

# Lower semicontinuity of solutions for order-perturbed parametric vector equilibrium problems

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**Abstract** The lower semicontinuity of the (weak) efficient solution mappings for parametric vector equilibrium problems under more weaker assumptions is established. Some examples are developed to illustrate our results are real generalization different from recent ones in the literature and to describe the essential conditions of the latest results in the references are not real essential.

**Keywords** Order-perturbed parametric vector equilibrium problems · Lower semicontinuity · Cone lower semicontinuity · Efficient solutions

**Mathematics Subject Classification** 49K40 · 90C31

## Introduction

Several classes of problems, including the vector variational inequality problem, the vector complementarity problem, the vector optimization problem and the vector saddle point problem, have been unified as a model of the vector equilibrium problem, which has been intensively studied in the literature (see [1–16]). One of the important topics in optimization theory is the stability analysis of the solution mappings for vector equilibrium problems. Stability may be understood as some types of lower or upper semicontinuity. Recently, the semicontinuity, especially the lower semicontinuity, of the solution mappings for parametric vector equilibrium problems has been intensively studied in various directions (see [1, 2, 4, 6, 10, 15] and references therein).

Anh and Khanh first obtained the semicontinuity of the solution mappings of parametric multivalued vector quasi-equilibrium problems (see [1]), and then obtained verifiable sufficient conditions for solution sets of general quasi-variational inclusion problems to have these semicontinuity-related properties and discussed in detail a traffic network problem as a sample for employing the main results in practical situations (see [2]), and latter established sufficient conditions for lower and Hausdorff lower semicontinuity, upper semicontinuity, and continuity of solution mappings of parametric quasi-equilibrium problems in topological vector spaces (see [3]).

Gong and Yao, by virtue of a density result and a scalarization technique, first discussed the lower semicontinuity of the set of efficient solutions to parametric vector equilibrium problems with monotone bifunctions (see [4]), and studied the continuity of the solution mapping to parametric weak vector equilibrium problems (see [5]), recently, established the lower semicontinuity of solutions to the parametric

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generalized strong vector equilibrium problem without the assumptions of monotonicity of the objective mapping and compactness of the constraint mapping (see [6]). Huang et al. used local existence results to establish the lower semicontinuity of solution mappings for parametric implicit vector equilibrium problems (see [7]). By using a new proof which is different from the ones of [4, 5], Chen et al. established the lower semicontinuity and continuity of the solution mappings to a parametric generalized vector equilibrium problem (see [9]). Li and Fang investigated the lower semicontinuity of the solutions mapping to parametric generalized Ky Fan inequality under a weaker assumption than  $C$ -strict monotonicity (see [10]). Recently, by using an idea of [4], Li et al. established the continuity of solution mappings to a parametric generalized strong vector equilibrium problem for set-valued mappings under an assumption which is different from the  $C$ -strict monotonicity (see [11]). Kimura and Yao discussed the semicontinuity of solution mappings of parametric vector quasi-equilibrium problems (see [13]). Cheng and Zhu obtained a lower semicontinuity result of the solution mapping to weak vector variational inequalities in finite-dimensional spaces by using the scalarization method (see [14]).

Zhang et al. obtained the lower semicontinuity of solution mappings for parametric vector equilibrium problems under the Höder-related assumptions [15]. Wangkeeree et al. extended the results in [15] to the case of set-valued mappings on parametric strong vector equilibrium problems (see [16]).

However, all these results are with respect to the fixed order relationship in the object space, that is, the cone partial order is not perturbed by the parameters. Motivated by the idea of variational domination structure, the main results in [15] will be obtained under more weaker conditions in this paper.

The organization of this work is as follows. In Sect. 2, we introduce the efficient solutions to parametric vector equilibrium problems with the cone partial order being perturbed by the parameters and recall some basic notions. In Sect. 3, we discuss the lower semicontinuity of the (weak) efficient solution mappings for parametric vector equilibrium problems in the case of weakly conditions. Some examples are given to illustrate that the assumptions of the main results in our work or in [15] are only sufficient for some special problems and indicate also that our outcomes are real extension from the corresponding ones in [15].

## Preliminaries

Let  $X$  and  $Z$  be two metric spaces, and let  $Y$  be a metric vector space and  $C$  be a pointed closed convex cone in  $Y$  with nonempty interior  $\text{int}C$ ; the zero element in  $Y$  is denoted by  $\theta$ . Let  $A$  be a nonempty subset of  $X$  and  $F$  be a

vector-valued mapping from  $A \times A$  into  $Y$ . A vector equilibrium problem, in short (VEP), is described as:

Find  $x \in A$  such that  $F(x, y) \notin -C \setminus \{\theta\}$ , for all  $y \in A$

A point  $x \in A$  is said to be an efficient solution to (VEP) if

$F(x, y) \notin -C \setminus \{\theta\}$ , for all  $y \in A$

When the subset  $A$  of  $X$  and the function  $F$  are perturbed by the parameter  $\lambda \in \Lambda$ , where  $\Lambda \subset Z$ , a parametric vector equilibrium problem, in short (PVEP), is a problem as following:

Find  $x \in A(\lambda)$  such that  $F(x, y, \lambda) \notin -C \setminus \{\theta\}$ , for all  $y \in A(\lambda)$

where  $A : \Lambda \rightarrow 2^X \setminus \{\emptyset\}$  is a set-valued mapping, and  $F : B \times B \times \Lambda \subset X \times X \times Z \rightarrow Y$  is a vector-valued mapping with  $A(\Lambda) = \cup_{\lambda \in \Lambda} A(\lambda) \subset B$ .

A point  $x \in A(\lambda)$  is said to be an efficient solution to (PVEP) if

$F(x, y, \lambda) \notin -C \setminus \{\theta\}$ , for all  $y \in A(\lambda)$

In this work, we consider (PVEP) with cone  $C$  being also perturbed by parameter  $\lambda \in \Lambda$ , described as follows:

Find  $x \in A(\lambda)$  such that  $F(x, y, \lambda) \notin -C(\lambda) \setminus \{\theta\}$ , for all  $y \in A(\lambda)$

where, for each  $\lambda \in \Lambda$ ,  $C(\lambda)$  is a pointed closed convex cone in  $Y$  with nonempty interior, that is to say,  $C : \Lambda \rightarrow 2^Y$  is a cone-valued mapping. In this case, we call the (PVEP) as parametric vector order-perturbed equilibrium problem, in short (PVOPEP).

A point  $x \in A(\lambda)$  is called an efficient solution to (PVOPEP) if

$F(x, y, \lambda) \notin -C(\lambda) \setminus \{\theta\}$ , for all  $y \in A(\lambda)$

The set of efficient solutions to (PVOPEP) is denoted by  $S(\lambda)$ , i.e.,

$S(\lambda) := \{x \in A(\lambda) | F(x, y, \lambda) \notin -C(\lambda) \setminus \{\theta\}, \forall y \in A(\lambda)\}$

It is easy to see that  $S$  is a set-valued mapping  $S : \Lambda \rightarrow 2^X$ . Throughout this work, we always assume that  $S(\lambda) \neq \emptyset$ ; for all  $\lambda \in \Lambda$ . Next, we recall some basic definitions and their properties which will be needed in the following.

$B_X(\lambda, \delta)$  denotes the open ball with center  $\lambda$  and radius  $\delta > 0$  in a metric space  $X$ ,  $d_X(\cdot, \cdot)$  denotes the distance in  $X$ , and the distance from  $x$  to the set  $A \subset X$  is denoted by  $d_X(x, A)$ .

**Definition 2.1** ([17]) A set-valued mapping  $S : \Lambda \rightarrow 2^X$  is said to be

1. lower semicontinuous (l.s.c.) at  $\lambda_0 \in \Lambda$  if for any open set  $V$  satisfying  $V \cap S(\lambda_0) \neq \emptyset$ , there exists  $\delta > 0$  such that for every  $\lambda \in B_\Lambda(\lambda_0, \delta)$ ,  $V \cap S(\lambda) \neq \emptyset$ ;



2. upper semicontinuous (*u.s.c.*) at  $\lambda_0 \in A$  if for any open set  $V$  satisfying  $S(\lambda_0) \subset V$ , there exists  $\delta > 0$  such that for every  $\lambda \in B_A(\lambda_0, \delta)$ ,  $S(\lambda) \subset V$ ;
3. *l.s.c.* (resp., *u.s.c.*) on  $A$  if it is *l.s.c.* (resp., *u.s.c.*) at each  $\lambda \in A$ ;
4. continuous on  $A$  if it is both *l.s.c.* and *u.s.c.* on  $A$ .

**Proposition 2.1** ([18, 19])

1.  $S:A \rightarrow 2^X$  is *l.s.c.* at  $\lambda_0 \in A$  if and only if for any sequence  $\{\lambda_n\} \subset A$  with  $\lambda_n \rightarrow \lambda_0$  and any  $x_0 \in S(\lambda_0)$ , there exists  $x_n \in S(\lambda_n)$  such that  $x_n \rightarrow x_0$ .
2. If  $S$  has compact values (i.e.,  $S(\lambda)$  is a compact set for each  $\lambda \in A$ ), then  $S$  is *u.s.c.* at  $\lambda_0 \in A$  if and only if for any sequence  $\{\lambda_n\} \subset A$  with  $\lambda_n \rightarrow \lambda_0$  and for any  $x_n \in S(\lambda_n)$ , there exist  $x_0 \in S(\lambda_0)$  and a subsequence  $\{x_{n_k}\}$  of  $\{x_n\}$  such that  $x_{n_k} \rightarrow x_0$ .

**Definition 2.2** A vector-valued mapping  $f: X \rightarrow Y$  is called cone lower semicontinuity (*c.l.s.c.*) at  $x_0 \in X$  if for each open set  $V$  of  $f(x_0)$ , there exists a neighborhood  $U$  of  $x_0$ , such that  $f(x) \subset V + C$ , for all  $x \in U$ , where  $C \subset Y$  is a cone.

**Definition 2.3** Let  $A$  be a topology space,  $Y$  be a topology vector space, and  $C:A \rightarrow 2^Y$  be a cone-valued mapping, if for every  $\lambda \in A$ ,  $C(\lambda)$  is a closed convex pointed cone in  $Y$ . The closed unit ball with center  $\theta$  in  $Y$  is denoted by  $B_Y(\theta)$ . We call the cone-valued mapping  $C$  is an upper semicontinuous cone-valued mapping, if for each  $\lambda \in A$  and each open set  $U$  in  $Y$  with  $U \supset C(\lambda) \cap B_Y(\theta)$ , there exists an open set  $V$  of  $\lambda$  such that  $U \supset C(\lambda') \cap B_Y(\theta)$  for every  $\lambda' \in V$ .

**The main results**

In this section, we present the lower semicontinuity of the solution mapping to (*PVOPEP*).

**Theorem 3.1.** Suppose that the following conditions are satisfied:

- ①  $A(\cdot)$  is continuous with compact values on  $A$ .
- ②  $F(\cdot, \cdot, \cdot)$  is *c.l.s.c.* on  $B \times B \times A$ .
- ③  $C(\cdot)$  is an upper semicontinuous cone-valued mapping on  $A$ .
- ④ If  $A(\lambda) \setminus S(\lambda) \neq \phi$  for each  $\lambda \in A$ , then for each  $\lambda \in A$ , for each  $x \in A(\lambda) \setminus S(\lambda)$ , there exist  $y \in S(\lambda)$  and a positive function  $M:A \rightarrow (0, +\infty)$  which is upper semicontinuous on  $A$ , such that

$$d_X(x, y) \leq M(\lambda) \cdot d_Y(F(x, y, \lambda), Y \setminus -\text{int}C(\lambda))$$

Then,  $S(\cdot)$  is *l.s.c.* on  $A$ .

*Proof* Suppose to the contrary that there exists  $\lambda_0 \in A$  such that  $S(\cdot)$  is not *l.s.c.* at  $\lambda_0$ . Then, there exists a sequence  $\{\lambda_n\} \subset A$  with  $\lambda_n \rightarrow \lambda_0$  and  $x_0 \in S(\lambda_0)$  with some open set  $V_1$  of  $x_0$ , such that for any  $x'_n \in S(\lambda_n)$ ,  $x'_n \notin V_1$ .

From  $x_0 \in S(\lambda_0)$ , we have  $x_0 \in A(\lambda_0)$  and

$$F(x_0, y, \lambda_0) \notin -C(\lambda_0) \setminus \{\theta\}, \quad \forall y \in A(\lambda_0) \tag{1}$$

Since  $A(\cdot)$  is *l.s.c.* at  $\lambda_0$ , there exists a sequence  $\{x_n\} \subset A(\lambda_n)$  such that  $x_n \in V_1$ . Then, for the above open set  $V_1$ , there exists a positive integer  $N$ ; such that  $x_n \in V_1$ , for all  $n \geq N$ . Obviously, we have  $x_n \in A(\lambda_n) \setminus S(\lambda_n)$  for all  $n \geq N$ . For the sake of convenience, we consider  $n$  as still from one to infinity. By ④, for each  $x_n \in A(\lambda_n) \setminus S(\lambda_n)$ , there exist  $y_n \in S(\lambda_n)$  and a positive function  $M:A \rightarrow (0, +\infty)$  which is upper semicontinuous on  $A$ , such that

$$d_X(x_n, y_n) \leq M(\lambda_n) \cdot d_Y(F(x_n, y_n, \lambda_n), Y \setminus -\text{int}C(\lambda_n)) \tag{2}$$

Since  $y_n \in A(\lambda_n)$ , it follows from the upper semicontinuity and compactness of  $A(\cdot)$  at  $\lambda_0$  that there exist  $y_0 \in A(\lambda_0)$  and a subsequence  $\{y_{n_i}\}$  of  $\{y_n\}$  such that  $y_{n_i} \rightarrow y_0$ . In particular, from (2), we have

$$d_X(x_{n_i}, y_{n_i}) \leq M(\lambda_{n_i}) \cdot d_Y(F(x_{n_i}, y_{n_i}, \lambda_{n_i}), Y \setminus -\text{int}C(\lambda_{n_i})) \tag{3}$$

Since the distance function  $d(\cdot, \cdot)$  is continuous,  $F(\cdot, \cdot, \cdot)$  is *c.l.s.c.*,  $M(\cdot)$  is *u.s.c.*, and  $C(\cdot)$  is an upper semicontinuous cone-valued mapping, then let  $i \rightarrow +\infty$  on both sides of (3), we have

$$d_X(x_0, y_0) \leq M(\lambda_0) \cdot d_Y(F(x_0, y_0, \lambda_0), Y \setminus -\text{int}C(\lambda_0)) \tag{4}$$

If  $x_0 \neq y_0$ , by (4), we can obtain  $M(\lambda_0) \cdot d_Y(F(x_0, y_0, \lambda_0), Y \setminus -\text{int}C(\lambda_0)) \geq d_X(x_0, y_0) > 0$

From  $M(\lambda_0) > 0$ , we have

$$d_Y(F(x_0, y_0, \lambda_0), Y \setminus -\text{int}C(\lambda_0)) > 0$$

Thus, we have

$$F(x_0, y_0, \lambda_0) \in -\text{int}C(\lambda_0)$$

which contradicts (1) as we see by taking  $y = y_0$ . Therefore  $x_0 = y_0$ . This is impossible by the contradiction assumption. Thus, the proof is complete.  $\square$

*Remark 3.1* The following examples indicate that the assumption ④ in Theorem 3.1 in our work (or the assumption (iii) of Theorem 3.1 in [15]) cannot be applied to the case when  $A(\lambda) \setminus S(\lambda) = \phi$ , for some  $\lambda \in A$ . So the assumption ④ of Theorem 3.1 in our paper (or the assumption (iii) of Theorem 3.1 in [15]) is not essential.

*Example 3.1* Let  $X = Z = R, A_1 = [\frac{1}{4}, \frac{2}{3}]$ ,  $A_2 = [2, 3]$ ,  $A(\lambda) = B = [0, 1]$ ,  $Y = R^2$ ,  $C(\lambda) = R^2_+$  and

$F(x, y, \lambda) = (\lambda(y - x), \lambda(y - x))$ . It follows from a direct computation that

$$S_1(\lambda) = \{0\}, \forall \lambda \in A_1 \text{ and } S_2(\lambda) = \{0\}, \forall \lambda \in A_2$$

For each  $\lambda \in A_1 \cup A_2$ , for every  $x \in A(\lambda) \setminus S(\lambda) = (0, 1]$ , taking  $y = 0 \in S(\lambda)$ ; we have

$$d_X(x, y) = x \text{ and } d_Y(F(x, y, \lambda), Y \setminus -\text{int}C(\lambda)) = \lambda x$$

Obviously,

$$d_X(x, y) \geq d_Y(F(x, y, \lambda), Y \setminus -\text{int}C(\lambda)), \forall \lambda \in A_1$$

$$d_X(x, y) \leq d_Y(F(x, y, \lambda), Y \setminus -\text{int}C(\lambda)), \forall \lambda \in A_2$$

From above two inequalities, it easy to see that the assumption (iii) in Theorem 3.1. in [15] is violated when  $\lambda \in A_1$ . However,  $S_1(\cdot)$  and  $S_2(\cdot)$  are all continuous on  $A_1$  and  $A_2$ , respectively. If we take  $M: A_1 \rightarrow (0, +\infty)$ , defined by  $M(\lambda) = 4, \forall \lambda \in A_1 = [\frac{1}{4}, \frac{2}{3}]$ , then

$$d_X(x, y) \leq M(\lambda) \cdot d_Y(F(x, y, \lambda), Y \setminus -\text{int}C(\lambda)), \forall \lambda \in A_1$$

Thus, the example 3.1 satisfies all the assumptions of Theorem 3.1. in our work, and then, it is obtained that  $S(\cdot)$  is *l.s.c.* on  $A$ . Consequently, Theorem 3.1. in our work is real extension from Theorem 3.1. in [15].  $\square$

**Example 3.2** Let  $X = Y = R$ ,  $C(\lambda) = R_+$ ,  $A = (0, 1]$ ,  $A(\lambda) = B = [\lambda^2, 1 + \lambda]$  and  $F(x, y, \lambda) = \lambda(y - x)$ . It follows from a direct computation that  $S(\lambda) = \{\lambda^2\}$ ,  $\forall \lambda \in A$ . And it is easy to check that  $S(\cdot)$  is continuous on  $A$ . For any  $\lambda \in (0, 1]$ , for each  $x \in A(\lambda) \setminus S(\lambda) = (\lambda^2, 1 + \lambda]$ , taking the unique element  $y = \lambda^2 \in S(\lambda)$ , we have

$$\begin{aligned} x - \lambda^2 &= d_X(x, y) \geq d_Y(F(x, y, \lambda), Y \setminus -\text{int}C(\lambda)) \\ &= \lambda(x - \lambda^2) \end{aligned}$$

Obviously, the assumption (iii) of Theorem 3.1. in [15] is violated, but if taking  $M: A \rightarrow (0, +\infty)$  as following:

$M(\lambda) = \frac{a}{\lambda}, \forall \lambda \in A$  where  $a$  is a constant number and  $a \geq 1$ .

Thus, the example 3.2 satisfies all the assumptions of Theorem 3.1. in our work, and it follows from Theorem 3.1. in our work that  $S(\cdot)$  is *l.s.c.* on  $A$ . But the Theorem 3.1. in [15] is invalid.

**Example 3.3** Let  $X = Y = Z = R$ ,  $C(\lambda) = R_+$ ,  $A = [0, 1]$ ,  $A(\lambda) = B = [1 - \lambda^2, 1 + \lambda^2]$  and  $F(x, y, \lambda) = x(1 + \lambda - y)$ .

It follows from a direct computation that  $S(\lambda) = [1 - \lambda^2, 1 + \lambda^2] = A(\lambda)$ . For any  $\lambda \in [0, 1]$ ,  $A(\lambda) \setminus S(\lambda) = \emptyset$ . Obviously, we cannot take any  $x \in A(\lambda) \setminus S(\lambda)$ . So the condition (iii) of Theorem 3.1 in [15] (or the assumption ④ in Theorem 3.1 in our work) cannot be applied. But it is easy to check that  $S(\cdot)$  is continuous on  $A$ .

Our approach can also be applied to study the lower semicontinuity of the weak solution mappings. A point  $x \in A(\lambda)$  is called a weak efficient solution to (PVOPEP) if  $F(x, y, \lambda) \notin -\text{int}C(\lambda), \forall y \in A(\lambda)$

The set of weak efficient solutions to (PVOPEP) is denoted by  $S_W(\lambda)$ , i.e.,

$$S_W(\lambda) := \{x \in A(\lambda) | F(x, y, \lambda) \notin -\text{int}C(\lambda), \forall y \in A(\lambda)\}$$

We can also obtain the following theorem on the lower semicontinuity of the weak efficient solution map to (PVOPEP) with a trivial adaptation of the proof.

**Theorem 3.2** Suppose that the following conditions are satisfied:

- ①  $A(\cdot)$  is continuous with compact values on  $A$ .
- ②  $F(\cdot, \cdot, \cdot)$  is *c.l.s.c.* on  $B \times B \times A$ .
- ③  $C(\cdot)$  is an upper semicontinuous cone-valued mapping on  $A$ .
- ④ If  $A(\lambda) \setminus S_W(\lambda) \neq \emptyset$ ; for each  $\lambda \in A$ , then for each  $\lambda \in A$ , for each  $x \in A(\lambda) \setminus S_W(\lambda)$ , there exist  $y \in S_W(-\lambda)$  and a positive function  $M: A \rightarrow (0, +\infty)$  which is upper semicontinuous on  $A$ , such that

$$d_X(x, y) \leq M(\lambda) \cdot d_Y(F(x, y, \lambda), Y \setminus -\text{int}C(\lambda))$$

Then,  $S_W(\cdot)$  is *l.s.c.* on  $A$ .

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