Detection of small delamination in mullite/Si/SiC model EBC system by pulse thermography

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Received: January 20, 2019; Revised: February 28, 2019; Accepted: March 22, 2019

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Abstract: The pulse thermography (PT) technique was applied to the detection of the delamination of a multi-layered coating system composed of mullite/Si onto a reaction-bonded SiC substrate. The potential evaluation was carried out in order to detect internal delamination in multi-layered material system. Moreover, the observation of the cross sections and 3D views obtained by X-ray computed tomography (CT) indicated that the delamination occurred at the interface between the top coat and the bond coat layers. The changes in the temperature distribution obtained by PT indicated the existence of a delamination area in the top coat layer of the mullite. In particular, the lower temperature region corresponded to the delamination area. The experimental results confirmed that the PT technique is effective with respect to the internal delamination of multi-layered coating system.

Keywords: environmental barrier coatings (EBCs); delamination; pulse thermography (PT); multi-layer; image processing

1 Introduction

Environmental barrier coatings (hereafter denoted as EBCs) are key technology for surface protection of SiC fiber reinforced SiC matrix (hereafter denoted as SiC/SiC) composites from combustion atmosphere in an advanced aero gas turbine engine. Multi-layered EBC systems typically consist of silicate top coat layer and Si-based bond coat layer [1–8]. The coefficient of thermal expansion (hereafter denoted as CTE) of silicate top coat layer ($(5\text{--}8)\times 10^{-6}\text{ K}^{-1}$ [9]) is usually larger than that of Si bond coat layer ($\sim 4\times 10^{-6}\text{ K}^{-1}$

[10,11]) and SiC/SiC substrate ($(4.5\text{--}5.5)\times 10^{-6}\text{ K}^{-1}$ [2]). Mismatch of CTE among top coat layer, bond coat layer, and substrate generates in-plane thermal tensile stress in a silicate top coat layer and it causes mud cracks in top coat layer. In the case of air plasma sprayed coatings, shrinkage of metastable silicates during crystallization from amorphous to crystalline of silicate also leads to severe damage after heating [12,13]. In addition, degradations of EBC layer such as melting of Si and formation of cristobalite in bond coat occur [5–8,13–16] because combustion environment in aero gas turbine engine is 1300–1700 °C with 10–30 atm and the partial pressure of H₂O is 1–3 atm [17], and H₂O reaches Si bond coat layer through the mud cracks in top coat layer. The formation of cristobalite causes the large tensile stress with 2–3 GPa because

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the transformation with volume shrinkage (~ 4.5 vol%) occurs at 220°C [7,14] and they become a driving force of delamination induced by cracks in Si bond coat layer and the interface between bond coat layer and top coat layer.

A simple method for detection of delamination after heat exposure as well as as-deposited coatings is required to prevent degradation taken place by excessive heating and chemical reactions. For detection of the delamination area in EBC, determination of shape and dimensions of delamination, spectroscopic nondestructive evaluation, ultrasonic method, thermal imaging method, and X-ray computed tomography (CT) have been applied [18–20]. Spectroscopic nondestructive evaluation and X-ray CT method require complicated experimental systems. In addition, the relation between spatial resolution and inspection time of these procedures is trade-off and these procedures are not suitable for in-line inspection. Ultrasonic is used as a simple method to detect defects; however, wave decay makes the evaluation difficult. Among them, thermal imaging method seems attractive because of simple detection in ambient air condition. This method is successfully applied for delamination problem of laminated fiber reinforced ceramics [21] and impact damages of composites with EBC are evaluated by thermal imaging method [20]. However, direct application of thermography technique for detection of the internal damages and delamination in EBC system is not examined yet.

In the present study, major attention has been paid to evaluate potential of pulse thermography (hereafter denoted as PT) techniques for detection of EBC delamination from substrate with high spatial resolution condition. Thermal images and temperature profiles of EBC system measured by PT are discussed in terms of microstructures and unsteady state heat conduction.

2 Experimental procedure

2.1 Specimen preparation

This study focused on evaluating the potential application of the PT technique in the detection of delamination evolved in multi-layered ceramics with a high spatial resolution using a model EBC system composed of mullite/Si/reaction bonded SiC (RBSC), which is a typical model material. RBSC was selected as the substrate to neglect the heterogeneity of the SiC/SiC substrate, e.g., the anisotropic thermal conductivity

originating from the fiber arrangement, pore within matrix, and inter-bundles. The RBSC contained ~ 80 – 90 vol% of dispersed small α -SiC particles (diameter 100 – $800\ \mu\text{m}$) and ~ 10 – 20 vol% of free-silicon (Si). The RBSC block was cut into disk-shape pieces with a diameter of ~ 12.7 mm and a thickness of ~ 2 mm. Thereafter, one side was carefully polished up to a diamond paste finish of $0.5\ \mu\text{m}$, and then fully cleaned with acetone under ultrasonic vibration to remove the adherent. The Si bond coat layer (purity $\sim 99\%$) was deposited by using a vacuum plasma spray (VPS) process. The thickness of the Si bond coat layer was $\sim 60\ \mu\text{m}$. The mullite top coat layer was also deposited on the surface of the Si bond coat layer by implementing the VPS process, and the thickness was $\sim 240\ \mu\text{m}$. Mullite/Si/RBSC EBC system was heat-exposed in air at 1450°C for 2 h by using an electric furnace. After the heat exposure, the appearance of the specimen was carefully observed. Subsequently, the specimen was embedded into an epoxy mount. Moreover, the specimens were cut and polished through a standard metallurgical process. Finally, the surface and polished sections were observed through optical and scanning electron microscopes (SEM, TM-3000, Hitachi, Tokyo, Japan).

2.2 X-ray CT and PT

PT was used to detect the delamination in the EBC system after heat exposure at 1450°C for 2 h. Figure 1 shows a schematic of the experimental setup.

The system was composed of two pulse light sources, high-speed infrared (IR) camera (SC7000, FLIR, France) with the frame rate of 40 ms, and an image processing computer. The InSb IR camera had 640×512 pixels with a special resolution of 20 mK (at 25°C). In this study, the close-up lens (observation area $5\ \text{mm} \times 5\ \text{mm}$) was applied to the detection of small delamination (typically ~ 100 – $200\ \mu\text{m}$). The pulse source had a nominal output power of 500 W with a pulse time of $\Delta t \approx 50$ ms. The samples were heated by pulse flash, which was synchronized with the IR camera (detection wavelength range: 1.5 – $5.1\ \mu\text{m}$). To obtain accurate temperature profiles, the IR camera was calibrated by using a black body plastic sheet before the measurements. The observation and evaluation areas were approximately $6\ \text{mm} \times 5\ \text{mm}$ and $3\ \text{mm} \times 3\ \text{mm}$ on the surface. The experiments were conducted under ambient air and room temperature conditions. The temperature was measured for 10 s

Fig. 1 Experimental setup for PT measurement.

after flash heating. In the present study, the bottom of specimen (substrate) was heated and temperature profiles for the surface were recorded to apply this method as an industrial non-destructive evaluation method.

We also obtained a 3D reconstructed image of the same EBC system used for PT by X-ray CT. In the experiments, a conventional X-ray scanner (InspecXio, SMX-225CT, Shimadzu Corp., Kyoto, Japan) was used. The voltage and tube current were set to 95 kV and 40 μ A, respectively. Additionally, the field of view was set to $\sim 13 \text{ mm} \times 13 \text{ mm} \times 13 \text{ mm}$. The number of images with a resolution of 512×512 pixels was 600. Under these conditions, one pixel corresponded to $\sim 25 \text{ }\mu\text{m}$. The 2D view was reconstructed by using image processing software (myVGL, Volume Graphixs GmbH, Germany).

3 Experimental results

In the present study, only microstructural characterization for cracking behavior of EBC system is conducted. The detailed characterization of microstructural change of mullite/Si/RBSC EBC system used in the present study by heat exposure has already been reported elsewhere [13,14]. Figure 2 shows the specimen surface used in this study. The mud-cracking pattern was clearly observed at the top coat layer surface. Typically, one piece of the cracked top coat layer segment was in the range of $0.2\text{--}1.8 \text{ mm}^2$. The polished cross sections are shown in Fig. 3 after heat exposure at 1450°C for 2 h. Two types of transverse cracks were observed in the top coat layer, namely a tunnel crack and a short

Fig. 2 Typical surface microstructure of EBC system after heat exposure at 1450°C for 2 h.

through-thick crack in the top coat layer. The former crack propagated along the through-the-thickness direction in the top coat layer, and deflected at the interface between the top coat and the Si bond coat layers. After deflection, the deflected crack in the Si bond coat layer propagated parallel along the interface. The propagated length was approximately $50\text{--}150 \text{ }\mu\text{m}$ through the cracks. The crack opening displacement (COD) and u defined in Fig. 3, became smaller with the separation from the outermost surface of the top coat layer. This behavior originated from the constraint effect of the mullite top coat layer by the Si bond coat layer. During the exposure to heat, the mullite top coat layer shrank by crystallization [15,17]. Upon cooling down from the temperature reached during the heat exposure, the cracks tended to open because the coefficient of thermal expansion (CTE) of the top coat layer was higher than that of the RBSC substrate. The top coat layer was subjected to an in-plane tensile strain,

Fig. 3 (a) Transverse section of EBC system after heat exposure at 1450 °C for 2 h, (b) an enlarged view of Fig. 3(a).

and this strain caused the shrinkage of the layer. In comparison to the outermost surface, the interface between the mullite top coat layer and the Si bond coat layer was constrained because of the smaller thermal expansion coefficient of RBSC ($\sim 4.5 \times 10^{-6} \text{ K}^{-1}$ [10,11, 22]) than that of the top coat layer ($(5\text{--}6) \times 10^{-6} \text{ K}^{-1}$ [9]). As a result, the shrinkage of the layer near the interface was suppressed.

Figures 4(a) and 4(b) show the xy -plane at the top coat/bond coat interface and xz -plane, which corresponded line between A and B of the model EBC system obtained by X-ray CT. Under the test conditions applied in this study, the micro-damages, such as cracks and pores at the inter-splats, were not detectable because the dimensions of these micro-damages were smaller than the spatial resolution. However, a delamination region at the interface was clearly observed. The contrast was sufficient enough to distinguish the crack and solid (mullite). The segments were not fully bonded with a Si bond coat layer, but rather delaminated at the interface between the mullite and Si bond coat layer. The penetration of the crack into the bond coat layer was not observed. These characteristics were similar to those identified by direct microstructural observation. Additionally, invisible partial delamination was detected by X-ray CT at the edges of the segment.

Figures 5(a) and 5(b) show the typical temperature distribution of the top coat layer and we defined the

Fig. 4 Example of X-ray CT image after cracking and partial delamination: (a) xy -plane, (b) xz -plane between line A and B in (a).

Fig. 5 (a) Typical examples of temperature distribution on surface of EBC; (b) enlarged view of (a); (c) temperature difference approximately in middle of segment and around crack.

areas with temperatures of 25.5, 25.4, and 25.3 °C as a crack, higher temperature area, and lower temperature area, respectively. Figure 5(c) shows the surface

temperature difference between points A and B in Fig. 5(a). It is clear that the lower temperature area, surrounded by a dotted line, spreads on both sides of the transverse cracks. In comparison to these lower temperature areas, the temperature at the center of the segmented area was always higher than the temperature at the edges.

When maximum temperature difference is defined as $T_{\text{dif,max}}$, the temperature difference with $T_{\text{dif,max}}/2$ is maintained for ~ 100 ms after pulse irradiation. Thus, the temperature profile should have been collected within ~ 100 ms after flush heating, since the larger temperature difference achieved a larger resolution for carrying out an evaluation by the PT method. An example of the surface temperature change within the selected area (between A and B shown in Fig. 5) on the top coat layer is shown in Fig. 6.

The result clearly shows that the lower temperature area with the length of ~ 25 pixels around both sides of the crack exists, and it agrees with the thermal images of PT. At the lower temperature area, the temperature was always lower than that at the higher temperature spot. In the present experiment, the pulse light illumination was carried out from the back of the RBSC surface, and this temperature difference appeared as a result of the heat transfer difference in the through-the-thick direction between points A and B. Because the constituent materials were the same for

both points, this temperature difference was attributed to the difference of thermal conductivity or the thermal diffusion path. Microscopically, the thermal conductivity path in the RBSC exhibited tortuosity because of the dispersed SiC particles and the connected Si phase. In comparison to the size of the lower temperature area, the size of the SiC particle was sufficiently small. Moreover, it is important that we were able to treat RBSC as a homogeneous material. However, when delamination occurred, thermal resistance appeared and the apparent thermal conductivity in the through-the-thick direction became lower because delamination area is filled with air with the thermal conductivity of $0.0254 \text{ W}/(\text{m}\cdot\text{K})$ [23]. Thermal images exhibiting clear contrast could be observed in the PT study for laminates, since the temperature at the delamination region was lower than the temperature in the non-delamination area [24,25]. Consequently, it is likely that the temperature of the delaminated area became lower. However, it is difficult to determine the distribution or length of the delamination in the EBC system from the temperature profile, since it did not exist in the baseline in order to determine the high temperature and low temperature regions, i.e., the delaminated and non-delaminated areas. An example of a method of determining the baseline is discussed below.

4 Discussion

To quantitatively evaluate the delamination in the EBC, appropriate data processing is required for application to the detection by the PT. Moreover, because of the low S/N ratio, it is impossible to select a quantitative baseline in order to divide the high temperature and low temperature regions.

The temperature profile shows the temperature of the surface, which was collected by a 1×1 pixel. The resolution was $\sim 10 \mu\text{m}$, since the observation area was approximately $6 \text{ mm} \times 5 \text{ mm}$, and the InSb IR camera had 640×512 pixels. Generally, the S/N ratio decreased as the area of the averaging temperature decreased. Thus, it is likely that an optimal resolution existed. In this study, the data processing area was defined as ΔS with an $n \times n$ pixel ($n = 1, 2, 4, 6, 8, 10$). Moreover, ΔS shows the averaging temperature of the designated area. For example, if the temperature is collected by a 1×1 pixel ($n = 1$), then ΔS is $100 \mu\text{m}^2$. The resolution $f(\Delta S)$

Fig. 6 Normalized temperature distribution around mud crack.

is expressed as follows:

$$f(\Delta S) = \frac{1}{\Delta S} \left(\int_{x=0}^{\infty} T(\Delta x) dx + \int_{y=0}^{\infty} T(\Delta y) dy \right) \\ = \frac{1}{\Delta S} \iint_{x=0, y=0} T(\Delta x, \Delta y) dx dy \quad (1)$$

Equation (1) represents the relationship between the area of the average temperature and the resolution. The resolution increases as the value of ΔS decreases.

Figure 7(a) shows the normalized temperature profile depending on the area of the averaging temperature. The temperature difference between each ΔS (Fig. 7(b)) clearly decreases as the area of the averaging temperature increases. Moreover, the relatively high temperature point for all the data, which was approximately positioned in the middle of the distance between A and B, likely indicates the crack in Fig. 5. The temperature difference decreased significantly above the 4×4 pixel, which means

Fig. 7 Example of data processing: (a) relationship between normalized temperature and area of averaging temperature, and (b) normalized temperature difference between crack and low temperature point.

that the temperature resolution decreased dramatically above this point. Furthermore, the normalized temperature reached almost 0 for the 2×2 and 4×4 pixels in the vicinity of the crack and the distance between the cracks (25.5°C). Additionally, the lower temperature area (25.3°C) was approximately 10–25 pixels. Thus, if the distance between the crack and the low temperature point is selected as an example of determining the temperature baseline, then the low temperature would be spread 100–200 μm around the crack.

Figure 8 shows the distribution of the delamination crack length, which is measured by the typical cross section of the heat-exposed EBC system. The definition of the crack length λ_L and λ_R is also shown in Fig. 8. For both cracks, the crack length was approximately 50–150 μm . Therefore, the lower temperature region shown in Figs. 5 and 6 exhibits the existence of these cracks. Therefore, the optimal area of the averaging temperature likely existed within $\Delta S = 400\text{--}1600\ \mu\text{m}^2$. Moreover, it was possible to detect the delamination in the EBC system by using the PT with the optimal area of the averaging temperature. In this study, it was clearly revealed that the range of the delamination length in the EBC system could be evaluated by the PT and by choosing the area of the averaging temperature (ΔS).

During measuring of temperature profiles of EBC system by PT, heat conduction is unsteady state because temperature difference among crack, higher temperature region, and lower temperature region appears only after $\sim 100\text{ ms}$ (see Fig. 5(c)). Figure 9(a) shows models for unsteady state heat conduction. Then, mullite/Si/RBSC model and mullite/air/RBSC are simulations for non-defect region and defect region in EBC system, respectively. The detailed calculation for unsteady state heat conduction in these models is described in Appendix. Figure 9(b) exhibits temperature profile for non-defect region and defect region. For non-defect

Fig. 8 Distribution of delamination crack length.

Fig. 9 (a) Analytical EBC model for temperature distribution evolved by PT, (b) dimensionless temperature profile in EBC system as a function of dimensionless coordinate system.

region, heat conduction from RBSC to mullite occurs after heating and the temperature profiles after 100 ms from heating is similar to steady state. For defect region, air acts as barrier for heat conduction. Thus, heat conduction from RBSC to mullite is inhibited. These simulations suggest that heat in EBC system is mainly conducted in non-defect region and a heat conduction decay occurs by the existence of defect region. In addition, the temperature difference between unsteady state and steady state disappears after approximately 100 ms and this is in good agreement with the result shown in Fig. 5. Thus, the delamination in mullite/Si/RBSC system is detectable until $\sim 100\text{ ms}$ after heating by PT to determine optimal processing area of the averaging temperature (ΔS) for the analysis of temperature profile. To apply PT as an industrial inspection method for EBC system, the determination of optimal area of the averaging temperature for EBC systems is required. In other words, to determine the optimal processing area, PT method is useful for other multi-layered materials composed of the different thermal properties.

5 Conclusions

In this study, an evaluation method of delamination in multi-layered material composed of different thermal conductivity and thickness by PT was proposed. As an example, the delamination behavior of the top coat layer in the mullite/Si/RBSC EBC system was evaluated by PT and X-ray CT. To determine the moderate processing area of ΔS , the EBC system used in this study had a mullite top coat layer delamination with a size of 100–200 μm , and the delamination area of the mullite top coat layer from the RBSC substrate was detectable by the PT, because the analysis resolution for the temperature distribution, which evolved by heating, depended on ΔS . The temperature distribution in the mullite top coat layer agrees with the delamination detected by the X-ray CT method and the temperature difference between the mullite top coat layers. In addition, the temperature difference between non-defect region and defect region measured by PT is in good agreement with it simulated by unsteady state heat conduction in EBC system. For detection of determination, the moderate processing area of ΔS is important. This study focused on detection of partial interface delamination in multi-layered ceramics; however, PT method is applicable to other multi-layered systems for detection of delamination.

Appendix

To simulate heat conduction in EBC system used in the present study, the calculation of unsteady state heat conduction for multilayered materials proposed by Sun and Wichman [26] is applied to models shown in Fig. 9(a). d_1 , d_2 , and d_3 are the thickness, k_1 , k_2 , and k_3 are the thermal conductivity, and α_1 , α_2 , and α_3 are thermal diffusivity of RBSC, Si, and mullite, respectively. At time $t = 0$, the temperature at the top surface of mullite and the bottom surface of RBSC become T_0 and T_1 . Here, dimensionless temperature (θ), coordinate system (ξ), time (τ), and thermal diffusivity (δ) are defined to analyze unsteady state heat conduction as follows: $\theta = (T - T_0) / (T_1 - T_0)$, $\xi_1 = x / d_1$, $\xi_2 = (x - d_1) / d_2$, $\xi_3 = [x - (d_1 + d_2)] / d_3$, $\tau = t / t_0$, $\delta_i = \alpha_i t_0 / d_i^2$ for $i = 1, 2, 3$. t_0 is reference time and set as 10 s. Heat flux (κ_i) is defined as $\kappa_i = k_i / d_i$ for $i = 1, 2, 3$. Then, initial condition for heat conduction is written as follows [26]:

$$\frac{\partial \theta}{\partial \tau} = \delta_i \frac{\partial^2 \theta}{\partial \xi_i^2} \quad (\text{in the } i\text{th layer}) \quad (\text{A1})$$

$$\theta|_{\xi_1=0} = 1, \theta|_{\xi_1=1} = 0 \quad (\text{boundary condition}) \quad (\text{A2})$$

$$\theta|_{\xi_1 \rightarrow 1^-} = \theta|_{\xi_2 \rightarrow 0^+} = 0 \quad (\text{continuity of temperature}) \quad (\text{A3})$$

$$-\kappa_1 \frac{\partial \theta}{\partial \xi_1} \bigg|_{\xi_1 \rightarrow 1^-} = -\kappa_2 \frac{\partial \theta}{\partial \xi_2} \bigg|_{\xi_2 \rightarrow 0^+} \quad (\text{continuity of flux}) \quad (\text{A4})$$

$$\theta|_{\xi_2 \rightarrow 1^-} = \theta|_{\xi_3 \rightarrow 0^+} = 0 \quad (\text{continuity of temperature}) \quad (\text{A5})$$

$$-\kappa_2 \frac{\partial \theta}{\partial \xi_2} \bigg|_{\xi_2 \rightarrow 1^-} = -\kappa_3 \frac{\partial \theta}{\partial \xi_3} \bigg|_{\xi_3 \rightarrow 0^+} \quad (\text{continuity of flux}) \quad (\text{A6})$$

$$\theta|_{\tau=0} = 0 \quad (\text{initial condition}) \quad (\text{A7})$$

Then, θ is depending on ξ_i and τ , and described as following equations [26]:

$$\begin{aligned} \theta(\xi_i, \tau) = & [1 - (\Delta\theta)_1 \xi_1] \\ & - \sum_{n=0}^{\infty} A_n \exp(-\lambda_n^2 \delta_1 \tau) X_n^1(\xi_1) \quad (\text{layer 1}) \end{aligned}$$

$$\begin{aligned} \theta(\xi_i, \tau) = & [1 - (\Delta\theta)_1 - (\Delta\theta)_2 \xi_2] \\ & - \Delta_1 \sum_{n=0}^{\infty} A_n \exp(-\lambda_n^2 \delta_2 \tau) X_n^2(\xi_2) \quad (\text{layer 2}) \quad (\text{A8}) \end{aligned}$$

$$\begin{aligned} \theta(\xi_i, \tau) = & [1 - (\Delta\theta)_1 - (\Delta\theta)_2 - (\Delta\theta)_3 \xi_3] \\ & - \Delta_2 \sum_{n=0}^{\infty} A_n \exp(-\lambda_n^2 \delta_3 \tau) X_n^3(\xi_3) \quad (\text{layer 3}) \end{aligned}$$

Then, λ_n is eigenvalue and satisfy:

$$\tan(\lambda_n) = - \frac{\left[\Delta_1 \tan\left(\sqrt{\frac{\delta_1}{\delta_2}} \lambda_n\right) + \Delta_2 \tan\left(\sqrt{\frac{\delta_1}{\delta_3}} \lambda_n\right) \right]}{\left[1 - (\Delta_2 / \Delta_1) \tan\left(\sqrt{\frac{\delta_1}{\delta_2}} \lambda_n\right) \tan\left(\sqrt{\frac{\delta_1}{\delta_3}} \lambda_n\right) \right]} \quad (\text{A9})$$

and $X_n^i(\xi_i)$ is eigenfunction and described as

$$X_n^1(\xi_1) = \sin(\lambda_n \xi_1) \quad (\text{A10})$$

$$\begin{aligned} X_n^2(\xi_2) = & \cos(\lambda_n) \sin\left(\sqrt{\frac{\delta_1}{\delta_2}} \lambda_n \xi_2\right) \\ & + \frac{\sin(\lambda_n)}{\Delta_1} \cos\left(\sqrt{\frac{\delta_1}{\delta_2}} \lambda_n \xi_2\right) \quad (\text{A11}) \end{aligned}$$

$$X_n^3(\xi_3) = \bar{\alpha}_n \sin\left(\sqrt{\frac{\delta_1}{\delta_3}} \lambda_n \xi_3\right) + \bar{\beta}_n \cos\left(\sqrt{\frac{\delta_1}{\delta_3}} \lambda_n \xi_3\right) \quad (\text{A12})$$

where $\bar{\alpha}_n$ and $\bar{\beta}_n$ are shown as follows:

$$\bar{\alpha}_n = \cos(\lambda_n) \cos\left(\sqrt{\frac{\delta_1}{\delta_2}} \lambda_n\right) - \frac{\sin(\lambda_n) \sin\left(\sqrt{\frac{\delta_1}{\delta_2}} \lambda_n\right)}{\Delta_1} \quad (\text{A13})$$

$$\bar{\beta}_n = \frac{\Delta_1 \cos(\lambda_n) \sin\left(\sqrt{\frac{\delta_1}{\delta_2}} \lambda_n\right)}{\Delta_2} + \frac{\sin(\lambda_n) \cos\left(\sqrt{\frac{\delta_1}{\delta_2}} \lambda_n\right)}{\Delta_2} \quad (\text{A14})$$

Parameter A_n is also depending on λ_n and described below:

$$A_n = \frac{\kappa_2 \kappa_3}{\lambda_n M_n} \quad (\text{A15})$$

$$M_n = \frac{\kappa_2 \kappa_3}{2} + \frac{\kappa_1 \kappa_3}{2} \left[\frac{\cos^2(\lambda_n) + \sin^2(\lambda_n)}{H_1^2} \right]$$

Table A2 Calculation parameters for mullite/Si/RBSC model (non-defect region)

n	λ_n	$\bar{\alpha}_n$	$\bar{\beta}_n$	M_n	A_n
1	1.570	-0.03701	0.1667	3.030E+10	1.083
2	3.524	-0.8875	-0.09575	1.333E+11	0.1097
3	4.722	0.1226	-0.1637	3.199E+10	0.3410
4	7.000	0.6168	0.1601	8.099E+10	0.09084
5	7.925	-0.2551	0.1530	3.820E+10	0.1701
6	10.37	-0.3417	-0.1862	4.690E+10	0.1059
7	11.24	0.4739	-0.1194	5.824E+10	0.07867
8	13.61	0.1641	0.1929	3.520E+10	0.1075
9	14.67	-0.7162	0.0475	9.519E+10	0.03688
10	16.77	-0.0591	-0.1954	3.220E+10	0.09538
11	18.17	0.8453	0.0557	1.225E+11	0.02314
12	19.91	-0.0284	0.1961	3.188E+10	0.08114
13	21.67	-0.7788	-0.1550	1.112E+11	0.02137
14	23.06	0.1236	-0.1926	3.361E+10	0.06644
15	25.11	0.5733	0.2204	7.758E+10	0.02644
16	26.26	-0.2526	0.1774	3.928E+10	0.04993
17	28.46	-0.3397	-0.2500	5.103E+10	0.03546
18	29.55	0.4217	-0.1371	5.266E+10	0.03310
19	31.68	0.1707	0.2593	4.016E+10	0.04048
20	32.95	-0.5837	0.0576	7.217E+10	0.02166

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$$+ \frac{\kappa_1 \kappa_2}{2} (\bar{\alpha}_n^2 + \bar{\beta}_n^2) \quad (\text{A16})$$

For calculation of λ_n , A_n , and M_n , the value of n is set as 20 in accordance with similar calculation conducted by Sun and Wichman [26]. The values in Table A1 is used for these calculations and parameters such as λ_n , $\bar{\alpha}_n$, $\bar{\beta}_n$, A_n , and M_n are also summarized in Tables A2 and A3.

Table A1 Basic properties of components in mullite/Si/RBSC EBC system [10,11,22,27–29]

	Thickness (μm)	Density (g/cm^3)	Thermal conductivity ($\text{W/(m}\cdot\text{K)}$)	Heat capacity ($\text{J/(kg}\cdot\text{K)}$)
Mullite	240	3.2	6	781
Si	60	2.33	124	712
RBSC	2000	3	220	687

Table A3 Calculation parameters for mullite/air/RBSC model (defect region)

n	λ_n	$\bar{\alpha}_n$	$\bar{\beta}_n$	M_n	A_n
1	61.3	-0.03893	0.02773	1.260E+07	1.308E-02
2	122.5	-0.9998	-0.7670	1.660E+09	4.965E-05
3	183.7	-0.08307	-0.05646	2.147E+07	2.561E-03
4	245.0	0.9990	1.533	1.704E+09	2.420E-05
5	306.3	0.005283	0.1350	1.062E+07	3.106E-03
6	367.5	-0.9978	-2.297	1.775E+09	1.548E-05
7	428.6	-0.2254	0.5348	1.007E+08	2.341E-04
8	489.9	0.9823	-0.6403	1.599E+09	1.289E-05
9	551.4	-0.05047	-0.3407	1.727E+07	1.060E-03
10	612.5	-0.9939	-3.813	2.003E+09	8.234E-06
11	673.6	-0.2683	0.2460	1.294E+08	1.159E-04
12	734.8	0.9452	-2.683	1.658E+09	8.290E-06
13	796.1	0.2895	-0.05205	1.473E+08	8.613E-05
14	857.4	-0.9677	-1.596	1.608E+09	7.325E-06
15	918.6	-0.3106	-0.1745	1.687E+08	6.516E-05
16	979.9	0.9619	2.324	1.664E+09	6.195E-06
17	1041	0.4241	-1.053	3.331E+08	2.913E-05
18	1102	-0.7654	8.966	3.051E+09	3.004E-06
19	1164	0.03990	0.5470	2.046E+07	4.242E-04
20	1225	0.9756	7.485	3.018E+09	2.732E-06

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