



Commentary on “From unidimensional to multidimensional inequality: a review”

Kristof Bosmans¹

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Abstract

This commentary on the review by Andreoli and Zoli questions the requirement of consistency between aggregation procedures and suggests that multidimensional inequality measurement benefits from an explicit account of allocative efficiency.

Keywords Multidimensional inequality · Allocative efficiency · Pigou-Dalton principle

Francesco Andreoli and Claudio Zoli provide a useful review of the literature on multidimensional inequality measurement with a strong focus on dominance criteria. Their review illuminates a literature that has made a lot of progress, but that still rests on shaky foundations.

1 Two procedures, equal in relevance?

Section 4 of the review presents the conditions under which two alternative procedures to aggregate multidimensional distributions are consistent with one another. Procedure 1 first aggregates across individuals for each attribute and next aggregates the obtained attribute indices. Procedure 2 first aggregates across attributes for each individual and next aggregates the obtained individual indices. But are these two procedures equally relevant? And do we want them to be consistent with one another?

I believe that procedure 2 is in general the more relevant of the two procedures. The procedure follows the standard approach in welfare economics, where one first aggregates each individual’s bundle of goods using the individual’s well-being function and next aggregates the obtained well-being levels. It is appropriate of procedure 2 to place the emphasis on the individual level. Indeed, our interest does not lie in the spread of the entries of some matrix, but rather in the inequality between individuals whose well-being happens to depend on multiple attributes.

Procedure 1 is relevant in those rare cases in which it also emphasizes the individual level. Let us consider two such cases.

✉ Kristof Bosmans
k.bosmans@maastrichtuniversity.nl

¹ Department of Microeconomics and Public Economics, Maastricht University, Maastricht, The Netherlands

First, consider the case of welfare comparisons under risk. In this setting, each attribute distribution represents an income distribution corresponding to a state of the world. Because only one attribute distribution is ultimately realized, the focus is on what matters to individuals, irrespective of the order of aggregation. The difference between procedure 1 (“ex post”) and procedure 2 (“ex ante”) lies elsewhere, viz., in whether the risk assessment takes place in the first or second step of the aggregation. As the two procedures have the same aim of providing welfare comparisons under risk, the requirement of consistency between the procedures is justified, and has indeed received ample attention.

Second, consider the case where procedure 2 is deemed ideal, but only data on the marginal distributions is available. Then procedure 1 constitutes a plausible compromise. Consideration of consistency between the two procedures is again justified, to gauge the extent to which the compromise is able to approach the ideal.

It is difficult, by contrast, to motivate procedure 1 from a “dashboard” perspective. A dashboard offers to the user a whole range of indices, for example inequality or welfare indices for each attribute distribution, as in the first step of procedure 1. But the essence of a dashboard is to respect the richness of the information and to resist summarizing it too much. Hence, one may question the appropriateness of additionally aggregating across the different attribute indices, as the second step of procedure 1 prescribes.

In sum, I believe that procedure 2 is in general more suitable than procedure 1 to assess multidimensional inequality. There are, however, exceptional cases in which procedure 1 is warranted, two of which are considered above. In such cases, consistency between the two procedures should only be required if the procedures moreover aim at answering the same question. Unless this condition is met, consistency is an overly demanding requirement that unduly restricts the available ethical positions.

2 Allocative efficiency, the missing ingredient

An essential but often overlooked aspect of the multidimensional setting is that exchanges of goods between individuals may be mutually beneficial. Drawing heavily from Bosmans et al. (2015, 2018), I argue that an explicit account of allocative efficiency helps to clarify two closely related topics that figure prominently in the review: inequality-welfare consistency and multidimensional transfer principles. My discussion assumes procedure 2.

In the unidimensional setting, welfare is a function of two components: the inequality level and the average. With the average fixed, welfare and inequality always move in opposite directions, as prescribed by inequality-welfare consistency. In the multidimensional setting, by contrast, welfare is a function of three components: the inequality level, the vector of attribute averages, and the efficiency level (Bosmans et al. 2015, footnote 16). With the attribute averages fixed, welfare and inequality may very well move in the same direction if efficiency changes as well, thus violating inequality-welfare consistency. Example 5.2 in the review presents such an instance where welfare, inequality and efficiency all move in the same direction. Andreoli and Zoli are right to question the validity of inequality-welfare consistency in the multidimensional setting.

The introduction of efficiency considerations also helps to uncover the shortcomings of various multidimensional transfer principles. In Bosmans et al. (2015), we show that the transfers recommended by multidimensional majorization always increase efficiency, but may either decrease or increase inequality. Example 5.2 illustrates a case where inequality

increases due to such transfers. We conclude that multidimensional majorization may qualify as an efficiency criterion, but is patently unfit to serve as an inequality criterion.

In Bosmans et al. (2015), we moreover show that if goods are complements, then correlation decreasing transfers always decrease efficiency, but affect overall inequality ambiguously. The intuition behind this ambiguity is as follows. The transfers reduce the inequality between the two individuals involved. But since this inequality reduction comes with a loss in efficiency, the effect is akin to a leaky-bucket transfer. And the effect of a leaky-bucket transfer on inequality (and welfare) is ambiguous. Similar ambiguity problems arise for the transfers implicit in the non-increasing increments property discussed in the review.

Finally, consider the prize majorization criterion to which the review devotes considerable attention. The transfers underlying this criterion again demand acceptance of potentially large efficiency losses. This is revealed by the restrictive form of the welfare functions that appear in Theorems 5.3 and 5.4: these welfare functions impose that the goods are perfect substitutes and thereby eliminate all efficiency considerations.

The multidimensional setting still lacks a transfer principle that truly captures the idea of the unidimensional Pigou-Dalton principle. The transfers recommended by the unidimensional Pigou-Dalton principle unambiguously decrease inequality, without affecting the other component of welfare, i.e., the average. For the multidimensional setting, we need a principle that recommends transfers that unambiguously decrease inequality, without affecting the attribute averages and the efficiency level. See Bosmans et al. (2018) for an attempt.

3 Concluding remark

As the title of the review suggests, the favored approach in the literature has been to extend criteria from the unidimensional setting to the multidimensional setting. This approach of moving up from the specific to the general has proved useful. But it has also been a source of confusion, as exemplified by the problems discussed in the previous section.

I suggest a complementary approach that moves in the opposite direction, from the general to the specific. The idea would be to start from a general setting with multiple goods and heterogenous, ordinal and noncomparable well-being rankings, as in, among others, Fleurbaey and Maniquet (2011), Piacquadio (2017) and Bosmans et al. (2018). The setting of the review can then be obtained as the specific case in which all individual well-being rankings are identical. By moving down from an informationally richer setting, instead of up from an informationally poorer one, the risk of neglecting important aspects is reduced.

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