Preface–Special Issue on Mathematical Optimization: Past, Present and Future

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Mathematical optimization is one of the foundations of fields such as operations research, computational mathematics, scientific and engineering computing, machine learning, and data sciences. Typical tasks usually include formulating appropriate mathematical models to describe related practical problems, designing suitable numerical methods to find optimal solutions, and exploring the theoretical properties of the models and algorithms. The evolution of modern computer architecture and the popularity of big/complex/smart data had a significant impact on mathematical optimization. The success of large-scale optimization in machine learning and signal processing certainly provides a very exciting paradigm on “Modeling + algorithms + computing power”. The challenges include not only computational complexity but also memory access and communication. They are intrinsically and extensively linked through various models, theories, algorithms, and applications. For such new and special problems, on the theoretical side, we are running out of applicable theories, and on the computational side, we are stuck in a deep local optimum. Therefore, new ideas, approaches, and techniques are urgently required. Techniques such as stochastic and robust methods and their hybridization with traditional methods are relevant. Making the best use of vastly parallel computation to qualitatively transform optimization is promising. How can optimization help machine learning, signal processing, statistics, and engineering? What is missing in optimization in these fields? What are these fields’ greatest requirements from optimization? How can the structures of various practical problems be exploited in model construction, algorithm design, and their analysis?

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Answers to these questions will help us identify important research issues that arise from existing or emerging applications.

This special issue reviews some major developments in the past and some current state-of-the-art methodologies in order to forge new frontiers and foster the future growth and improvement of mathematical optimization. Our objectives are as follows: (a) to discuss the opportunities and challenges in optimization facing the rapid growth of computational power; (b) to highlight the fundamental importance of optimization techniques as a powerful computational and analytical tool for machine learning, signal processing, statistics, engineering, and related fields. This special issue comprises five survey papers on optimization models and algorithms for manifold optimization, deep learning, medical image reconstruction, electric power optimization, and machine learning. These papers are summarized below.

Jiang Hu, Xin Liu, Zai-Wen Wen, and Ya-Xiang Yuan provide an introduction to manifold optimization. The non-convexity from the manifold constraints is one of the main difficulties. The applications, algorithms, and theory for manifold optimization are presented.

Ruo-Yu Sun reviews the theory and algorithms for optimization in deep learning from three perspectives: (i) issues related to the neural network models, such as the gradient explosion/vanishing, careful initialization, normalization methods, and skip connections; (ii) algorithms: the stochastic gradient descent and adaptive gradient methods; (iii) theory on finding global optimal solutions such as global landscape, mode connectivity, lottery ticket hypothesis, and neural tangent kernel.

Dong Han and Xiao-Jiao Tong summarize optimization methods and modeling techniques of power system optimization associated with system planning, operation, and control. A few topics on the modeling and calculation of the Energy Internet are discussed according to the design structure and operational characteristics.

Hai-Miao Zhang and Bin Dong discuss the development of modeling in image reconstruction in the past three decades: handcraft modeling, handcraft + data-driven modeling, and deep learning models. They review some recently developed deep learning models for medical image reconstruction and share their understanding of deep neural networks from the perspective of representation learning and differential equations.

Zhou-Chen Lin explains different goals between the machine learning and the optimization communities. He gives suggestions on how optimization and machine learning can help each other better.

All papers in this special issue have been peer-reviewed according to the standard of the journal. We thank all authors for their timely response and careful revisions. We would also like to express our gratitude to all the reviewers for providing their expertise and constructive comments.

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