^{i2\pi(0.72)}, 0.92e^{i2\pi(0.32)}\right)\right)$
$\mathcal{A}_{Al-4}$	$\left(\left[\dot{S}_1, \dot{S}_4\right], \left(0.83e^{i2\pi(0.73)}, 0.80e^{i2\pi(0.87)}\right)\right)$	$\left(\left[\dot{S}_1, \dot{S}_4\right], \left(0.97e^{i2\pi(0.84)}, 0.44e^{i2\pi(0.35)}\right)\right)$	$\left(\left[\dot{S}_2, \dot{S}_4\right], \left(0.63e^{i2\pi(0.83)}, 0.83e^{i2\pi(0.23)}\right)\right)$	$\left(\left[\dot{S}_3, \dot{S}_5\right], \left(0.93e^{i2\pi(0.73)}, 0.43e^{i2\pi(0.33)}\right)\right)$

Table 14 Comparison between explored and existing operators

Methods	Operator	Score values	Ranking
Liu and Jin (2012)	Averaging	$CannotbeClassified$	–
	Geometric	$CannotbeClassified$	–
Lu and Wei (2017)	Averaging	$CannotbeClassified$	–
	Geometric	$CannotbeClassified$	–
Liu et al. (2019b)	Averaging	$CannotbeClassified$	–
	Geometric	$CannotbeClassified$	–
Garg and Rani (2020b)	Averaging	$CannotbeClassified$	–
	Geometric	$CannotbeClassified$	–
Proposed work for $q = 2$ (Complex Pythagorean uncertain linguistic information)	Averaging	$CannotbeClassified$	–
	Geometric	$CannotbeClassified$	–
Proposed work for $q = 3$ (Complex q-rung orthopair uncertain linguistic information)	Averaging	$\underset{.}{S}(\mathcal{A}_{Al-1}) = \dot{S}_{1.6055}$, $\underset{.}{S}(\mathcal{A}_{Al-2}) = \dot{S}_{1.4755}$, $\underset{.}{S}(\mathcal{A}_{Al-3}) = \dot{S}_{0.9060}$, $\underset{.}{S}(\mathcal{A}_{Al-4}) = \dot{S}_{1.0878}$	$\mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-4} \geq \mathcal{A}_{Al-3}$
	Geometric	$\underset{.}{S}(\mathcal{A}_{Al-1}) = \dot{S}_{0.8657}$, $\underset{.}{S}(\mathcal{A}_{Al-2}) = \dot{S}_{0.7480}$, $\underset{.}{S}(\mathcal{A}_{Al-3}) = \dot{S}_{0.04259}$, $\underset{.}{S}(\mathcal{A}_{Al-4}) = \dot{S}_{0.1815}$	$\mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-4} \geq \mathcal{A}_{Al-3}$

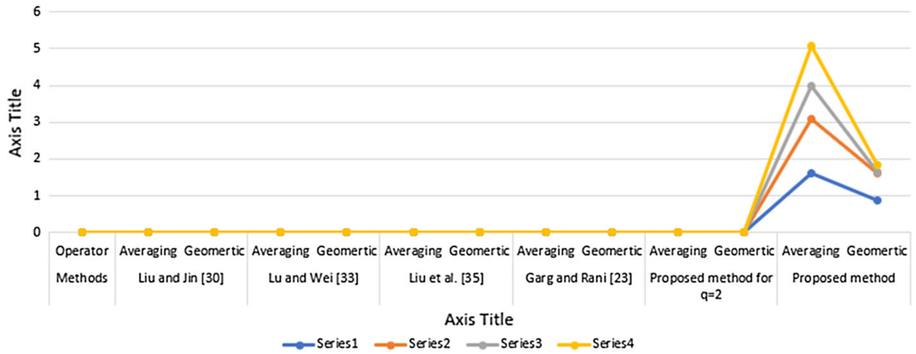

Fig. 3 Graphical representation using the information of Table 14

information, which is discussed in Table 8, are understood with the help of Fig. 3. For further improvement of this manuscript, we consider the example from Ref. (Liu and Jin 2012), and solve it by using the established operator and existing operators, whose discussion is explained below.

Example 3 This example is taken form Ref. (Liu and Jin 2012), example 5. The authors have chosen four alternatives and four attributes and their weight vectors is follow as: $\omega^{w} = (0.32, 0.26, 0.18, 0.24)^{T}$. For these information, the decision matrix which is taken form Ref. (Liu and Jin 2012) is discussed below. The authors chose the intuitionistic uncertain linguistic information, which is discussed in Table 15, and solved it by using some existing methods (Liu and Jin 2012; Lu and Wei 2017; Liu et al. 2019b).

Form the above analysis, its clear that $e^{i2\pi(0.0)} = e^{0} = 1$, then with the information of the Table 15, we converted it to the information of the Table 16, which is in the form of polar co-ordinates.

The aggregated values of the normalized decision matrix, whose information is given in Table 16, are discussed in Table 17. The comparison of the elaborated approach in this manuscript is examined with the help of some existing operators as discussed in Table 17, whose information is discussed in Table 16, which is stated below.

Form the above discussion, we obtained the result in the cities which is more affected by COVID-19 are $\mathcal{A}_{Al-1}$ and $\mathcal{A}_{Al-2}$, by using weighted-averaging and weighted-geometric aggregation operators, which is Islamabad and Rawalpindi areas. It is required by the government of Pakistan to supply the necessities' of the people in the duration of lockdown and strictly inform the people of the affected city to stay at home to save their lives. The graphical interpretation of the information, which is discussed in Table 17, can be understood with the help of Fig. 4.

From the above discussions, we get the result; our established approach is more refillable and extensive consistence then existing methods (Liu and Jin 2012; Lu and Wei 2017; Liu et al. 2019b), due to its constraints. Therefore, the established approaches in this manuscript are more reliable and more efficient then CIFS and CPFS to cope with uncertain and awkward information in realistic decision theory.

Table 15 Decision matrix, whose information is in the form of intuitionistic uncertain linguistic numbers

Symbols	$\mathcal{C}_{AT-1}$	$\mathcal{C}_{AT-2}$	$\mathcal{C}_{AT-3}$	$\mathcal{C}_{AT-4}$
$\mathcal{A}_{AI-1}$	$([\dot{S}_5, \dot{S}_5], (0.2, 0.7))$	$([\dot{S}_2, \dot{S}_3], (0.4, 0.6))$	$([\dot{S}_5, \dot{S}_6], (0.5, 0.5))$	$([\dot{S}_3, \dot{S}_4], (0.2, 0.5))$
$\mathcal{A}_{AI-2}$	$([\dot{S}_4, \dot{S}_5], (0.4, 0.6))$	$([\dot{S}_5, \dot{S}_5], (0.4, 0.5))$	$([\dot{S}_3, \dot{S}_4], (0.1, 0.8))$	$([\dot{S}_4, \dot{S}_4], (0.5, 0.5))$
$\mathcal{A}_{AI-3}$	$([\dot{S}_3, \dot{S}_4], (0.2, 0.7))$	$([\dot{S}_4, \dot{S}_4], (0.2, 0.7))$	$([\dot{S}_4, \dot{S}_5], (0.3, 0.7))$	$([\dot{S}_4, \dot{S}_5], (0.2, 0.7))$
$\mathcal{A}_{AI-4}$	$([\dot{S}_6, \dot{S}_6], (0.5, 0.4))$	$([\dot{S}_2, \dot{S}_3], (0.2, 0.8))$	$([\dot{S}_3, \dot{S}_4], (0.2, 0.6))$	$([\dot{S}_3, \dot{S}_3], (0.3, 0.6))$

Table 16 Decision matrix, whose information is in the form of complex intuitionistic uncertain linguistic numbers

Symbols	$\mathcal{C}_{AT-1}$	$\mathcal{C}_{AT-2}$	$\mathcal{C}_{AT-3}$	$\mathcal{C}_{AT-4}$
$\mathcal{A}_{AI-1}$	$\left([\dot{S}_5, \dot{S}_5], \left(0.2e^{i2\pi(0.0)}, 0.7e^{i2\pi(0.0)}\right)\right)$	$\left([\dot{S}_2, \dot{S}_3], \left(0.4e^{i2\pi(0.0)}, 0.6e^{i2\pi(0.0)}\right)\right)$	$\left([\dot{S}_5, \dot{S}_6], \left(0.5e^{i2\pi(0.0)}, 0.5e^{i2\pi(0.0)}\right)\right)$	$\left([\dot{S}_3, \dot{S}_4], \left(0.2e^{i2\pi(0.0)}, 0.5e^{i2\pi(0.0)}\right)\right)$
$\mathcal{A}_{AI-2}$	$\left([\dot{S}_4, \dot{S}_5], \left(0.4e^{i2\pi(0.0)}, 0.6e^{i2\pi(0.0)}\right)\right)$	$\left([\dot{S}_5, \dot{S}_5], \left(0.4e^{i2\pi(0.0)}, 0.5e^{i2\pi(0.0)}\right)\right)$	$\left([\dot{S}_3, \dot{S}_4], \left(0.1e^{i2\pi(0.0)}, 0.8e^{i2\pi(0.0)}\right)\right)$	$\left([\dot{S}_4, \dot{S}_4], \left(0.5e^{i2\pi(0.0)}, 0.5e^{i2\pi(0.0)}\right)\right)$
$\mathcal{A}_{AI-3}$	$\left([\dot{S}_3, \dot{S}_4], \left(0.2e^{i2\pi(0.0)}, 0.7e^{i2\pi(0.0)}\right)\right)$	$\left([\dot{S}_4, \dot{S}_4], \left(0.2e^{i2\pi(0.0)}, 0.7e^{i2\pi(0.0)}\right)\right)$	$\left([\dot{S}_4, \dot{S}_5], \left(0.3e^{i2\pi(0.0)}, 0.7e^{i2\pi(0.0)}\right)\right)$	$\left([\dot{S}_4, \dot{S}_5], \left(0.2e^{i2\pi(0.0)}, 0.7e^{i2\pi(0.0)}\right)\right)$
$\mathcal{A}_{AI-4}$	$\left([\dot{S}_6, \dot{S}_6], \left(0.5e^{i2\pi(0.0)}, 0.4e^{i2\pi(0.0)}\right)\right)$	$\left([\dot{S}_2, \dot{S}_3], \left(0.2e^{i2\pi(0.0)}, 0.8e^{i2\pi(0.0)}\right)\right)$	$\left([\dot{S}_3, \dot{S}_4], \left(0.2e^{i2\pi(0.0)}, 0.6e^{i2\pi(0.0)}\right)\right)$	$\left([\dot{S}_3, \dot{S}_3], \left(0.3e^{i2\pi(0.0)}, 0.6e^{i2\pi(0.0)}\right)\right)$

Table 17 Comparison between explored and existing operators

Methods	Operator	Score values	Ranking
Liu and Jin (2012)	Averaging	$\underset{\cdot}{S}(\mathcal{A}_{Al-1}) = \dot{S}_{-0.6739}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-2}) = \dot{S}_{-0.4314}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-3}) = \dot{S}_{-0.9894}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-4}) = \dot{S}_{-0.7035}$	$\mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-4} \geq \mathcal{A}_{Al-3}$
	Geometric	$\underset{\cdot}{S}(\mathcal{A}_{Al-1}) = \dot{S}_{-0.6159}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-2}) = \dot{S}_{-0.5928}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-3}) = \dot{S}_{-0.9747}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-4}) = \dot{S}_{-0.5928}$	$\mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-4} \geq \mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-3}$
Lu and Wei (2017)	Averaging	$\underset{\cdot}{S}(\mathcal{A}_{Al-1}) = \dot{S}_{-0.577}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-2}) = \dot{S}_{-0.3873}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-3}) = \dot{S}_{-0.9067}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-4}) = \dot{S}_{-0.5949}$	$\mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-4} \geq \mathcal{A}_{Al-3}$
	Geometric	$\underset{\cdot}{S}(\mathcal{A}_{Al-1}) = \dot{S}_{-0.55}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-2}) = \dot{S}_{-0.57}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-3}) = \dot{S}_{-0.89}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-4}) = \dot{S}_{-0.561}$	$\mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-4} \geq \mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-3}$
Liu et al. (2019b)	Averaging	$\underset{\cdot}{S}(\mathcal{A}_{Al-1}) = \dot{S}_{-0.391}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-2}) = \dot{S}_{-0.2743}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-3}) = \dot{S}_{-0.682}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-4}) = \dot{S}_{-0.3951}$	$\mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-4} \geq \mathcal{A}_{Al-3}$
	Geometric	$\underset{\cdot}{S}(\mathcal{A}_{Al-1}) = \dot{S}_{-0.39}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-2}) = \dot{S}_{-0.437}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-3}) = \dot{S}_{-0.67}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-4}) = \dot{S}_{-0.44}$	$\mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-4} \geq \mathcal{A}_{Al-3}$
Garg and Rani (2020b)	Averaging	$\underset{\cdot}{S}(\mathcal{A}_{Al-1}) = \dot{S}_{-0.6739}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-2}) = \dot{S}_{-0.4314}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-3}) = \dot{S}_{-0.9894}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-4}) = \dot{S}_{-0.7035}$	$\mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-4} \geq \mathcal{A}_{Al-3}$
	Geometric	$\underset{\cdot}{S}(\mathcal{A}_{Al-1}) = \dot{S}_{-0.6159}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-2}) = \dot{S}_{-0.5928}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-3}) = \dot{S}_{-0.9747}$, $\underset{\cdot}{S}(\mathcal{A}_{Al-4}) = \dot{S}_{-0.5928}$	$\mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-4} \geq \mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-3}$

Table 17 continued

Methods	Operator	Score values	Ranking
Proposed work for $q = 2$ (Complex Pythagorean uncertain linguistic information)	Averaging	$\underset{.}{S}(\mathcal{A}_{Al-1}) = \dot{S}_{-0.577}$, $\underset{.}{S}(\mathcal{A}_{Al-2}) = \dot{S}_{-0.3873}$, $\underset{.}{S}(\mathcal{A}_{Al-3}) = \dot{S}_{-0.9067}$, $\underset{.}{S}(\mathcal{A}_{Al-4}) = \dot{S}_{-0.5949}$	$\mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-4} \geq \mathcal{A}_{Al-3}$
	Geometric	$\underset{.}{S}(\mathcal{A}_{Al-1}) = \dot{S}_{-0.55}$, $\underset{.}{S}(\mathcal{A}_{Al-2}) = \dot{S}_{-0.57}$, $\underset{.}{S}(\mathcal{A}_{Al-3}) = \dot{S}_{-0.89}$, $\underset{.}{S}(\mathcal{A}_{Al-4}) = \dot{S}_{-0.561}$	$\mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-4} \geq \mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-3}$
Proposed work for $q = 3$ (Complex q-rung orthopair uncertain linguistic information)	Averaging	$\underset{.}{S}(\mathcal{A}_{Al-1}) = \dot{S}_{-0.391}$, $\underset{.}{S}(\mathcal{A}_{Al-2}) = \dot{S}_{-0.2743}$, $\underset{.}{S}(\mathcal{A}_{Al-3}) = \dot{S}_{-0.682}$, $\underset{.}{S}(\mathcal{A}_{Al-4}) = \dot{S}_{-0.3951}$	$\mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-4} \geq \mathcal{A}_{Al-3}$
	Geometric	$\underset{.}{S}(\mathcal{A}_{Al-1}) = \dot{S}_{-0.39}$, $\underset{.}{S}(\mathcal{A}_{Al-2}) = \dot{S}_{-0.437}$, $\underset{.}{S}(\mathcal{A}_{Al-3}) = \dot{S}_{-0.67}$, $\underset{.}{S}(\mathcal{A}_{Al-4}) = \dot{S}_{-0.44}$	$\mathcal{A}_{Al-1} \geq \mathcal{A}_{Al-2} \geq \mathcal{A}_{Al-4} \geq \mathcal{A}_{Al-3}$

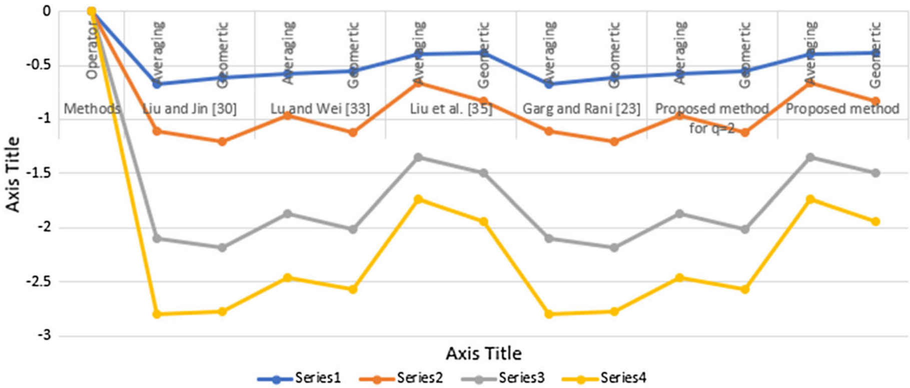

Fig. 4 Graphical representation using the information of Table 17

6 Conclusion

Various theories have developed in the environment of fuzzy sets. But, one of the most important theory is not explored till date, which is effectively dealing with some issues, no notions deal with such kinds of issues. For instance, when a decision maker gives the complex q-rung orthopair uncertain linguistic types of information. For coping such kinds of issues, in this paper, the theory of complex q-rung orthopair uncertain linguistic set (CQROULS) is a combination of complex q-rung orthopair fuzzy set (CQROFS) and uncertain linguistic variable set (ULVS) is a proficient technique to express uncertain and awkward information in real decision theory is explored. CQROULS contains uncertain linguistic variable, truth, and falsity grades, which gives extensive freedom to a decision makers for taking a decision is compared to CQROFS and their special cases. CQROULS can examine the qualitative assessment of decision makers and gives them extensive freedom in reflecting their belief about allowable truth grades. Based on the established operational laws and comparison methods for CQROULSs, the notions of complex q-rung orthopair uncertain linguistic weighted averaging aggregation operator and complex q-rung orthopair uncertain linguistic weighted geometric aggregation operator are explored. Some special cases and the desirable properties of the explored operators are also established and studied. Additionally, the VIKOR method based on CQROULSs are also explored and verified it with the help of numerical example. Moreover, based on the above analysis, we establish a method to solve the multi-attribute group decision making problems, in which the evaluation information is shown as CQROULNs. Finally, we solve some numerical examples using some decision making steps and explain the verity and proficiency of the explored operators by comparing with other methods, the advantages and graphical interpretation of the explored work are also discussed.

In the future, we will evaluate some more aggregation operators (Ullah et al. 2018a, 2020; Ghorabaee et al. 2017; Shen and Wang 2018), similarity measures (Jana et al. 2020a, b; Liu et al. 2020b; Wang and Zhang 2012; Wang et al. 2012; Zhang et al. 2020b, c, d, e; Zhan et al. 2020a, b; Jiang et al. 2020), and different methods (He et al. 2019; Garg et al. 2020; Ali and Mahmood 2020a, b; Jan et al. 2020; Ali et al. 2020; Mahmood et al. 2019; Ullah et al. 2018b, 2019b; Quek et al. 2019) solved by using the complex q-rung orthopair uncertain linguistic information.

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