

Vague ranking of fuzzy numbers

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Abstract In a lot of scientific models in the real world, we confront with comparing fuzzy numbers as decision-making procedures and etc. It will be interest, if we know that, comparison discuss is sometimes ambiguous. Hence, this article focus on ranking fuzzy numbers with protection ambiguity. Our idea for this work is based on this claim that ranking of two fuzzy numbers should be a vague value. However, we utilize the notion of max and min fuzzy simultaneously.

Keywords Fuzzy numbers · Ranking · Vague value

Introduction

In variety of application domains, such as decision making [26], risk assessment [13], linear programming [20], linear systems [12], and artificial intelligence [6], ranking fuzzy numbers are used. This topic has been studied by many researchers. Some researchers employed a distance for ordering of fuzzy numbers such as Abbasbandy and Asady [1], Yao and Wu [25], Allahviranloo and Adabitarbar Firozja [4], Deng [21] and Janizade-Haji et al. [14]. Some researchers as [2, 15, 16] presented a defuzzification method for ranking fuzzy

numbers. Vincent and Luu in [22] proposed improve their ranking method for fuzzy numbers with integral values. In [7], Deng by using ideal solutions showed a ranking approach. Fortemps and Roubens [11] introduced a ranking method based on area compensation. Some of the other researchers such as Adabitarbar firozja et al. [3], Ezzati et al. [9, 10] and Modarres and Sadi-Nezhad [18] proposed a function for ranking. Wang et al. [23] defined the maximal and minimal reference sets and then proposed the ranking method based on deviation degree and relative variation of fuzzy numbers and subsequent Asady in [5] proposed a revised method of ranking LR fuzzy number based on deviation degree with Wang's method. Wang and Luo in [24] presented a ranking approach with positive and negative ideal points. Mahmodi Nejad and Mashinchi [17], introduced ranking fuzzy numbers based on the areas on the left and right sides of fuzzy number. In this paper, we provide a method for calculating the amount of vague value ranking fuzzy numbers.

The paper is organized as follows: The background on fuzzy concepts is presented in Sect. 2. A vague ranking of two fuzzy numbers with its properties is introduced in Sect. 3. Subsequently, in Sect. 4 some examples are presented. Finally, conclusion are drawn in Sect. 5.

Background

There are several definitions of a fuzzy number. In this paper we use the following definition.

Definition 1 [8] A set \tilde{A} is a generalized left right fuzzy numbers (GLRFN) and denoted as $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$, if its membership function satisfies the following:

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$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\frac{a_2 - x}{a_2 - a_1}\right), & a_1 \leq x \leq a_2, \\ 1, & a_2 \leq x \leq a_3, \\ R\left(\frac{x - a_3}{a_4 - a_3}\right), & a_3 \leq x \leq a_4, \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

where L and R are strictly decreasing functions defined on $[0, 1]$ and satisfying the conditions:

$$\begin{aligned} L(t) = R(t) = 1 & \quad \text{if } t \leq 0 \\ L(t) = R(t) = 0 & \quad \text{if } t \geq 1 \end{aligned} \quad (2)$$

Remark 1 Trapezoidal fuzzy numbers (TrFN) are special cases of GLRFN with $L(t) = R(t) = 1 - t$ and we show it as $\tilde{A} = (a_1, a_2, a_3, a_4)$.

Definition 2 A α -level interval of fuzzy number \tilde{A} is denoted as:

$$[\tilde{A}]^\alpha = [A_l(\alpha), A_r(\alpha)] = [a_2 - (a_2 - a_1)L_A^{-1}(\alpha), a_3 + (a_4 - a_3)R_A^{-1}(\alpha)] \quad (3)$$

Remark 2 Suppose, $\lambda \in R$ then

$$\begin{aligned} \tilde{A} + \lambda &= (a_1 + \lambda, a_2 + \lambda, a_3 + \lambda, a_4 + \lambda)_{LR} \\ \lambda \tilde{A} &= \begin{cases} (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4)_{LR} & \lambda \geq 0 \\ (\lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1)_{LR} & \lambda < 0 \end{cases} \end{aligned} \quad (4)$$

Definition 3 [3] Let $[A]^\alpha = [A_l(\alpha), A_r(\alpha)]$ and $[B]^\alpha = [B_l(\alpha), B_r(\alpha)]$; $\alpha \in [0, 1]$ be two α -cuts of fuzzy numbers. We get

$$[\max\{A, B\}]^\alpha = \max\{[A]^\alpha, [B]^\alpha\} = [\max\{A_l(\alpha), B_l(\alpha)\}, \max\{A_r(\alpha), B_r(\alpha)\}]$$

and

$$[\min\{A, B\}]^\alpha = \min\{[A]^\alpha, [B]^\alpha\} = [\min\{A_l(\alpha), B_l(\alpha)\}, \min\{A_r(\alpha), B_r(\alpha)\}].$$

This definition is showed in Fig. 1.

Definition 4 [3] Let $U = \{u_1, u_2, u_3, \dots, u_n\}$, a vague set A in U is characterized by a truth-membership function $t_A : U \rightarrow [0, 1]$ and a false-membership function $f_A : U \rightarrow [0, 1]$, where $t_A(u_i)$ is a lower bound on the grade

of membership of u_i derived from the evidence for u_i , $f_A(u_i)$ is a lower bound on the negation of u_i derived from the evidence against u_i , and $t_A(u_i) + f_A(u_i) \leq 1$. The grade of membership of u_i in the vague set A is vague value where bounded by a subinterval $[t_A(u_i), 1 - f_A(u_i)]$ of $[0, 1]$. Simply expressed, $A(u_i) = [t_A(u_i), 1 - f_A(u_i)]$.

For an arbitrary element $a \in [0, 1]$, we assume that a is the same as $[a, a]$, namely, $a = [a, a]$. For any $A = [a_1, a_2]$ and $B = [b_1, b_2]$, we can popularize operators such $+$ and $-$ and have $A + B = [a_1 + b_1, a_2 + b_2]$, $A - B = [a_1 - b_2, a_2 - b_1]$. Furthermore, we have $A = B \Leftrightarrow a_1 = b_1, a_2 = b_2$, $A \leq B \Leftrightarrow a_1 \leq b_1, a_2 \leq b_2$ and $A < B \Leftrightarrow a_1 < b_1, a_2 < b_2$.

Vague ranking of two fuzzy numbers

Given $\tilde{A}, \tilde{B} \in E_{LR}$, are two fuzzy numbers. Regarding to many methods and shortcoming in ranking for fuzzy numbers, it is show that ranking is not deterministic. In other words, we know if $\text{supp}(\tilde{A}) \cap \text{supp}(\tilde{B}) \neq \phi$ then we can not define a crisp rank for \tilde{A}, \tilde{B} . Therefore, we claim that ranking of two fuzzy numbers should be a vague value. Some researcher are used the max or min notion for ranking the fuzzy number. But, we utilize the notion of max and min simultaneously. It is trivial maximum of two fuzzy numbers is greater than or equal both of them and minimum of two fuzzy numbers is less than or equal both of them. Therefore, we will present true rate $\tilde{A} \leq \tilde{B}$ as $t_{A \leq B}$ and false rate $\tilde{A} \leq \tilde{B}$ as $f_{A \leq B}$ in ranking \tilde{A} and \tilde{B} as follows:

$$\begin{aligned} t_{A \leq B} &= S\{\{\max\{\tilde{A}, \tilde{B}\} \setminus \tilde{A}\} \cup \{\min\{\tilde{A}, \tilde{B}\} \setminus \tilde{B}\}\} \\ f_{A \leq B} &= S\{\{\max\{\tilde{A}, \tilde{B}\} \setminus \tilde{B}\} \cup \{\min\{\tilde{A}, \tilde{B}\} \setminus \tilde{A}\}\} \end{aligned} \quad (5)$$

where the signs \setminus and $S\{\cdot\}$ show subtract of Venn diagram and area, respectively. For this purpose, consider the following Fig. 2.

Where geometrically, $t_{A \leq B}$ and $f_{A \leq B}$ defined above is as follows:

$$\begin{aligned} t_{A \leq B} &= \frac{S(A < B)}{S}, & f_{A \leq B} &= \frac{S(B < A)}{S}, & S &= S(A < B) \\ & & & & & + S(B < A) + S(A = B) \end{aligned} \quad (6)$$

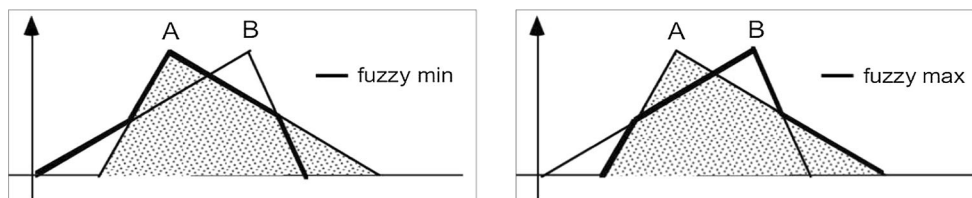


Fig. 1 Fuzzy max and fuzzy min of triangular fuzzy numbers

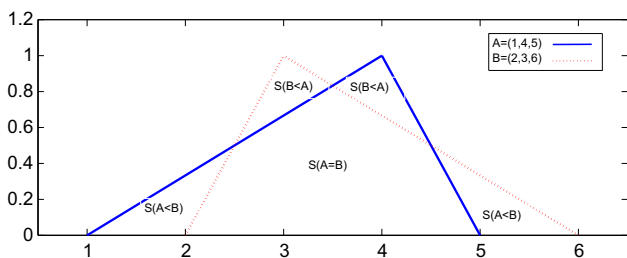


Fig. 2 Comparison of two fuzzy numbers \tilde{A}, \tilde{B}

$S(A < B)$ is part of surface B that is located on the right side of A or part of surface A that is located on the left side of B and $S(A = B)$ is common surface of A and B .

Remark 3 For $\tilde{A}, \tilde{B} \in E_{LR}$, $t_{A \preceq B} + f_{A \preceq B} \leq 1$.

Proof

$$t_{A \preceq B} + f_{A \preceq B} = \frac{S(A < B)}{S} + \frac{S(B < A)}{S} = \frac{S(A < B) + S(B < A)}{S(A < B) + S(B < A) + S(A = B)} \leq 1$$

As mentioned in above we used the notion area for ranking. Hence, we introduce two theorems in below. \square

Theorem 1 [19] If points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be arbitrarily coordinates are triangular vertexes in anti-clock wise sense then the area of triangle $\triangle ABC$ is determined as follows:

$$S_{\triangle ABC} = \frac{1}{2} \left\{ \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + \begin{vmatrix} x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} \right\} \quad (7)$$

Theorem 2 [19] The area of any regular polygon with $P_j(x_j, y_j)$, $j = 1, \dots, n$ vertex in anti-clock wise sense is as follows:

$$S_{P_1 P_2 \dots P_n} = \frac{1}{2} \left\{ \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + \dots + \begin{vmatrix} x_n & y_n \\ x_1 & y_1 \end{vmatrix} \right\} \quad (8)$$

Definition 5 Assume that \tilde{A} and \tilde{B} be two GLRFNs, validity rating of $A \preceq B$ is belong to interval $[t_{A \preceq B}, 1 - f_{A \preceq B}]$ where $t_{A \preceq B}$ minimum accuracy and $1 - f_{A \preceq B}$ maximum accuracy and we show with vague value of rank $\tilde{A} \preceq \tilde{B}$ with $VR(A \preceq B)$ where

$$VR(A \preceq B) = [t_{A \preceq B}, 1 - f_{A \preceq B}] \quad (9)$$

Some properties

For \tilde{A} and $\tilde{B} \in E_{LR}$ and $\lambda \in R$:

Proposition 1 $VR(A \preceq B)$ is a vague value.

Proof With Remark 1. proof is evident.

Proposition 2 $t_{A \preceq B} = f_{B \preceq A}$, $f_{A \preceq B} = t_{B \preceq A}$.

Proof With Eq. (6) proof is evident.

Proposition 3 $VR(A \preceq B) = 1 - VR(B \preceq A)$

Proof Regarding to Eq. (9) and Proposition 2. $1 - VR(B \preceq A) = [1, 1] - [t_{B \preceq A}, 1 - f_{B \preceq A}] = [f_{B \preceq A}, 1 - t_{B \preceq A}] = [t_{A \preceq B}, 1 - f_{A \preceq B}] = VR(A \preceq B)$

Proposition 4 $VR(\lambda A \preceq \lambda B) = \begin{cases} VR(A \preceq B) & 0 \leq \lambda, \\ VR(B \preceq A) & \text{otherwise.} \end{cases}$

Proof Regarding to the Eqs. (9) and (6); if $\lambda \geq 0$ $VR(\lambda A \preceq \lambda B) = [t_{\lambda A \preceq \lambda B}, 1 - f_{\lambda A \preceq \lambda B}] = [t_{A \preceq B}, 1 - f_{A \preceq B}] = VR(A \preceq B)$ And if $\lambda < 0$

$$VR(\lambda A \preceq \lambda B) = [t_{\lambda A \preceq \lambda B}, 1 - f_{\lambda A \preceq \lambda B}] = [t_{B \preceq A}, 1 - f_{B \preceq A}] = VR(B \preceq A)$$

Proposition 5 $VR(\lambda + A \preceq \lambda + B) = VR(A \preceq B)$.

Proof Regarding Eqs. (9) and (6)

$$VR(\lambda + A \preceq \lambda + B) = [t_{\lambda + A \preceq \lambda + B}, 1 - f_{\lambda + A \preceq \lambda + B}] = [t_{A \preceq B}, 1 - f_{A \preceq B}] = VR(A \preceq B).$$

Proposition 6 If $a_4 \leq b_1$ then $VR(\tilde{A} \preceq \tilde{B}) = [1, 1]$.

Proof Regarding to Eqs. (9) and (6) proof is evident.

Proposition 7 If \tilde{A} and \tilde{B} are two GLRFNs then only one of the following relationship is established:

$VR(A \preceq B) = VR(B \preceq A)$, $VR(A \preceq B) < VR(B \preceq A)$ and $VR(A \preceq B) > VR(B \preceq A)$.

Proof If $t_{A \preceq B} = t_{B \preceq A}$ then with Proposition 2, $f_{A \preceq B} = f_{B \preceq A}$ therefore $VR(A \preceq B) = VR(B \preceq A)$. If $t_{A \preceq B} < t_{B \preceq A}$ then with Proposition 2, $f_{A \preceq B} < f_{B \preceq A}$ therefore $VR(A \preceq B) < VR(B \preceq A)$. If $t_{A \preceq B} > t_{B \preceq A}$ then with Proposition 2, $f_{A \preceq B} > f_{B \preceq A}$ therefore $VR(A \preceq B) > VR(B \preceq A)$.

Definition 6 If \tilde{A} and \tilde{B} are two GLRFNs, validity rating of $A \preceq B$ is belong to interval $VR(A \preceq B) = [t_{A \preceq B}, 1 - f_{A \preceq B}]$ where $t_{A \preceq B}$ minimum accuracy and $1 - f_{A \preceq B}$ maximum accuracy. With Proposition 9, we define ranking method as follows:

1. If $VR(A \preceq B) = VR(B \preceq A) = [0.5, 0.5]$, then can be said $\tilde{A} \approx \tilde{B}$.
2. If $VR(A \preceq B) = VR(B \preceq A) = [0, 1]$, then can be said $\tilde{A} = \tilde{B}$.
3. If $VR(A \preceq B) < VR(B \preceq A)$ then can be said $\tilde{B} \preceq \tilde{A}$.
4. If $VR(A \preceq B) = [1, 1]$ or $VR(B \preceq A) = [0, 0]$ then can be said $\tilde{A} < \tilde{B}$.

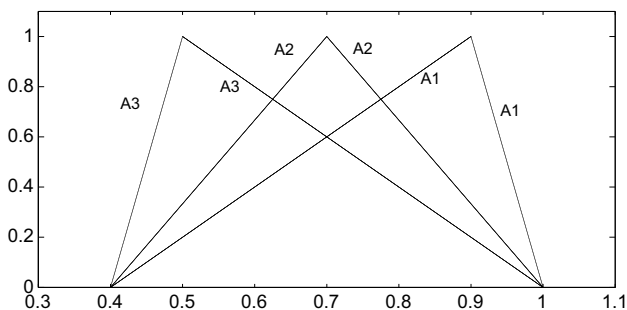


Fig. 3 Set 1- $\{\tilde{A}_1, \tilde{A}_2, \tilde{A}_3\}$

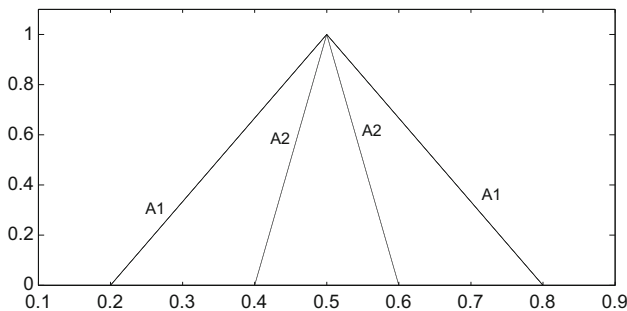


Fig. 4 Set 2- $\{\tilde{A}_1, \tilde{A}_2\}$

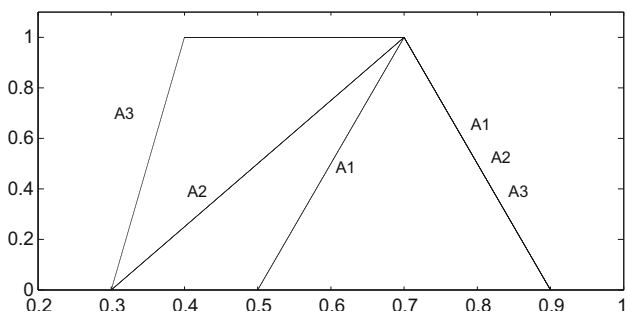


Fig. 5 Set 3- $\{\tilde{A}_1, \tilde{A}_2, \tilde{A}_3\}$

Numerical examples

For description the proposed method some examples constructed as follow [4, 21].

Set 1 $A_1 = (0.4, 0.9, 1)$, $A_2 = (0.4, 0.7, 1)$, $A_3 = (0.4, 0.5, 1)$ where show in Fig. 3.

$VR(A_2 \leq A_1) = [0.4, 1]$, with Proposition 3 $VR(A_1 \leq A_2) = [0, 0.6]$ therefore according to Definition 2, $A_1 \succeq A_2$.

$VR(A_3 \leq A_1) = [0.82, 1]$ and obviously $VR(A_1 \leq A_3) = [0, 0.18]$ therefore, $A_1 \succeq A_3$.

$VR(A_3 \leq A_2) = [0.4, 1]$ and it is trivial that $VR(A_2 \leq A_3) = [0, 0.6]$ therefore, $A_2 \succeq A_3$.

Set 2 $A_1 = (0.2, 0.5, 0.8)$, $A_2 = (0.4, 0.5, 0.6)$, where show in Fig. 4.

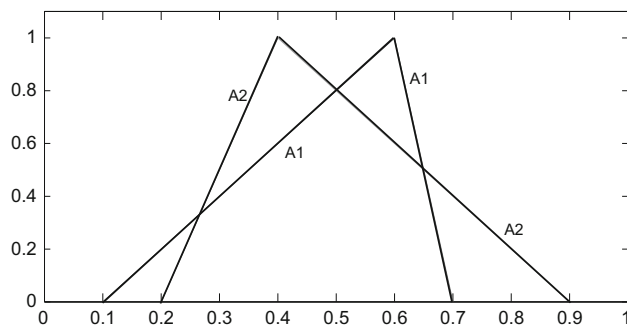


Fig. 6 Set 4- $\{\tilde{A}_1, \tilde{A}_2\}$

$VR(A_1 \leq A_2) = [0.5, 0.5]$ therefore $VR(A_2 \leq A_1) = [0.5, 0.5]$ hence $A_1 \approx A_2$.

Set 3 $A_1 = (0.5, 0.7, 0.9)$, $A_2 = (0.3, 0.7, 0.9)$, $A_3 = (0.3, 0.4, 0.7, 0.9)$, where show in Fig. 5.

$VR(A_3 \leq A_2) = [0.333, 1]$, $VR(A_3 \leq A_1) = [0.56, 1]$ and $VR(A_2 \leq A_1) = [0.333, 1]$ as before, it follows that, $A_1 \succeq A_2$, $A_2 \succeq A_3$ and $A_1 \succeq A_3$.

And we consider another example for comparing the current method with

Set 4 $A_1 = (0.1, 0.6, 0.7)$, $A_2 = (0.2, 0.4, 0.9)$ where show in Fig. 6.

$VR(A_2 \leq A_1) = [0.1931417, 0.83182047]$, that shows $A_2 \succeq A_1$.

Conclusions

In this paper, we showed that ranking of two fuzzy numbers should be a vague value. For this reason, we utilize the notion of max and min simultaneously in order to determining the ambiguity rate in ranking of two fuzzy numbers. It is shown that this approach verifies some properties as stability, transition and complement.

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