Optimal Control Approach for the COVID-19 Pandemic in Bahia and Santa Catarina, Brazil

Igor M. L. Pataro¹ · Marcelo M. Morato² · Marcus V. Americano da Costa³ · Julio E. Normey-Rico²

Received: 18 March 2021 / Revised: 26 May 2021 / Accepted: 15 June 2021 / Published online: 8 July 2021
© Brazilian Society for Automatics–SBA 2021

Abstract

The COVID-19 pandemic is the most profound crisis of the twenty-first century. The SARS-CoV-2 virus was first registered in Brazil on March 2020, and its social and economic repercussions have been catastrophic. This paper investigates how to apply model predictive control (MPC) algorithms to plan appropriate social distancing policies that mitigate the pandemic effects. We consider MPC applications for the states of Bahia and Santa Catarina (Brazil), two regions of very different social and cultural demographics. We use Susceptible-Infected-Recovered-Deceased model to describe the pandemic dynamics in these two states, for which parameters are identified using a constrained optimization procedure. The control input to the process is a social isolation guideline passed to the population. Two MPC frameworks are developed and discussed: (a) a centralized approach, which coordinates a single predictive control policy for both states, and (b) a distributed strategy, for which a single MPC problem is solved for each state. We provide a series of simulation results in order to illustrate and compare the results obtained with both these MPC strategies. Discussions are drawn regarding the effectiveness of MPC to guide social distancing measures during pandemics and which approach (distributed, centralized) is more convenient, regarding different conditions.

Keywords Model predictive control · COVID-19 · Social isolation · SIRD model · Identification

1 Introduction

The COVID-19 pandemic is the definite crisis of the twenty-first century. The SARS-CoV-2 virus was first registered in humans in China, by the end of 2019. This virus causes a severe acute respiratory syndrome, which may lead to death. The spread of the contagion has been very rapid around the globe; by mid-June 2020, this disease had caused the death of over 410,000 people. Vaccines are currently being developed, but are previewed to be ready only by mid-2021 (Lurie et al., 2020). Therefore, in order to address and mitigate the pandemic effects, global scientific efforts are being provided (Bedford et al., 2019), while countries have adopted social distancing measures, seeking to avoid the spread of the virus (Adam, 2020).

Much more than presenting drastic effects on health systems, this pandemic has also caused social and economical backlashes, especially in countries with larger social inequalities. In Brazil, one of the leading countries in numbers of COVID-19 cases and deaths, catastrophic outcomes are already felt (The Lancet, 2020). The effects of the virus on

An early version of paper was presented at XXIII Congresso Brasileiro de Automática, CBA 2020. Igor M. L. Pataro, Marcelo M. Morato and J. E. Normey-Rico acknowledge the financial support of National Council for Scientific and Technological Development (CNPq, Brazil) under Grants 304032/2019-0 and 201143/2019-4.

✉ Igor M. L. Pataro
ilp428@inlumine.ual.es

Marcelo M. Morato
marcelomnzm@gmail.com

Marcus V. Americano da Costa
marcus.americano@ufba.br

Julio E. Normey-Rico
julio.normey@ufsc.br

¹ Department of Informatics, CIESOL, University of Almeria (UAL), Almería, Spain

² Renewable Energy Research Group (GPER), Department of Automation and Systems (DAS), Federal University of Santa Catarina (UFSC), Florianópolis, SC, Brazil

³ Department of Chemical Engineering (DEQ), Federal University of Bahia, 02 Professor Aristides Novis St., Salvador, BA, Brazil

populations with poorer access to health systems and sanitation facilities¹ are strikingly stronger (San Lau et al., 2020).

In this paper, we consider the COVID-19 pandemic context in Brazil (Werneck & Carvalho, 2020), a continent-sized country with 26 federated states, which have been choosing different social distancing measures since mid-March 2020. The federal government is reluctant to implement nationwide policies, claiming that the adverse economic effects are too steep and that social distancing is a wrong choice (The Lancet, 2020). The government suggests that the economy cannot stop and that herd immunity and early treatment are the most viable solutions to this pandemic, while vaccines are not available on a large scale. However, the expectations and scenarios previewed in recent literature are ruinous, showing that planning social distance can be an essential alternative to decreasing the COVID-19 transmission (Rocha Filho et al., 2020; Morato et al., 2020a).

Along the year of 2020, no coordinated social distancing measure was formally discussed or implemented by the federal government, only local measures were enacted, coordinated by states or municipalities. Therefore, in order to account for locations that have been following very different paths regarding COVID-19, the data from two states are considered: from (i) Bahia (BA), which lies on the north-east sea-side and is more extensive than Spain (in the total surface), and from (ii) Santa Catarina (SC), which is in the south of the country and is three times larger than Belgium. We highlight that these states have very distinct social, historical, and cultural backgrounds and have exhibited different behaviors facing COVID-19.

The central concept behind social distance is to prevent health systems from becoming saturated due to large amounts of COVID-19 patients being treated simultaneously. With social distancing policies, hospital bed shortages do not occur since the large original peak of infections becomes distributed over time. Even though a robust public health system is available in Brazil, many states were already exhibiting a near-collapse situation by May 2020, with over 95 % of intense care unit (ICU) hospital beds occupied with COVID-19 patients. The first wave peaked in late July 2020, and since then, until early November 2020, the number of cases has started to decline. However, the relaxation of government measures and the population disengagement contributed to the new second wave, which already accounts for a higher number of cases than the first.

What concerns the states of BA and SC, although they represent different demographic circumstances, both regions present similar situation regarding the ICU beds occupation. By the end of February 2021, BA had roughly 80 % of ICU occupancy, and SC has estimated at 89 % of occupancy, even though both states had applied strict social isolation rules since the initial stage (around March 2020). A fundamental issue regarding social distancing is to perform these interventions at the right moments and for the correct duration. Well-designed distancing policies should help mitigate the contagion spread, avoiding the health systems' saturation, minimizing social and economic side effects.

Motivated by the previous discussion, the determination of optimal social distancing policies is investigated in this work regarding BA and SC applications. For this goal, we use a model predictive control (MPC) (Camacho & Bordons, 2013) framework. MPC is an interesting option to plan social distancing measures, since it can conveniently consider the effect of lockdown/quarantine as the constraints of a minimization problem (regarding the number of infected individuals). Furthermore, we compare and discuss the differences regarding the use of a centralized MPC scheme, which generates a single control law for both states, and of a distributed MPC, which solves two separate procedures, with individual laws for each state.²

The main focus of this paper is to debate the differences between these two MPC approaches, providing assessments on which situations each framework is more suitable. Accordingly, the main contributions of this paper are the following:

- An optimization procedure is developed in order to minimize a least-squares criterion and estimate the virus spread model parameters, considering both states (BA, SC). Uncertainty in the available data sets is considered (Sect. 3).
- Based on the identified models, two different MPC strategies are designed in order to determine when to apply (or not) social distancing measures (Sect. 4): a centralized and a distributed approach.
- Simulation results are provided in order to illustrate the obtained results; discussions are drawn regarding how these optimal control schemes can be used to guide social distancing in pandemic situations of large countries and continental regions (Sects. 5 and 6). The debated is formally based on comparisons regarding the COVID-19

¹ A very illustrative example of these differences can be seen in the city of São Paulo: the city hall released a technical note by the end of April 2020, stating that the observed mortality rate is 10 times larger in neighborhoods of the city with worse social conditions and precarious housing. See https://www.prefeitura.sp.gov.br/cidade/secretarias/upload/saude/PMSP_SMS_COVID19_Boletim%20Quinzenal_20200430.pdf.

² We note that the centralized and decentralized concepts presented in this work are not related to the classical control theory's definition. Instead, we present these approaches to describe how the optimal social distancing measures, computed by the MPC strategy, will be implemented: either in a unique way for all Brazilian states (central controller) or differently for each state (decentralized controller).

spread in both states, BA and SC, and how the two different strategies are able to mitigate the viral spread.

MPC schemes have been with Susceptible-Infected-Recovered-Deceased (SIRD) models specially adapted for the COVID-19 pandemic (Bastos & Cajueiro, 2020). These models explicitly embed the effects of social distancing measures through an additional input variable ψ (details are provided in Sect. 2).

Furthermore, we stress that this paper is different from previous works of Morato et al. (2020a, 2020b): while we evaluated the effectiveness of MPC to guide social distancing measures in pandemic conditions (under different formalism), in the work of Morato et al. (2020a), we debate how an optimal On-Off MPC can provide a decrease on the spread of the virus. Differently, in work of Morato et al. (2020b), we pursue a parameterized MPC framework, which generates a social distancing input coherent with possible implementation rules (i.e., use of masks, closing public transport, etc.).

1.1 Problem Statement

In this paper, we assume that MPC is indeed a viable alternative to guide social distancing (as demonstrated by the prior works), and we focus on investigating national coordination, debating which kind of implementation (distributed or centralized) is more effective. We assume that the centralized approach is defined as national social distancing guidance applied in every state. At the same time, a distributed control would guide the social distancing index differently for each state.

The methodology assumed in this work considers that the centralized MPC is applied for a SIMO decoupled system, wherein the system outputs are the SIRD models of all Brazilian states, and the input is a **uniform** social distance guideline. Further, the decentralized MPC (a SIMO system as well) is a **non-uniform** input social distance measure for each Brazilian state. Notice that the differences between CMPC and DMPC are: (i) the inputs (uniform or non-uniform) and (ii) the outputs (all state SIRD models or individual SIRD model). The objective is to compare both scenarios, considering the country's particularities and the pandemic status in each state, and then discuss a suitable solution to be used by the Brazilian government.

We emphasize that in our previous works, no discussions were provided with regard to the segmentation. This is an essential issue to be taken into account in the case of large countries, such as Brazil. Accordingly, we analyze the contagion spread using realistic simulation based on real data from two states, which are used as a case study to investigate the proposed predictive control framework. We note that, as presented by Morato et al. (2020b), a social distancing coordination law based on input-parametrized control laws that

correspond to associated government guidelines is certainly viable, and the proposed control strategies presented herein are realistic alternatives for coping with the COVID-19 spread. The discussion herein presented examines the viability of a national uniform measure or a non-uniform measure for each state. The investigation is encouraged mainly due to the fact of notable differences in each region in Brazil. We foster this debate to find the better alternative for facing the pandemic spread in the country since the discussion can still be fruitful. Our solution is to offer mathematical models, optimization approaches, and control theory as scientific tools to help combat the COVID-19.

2 SARS-CoV-2 Pandemic Spread Model

Recent literature (Peng et al., 2020; Kucharski et al., 2020) demonstrates that the infection rate and evolution dynamics of the SARS-CoV-2 virus can be adequately described by Susceptible-Infected-Recovered-Deceased (SIRD) models. In this section, the SIRD model from the research of Keeling et al. (2008) is detailed. An additional dynamic variable models the population's response to isolation policies, as proposed by Bastos and Cajueiro (2020), Morato et al. (2020a).

The SIRD model type including the social distancing index, as in Eq. (1), describes a contagion spread in a population which is split into four non-intersecting classes:

- Susceptible people³ $S(t)$, who are prone to contract the virus;
- Infected individuals $I(t)$, which are currently sick;
- Recovered people $R(t)$, who have already recovered from the SARS-CoV-2;
- and Deceased individuals $D(t)$, who have died due to the contagion.

$$[\text{SIRD} + \psi] \quad \times \quad \begin{cases} \frac{dS(t)}{dt} = -(1 - \psi(t)) \frac{\beta(t)I(t)S(t)}{N(t)}, \\ \frac{dI(t)}{dt} = (1 - \psi(t)) \frac{\beta(t)I(t)S(t)}{N(t)} - \frac{\gamma(t)I(t)}{1 - \rho(t)}, \\ \frac{dR(t)}{dt} = \gamma(t)I(t), \\ \frac{dD(t)}{dt} = \frac{\rho(t)}{1 - \beta(t)} \gamma(t)I(t), \end{cases} \quad (1)$$

³ In this paper, we do not consider the effects of demographic variations. Despite recent discussion regarding the possibilities of reinfection (Del Rio & Malani, 2020), we assume that the recovered individuals will not be reinfected (at least for simplicity purposes), i.e., an individual does not contract the disease twice. This assumption is, therefore, suitable for the time window used in this work.

In Eq. (1), β stands for the probability of disease transmission per individual; γ stands for the recovery rate, affecting the number of individuals that “leave” from the infected class; ρ denotes the observed mortality rate of the virus. Furthermore, we assume that the epidemiological parameters are piecewise time-varying, which may slightly vary according to the pandemic stage and correct minor uncertainties in the available data used in the identification procedure. This assumption follows recent immunology discussion and theoretical results, e.g., Sun et al. (2020), Dowd et al. (2020), He et al. (2020). Moreover, the model properties (for instance, non-null and greater than 0 particular behavior of SIRD states) are comprehensively discussed by Morato et al. (2020b).

Social distancing measures are expressed through $\psi(t)$, which denotes the average amount of people circulating freely, i.e., $\psi = 1$ stands for a complete isolation condition (100 % quarantine, when contacts are reduced to zero), whereas $\psi = 0$ means no social distancing. Although the condition $\psi = 1$ is not practically viable, we use the whole interval to formulate the control problem. However, the results show that the maximal ψ is near 0.5.

The size of the total population exposed is denoted $N(t)$; it holds that $N(t) = N_0 - D(t)$, being N_0 is the initial population size (prior to the contagion). In this work, we assume that the natural deaths balance the number of newborns. We note that the expression $\beta(t)I(t)/N(t)$ gives the average number of contacts sufficient for viral transmission to one susceptible individual per unit of time. In contrast, $\beta(t)I(t)/N(t))S(t)$ gives the number of new cases concerning the number of susceptible individuals (those that are “available for infection”), per unit of time.

An essential concept in epidemiology theory is the effective reproduction number, usually denoted by $R_t(t)$. This index can measure the average effective potential of transmission of a disease at a given moment t . In other words, it represents the expected cases that a single primary can generate in a population, taking in account that susceptible individuals vary along time. From a system theory viewpoint, R_t represents the epidemic velocity. If $R_t > 1$, the infection is spreading, and the number of infected people increases, occurring typically at the beginning of the epidemic. If $R_t < 1$, it means that more individuals “leave” from the infected class, either recovering or dying and, thereby, the epidemic is ceasing. This effective reproduction number R_t is affected by different factors, including the virus’s biological characteristics and government policies to control the number of susceptible people, which can be reduced by social distancing.

In order to calculate $R_t(t)$, we assume that, at the beginning of the pandemic, $S \approx N$. Considering the parameters $\beta(t)$, $\gamma(t)$ and $\rho(t)$ from Eq. 1, $R_t(t)$ is, then, approximately given as follows, being ψ the observed social distancing factor:

$$R_t(t) \approx \frac{(1 - \psi(t))\beta(t)(1 - \rho(t))}{\gamma(t)}. \quad (2)$$

The following inequality can be directly checked to verify whether the disease is spreading in a given moment t : $\frac{(1 - \psi(t))\beta(t)(1 - \rho(t))}{\gamma(t)} < 1$. The social distancing ratio $\psi(t)$ directly affects the contagion spread, which is a fundamental aspect of the control strategy proposed herein.

Aiming to consider the effect of public health policies enacted by local governments to mitigate the effects of the COVID-19 pandemic, a model is included for the dynamics of $\psi(t)$, which concatenates to the SIRD dynamics. The time-varying parameter ψ models not only social isolation but also incentives to use masks and other measures which contain the contagion spread. The dynamics for ψ are obtained according to the first-order heuristics proposed and validated by Bastos and Cajueiro (2020), Morato et al. (2020a), Pataro et al. (2021) in order to model the popular response to the enacted social isolation measures decreed by the government. The idea is to associate the government acts $u(t)$ to the social distancing index $\psi(t)$, which is here represented as:

$$\frac{d\psi(t)}{dt} = \frac{1}{\varrho}(u(t)\psi_\infty - \psi(t)), \quad (3)$$

for which $u(t)$ is the control variable defined within $[0, 1]$ that sets the social distancing goal. Note that $\psi(t)$ converges to ψ_∞ with a time constant of ϱ , with $u = 1$. It follows that ψ_∞ is a factor representing the maximal observed effect of social distancing in a given place. For larger values of ψ_∞ (closer to 1), when tough quarantine measures are enacted (u closer to 1), the SIRD+ ψ model dynamics (with $\psi(t) \rightarrow \psi_\infty$) are slowed down, exhibiting a smaller peak of infections and number of deaths. Furthermore, larger ψ values directly influence the transmission spread factor R_t . The main control goal is to ensure $\frac{(1 - \psi(t))\beta(t)(1 - \rho(t))}{\gamma(t)} < 1$, in such way that the contagion ceases.

3 Identification Procedure

For practical purposes, model parameters are estimated using real data employed in a similar technique to the estimation scheme presented by Morato et al. (2020b).

A system composed of two layers, including analytical solutions and ordinary least square (OLS) methods, is applied to estimate parameters $\beta(t)$, $\gamma(t)$, and $\rho(t)$ from Eq. 1. For the considered application, the identification procedure is performed over the official data provided by the Brazilian Ministry of Health, considering the first confirmed case in each state (06/03/20 for BA and 13/03/2020 for SC) until the last data point of 16/06/2020. The complete data set can

be found in the open-source repository *Brasil.IO*.⁴ We note that the official data (disclosed daily) correspond to the total (cumulative) number of infections, recoveries, and deaths. In order to compute the number of daily (active) infections, we use $I(t) = I_c(t) - R(t) - D(t)$, being $I_c(t)$ the cumulative number of infections. Other identification procedures can be seen in works of Morato et al. (2020a), Bastos and Cajueiro (2020).

Firstly, we stress that the SIRD+ ψ model is identified considering the estimated social distancing indexes observed in Brazil's different states, as presented by Jorge et al. (2020). It must be noted that the optimization procedure considers the actual social distancing index as a model input variable. To avoid strong fluctuations into the parameter identification algorithm, we consider the ψ mean value in the chosen time window $(t_{\text{opt}} - t_i)$. Furthermore, for the sake of simplicity of the optimization algorithm formulation, the differential equations $dI(t)/dt$ and $dD(t)/dt$ are modified to yield linear dependence with respect to the identified parameters, as follows:

$$\begin{aligned}\frac{dI(t)}{dt} &= \frac{(1 - \psi(t))\beta(t)I(t)S(t)}{N(t)} - \gamma(t)I(t) - \alpha(t)I(t), \\ \frac{dD(t)}{dt} &= \alpha(t)I(t),\end{aligned}\quad (4)$$

being $\alpha(t) = \rho(t)\gamma(t)/(1 - \rho)$.

Then, an optimization problem is formulated, taking into account the minimum square error between real data (as disclosed by the Brazilian Ministry of Health) and the estimated SIRD+ ψ dynamic model, regarding parameters $\beta(t)$, $\gamma(t)$, and $\alpha(t)$. For each variable $I(t)$, $R(t)$ and $D(t)$, the following estimation error is established:

$$E_I(t) = (I(t) - \hat{I}(t, \beta(t), \gamma(t), \alpha(t)))^2, \quad (5)$$

$$E_R(t) = (R(t) - \hat{R}(t, \beta(t), \gamma(t), \alpha(t)))^2, \quad (6)$$

$$E_D(t) = (D(t) - \hat{D}(t, \beta(t), \gamma(t), \alpha(t)))^2, \quad (7)$$

being $\hat{I}$, $\hat{R}$ and $\hat{D}$ the variables estimated with the SIRD+ ψ model. With these variables, the complete OLS optimization problem is

$$\min_{\beta(t), \gamma(t), \alpha(t)} \sum_{i=t_i}^{i=t_i+t_{\text{opt}}} (k_1 E_I(i) + k_2 E_R(i) + k_3 E_D(i)), \quad (8)$$

$$\text{s.t.: } 0 \leq \beta(t) \leq \frac{0.65}{(1 - \psi(i))}, \quad (9)$$

$$0 \leq \gamma(t) \leq 0.7, \quad (10)$$

$$0 \leq \alpha(t) \leq 0.2. \quad (11)$$

This optimization begins defining the initial conditions $\beta(t_i) = 0.5$, $\gamma(t_i) = 0.5$ and $\alpha(t_i) = 0.1$. The tuning parameters k_1 , k_2 and k_3 are taken as positive weighting values, used to normalize the total optimization cost regarding E_I , E_R and E_D .

Therefore, to solve this problem, we consider a sliding optimization horizon of fixed size $(t_{\text{opt}} - t_i + 1)$. This window is moving along the available data. For example, assuming that the time window is 5 days, the identification procedure is performed considering the day $t_i = 1$ to $t_{\text{opt}} = 5$; then, the procedure is repeated for the day $t_i = 6$ to $t_{\text{opt}} = 10$, and so on. We proposed this scheme, considering that the available data may not represent the real trend of the epidemic dynamics due to different uncertainties regarding the reported cases. This issue may deteriorate the overall parameters estimation and the obtained model prediction. Moreover, the beginning data points usually embed substantial variations in the number of cases reported due to the absence of testing when the viral spread starts. Also, as a natural consequence of pandemic, infections, recovered and mortality rates start with substantial variations at the beginning of the spread, until convergence to a steadier behavior is observed.

The horizon window $(t_{\text{opt}} - t_i + 1)$ is smaller than the total available amount of data and rolls along the complete number of daily samples. We have found that the best model-data fitting results are achieved with windows between 5 and 10 days,⁵ which is coherent with the viral dynamics, since the average incubation period of the SARS-CoV-2 virus is of 5 days (and, at most, 14 days). Hence, we identify piecewise constant parameter values $\beta(t)$, $\gamma(t)$ and $\alpha(t)$, for each window $(t_{\text{opt}} - t_i)$. The procedure starts with $t_i = 1$ as the first day of available and follows with $t_i^+ = t_i + t_{\text{opt}} + 1$; the optimal parameters identified on the previous window ($\beta_{\text{opt}}(t)$, $\gamma_{\text{opt}}(t)$ and $\alpha_{\text{opt}}(t)$) are set as initial conditions for following loop, as in any moving-horizon optimization strategy.

4 An Optimal Social Distancing Method

Based on the SIRD+ ψ model detailed in Sect. 2 and the parameter estimates found through the identification procedure from Sect. 3, two different optimal control procedures are proposed, aiming to guide social distancing policies in SC and BA. These procedures are set within an MPC framework, in centralized and distributed paradigms, as detailed in the sequel.

We recall that MPC is formulated in a discrete-time paradigm. Therefore, since new measurements of infections and deaths are available every day, and the contagion dynam-

⁴ Refer to <https://brasil.io/dataset/covid19/>.

⁵ In practice, we use a fixed horizon size of 6 days, as discussed in Sect. 5.

Fig. 1 Identification procedure and control scheme for CMPC and DMPC control strategies

ics is slow (in the order of days), the SIRD model and the social distancing dynamics from Eqs. (1) and (3) are Euler-discretized with $T_s = 1$ day. The discrete sampling instants are denoted as $k = \frac{t}{T_s}$.

The MPC procedures are designed through a minimization problem, where performance goals are delimited as quadratic maps. Regarding the COVID-19 situation, the control objective is evident: minimize the number of active infections ($I(k)$) while altogether reducing the social distancing efforts ($u(k)$). We stress that long rigid social distancing provokes devastating economic and psychological effects, and thus, such measures should be kept for the smallest duration possible, as treated in the research of Eichenbaum et al. (2020).

One cannot expect to increase or decrease social isolation instantaneously. As observed in practice, the population takes some time to respond to new social isolation measures, adapting to the enacted paradigm. Therefore, in consonance with the dynamic Eq. (3) and with real isolation policies put in practice in Brazil, we consider that the control action u can vary ± 0.15 per day, which means that the actual isolation factor will increase/decrease with a rate of, at most, 15 %/day.

We note that this is a preliminary assumption since the actual implemented social isolation policy should be a “translation” of the control signal u into a feasible set of actions. These actions could represent different guidelines, such as total isolation, with no one leaving their homes (for $u = 1$), a partial isolation, with people allowed to leave only for short periods, with masks (for $u = 0.9$), and so forth, until a total “relaxed” condition (for $u = 0$).

Figure 1 summarizes the identification procedure and both control approaches presented in the sequel.

4.1 Centralized MPC

Based on the problem and constraints detailed above, the first control procedure proposed is set as a single, centralized MPC (CMPC) algorithm. This approach considers the evolution of the contagion in both states, BA and SC, and, thereby, determines a single control action u which guides the social isolation policies in both states. Such optimization, with cost function J_{CMPC} , is expressed as follows:

$$\min_{U(k)} \sum_j \sum_{i=1}^{N_p} \frac{(I_j(k+i)^T q_I I_j(k+i))}{n_j^2} \quad (12)$$

$$+ \sum_{i=1}^{N_p} (u(k+i-1)^T q_u u(k+i-1)), \quad (13)$$

$$\text{s.t.: Discrete SIRD Models } \forall i \in \mathbb{N}_{[1, N_p]}, \quad (14)$$

$$0 \leq u(k+i-1) \leq 1, \quad (15)$$

$$-0.15 \leq u(k+i) - u(k+i-1) \leq 0.15, \quad (16)$$

$$0_{2 \times 1} \leq \begin{bmatrix} I_j(k+i) \\ D_j(k+i) \end{bmatrix} \leq \begin{bmatrix} n_j^{\text{ICU}} \\ n_j \end{bmatrix} \forall j, \quad (17)$$

wherein N_p is a prediction horizon; the subscript j indicates the state (i.e., I_{BA} stands for the infections in Bahia), n_j stands for the total population size of the j -th state, n_j^{ICU} represents the total ICU beds available in the state and $U(k)$ represents the sequence of control actions inside the prediction horizon, i.e., $U(k) = \text{col}\{u(k) u(k+1) \dots u(k+N_p-1)\}$. The weights q_I and q_u determine the trade-off between conflicting objectives of minimizing the spread and reducing social isolation efforts. Note that the term n_j^2 is used in the first sum of J_{CMPC} to normalize the two terms of the cost function. Notice that the main goal is to minimize the number of infection for both states I_j along the prediction horizon and also the social isolation policies index u , considering the maximum and minimum limits of u , and guarantying that the number of infections will not overpass the number of beds occupancy n^{ICU} and the fatal cases will not overpass n .

4.2 Distributed MPC

A distributed MPC (DMPC) formulation is much like the one in Eq. (13), but considers the SIRD+ ψ model for just a single state and, thereby, finds an individual control law (u_j) for the referenced state. This optimization procedure, with cost function J_{DMPC} , is:

$$\min_{U(k)} \sum_{i=1}^{N_p} \frac{(I_j(k+i)^T q_I I_j(k+i))}{n_j^2} \quad (18)$$

$$+ \sum_{i=1}^{N_p} (u_j(k+i-1)^T q_{u_j} u_j(k+i-1)),$$

$$\text{s.t.: Discrete SIRD Model } (j) \forall i \in \mathbb{N}_{[1, N_p]}, \quad (19)$$

$$0 \leq u_j(k+i-1) \leq 1, \quad (20)$$

$$-0.15 \leq u_j(k+i) - u_j(k+i-1) \leq 0.15, \quad (21)$$

$$0_{2 \times 1} \leq \begin{bmatrix} I_j(k+i) \\ D_j(k+i) \end{bmatrix} \leq \begin{bmatrix} n_j^{ICU} \\ n_j \end{bmatrix}. \quad (22)$$

wherein the parameters are those defined for the CMPC framework. Notice that the notation j associates the parameters and variables to each j -th state; in this case, $j = \text{BA}$ or $j = \text{SC}$. Thus, the objective function is to separate for each controller in each state. The CMPC approach, regarding Brazil, would stand for a single social isolation guide to all 26 states. On the other hand, the design of the DMPC would represent the social isolation defined through individual state guidelines, unrelated to the usual decentralized concept in which a coupled problem is solved in a decentralized way. For simplicity, although the whole approach presented here can be extrapolated for other scenarios, this work considers only the states of Bahia and Santa Catarina in the following results. We propose investigating the particularities of each approach.

The CMPC provides a uniform social distance guideline for all Brazilian states as a suitable solution to face the pandemic. In this case, the control problem is formulated considering only one input $u(t)$, giving a unique social distancing for the entire country, consequently treating the problem as the SIMO system. Differently, the DMPC handles the pandemic spread individually for each state, wherein each state can provide its social distancing guidance (e.g., $u(t)_{\text{BA}}$, $u(t)_{\text{SC}}$) considering its individual SIRD model. It is worth mentioning that the SIRD models are assumed decoupled for both the CMPC and DMPC. The purpose of offering the DMPC approach is that each state can apply its own measures accordingly with its pandemic spread, the number of infections, recovered, and deaths.

Remark 1 In recent papers (Morato et al., 2020a; Köhler et al. 2020; Morato et al., 2020b), the issue of MPC regarding COVID-19 has also been discussed. Notice that the differences between the CMPC/DMPC formulations presented in this paper to those in the references are the following: (i) the control signal presented by Köhler et al. (2020) is a factor that multiplies the contagion transmission factors β and γ , while in this paper and in Author's previous work (Morato et al., 2020a, 2020b), it goes through a dynamic model regarding ψ ; (ii) both previous papers consider uncertainty and approach the problem using a robust design procedure, which is out of the scope of this paper since we do not have sufficient data for consistent robust parameters estimates; (iii) the CMPC/DMPC approaches consider slew rate constraints on the control signal u , which had not yet been tested (the previous references considered that the social isolation reference could vary arbitrarily at each future sample $k+i$); and, (iv) in (Morato et al., 2020b) the authors propose a control

Table 1 Optimization weights

Parameter	k_1	k_2	k_3
Value	1	10	2

framework to cope with the pandemic throughout the entire country, while the present work analyzes how the control strategy can be implemented in practice regarding the application of the measures, whether using the uniform (CMPC) or distributed (DMPC) guidance. This discussion is still under debate in the public sector, which is attractive for engineering study and research, besides helping to face the health crisis caused by COVID-19.

5 Main Results

We proceed by depicting the results concerning the identification procedure and the obtained control results. The following results were obtained with the aid of MATLAB software, Yalmip toolbox and fmincon solver. We use available data from March 06, 2020, to February 23, 2021.

5.1 Model Identification

We proceed by depicting the results concerning the identification procedure and the obtained control results. The following results were obtained with the aid of MATLAB software, Yalmip toolbox, and fmincon solver. Table 1 depicts the optimization weights used during the identification procedure, initially tuned to take into account the magnitude of each variable I , R , D and then slightly adjusted experimentally to provide better fit results.

Remark 2 At the beginning of the pandemic in SC, there was a considerable lack of reported recovered cases (until 05/05/20), inconsistent concerning the active infections and R_t . However, this does not affect the overall forecasts due to the moving-horizon optimization strategy and does not affect the obtained control results.

In order to perform the identification procedure, we consider the actual social distancing index for both states. We use the historical data for the observed social isolation factor available in InLoco (2020). Figure 2 depicts the daily and the mean ψ value within time windows of 6 days for both states.

Therefore, using the collected data and the mean ψ values, we perform the model parameters identification procedure. Figures 3 and 4 present the identified model parameters. Notice that the identified values are piecewise divided in 6 days, according to the proposed optimization time window.

In order to demonstrate the effectiveness of the identification approach, we present a forecasting scenario. In this case,

Fig. 2 Social distancing index for states of Bahia and Santa Catarina

Fig. 3 Identified parameters for state of Bahia

Fig. 4 Identified parameters for state of Santa Catarina

as will be treated in the MPC implementation, we keep constant the last identified parameter values as possible future information for the model dynamic. Thus, we perform the identification procedure using data from March 06, 2020, until February 11, 2021. The remaining information data from February 12, 2021, to February 23, 2021, are used as the validation stage. Figures 5 and 6 depict the model-data fitting results and the forecasting scenario for the whole data sets, accounting for the cumulative number of cases, active infections, recovered and fatal cases. We also include an error margin of 5% for the identified model curves. Moreover, Fig. 7 presents the estimated effective reproduction number R_t .

The identification procedure yields a quite good parameter estimation since the simulated SIRD+ ψ model uses official data for a time window of 6 days, which leads to small vari-

ations between the series. As can be seen, the forecasting procedure can provide a reliable estimate of the model curves for a 14-day forecast, which demonstrates the quality of the identified parameters (see Figs. 5 and 6). We stress that the coefficient of determination R_{cd} of the obtained identification results is very close to 1, as presented in Table 2.

The model parameters obtained for the last data set window, which is from February 05, 2021, until February 11, 2021, are presented in Table 3.

It is worth mentioning that different pandemic stages are seen in each state. From Figs. 5, 6 and 7, we can see that a “second wave” with more cases has been seen in Santa Catarina. Moreover, it can be noticed that the identification procedure works very well in the situations with a descending tendency, evidenced in the periods from the end of August 2020 until January 2021 for the state of Bahia, and from the beginning of August 2020 until November 2020 for the state of Santa Catarina. These results are also reflected in R_t index values, which indicate that the identified epidemiological parameters can be used to compute the effective reproduction number in the course of the pandemic with fidelity. As can be seen in Fig. 7, when $R_t > 1$ the number of cases is increasing, including during the second wave, as can be seen in Figs. 5 and 6 at the end of October 2020.

We note that the SIRD+ ψ model parameters used for control are those for the last available window, as given in Table 3. Since a window of 6 days is shown to be sufficient to estimate the SIRD+ ψ model parameters with model-fitting efficiency (R_{cd} coefficient close to 1), the most adequate control procedure is to adjust the model of the MPC controller iteratively, as time progresses. This kind of procedure allows one to incorporate the variability of the SIRD parameters, which is inherent to the SARS-CoV-2 viral spread dynamics. We cannot proceed with such a paradigm since we consider the control action being deployed through the future (for which we have no data). Thus, we keep the last available SIRD parameters as those used for control purposes.

5.2 Control Results

Considering the given parameters for the SIRD+ ψ model, the control results are presented. The values for the social isolation response dynamics of Eq. (3) are borrowed from the work of Morato et al. (2020a). As presented in Table 4, the maximal social isolation factors were retrieved from recent technical notes from these states. The time constant in Eq. 3 is related to the average time the population takes to respond to new social distancing measures. We have investigated this issue with care in the paper of Morato et al. (2020a), and corresponding discussions are presented therein. For brevity, we only make reference to the prior in this paper.

The MPC strategies are synthesized with a prediction horizon of $N_p = 14$ days. This prediction horizon is coherent

Fig. 5 Identification procedure: State of Bahia

Fig. 6 Identification procedure: State of Santa Catarina

Fig. 7 Identification procedure: R_t Table 2 Coefficient of determination R_{cd}

	BA	SC
Cumulative cases	0.9999	0.9999
Active infections	0.9583	0.9643
Recovered cases	0.9999	0.9998
Fatal cases	0.9999	0.9999

Table 3 Model parameters

States		$\beta(t)$	$\gamma(t)$	$\alpha(t)$
BA	From February 05th, 2021 to February 11th 2021	0.3034	0.1340	0.023
SC		0.1956	0.0839	0.011

Table 4 Social isolation response

State	ϱ	ψ_{∞}
BA	1.66 days	0.563
SC	1.66 days	0.514

with the COVID-19 dynamics since the incubation period of the SARS-CoV-2 virus is of, at most, 14 days. In all the following simulation results, the MPC weights q_I and q_u are chosen, respectively, as 0.5 each, so that these predictive controllers try to find a “balance” between minimizing infections and relaxing social isolation measures.⁶

The achieved results are obtained considering an initial condition regarding the available data at 23/02/2021. As of this date, we note that the state of Bahia (BA) has many infections (the ICU beds at BA are almost full), while the state of Santa Catarina (SC) has already passed through the infection peak. The control strategy is assumed to act from 24/02 to mitigate the backlashes. The results indicate what **could still be done** to avoid the expected catastrophic results

if no stronger health policy is employed. Through the sequel, NC denotes the results with “no control,” i.e., with $u = 0$ and, thus, with no social isolation, i.e., $\psi \rightarrow 0$.

In Fig. 8, the derived control laws are presented and compared to the social isolation factors ψ . The CMPC strategy is presented on the upper subplot, while the DMPC is shown in the lower subplot. The CMPC control input is a single social distancing guideline u , with a corresponding ψ variable for both states. We consider, in this case, ψ_{∞} as the maximal historical social isolation index of Bahia (which is smaller than the one observed for SC). This figure depicts how the generated CMPC law proceeds by increasing social distancing until the peak of infections is minimized and, then, progressively loosening the quarantine measures as the pandemic ceases in both states. The DMPC law, as it tries to balance the local conditions of each state, shows a slower descend for the state of SC, while a faster descend for BA, since the latter state currently exhibits a closer-to-peak condition.

It must be stressed that we analyze the SIRD models as if there are no coupling effects between them. Anyhow, in practice, the 26 states in Brazil cannot pursue individual social isolation laws (as the DMPC approach) since their borders are not closed. The DMPC results only indicate that local conditions should be taken into account, but a centralized coordination (like the CMPC) is forcefully necessary to reduce the infections all over Brazil. It seems to us much more prudent if the federal government dispatches a coordinated social distancing health policy (following a CMPC method), while each state figures out their possible relaxations according to a DMPC approach and considers the infection level in

⁶ Remember that a normalization was applied in the terms of J .

Fig. 8 Control action and social response

Fig. 9 Control results: active infections

Fig. 10 Control results: deceased individuals

the frontiers states. The same idea can be extended to the cities coordinated by their states. However, at the moment, this study could not be carried out because there are no data available on population movement between states.

Regarding the depicted control laws, Figs. 9, 10, and 11 show, respectively, the evolution of the active infections, the total number of deaths, and the total recovered individuals due to COVID-19 in both states, over time. Firstly, these results indicate that the “no-control” (NC) condition would certainly be catastrophic, since infections are largely increasing in both states, which currently (23/02/2021) exhibit ICU occupancy levels of 75% (SC) and 85% (BA). We note that both CMPC and DMPC strategies are able to revert this pos-

sible catastrophe, mitigating the peak of infections in both states and saving more than 25% of lives in SC and 40% in BA. Since the DMPC takes into account local characteristics, a “stronger” social distancing guideline is planned for SC at first, since this state is currently farther away from the peak forecast. In the case of BA, since the peak forecast is closer, the DMPC relaxes social distancing measures sooner, which must be corrected later on with a strengthening of the quarantine guidelines around May–June/2021. The CMPC strategy, while taking into account the pandemic effects in both states, provides a social distancing measure with a slower decedent, which leads to pandemic scenario which lasts longer.

Fig. 11 Control results: recovered individuals

Therefore, it becomes evident that the DMPC is an interesting alternative for the local coordination of control policies, by municipalities and local governments, while a CMPC approach is also necessary from the viewpoint of a federal government. We indicate that a mixed strategy, with two control levels, could be the final and more conclusive alternative, with the federal government (CMPC) determining general indications of when to close airports, inter-state transports, and so on, and with local decision makers (DMPC) being responsible for decentralized and more meticulous decisions, such as closing the local transport and limiting economic trades. The DMPC and the CMPC strategies, by themselves, may be insufficient, since inter-state effects may deteriorate the control results obtained in the DMPC sense, while the CMPC sense may lead to a social distancing guideline which is too harsh.

As a final (yet critical) comment, we must discuss that an actual application of the proposed method (either CMPC or DMPC) depends on how the control signal can be translated into actual public health policies to be put into practice. This maneuver can be understood as actuator filtering of the control signal since abrupt daily variations on u make no sense regarding health policies. As an example, one cannot expect to determine relaxations (allow public transport) in one day to revert it in the following. Such parametrized paradigm has been discussed by Morato et al. (2020b), which offers an elegant solution to practically implement the MPC-generated inputs.

6 Discussions and Conclusions

In this paper, we investigated how predictive control and optimization-based procedures could be used to formulate social isolation guidelines for the COVID-19 pandemic in Brazil, taking into account the spread of the virus in the states of Bahia and Santa Catarina. Centralized and distributed MPC approaches based on SIRD models with parameters identified via least-square optimization were proposed in this work. The results indicate that strong quarantine/lockdown measures still have to be enacted for future scenarios before the vaccines are available for the entire population.

Below, we summarize some key points:

- The results corroborate the hypothesis formulated in work of Hellewell et al. (Hellewell et al. (2020)) and previously discussed by Morato et al. (2020a). Herd immunity cannot be considered a plausible solution for Brazil since it offers great risk and leads to high fatality due to its multiple social-economical issues.
- The control results show that a centralized and coordinated federal government action is necessary to set guidelines to the states, which can perform individual optimization procedures to determine when to relax quarantine measures. A forecast is presented, indicating that a coordinated social isolation public policy could save over 40% of lives in BA and 25% in SC. Nevertheless, a CMPC with uniform control action may have the performance deteriorated compared to a DMPC. Since each state has its particular pandemic stage and different government actions, these particularities must be considered to apply a coordinated social distance guideline. The presented

DMPC approach intends to address this issue, helping to improve the control performance, allowing each state to deal individually with the pandemic, and conceding that it uses different control tunings.

- Based on the proposed BA and SC scenarios, considering a large number of cities in each state and the strong coupling due to the proximity of the towns, a uniform government measure could be applied in this scenario. The provided measures and decrees given by the states must certainly be adapted for each city in order to be able to reach the desired social distance index. Hence, the whole state will achieve the optimal measure, but it may differ from one city to another.
- The SARS-CoV-2 contagion is an inherently complex phenomenon and is influenced by many factors. More extended and precise prediction of its dynamics is not possible, and, therefore, the quantitative results presented herein cannot be accounted for without considering the uncertainty margins. Anyhow, the qualitative results are robust. A correct control procedure should be based on a recurrent model tuning and re-calculation of the control law. Since the country has experienced a reluctance to implement more stringent social isolation measures formally (The Lancet, 2020), the pandemic's social and economic costs are already brutal and indicate to get even crueler.

Acknowledgements An early version of the paper was presented at XXIII Congresso Brasileiro de Automação (CBA 2020)

Author Contributions IMLP, MMM, MVAC, and JEN-R performed conceptualization; IMLP, MMM, MVAC, and JEN-R done methodology; IMLP, MMM, MVAC, and JEN-R were involved in formal analysis and investigation; IMLP and MMM contributed to writing—original draft preparation; IMLP, MMM, MVAC, and JEN-R writing—review and editing; IMLP and MMM done funding acquisition; IMLP and MMM contributed to resources; MVAC and JEN-R supervised the study.

Funding The authors acknowledge the financial support of National Council for Scientific and Technological Development (CNPq, Brazil) under Grants 304032/2019-0 and 201143/2019-4.

Declarations

Conflict of interest The authors declare no conflict of interest.

References

- Adam, D. (2020). The simulations driving the world's response to COVID-19: How epidemiologists rushed to model the coronavirus pandemic? *Nature*, 580(7803), 316–318.
- Bastos, S. B., & Cajueiro, D. O. (2020). Modeling and forecasting the early evolution of the COVID-19 pandemic in Brazil. Preprint [arXiv:2003.14288](https://arxiv.org/abs/2003.14288)
- Bedford, J., Farrar, J., Ihekweazu, C., Kang, G., Koopmans, M., & Nkengasong, J. (2019). A new twenty-first century science for effective epidemic response. *Nature*, 575, 130–136.
- Camacho, E. F., & Bordons, C. (2013). *Model predictive control*. Springer Science & Business Media.
- Del Rio, C., & Malani, P. N. (2020). Covid-19-new insights on a rapidly changing epidemic. *Jama*, 323, 1339–1340.
- Dowd, J. B., Andriano, L., Brazel, D. M., Rotondi, V., Block, P., Ding, X., et al. (2020). Demographic science aids in understanding the spread and fatality rates of COVID-19. *Proceedings of the National Academy of Sciences*, 117(18), 9696–9698.
- Eichenbaum, M. S., Rebelo, S., & Trabandt, M. (2020). The macroeconomics of epidemics. Working Paper 26882, National Bureau of Economic Research. <https://doi.org/10.3386/w26882>, <http://www.nber.org/papers/w26882>
- He, X., Lau, E. H., Wu, P., Deng, X., Wang, J., Hao, X., et al. (2020). Temporal dynamics in viral shedding and transmissibility of COVID-19. *Nature Medicine*, 26(5), 672–675.
- Hellewell, J., Abbott, S., Gimma, A., Bosse, N. I., Jarvis, C. I., Russell, T. W., et al. (2020). Feasibility of controlling covid-19 outbreaks by isolation of cases and contacts. *The Lancet Global Health*, 8(4), e488–e496.
- InLoco. (2020). Social isolation map covid-19 (in portuguese). Retrieved February 23, 2021, from <https://mapabrasileirodacovid.inloco.com.br/pt/>
- Jorge, D. C. P., Rodrigues, M. S., Silva, M. S., Cardim, L. L., da Silva, N. B., Silveira, I. H., Silva, V. A., Pereira, F. A., Pinho, S. T. R., Andrade, R. F. S., Ramos, P. I. P., & Oliveira, J. F. (2020). Assessing the nationwide impact of COVID-19 mitigation policies on the transmission rate of sars-cov-2 in brazil. Preprint <https://doi.org/10.1101/2020.06.26.20140780>
- Keeling, M., Rohani, P., & Pourbohloul, B. (2008). Modeling infectious diseases in humans and animals. *Clinical Infectious Diseases: An Official Publication of the Infectious Diseases Society of America*, 47, 864–865. <https://doi.org/10.1086/591197>.
- Köhler, J., Schwenkel, L., Koch, A., Berberich, J., Pauli, P., & Allgöwer, F. (2020). Robust and optimal predictive control of the COVID-19 outbreak. Preprint [arXiv:2005.03580](https://arxiv.org/abs/2005.03580)
- Kucharski, A. J., Russell, T. W., Diamond, C., Liu, Y., Edmunds, J., Funk, S., et al. (2020). Early dynamics of transmission and control of covid-19: A mathematical modelling study. *The Lancet Infectious Diseases*, 20(5), 553–558.
- Lurie, N., Saville, M., Hatchett, R., & Halton, J. (2020). Developing COVID-19 vaccines at pandemic speed. *New England Journal of Medicine*, 382(21), 1969–1973.
- Morato, M. M., Bastos, S. B., Cajueiro, D. O., & Normey-Rico, J. E. (2020). An optimal predictive control strategy for COVID-19 (SARS-CoV-2) social distancing policies in Brazil. *Annual Reviews in Control.*, <https://doi.org/10.1016/j.arcontrol.2020.07.001>.
- Morato, M. M., Pataro, I. M., Americano da Costa, M. V., & Normey-Rico, J. E. (2020). A parametrized nonlinear predictive control strategy for relaxing covid-19 social distancing measures in brazil. *ISA Transactions.*, <https://doi.org/10.1016/j.isatra.2020.12.012>.
- Pataro, I. M. L., Oliveira, J. F., Morato, M. M. et al. (2021). A control framework to optimize public health policies in the course of the COVID-19 pandemic. *Scientific Reports* 11, 13403. <https://doi.org/10.1038/s41598-021-92636-8>
- Peng, L., Yang, W., Zhang, D., Zhuge, C., & Hong, L. (2020). Epidemic analysis of covid-19 in china by dynamical modeling. medRxiv. <https://doi.org/10.1101/2020.02.16.20023465>
- Rocha Filho, T. M., Ganem dos Santos, F. S., Gomes, V. B., Rocha, T. A., Croda, J. H., Ramalho, W. M., & Araujo, W. N. (2020). Expected impact of covid-19 outbreak in a major metropolitan area in brazil. medRxiv. <https://doi.org/10.1101/2020.03.14.20035873>.

- San Lau, L., Samari, G., Moresky, R. T., Casey, S. E., Kachur, S. P., Roberts, L. F., et al. (2020). COVID-19 in humanitarian settings and lessons learned from past epidemics. *Nature Medicine*, 26(5), 647–648.
- Sun, P., Lu, X., Xu, C., Sun, W., & Pan, B. (2020). Understanding of COVID-19 based on current evidence. *Journal of Medical Virology*, 92(6), 548–551.
- The Lancet. (2020). Covid-19 in Brazil: So what? *The Lancet*, 395(10235), 1461. [https://doi.org/10.1016/S0140-6736\(20\)31095-3](https://doi.org/10.1016/S0140-6736(20)31095-3).
- Werneck, G. L., & Carvalho, M. S. (2020). The COVID-19 pandemic in Brazil: Chronicle of a health crisis foretold [Text in Portuguese]. *Cadernos de Saúde Pública*,. <https://doi.org/10.1590/0102-311x00068820>.

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.