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ORIGINAL RESEARCH

Common fixed point theorems for infinite families of contractive maps

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Abstract In this paper, we prove some fixed point theorems for infinite families of self-mappings of a complete metric space satisfying some new conditions of common contractivity. An example is presented to show the effectiveness of our results.

Keywords Fixed point · Family of contractive maps

Mathematics Subject Classification 47H10 · 54H25

Introduction and preliminaries

Fixed point theory constitutes an important and the core part of the subject of nonlinear functional analysis and is useful for proving the existence theorems for nonlinear differential and integral equations. The Banach contraction principle [3] is the simplest and one of the most versatile elementary results in fixed point theory, which is a very popular tool for solving existence problems in many branches of mathematical analysis. Several authors have extended the Banach's fixed point theorem in various ways. The family of contraction mappings was introduced and studied by Ćirić [7] and Tasković [11]. Also in the process, the study of existence of common fixed point for finite and infinite family of self-mapping has been carried out by many authors. For example, one may refer [1, 2, 4–6, 12–14].

Recently, some new results for the existence and uniqueness of fixed points were presented for the cases of partially ordered metric spaces, cone metric spaces and fuzzy metric spaces (for example, see [1, 15–18]). Also, the study of common fixed points for a family of contractive type maps has been paid attention, and many interesting fixed point results have been obtained (for example, see [2, 7–11]).

The aim of this paper is to define some new conditions of common contractivity for an infinite family of mappings and give some new results on the existence and uniqueness of common fixed points in the setting of complete metric space.

Here, we state some known definitions and facts. We refer for more details to [1, 7].

Definition 1 Let X be a nonempty set and let $\{T_n\}$ be a family of self-mappings on X. A point $x_0 \in X$ is called a common fixed point for this family iff $T_n(x_0) = x_0$, for each $n \in N$.

The following interesting theorem was given by Ćirić [7] for a family of generalized contractions.

Theorem 1 *Let (X, d) be a complete metric space and let $\{T_\alpha\}_{\alpha\in J}$ be a family of self-mappings of X. If there exists fixed $\beta \in J$ such that for each $\alpha \in J$:*

$$d(T_\alpha x, T_\beta y) \leq \lambda \max\{d(x,y), d(x,T_\alpha x), d(y,T_\beta y), \frac{1}{2}[d(x,T_\beta y) + d(y,T_\alpha x)]\}, \tag{1}$$

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for some $\lambda = \lambda(\alpha) \in (0, 1)$ *and all* $x, \ y \in X$, *then all* T_α *have a unique common fixed point, which is a unique fixed point of each* T_α, $\alpha \in J$.

Common fixed point theorems for a family of mappings

In this section, we prove existence of a unique common fixed point for a family of contractive type self-maps on a complete metric space.

Theorem 2 *Let* (X, d) *be a complete metric space and* $0 \leq a_{i,j} (i, j = 1, 2, \ldots)$ *satisfy*

(i) for each j, $\overline{\lim}_{i \longrightarrow \infty} a_{i,j} < 1$
(ii) $\sum_{n=1}^{\infty} A_n < \infty$ *where* $A_n = \Pi_{i=1}^{n} \frac{a_{i,i+1}}{1 - a_{i,i+1}}$.

If $\{T_n\}$ *is a sequence of self-maps on X satisfying*

$$d(T_i x, T_j y) \leq a_{i,j}[d(x, T_j y) + d(y, T_i x)], \tag{2}$$

for $x, y \in X; i, j = 1, 2, \ldots$ with $x \neq y$ and $i \neq j$ then all T_ns have a unique common fixed point in X.

Proof For any $x_0 \in X$, let $x_n = T_n(x_{n-1}), n = 1, 2, \ldots$, then using (2.1) we get

$$\begin{aligned} d(x_1, x_2) = d(T_1(x_0), T_2(x_1)) &\leq a_{1,2}[d(x_0, T_2(x_1)) + d(x_1, T_1(x_0))] \\ &\leq a_{1,2}[d(x_0, x_2) + d(x_1, x_1)] \\ &= a_{1,2} d(x_0, x_2) \\ &\leq a_{1,2}[d(x_0, x_1) + d(x_1, x_2)] \end{aligned}$$

which implies

$$(1 - a_{1,2}) d(x_1, x_2) \leq a_{1,2} d(x_0, x_1).$$

So

$$d(x_1, x_2) \leq \frac{a_{1,2}}{1 - a_{1,2}} d(x_0, x_1).$$

Also we have,

$$\begin{aligned} d(x_2, x_3) = d(T_2(x_1), T_3(x_2)) &\leq a_{2,3}[d(x_1, T_3(x_2)) + d(x_2, T_2(x_1))] \\ &\leq a_{2,3}[d(x_1, x_3) + d(x_2, x_2)] \\ &= a_{2,3} d(x_1, x_3) \\ &\leq a_{2,3}[d(x_1, x_2) + d(x_2, x_3)] \end{aligned}$$

implies

$$(1 - a_{2,3}) d(x_2, x_3) \leq a_{2,3} d(x_1, x_2).$$

So

$$\begin{aligned} d(x_2, x_3) &\leq \frac{a_{2,3}}{1 - a_{2,3}} d(x_1, x_2) \\ &\leq \frac{a_{1,2}}{1 - a_{1,2}} \cdot \frac{a_{2,3}}{1 - a_{2,3}} d(x_0, x_1). \end{aligned}$$

In general, we get

$$d(x_n, x_{n+1}) \leq \prod_{i=1}^{n} \frac{a_{i,i+1}}{1 - a_{i,i+1}} d(x_0, x_1) = A_n d(x_0, x_1). \tag{3}$$

Therefore, for $m, n \in N, \ m \geq n$, and using (2.2)

$$\begin{aligned} d(x_n, x_m) &\leq \sum_{k=n}^{m-1} d(x_k, x_{k+1}) \\ &\leq \sum_{k=n}^{m-1} \prod_{i=1}^{k} \frac{a_{i,i+1}}{1 - a_{i,i+1}} d(x_0, x_1) \\ &= \sum_{k=n}^{m-1} A_k d(x_0, x_1). \end{aligned}$$

Thus $\{x_n\}$ is a Cauchy sequence and by completeness of X, $\{x_n\}$ converges to x (say) in X.

So using (2.1), for any positive integer m we have

$$\begin{aligned} d(x, T_m x) &\leq d(x, x_n) + d(x_n, T_m x) \\ &= d(x, x_n) + d(T_n x_{n-1}, T_m x) \\ &\leq d(x, x_n) + a_{n,m}[d(x_{n-1}, T_m x) + d(x, T_n x_{n-1})]. \end{aligned}$$

Taking $\overline{\lim}$ as $n \longrightarrow \infty$, we get

$$d(x, T_m x) \leq \overline{lim} a_{n,m} d(x, T_m x),$$

and it follows that $d(x, T_m x) = 0$ which shows that x is a common fixed of $\{T_m\}$.

Now we prove the uniqueness of the common fixed point x. Suppose that y be another common fixed point of $\{T_k\}$. Since $\sum_{n=1}^{\infty} A_n < \infty$ so $\lim_{n \longrightarrow \infty} A_n = 0$ and therefore there exists an $i_0 \in N$ such that $a_{i_0, i_0+1} < \frac{1}{2}$. Thus, from (2.1) we have

$$\begin{aligned} d(x, y) &= d(T_{i_0} x, T_{i_0+1} y) \\ &\leq a_{i_0, i_0+1}[d(x, T_{i_0+1} y) + d(y, T_{i_0} x)] \\ &= a_{i_0, i_0+1}[d(x, y) + d(y, x)] \\ &< d(x, y) \end{aligned}$$

which implies that $x = y$. So the uniqueness is proved and the proof is complete.

Corollary 1 *In addition to hypotheses of Theorem 2, suppose that for every* $n \in N$, *there exists a* $k_n \in N$ *such that* $a_{n,k_n} < \frac{1}{2}$, *then every* T_n *has a unique fixed point in X.*

Proof Following the proof of Theorem 2, $\{T_n\}$ have a unique common fixed point $x \in X$. If y is another fixed point of a T_m then

$$\begin{aligned} d(x, y) = d(T_m y, T_{k_m} x) &\leq a_{k_m, m}[d(x, T_m y) + d(y, T_{k_m} x)] \\ &= a_{k_m, m}[d(x, y) + d(y, x)] \\ &< d(x, y), \end{aligned}$$

which implies $d(x,y)=0$. Therefore, $x=y$, which gives the desired result.

Example 1 Let $X=[0,1]$ be a complete metric space with the distance $d(x,y)=|x-y|, x,y\in X$, and $T_n: X\longrightarrow X$ be defined by

$$T_n(x)=\begin{cases}1, & 0<x\leq 1,\\ \frac{2}{3}+\frac{1}{n+2}, & x=0.\end{cases}$$

Let $a_{i,j}=\frac{1}{3}+\frac{1}{|i-j|+6}, i\neq j$, then for each $j,\overline{\lim}_{i\longrightarrow\infty}a_{i,j}<1$ and $A_n=\Pi_{i=1}^n\frac{a_{i,i+1}}{1-a_{i,i+1}}=\left(\frac{10}{11}\right)^n$, therefore $\sum_{n=1}^{\infty}\left(\frac{10}{11}\right)^n<\infty$. Now we prove that for each $x,y\in X$,

$$d(T_ix,T_jy)\leq a_{i,j}[d(x,T_jy)+d(y,T_ix)].$$

There are three possible cases:

1. $x\in(0,1]$, $y\in(0,1]$. Then

$$\begin{aligned}d(T_ix,T_jy)=&\ |T_ix-T_jy|=0\\ \leq&\ a_{i,j}(|x-1|+|y-1|)\\ =&\ a_{i,j}(d(x,T_jy)+d(y,T_ix)).\end{aligned}$$

2. $x\in(0,1]$, $y=0$. Then

$$\begin{aligned}d(T_ix,T_jy)&=|T_ix-T_j(0)|=\left|\frac{1}{3}-\frac{1}{j+2}\right|\leq\frac{1}{3}\\ &\leq\left(\frac{1}{3}+\frac{1}{|i-j|+6}\right)\left(\left|x-\frac{2}{3}-\frac{1}{j+2}\right|+|0-1|\right)\\ &=a_{i,j}[d(x,T_jy)+d(y,T_ix)].\end{aligned}$$

3. $x=y=0$, $i<j$. Then

$$\begin{aligned}d(T_ix,T_jy)&=|T_i(0)-T_j(0)|=\left|\frac{2}{3}+\frac{1}{i+2}-\frac{2}{3}-\frac{1}{j+2}\right|\\ &=\left|\frac{1}{i+2}-\frac{1}{j+2}\right|\\ &\leq\frac{1}{i+2}\\ &\leq\left(\frac{1}{3}+\frac{1}{|i-j|+6}\right)\left(\left|\frac{2}{3}+\frac{1}{j+2}\right|+\left|\frac{2}{3}+\frac{1}{i+2}\right|\right)\\ &=a_{i,j}[d(x,T_jy)+d(y,T_ix)].\end{aligned}$$

So all the conditions of Theorem 2 are satisfied and note that $x=1$ is the only fixed point for all T_n.

Theorem 3 *Let* (X,d) *be a complete metric space and* $0\leq a_{i,j}(i,j=1,2,\ldots)$, *satisfy*

(i) *for each* j, $\overline{\lim}_{i\longrightarrow\infty}a_{i,j}<1$,
(ii) $\sum_{n=1}^{\infty}A_n<\infty$ *where* $A_n=\Pi_{i=1}^n\frac{a_{i,i+1}}{1-a_{i,i+1}}$.

If $\{T_n\}$ *is a sequence of self-maps on X satisfying*

$$d(T_ix,T_jy)\leq a_{i,j}\max\{d(x,y),d(x,T_ix),d(y,T_jy),d(x,T_jy),d(y,T_ix)\},\tag{4}$$

for all $x,y\in X, i,j=1,2,\ldots$ *with* $x\neq y$ *and* $i\neq j$ *then all* T_n,*s have a unique common fixed point in X. Further, if* $x\in X$ *be unique common fixed point of* $\{T_n\}$,*s then x is a unique fixed point for all* T_n,s.

Proof For any $x_0\in X$, let $x_n=T_n(x_{n-1}), n=1,2,\ldots$, then using (2.3) we obtain

$$\begin{aligned}d(x_1,x_2)&=d(T_1(x_0),T_2(x_1))\\ &\leq a_{1,2}\max\{d(x_0,x_1),d(x_0,T_1x_0),d(x_1,T_2x_1),\\ &\quad d(x_0,T_2(x_1)),d(x_1,T_1(x_0))\}\\ &=a_{1,2}\max\{d(x_0,x_1),d(x_0,x_1),d(x_1,x_2),d(x_0,x_2),\\ &\quad d(x_1,x_1)\}\\ &=a_{1,2}\max\{d(x_0,x_1),d(x_1,x_2),d(x_0,x_2)\}\\ &\leq a_{1,2}(d(x_0,x_1)+d(x_1,x_2)).\end{aligned}$$

Therefore,

$$d(x_1,x_2)\leq\frac{a_{1,2}}{1-a_{1,2}}d(x_0,x_1).$$

Also

$$\begin{aligned}d(x_2,x_3)&=(T_2(x_1),T_3(x_2))\\ &\leq a_{2,3}\max\{d(x_1,x_2),d(x_1,T_2x_1),d(x_2,T_3x_2),\\ &\quad d(x_1,T_3(x_2)),d(x_2,T_2(x_1))\}\\ &\leq a_{2,3}\max\{d(x_1,x_2),d(x_2,x_3),d(x_1,x_3)\},\end{aligned}$$

which similar to the previous case we get

$$d(x_2,x_3)\leq\frac{a_{2,3}}{1-a_{2,3}}d(x_1,x_2).$$

Hence we have

$$d(x_2,x_3)\leq\frac{a_{1,2}}{1-a_{1,2}}.\frac{a_{2,3}}{1-a_{2,3}}d(x_0,x_1).$$

In general

$$d(x_n,x_{n+1})\leq\prod_{i=1}^{n}\frac{a_{i,i+1}}{1-a_{i,i+1}}d(x_0,x_1).\tag{5}$$

Therefore, for $m,n\in N,\ \ m\geq n$, and using (2.4)

$$\begin{aligned}d(x_n,x_m)&\leq\sum_{k=n}^{m-1}d(x_k,x_{k+1})\\ &\leq\sum_{k=n}^{m-1}\prod_{i=1}^{k}\frac{a_{i,i+1}}{1-a_{i,i+1}}d(x_0,x_1)\\ &=\sum_{k=n}^{m-1}A_kd(x_0,x_1).\end{aligned}$$

Thus $\{x_n\}$ is a Cauchy sequence and by completeness of X, $\{x_n\}$ converges to x (say) in X.

So for any positive integer m,

$$\begin{aligned} d(x, T_m x) &\leq d(x, x_n) + d(x_n, T_m x) = d(x, x_n) + d(T_n x_{n-1}, T_m x) \\ &\leq d(x, x_n) + a_{n,m} \max\{d(x_{n-1}, x), d(x_{n-1}, T_n x_{n-1}), \\ &\quad d(x, T_m x), d(x_{n-1}, T_m x), d(x, T_n x_{n-1})\} \\ &\leq d(x, x_n) + a_{n,m} \max\{d(x_{n-1}, x), d(x_{n-1}, x_n), d(x, T_m x), \\ &\quad d(x_{n-1}, T_m x), d(x, x_n)\}. \end{aligned}$$

Taking $\overline{\lim}$ as $n \longrightarrow \infty$, we get

$$d(x, T_m x) \leq \overline{\lim}_{n \longrightarrow \infty} a_{n,m} d(x, T_m x).$$

From condition (i), it follows that $d(x, T_m x) = 0$ gives x as a common fixed point of $\{T_m\}$.

Let y be another fixed point of $\{T_n\}$, then

$$\begin{aligned} d(x, y) &= d(T_n x, T_m y) \\ &\leq a_{n,m} \max\{d(x, y), d(x, T_n(x), d(y, T_m y), d(x, T_m y), \\ &\quad d(y, T_n x)\} \\ &\leq a_{n,m} d(x, y). \end{aligned}$$

Taking $\overline{\lim}$ as $n \longrightarrow \infty$, we get

$$d(x, y) \leq \overline{\lim}_{n \longrightarrow \infty} a_{n,m} d(x, y),$$

which is possible only when $x = y$. Hence x is the unique common fixed point of $\{T_n\}$. Further, if $y \in X$ is a unique fixed point of T_k, then according to $\overline{\lim}_{i \longrightarrow \infty} a_{i,k} < 1$, there exists an $i_k \in N$ such that $a_{i_k,k} < 1$. Thus, by (2.3) we have

$$\begin{aligned} d(x, y) &= d(T_{i_k} x, T_k y) \\ &\leq a_{i_k,k} \max\{d(x, y), d(x, T_{i_k} x), d(y, T_k y), d(x, T_k y), d(y, T_{i_k} x)\} \\ &\leq a_{i_k,k} d(x, y), \end{aligned}$$

which implies $d(x, y) = 0$ and hence $x = y$.

Theorem 4 *Let (X, d) be a complete metric space and* $0 \leq a_{i,j} + b_{i,j} < 1 (i, j = 1, 2, \ldots)$, *satisfy*

(i) *for each j,* $\overline{\lim}_{i \longrightarrow \infty} a_{i,j} < 1$ *and* $\overline{\lim}_{i \longrightarrow \infty} b_{i,j} < 1$,

(ii) $\sum_{n=1}^{\infty} A_n < \infty$ *where* $A_n = \prod_{i=1}^{n} \frac{b_{i,i+1}}{1 - a_{i,i+1}}$.

If $\{T_n\}$ *is a sequence of self-maps on X satisfying*

$$d(T_i x, T_j y) \leq a_{i,j} d(y, T_j y) \varphi(d(x, T_i x), d(x, y)) + b_{i,j} d(x, y), \tag{6}$$

for all $x, y \in X, i, j = 1, 2, \ldots$ *with* $x \neq y$ *and* $i \neq j$ *where* $\varphi : [0, \infty) \times [0, \infty) \longrightarrow [0, \infty)$ *is a continuous function such that* $\varphi(t, t) = 1$ *for all* $t \in [0, \infty)$ *then, all* T_n *have a unique common fixed point in X.*

Proof For any $x_0 \in X$, let $x_n = T_n(x_{n-1}), \quad n = 1, 2, \ldots$, then from (2.5) we obtain

$$\begin{aligned} d(x_1, x_2) &= d(T_1(x_0), T_2(x_1)) \\ &\leq a_{1,2} d(x_1, T_2 x_1) \varphi(d(x_0, T_1 x_0), d(x_0, x_1)) + b_{1,2} d(x_0, x_1) \\ &\leq a_{1,2} d(x_1, x_2) \varphi(d(x_0, x_1), d(x_0, x_1)) + b_{1,2} d(x_0, x_1) \\ &\leq a_{1,2} [d(x_1, x_2) + b_{1,2} d(x_0, x_1)], \end{aligned}$$

implies

$$(1 - a_{1,2}) d(x_1, x_2) \leq b_{1,2} d(x_0, x_1).$$

Hence we have

$$d(x_1, x_2) \leq \frac{b_{1,2}}{1 - a_{1,2}} d(x_0, x_1).$$

Also,

$$\begin{aligned} d(x_2, x_3) &= (T_2(x_1), T_3(x_2)) \\ &\leq a_{2,3} d(x_2, T_3 x_2) \varphi(d(x_1, T_2 x_1), d(x_1, x_2)) + b_{2,3} d(x_1, x_2) \\ &\leq a_{2,3} d(x_2, x_3) \varphi(d(x_1, x_2), d(x_1, x_2)) + b_{2,3} d(x_1, x_2) \\ &\leq a_{2,3} d(x_2, x_3) + b_{2,3} d(x_1, x_2). \end{aligned}$$

Then

$$\begin{aligned} d(x_2, x_3) &\leq \frac{b_{2,3}}{1 - a_{2,3}} d(x_1, x_2) \\ &\leq \frac{b_{1,2}}{1 - a_{1,2}} \times \frac{b_{2,3}}{1 - a_{2,3}} d(x_0, x_1). \end{aligned}$$

Generally we conclude that

$$d(x_n, x_{n+1}) \leq \prod_{i=1}^{n} \frac{b_{i,i+1}}{1 - a_{i,i+1}} d(x_0, x_1) = A_n d(x_0, x_1). \tag{7}$$

Therefore, for $m, n \in N, \quad m \geq n$, and using (2.6) we get

$$\begin{aligned} d(x_n, x_m) &\leq \sum_{k=n}^{m-1} d(x_k, x_{k+1}) \\ &\leq \sum_{k=n}^{m-1} A_k d(x_0, x_1). \end{aligned}$$

Now passing to limit $n, m \longrightarrow \infty$, we get $d(x_n, x_m) \longrightarrow 0$.

Thus $\{x_n\}$ is a Cauchy sequence and by completeness of X, $\{x_n\}$ converges to x in X that is $\lim_{n \longrightarrow \infty} x_n = x \in X$.

So, for any positive integer m,

$$\begin{aligned} d(x, T_m x) &\leq d(x, x_n) + d(x_n, T_m x) = d(x, x_n) + d(T_n x_{n-1}, T_m x) \\ &\leq d(x, x_n) + a_{n,m} d(x, T_m x) \varphi(d(x_{n-1}, T_n x_{n-1}), d(x_{n-1}, x)) \\ &\quad + b_{n,m} d(x_{n-1}, x) \\ &\leq d(x, x_n) + a_{n,m} d(x, T_m x) \varphi(d(x_{n-1}, x_n), d(x_{n-1}, x_n)) \\ &\quad + b_{n,m} d(x_{n-1}, x) \\ &\leq d(x, x_n) + a_{n,m} d(x, T_m x) + b_{n,m} d(x_{n-1}, x). \end{aligned}$$

Taking $\overline{\lim}$ as $n \longrightarrow \infty$, we get

$$d(x, T_m x) \leq \overline{lima}_{n,m} d(x, T_m x) < d(x, T_m x).$$

It follows that $d(x, T_m x) = 0$ gives x as a common fixed of $\{T_m\}$.

Let y be another common fixed point, then

$$\begin{aligned} d(x,y) = d(T_n x, T_m y) &\leq a_{n,m} d(y, T_m y)\varphi(d(x, T_n(x)), d(x,y)) \\ &\quad + b_{n,m} d(x,y) \\ &= a_{n,m} d(y,y)\varphi(d(x,x), d(x,y)) + b_{n,m} d(x,y) \\ &= b_{n,m} d(x,y). \end{aligned}$$

Taking $\overline{lim}$ as $n \longrightarrow \infty$, we get $x = y$. So, the uniqueness is proved. □

Author contributions All authors contributed equally and significantly in writing this paper. All authors read and approved the final manuscript.

Compliance with ethical standards

Conflict of interest The authors declare that they have no competing interests.

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