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ORIGINAL RESEARCH

Evaluating wind speed probability distribution models with a novel goodness of fit metric: a Trinidad and Tobago case study

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Abstract

Wind energy has been explored as a viable alternative to fossil fuels in many small island developing states such as those in the Caribbean for a long time. Central to evaluating the feasibility of any wind energy project is choosing the most appropriate wind speed model. This is a function of the metric used to assess the goodness of fit of the statistical models tested. This paper compares a number of common metrics then proposes an alternative to the application-blind statistical tools commonly used. Wind speeds at two locations are considered: Crown Point, Tobago; and Piarco, Trinidad. Hourly wind speeds over a 15-year period have been analyzed for both sites. The available data is modelled using the Birnbaum–Saunders, Exponential, Gamma, Generalized Extreme Value, Generalized Pareto, Nakagami, Normal, Rayleigh and Weibull probability distributions. The distributions were compared graphically and their parameters were estimated using maximum likelihood estimation. Goodness of fit was assessed using the normalised mean square error testing, Chi-squared testing, Kolmogorov–Smirnov, R-squared, Akaike information criteria and Bayesian information criteria tests and the distributions ranked. The distribution ranking varied widely depending on the test used highlighting the need for a more contextualized goodness of fit metric. With this in mind, the concept of application-specific information criteria (ASIC) for testing goodness of fit is introduced. This allows distributions to be ranked by secondary features which are a function of both the primary data and the application space.

Keywords Wind speed · Statistical analysis · Probability distributions · Goodness of fit · Application-specific information criteria (asic)

Abbreviations

R^2	Coefficient of determination
χ^2	Chi-squared statistic
D^*	Two-sample Kolmogorov–Smirnov statistic
ASIC	Application-specific information criteria
AIC	Akaike information criterion
BIC	Bayesian information criterion
NMSE	Normalised mean square error
PDF	Probability density function
SIDS	Small Island Developing States

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Introduction

Electricity costs in most Caribbean Small Island Developing States (SIDS) are amongst the highest in the world [1] with majority of electrical energy being produced from imported fossil fuels [2]. As a result, wind energy is increasingly being explored as an alternative source of energy [3] with feasibility studies showing great potential for various islands [4–7]. Conversely, the Caribbean countries are amongst the most wind storm-prone regions in the world suffering from 26 storm impacts in the last 4 years alone [8–10]. These storms have an acute impact on the economies of these small states [11]. Additionally, wind speeds can even impact upon the region's flora and fauna [12].

Given the importance of wind to Caribbean SIDS, it is necessary for the characteristics of the wind be studied closely. Energy studies, storm risk studies and aviation considerations, among others, require the wind to be modelled as accurately and comprehensively as possible. Many studies have aimed at characterizing or comparing wind speed distributions at different locations [13–21]. Several emphasize seasonal variations

while diurnal variations have also been examined [5]. However, these have either been located outside the Caribbean or have been limited in their exploration of candidate distributions. Furthermore, there is no consensus on the goodness of fit criterion which is most suitable for evaluating the appropriateness of the distribution to a particular application [22].

The paper examines the applicability of probability distributions commonly used to model wind speeds to data available from two locations in Trinidad and Tobago. The relative performance of these distributions is compared using goodness of fit tests. Additionally, the concept of application-specific information criteria (ASIC), is introduced as an improved method for distribution ranking in the case of wind energy studies. Section 2, "Description of Data,"describes the data used for this study, investigates the basic statistical properties and outlines the pre-processing required before utilisation of the data. Section 3, "Methodology" describes the candidate distributions, goodness-of-fit criteria used and the method for parameter estimation. Section 4, "Results and Discussion" displays the fit of the candidate distributions to the data graphically. Several goodness-of-fit tests are used to rank the performance of the distributions. In addition, expected wind energy output from a turbine is estimated and compared to the energy output calculated using the actual wind data. Finally, concluding remarks are given in Sect. 5.

Description of data

General

The locations given in Fig. 1, provide a useful opportunity for comparison as they are both greeted by the same north-easterly trade wind system [23], but are located at sites with differing geography. Crown Point is on a sheltered coast while Piarco is located in-land in an open plain. Piarco also receives some degree of sheltering by mountains to the North.

The dataset consists of the mean hourly wind speeds at Crown Point, Tobago and Piarco, Trinidad (locations indicated in Table 1 for the years 2000–2015, provided by the meteorological offices at airports at both locations and does not include wind direction or peak gust speed. The speeds were recorded in knots, rounded to the nearest knot, and are given in intervals of 1 h for each hour of the 24-h day for every day of the month. It should be noted that there are data points missing from both datasets. For Piarco, Trinidad, approximately 19 days of data from October 21 to November 9, 2009 are missing. There are also missing data points for a number of hours on other days. The total number of measurements is 133,083 out of a maximum possible number of 133,656 (0.4% missing data). For Crown Point, the data for the months of July to September, 2001, and August, 2011, are missing. Additionally, some days are missing data for an hour or a few hours. The total number of measurements is 123,429 out of a maximum possible number of 133,656 (7.7% missing data).

Table 1 Coordinates of wind measurement locations

Location	Latitude	Longitude	Elevation (m)
Crown Point, Tobago	11.1497N	60.8322W	12
Piarco, Trinidad	10.5953N	61.3372W	15

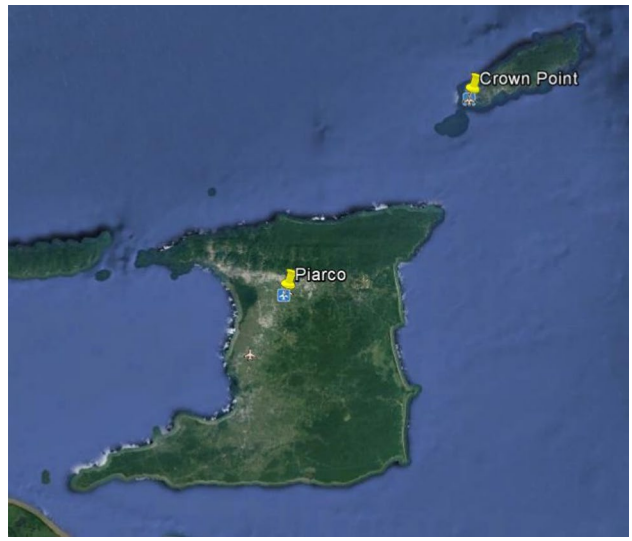


Fig. 1 Map of Trinidad and Tobago illustrating measurement locations [24]

Basic statistics

Before any pre-processing, the data were described by the statistics given in Tables 2 and 3. Data pre-processing included data inspection for possibly erroneous values:

- As seen in Table 2, at Piarco, the maximum wind speed recorded is 196 knots at 9:00 a.m. on July 3, 2014. The maximum wind speed on that day otherwise was 16 knots, and the average was 8 knots. A wind speed of 196 knots (363 km/h) is the equivalent of a Category 5 hurricane [25]. 196 knots surpasses the sustained wind speeds of the strongest hurricane on record, Hurricane Patricia in October 2015 [26]. Records from satellite-based models available online [27] confirm that the measurement is erroneous.
- Similarly, a speed of 94 knots was recorded as the maximum wind speed in 2009. This peak wind speed was recorded on June 12 at 11:00 a.m., followed by a measurement of 84 knots at 12:00 p.m.. The average wind speed on that day without these two measurements was 5.9 knots. These two wind speeds are the equivalent of a Category 2 Hurricane according to the

Table 2 Basic statistics of wind speeds at Piarco before pre-processing

Year	Number of readings	Max knots	Mean knots	Std. dev. knots	Kurtosis	Skewness
2000	8779	21	5.454	4.841	– 0.954	0.396
2001	8760	20	5.654	4.874	– 1.080	0.301
2002	8758	21	5.634	4.721	– 0.923	0.326
2003	8760	21	5.356	4.653	– 0.923	0.374
2004	8774	20	4.483	4.467	– 0.928	0.512
2005	8759	26	4.226	4.600	– 0.646	0.680
2006	8758	22	4.842	4.834	– 0.881	0.534
2007	8757	20	4.794	4.867	– 0.973	0.531
2008	8782	23	4.478	4.882	– 0.793	0.656
2009	8300	94	4.974	5.128	16.473	1.486
2010	8682	23	5.047	5.290	– 0.891	0.591
2011	8760	23	5.063	5.021	– 0.749	0.555
2012	8783	23	5.075	5.032	– 0.839	0.547
2013	8755	22	5.786	5.418	– 0.878	0.482
2014	8756	196	5.863	5.775	132.684	4.358
2015	2160	40	6.261	5.779	– 0.765	0.365
Total	133,083					
Average			5.187	5.012	8.558	0.793

Table 3 Basic statistics of wind speeds at Crown Point before pre-processing

Year	Number of readings	Max knots	Mean knots	Std. dev. knots	Kurtosis	Skewness
2000	8762	30	6.744	5.189	– 0.915	0.205
2001	6519	264	8.393	6.732	393.540	11.340
2002	8036	39	7.616	5.043	– 0.728	0.122
2003	8722	26	8.319	4.966	– 0.848	– 0.099
2004	8770	45	6.949	4.685	– 0.200	0.279
2005	8735	214	6.906	5.296	275.285	7.409
2006	8598	70	7.519	4.996	1.835	0.170
2007	8752	22	7.713	5.387	– 1.151	– 0.111
2008	7063	30	7.103	5.463	– 1.089	0.089
2009	3655	22	6.633	4.910	– 1.126	0.061
2010	8654	22	6.896	5.502	– 1.231	0.090
2011	8725	28	6.909	5.412	– 0.973	0.158
2012	8772	38	6.774	5.757	– 0.855	0.318
2013	8749	28	7.445	5.870	– 0.996	0.201
2014	8757	28	7.890	5.929	– 0.990	0.163
2015	2160	23	6.886	5.963	– 1.104	0.279
Total	123,429					
Average			7.293	5.444	41.153	1.292

Saffir–Simpson scale [25]. However, review of archived daily newspapers, for the next day make no mention of the event [28]. Again, no corroborating records were found to confirm these wind speeds [14] and as such, they were deemed erroneous.

– On February 16, 2015 a wind speed of 40 knots was noted at 9:00 a.m. This would be classified as tropical storm wind speeds according to the Saffir–Simpson scale. Online records corroborate the data for the rest of the day.
– At Crown Point, Tobago a maximum wind speed of 264 knots is recorded on November 9, 2001 at 11:00 p.m. This wind speed exceeds the world record maximum 3 second surface wind gust of 220 knots. As such, it can

safely be concluded that this is an erroneous record. Online records corroborate the data for the rest of the day.

– Similar methods to those utilised above were used to identify 214 knots recorded on November 20, 2005 at 2:00 p.m., 187 knots on April 20, 2001, 96 knots July 17, 2005, 70 knots at 9:00 p.m. on June 20, 2006, 62 knots on March 20, 2001, 45 knots January 3, 2004 and 39 knots on January 2, 2002 at 11:00 a.m.

All noted erroneous measurements were replaced with null values. Tables 4 and 5 reflects the wind statistics after pre-processing.

Table 4 Basic statistics of wind speeds at Piarco after pre-processing

Year	Number of readings	Max knots	Mean knots	Std. dev. knots	Kurtosis	Skewness
2000	8779	21	5.454	4.841	– 0.954	0.396
2001	8760	20	5.654	4.874	– 1.080	0.301
2002	8758	21	5.634	4.721	– 0.923	0.326
2003	8760	21	5.356	4.653	– 0.923	0.374
2004	8774	20	4.483	4.467	– 0.928	0.512
2005	8759	26	4.226	4.600	– 0.646	0.680
2006	8758	22	4.842	4.834	– 0.881	0.534
2007	8757	20	4.794	4.867	– 0.973	0.531
2008	8782	23	4.478	4.882	– 0.793	0.656
2009	8298	31	4.954	4.959	– 1.023	0.471
2010	8682	23	5.047	5.290	– 0.891	0.591
2011	8760	23	5.063	5.021	– 0.749	0.555
2012	8783	23	5.075	5.032	– 0.839	0.547
2013	8755	22	5.786	5.418	– 0.878	0.482
2014	8755	21	5.842	5.406	– 1.170	0.356
2015	2159	22	6.245	5.735	– 1.246	0.288
Total	133,079					
Average			5.183	4.975	– 0.931	0.475

Table 5 Basic Statistics of Wind Speeds at Crown Point after pre-processing

Year	Number of readings	Max	Mean	Std. dev.	Kurtosis	Skewness
2000	8762	30	6.744	5.189	– 0.915	0.205
2001	6516	25	8.318	5.475	– 0.998	0.046
2002	8035	22	7.612	5.031	– 0.895	0.094
2003	8722	26	8.319	4.966	– 0.848	– 0.099
2004	8769	30	6.945	4.667	– 0.660	0.223
2005	8733	26	6.872	4.715	– 0.892	0.049
2006	8597	36	7.512	4.951	– 0.937	– 0.055
2007	8752	22	7.713	5.387	– 1.151	– 0.111
2008	7063	30	7.103	5.463	– 1.089	0.089
2009	3655	22	6.633	4.910	– 1.126	0.061
2010	8654	22	6.896	5.502	– 1.231	0.090
2011	8725	28	6.909	5.412	– 0.973	0.158
2012	8772	38	6.774	5.757	– 0.855	0.318
2013	8749	28	7.445	5.870	– 0.996	0.201
2014	8757	28	7.890	5.929	– 0.990	0.163
2015	2160	23	6.886	5.963	– 1.104	0.279
Total	123,421					
Average			7.286	5.324	– 0.979	0.107

Methodology

Distribution fitting

Typically, wind data is modelled as a Weibull distribution, especially when the aim is to characterise the annual resource [5, 15, 29–32], however, a number of other candidate distributions have been catalogued [33]. For other applications such as statistical analysis of extreme wind speeds, the Weibull (or reverse Weibull) has also been recommended [34] while other distributions such as the generalised extreme value distribution [17] and the generalised pareto [19] have been explored. Agustin [20] encouraged using mixed distributions while confirming the applicability of Weibull. Sarkar et al. [35] identified the weakness of the Weibull distribution as its failure to describe the upper tail. The Rayleigh distribution has also been used as a probability model for wind speed [31], although some applications have found Weibull to be more accurate [32, 36]. Recent studies found autoregressive models [37] and maximum entropy distributions [38] to be better suited to wind speed applications than Weibull or Rayleigh. Alavi et al. [39] found that the Nakagami distribution performed well when compared to other distributions frequently used to model wind speed. Additionally, the Normal and Exponential Distributions were identified as potential candidates via visual inspection of the histogram shape. The Birnbaum–Saunders and Gamma distributions performed well when goodness of fit was assessed using the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) criteria, and were thus included in the comparative analysis (Figs. 2, 3).

Review of probability distribution functions

The equations defining the probability density functions (PDFs) for various candidate distributions of interest are given below. While by no means exhaustive, the distributions represent those commonly used in the literature.

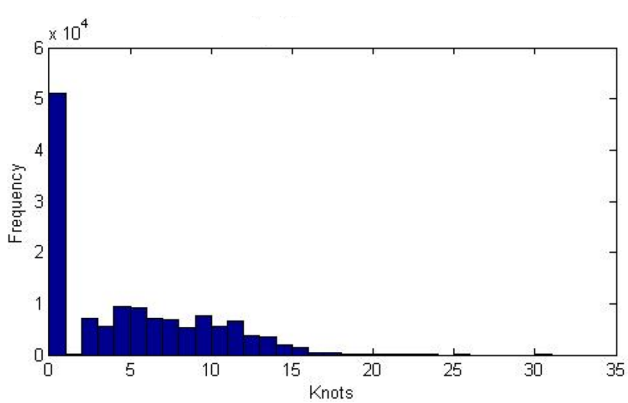


Fig. 2 Histogram of wind speeds at Piarco

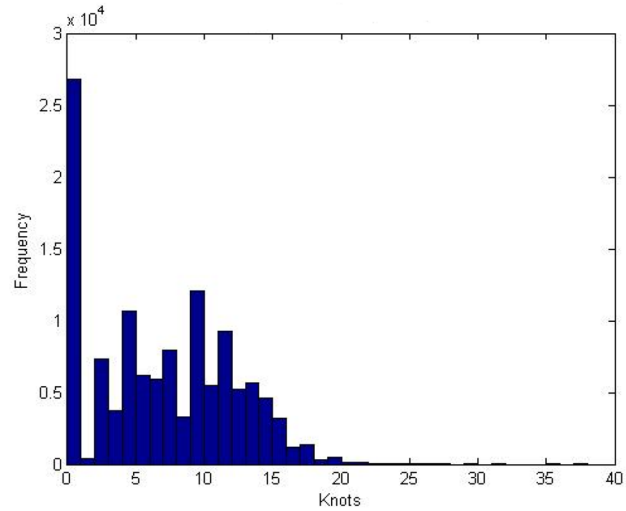


Fig. 3 Histogram of wind speeds at Crown Point

Birnbaum–Saunders

$$f(x) = \frac{\sqrt{\frac{x-\mu}{\beta}} + \sqrt{\frac{\beta}{x-\mu}}}{2\gamma(x-\mu)} \Phi\left(\frac{\sqrt{\frac{x-\mu}{\beta}} - \sqrt{\frac{\beta}{x-\mu}}}{\gamma}\right), \quad x > \mu, \tag{1}$$

where μ is the location parameter, $\gamma > 0$ is the shape parameter, $\beta > 0$ is the scale parameter and $\Phi(t) = \frac{1}{\sqrt{2\pi}} e^{\frac{t^2}{2}}$ [40].

Exponential

$$f(x) = \lambda e^{-\lambda x}, \quad x \geq 0, \tag{2}$$

where $\lambda > 0$ is the rate parameter [41].

Gamma

$$f(x) = \frac{x^{k-1} e^{-\frac{x}{\theta}}}{\theta^k \Gamma(k)}, \quad x \geq 0, \tag{3}$$

where k, $\theta \geq 0$, and $\Gamma(k) = (k-1)!$ is the gamma function [42, 43].

Generalized extreme value

$$f(x) = \frac{1}{\sigma} t(x)^{\xi+1} e^{-t(x)} \tag{4}$$

where

$$t(x) = \begin{cases} \left(1 + \left(\frac{x-\mu}{\sigma}\xi\right)\right)^{-\frac{1}{\xi}}, & \xi \neq 0, \\ e^{-\frac{x-\mu}{\sigma}}, & \xi = 0, \end{cases} \tag{5}$$

where $\mu \in \mathbf{R}$ is the location parameter, $\sigma > 0$ is the scale parameter and $\xi \in \mathbf{R}$ is the shape parameter [44].

Generalized Pareto

$$f(x) = \frac{1}{\sigma}\left(1 + \frac{x-\mu}{\sigma}\xi\right)^{-\left(\frac{\xi+1}{\xi}\right)} \tag{6}$$

where $\mu \in \mathbf{R}$ is the location parameter, $\sigma > 0$ is the scale parameter and $\xi \in \mathbf{R}$ is the shape parameter [45].

Nakagami

$$f(x) = 2\left(\frac{\mu}{\omega}\right)^{\mu}\frac{1}{\Gamma(\mu)}x^{(2\mu-1)}\mathrm{e}^{-\frac{\mu}{\omega}x^2} \tag{7}$$

where μ is the shape parameter and ω is the spread parameter, for $x > 0$ [46].

Normal

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\mathrm{e}^{-\frac{(x-\mu)^2}{2\sigma^2}}, \tag{8}$$

where μ is the mean and σ is the standard deviation [45].

Rayleigh

$$f(x) = \frac{x}{\sigma^2}\mathrm{e}^{\frac{-x^2}{2\sigma^2}}, \tag{9}$$

where σ is the scale parameter [47].

Weibull

$$f(x) = \begin{cases} \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1}\mathrm{e}^{-\left(\frac{x}{\lambda}\right)^k} & x \geq 0 \\ 0, & x < 0, \end{cases} \tag{10}$$

where $k > 0$ is the shape parameter and $\lambda > 0$ is the scale parameter [48].

Parameter estimation

Several techniques exist for parameter estimation (e.g., [22]). In this work, the parameters for these various distributions were estimated using the maximum likelihood method, which selects as its estimate the parameter value that maximizes the probability of the observed data [49]. This method is popularly used since the resulting estimators are generally asymptotically unbiased and consistent. They also offer the advantage of simplicity in implementation. While this method can be limited through the need to determine closed-form estimator solutions, for the distributions of interest in this work, these can be readily obtained [22].

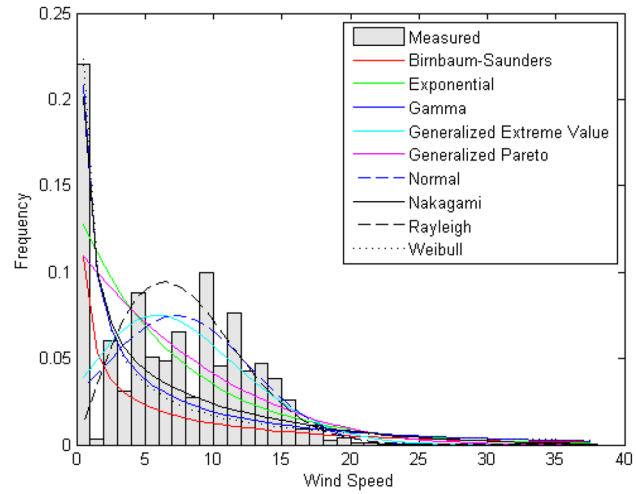


Fig. 4 Wind distribution at Crown Point

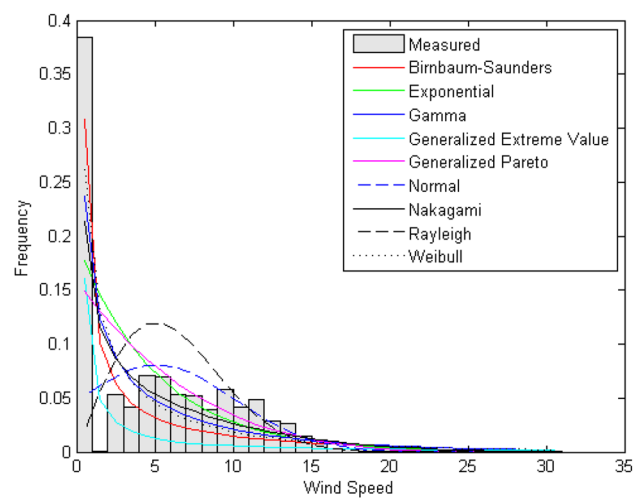


Fig. 5 Wind distribution at Piarco

Goodness of fit

After plotting the distributions using the estimated parameters, the goodness of fit of the distributions to the data profile were assessed using the following metrics:

- Normalised mean square error (NMSE).
- Chi-squared statistic.
- Two sample.
- Kolmogorov–Smirnov.
- Coefficient of determination (R^2).
- Akaike information criterion (AIC).
- Bayseian information criterion (BIC).

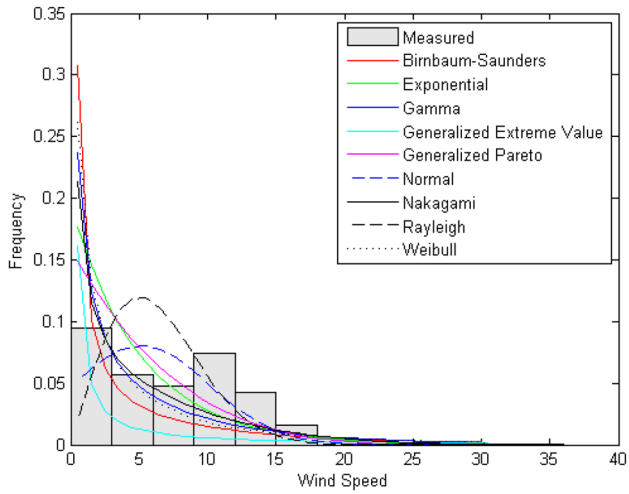


Fig. 6 Wind distribution at Crown Point (Bin Size 3)

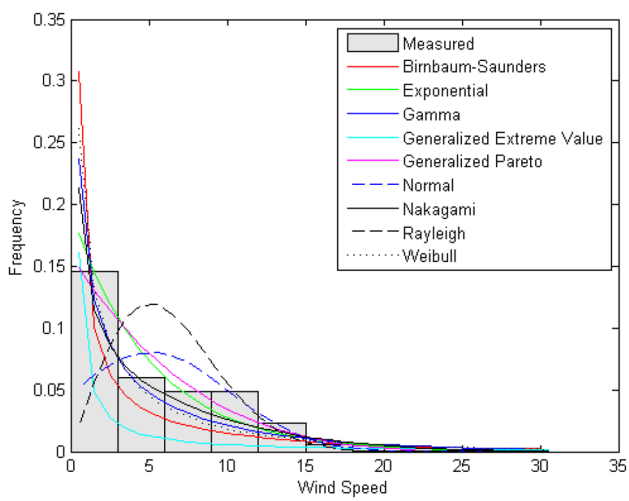


Fig. 7 Wind distribution at Piarco (Bin Size 3)

The normalised mean square error (NMSE)

The NMSE was calculated using the previous method with the following equation:

$$\mathrm{NMSE} = 1 - \sum_{i=1}^{n} \left(\frac{y_i - f_i}{y_i - \hat{f}_i} \right)^2 \tag{11}$$

Where y_n are the modelled values and f_n are the reference data.

The returned value varies between $-\infty$ (bad fit) to 1 (perfect fit) [50, 51].

The Chi-squared statistic

For testing the goodness of fit, the Chi-squared was used. The Chi-square statistic (χ^2) is calculated as follows:

$$\chi^2 = \sum_{i=1}^{N} \frac{(O_i - E_i)^2}{E_i}, \tag{12}$$

where O_i are the observed counts and E_i are the expected counts [49]. O_i was the estimated sample datasets calculated using the estimated pdf of each distribution. E_i was derived via the frequency histogram based on the measured data. N was determined by the number of bins used in the frequency histogram. A smaller Chi-squared statistic indicates a better fit.

The two-sample Kolmogorov–Smirnov test

The two-sample Kolmogorov–Smirnov test statistic was calculated as follows:

$$D^* = \max_x (|F_1(x) - F_2(x)|) \tag{13}$$

Table 6 Distribution ranking for Crown Point data (Bin Size 1)

Rank	NMSE	X2	KS	R^2 and R^2 Adj	LogL, AIC and BIC
1	Nakagami	Normal	Normal	Nakagami	Birn-Saun
2	Weibull	Gen. ext val	Gen. ext val	Gamma	Nakagami
3	Gamma	Rayleigh	Rayleigh	Weibull	Gamma
4	Exponential	Gen. Pareto	Gen. Pareto	Gen. Pareto	Weibull
5	Gen. Pareto	Exponential	Exponential	Exponential	Gen. Pareto
6	Rayleigh	Nakagami	Nakagami	Birn-Saun	Exponential
7	Normal	Gamma	Weibull	Gen. ext val	Gen. ext val
8	Gen. ext val	Weibull	Birn-Saun	Normal	Normal
9	Birn-Saun	Birn-Saun	Gamma	Rayleigh	Rayleigh

Table 7 Distribution ranking for Piarco data (Bin Size 1)

Rank	NMSE	X2	KS	R^2 and R^2 Adj	LogL, AIC and BIC
1	Birn-Saun	Normal	Normal	Birn-Saun	Gen. ext val
2	Weibull	Gen. Pareto	Gen. Pareto	Weibull	Birn-Saun
3	Gamma	Exponential	Rayleigh	Gamma	Nakagami
4	Nakagami	Nakagami	Exponential	Nakagami	Gamma
5	Exponential	Gamma	Nakagami	Gen. ext val	Weibull
6	Gen. Pareto	Rayleigh	Weibull	Exponential	Exponential
7	Rayleigh	Birn-Saun	Gen. ext val	Gen. Pareto	Gen. Pareto
8	Gen. ext val	Weibull	Birn-Saun	Normal	Normal
9	Normal	Gen. ext val	Gamma	Rayleigh	Rayleigh

Table 8 Distribution ranking for Crown Point data (Bin Size 3)

Rank	NMSE	X2	KS	R^2 and $R^2$2 Adj	LogL, AIC and BIC
1	Gen. Pareto	Normal	Normal	Gen. Pareto	Birn-Saun
2	Exponential	Gen. ext val	Gen. ext val	Exponential	Nakagami
3	Gen. ext val	Rayleigh	Rayleigh	Gen. ext val	Gamma
4	Normal	Gen. Pareto	Gen. Pareto	Nakagami	Weibull
5	Nakagami	Exponential	Exponential	Normal	Gen. Pareto
6	Rayleigh	Nakagami	Nakagami	Gamma	Exponential
7	Gamma	Gamma	Birn-Saun	Weibull	Gen. ext val
8	Weibull	Weibull	Weibull	Rayleigh	Normal
9	Birn-Saun	Birn-Saun	Gamma	Birn-Saun	Rayleigh

Table 9 Distribution ranking for Piarco data (Bin Size 3)

Rank	NMSE	X2	KS	R^2 and R^2 Adj	LogL, AIC and BIC
1	Exponential	Normal	Rayleigh	Exponential	Gen. ext val
2	Gen. Pareto	Gen. Pareto	Normal	Gen. Pareto	Birn-Saun
3	Weibull	Exponential	Gen. Pareto	Nakagami	Nakagami
4	Gamma	Nakagami	Weibull	Gamma	Gamma
5	Nakagami	Gamma	Gen. ext val	Weibull	Weibull
6	Birn-Saun	Rayleigh	Exponential	Birn-Saun	Exponential
7	Rayleigh	Birn-Saun	Nakagami	Normal	Gen. Pareto
8	Normal	Weibull	Birn-Saun	Rayleigh	Normal
9	Gen. ext val	Gen. ext val	Gamma	Gen. ext val	Rayleigh

where $F_1(x)$ is the proportion of $x1$ values less than or equal to x, and $F_2(x)$ is the proportion of $x2$ values less than or equal to x. The smaller the test statistic the better the fit [52].

Co-efficient of determination, R^2

The R^2 statistic was calculated as follows:

$$R^2 = 1 - \frac{SS_{\text{res}}}{SS_{\text{tot}}}, \tag{14}$$

where,

$$SS_{\text{res}} = \sum_i \left(y_i - f_i\right)^2, \tag{15}$$

and,

$$SS_{\text{res}} = \sum_i \left(y_i - \bar{y}\right)^2, \tag{16}$$

where y_i represents the dataset and f_i represents the modelled values. R^2 varies between − Inf (bad fit) to 1 (perfect fit) [53].

Akaike information criterion

The AIC statistic was calculated as follows:

$$\text{aic} = -2\log L(\theta) + 2k, \tag{17}$$

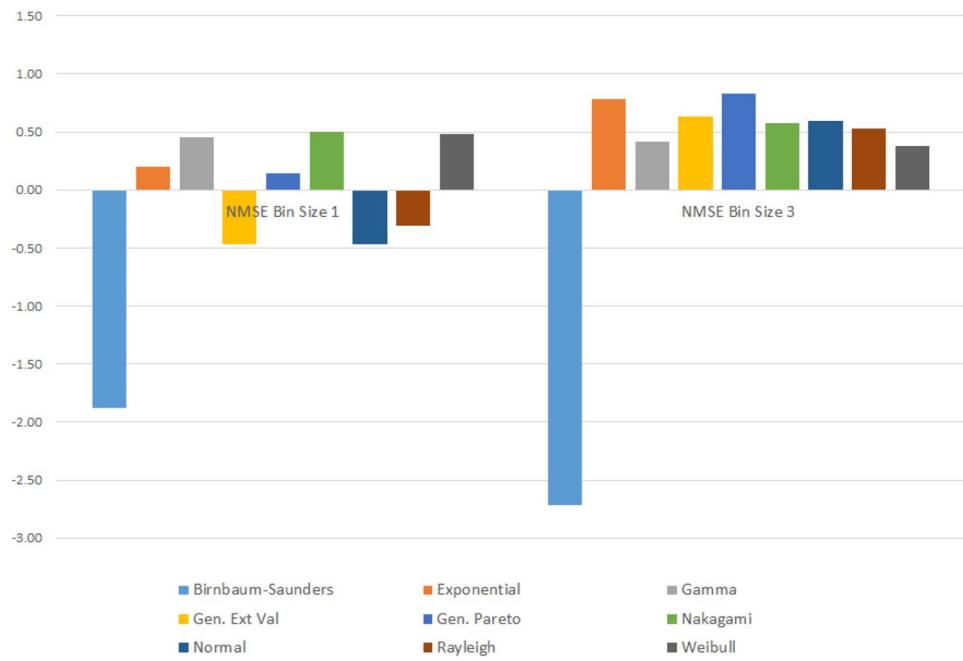


Fig. 8 Crown Point—NMSE

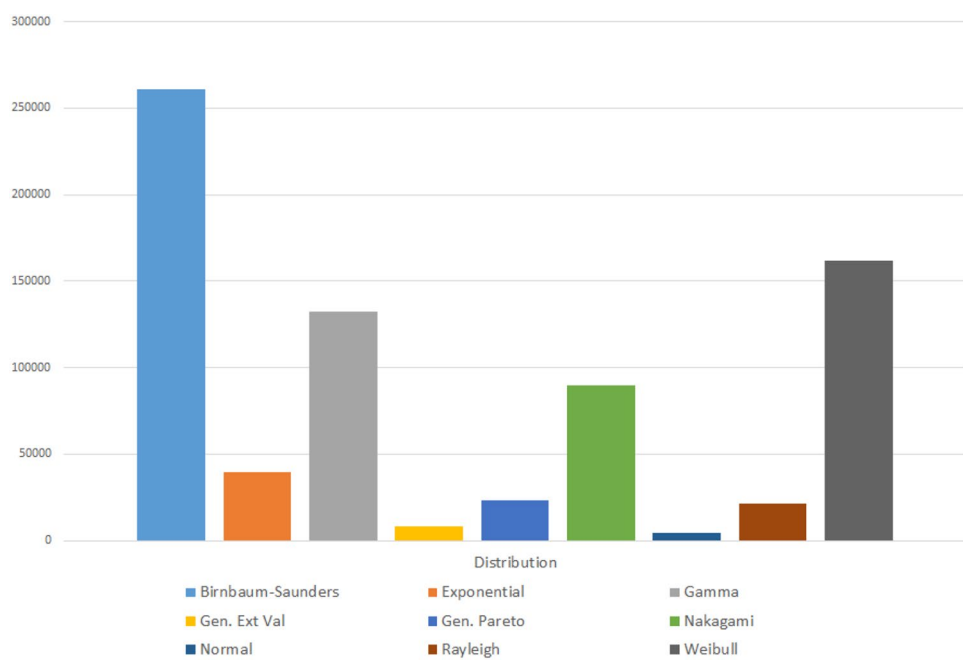


Fig. 9 Crown Point—Chi Squared

where $\log L(\theta)$ denotes the value of the maximized loglikelihood objective function for a model with k parameters. A smaller AIC statistic value indicates a better fit [54].

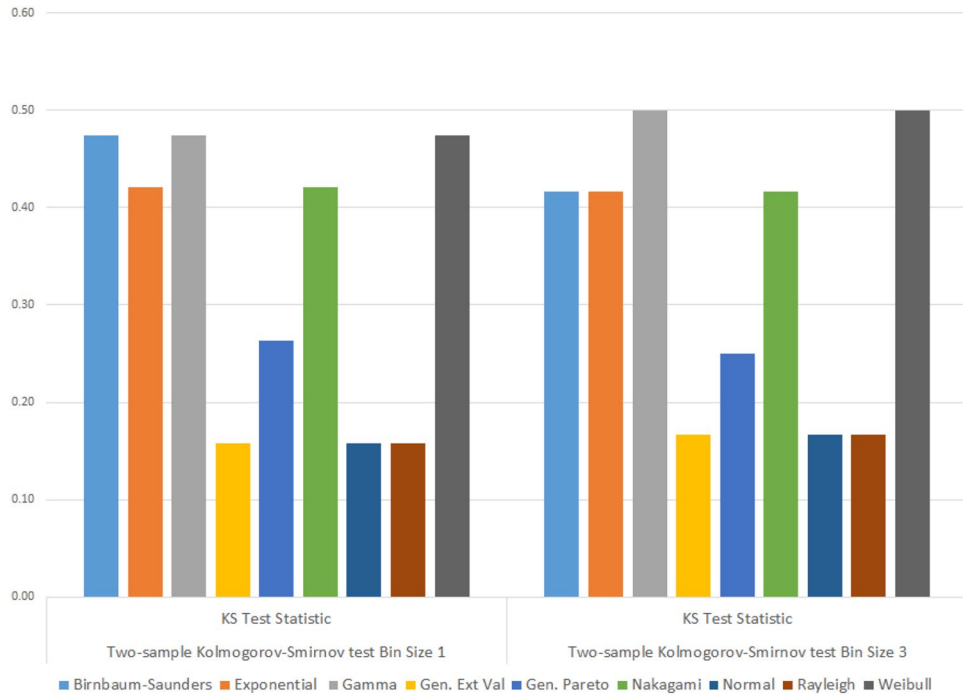
Bayesian information criterion

The BIC statistic was calculated as follows:

$$\text{bic} = -2\log L(\theta) + k\log(N), \tag{18}$$

where $\log L(\theta)$ denotes the value of the maximized loglikelihood objective function for a model with k parameters fit to N data points. A smaller BIC statistic value indicates a better fit [54] (Figs. 4, 5, 6, 7).

Fig. 10 Crown Point—KS



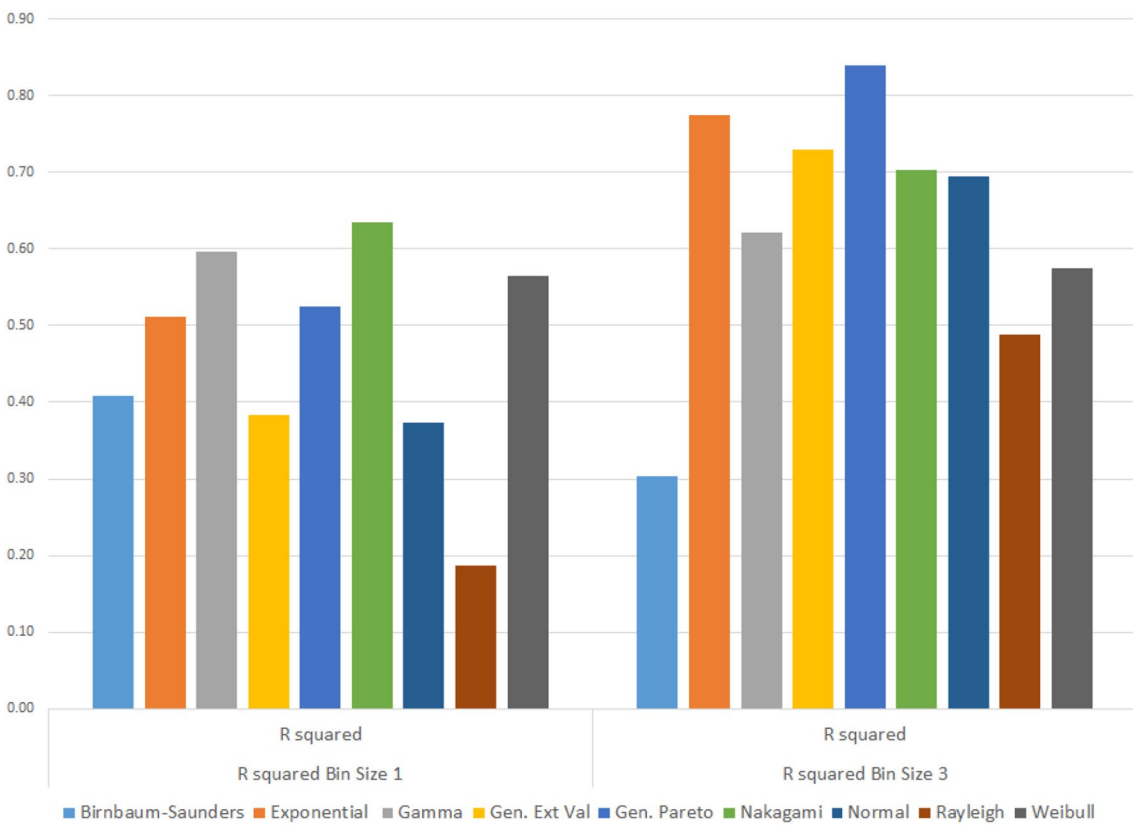


Fig. 11 Crown Point—R Squared

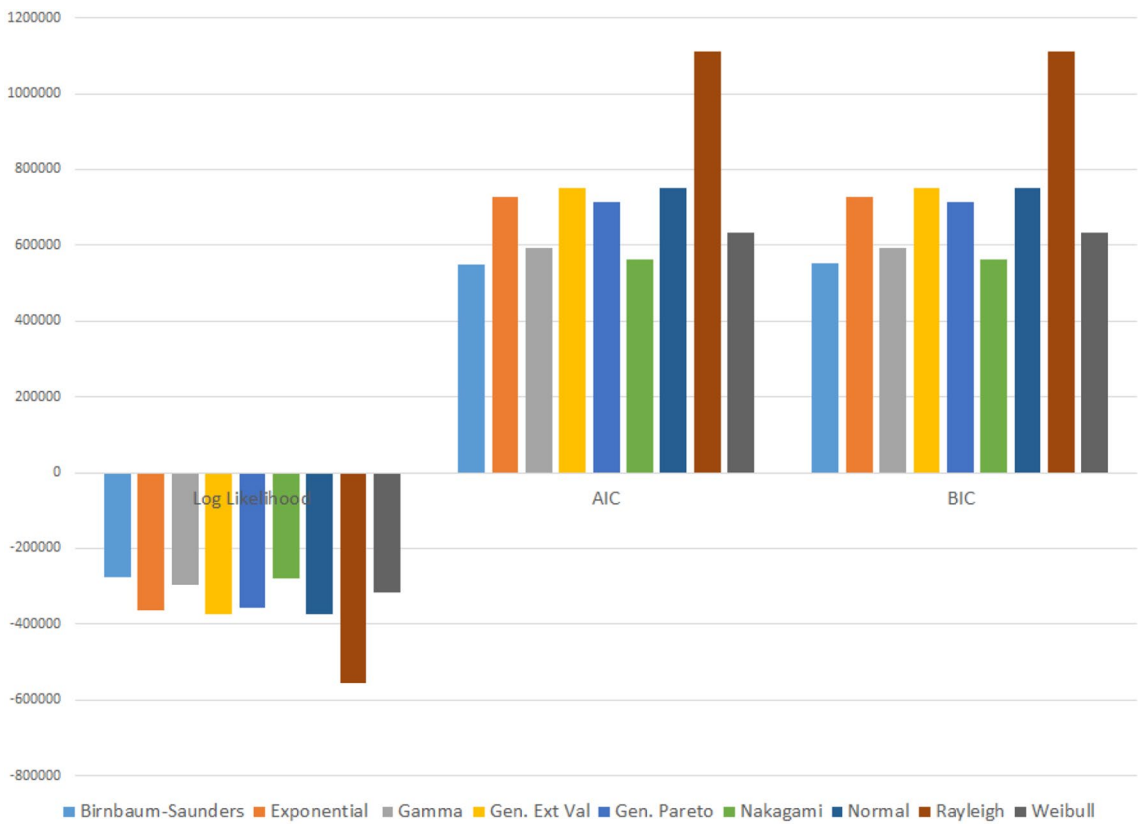


Fig. 12 Crown Point—Log Likelihood, AIC and BIC

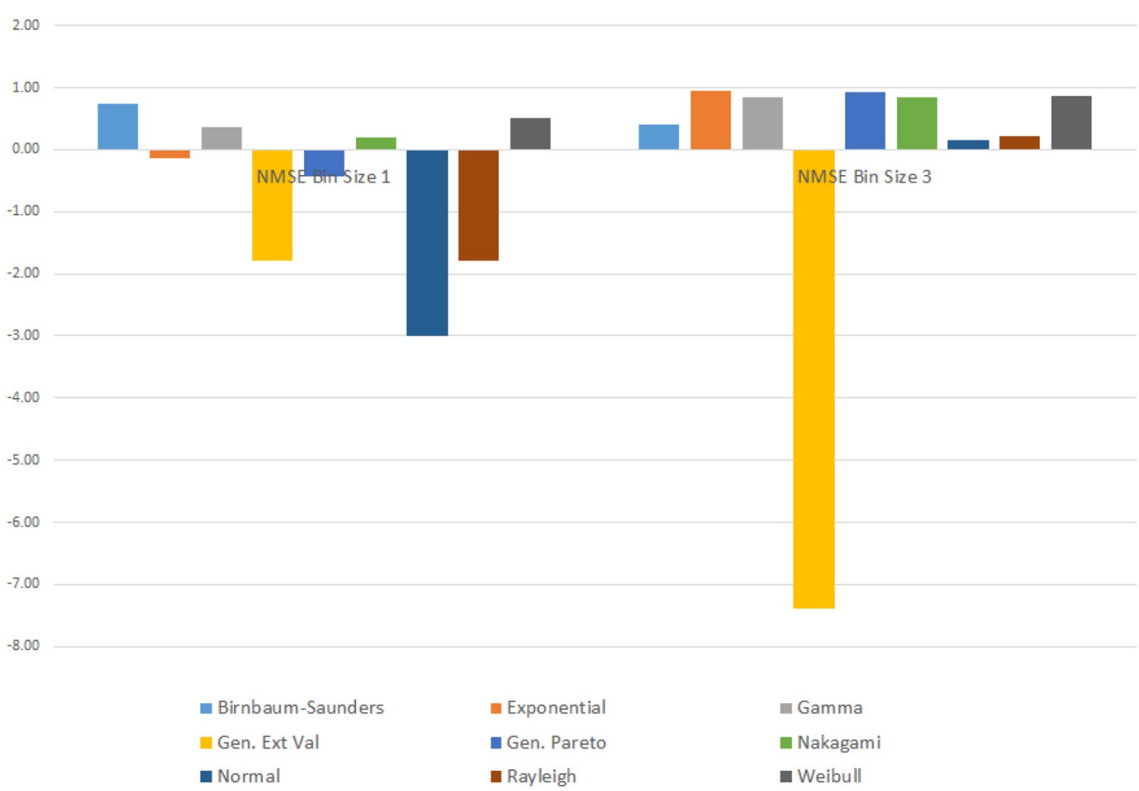


Fig. 13 Piarco—NMSE

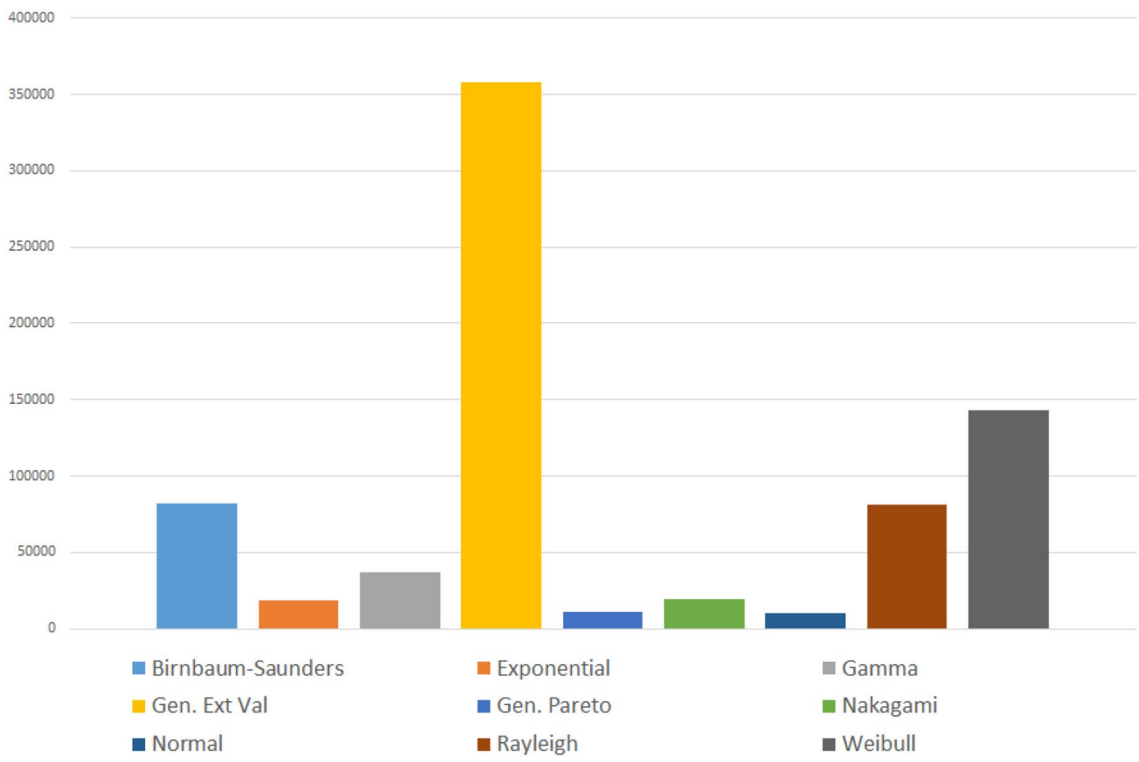


Fig. 14 Piarco—Chi Squared

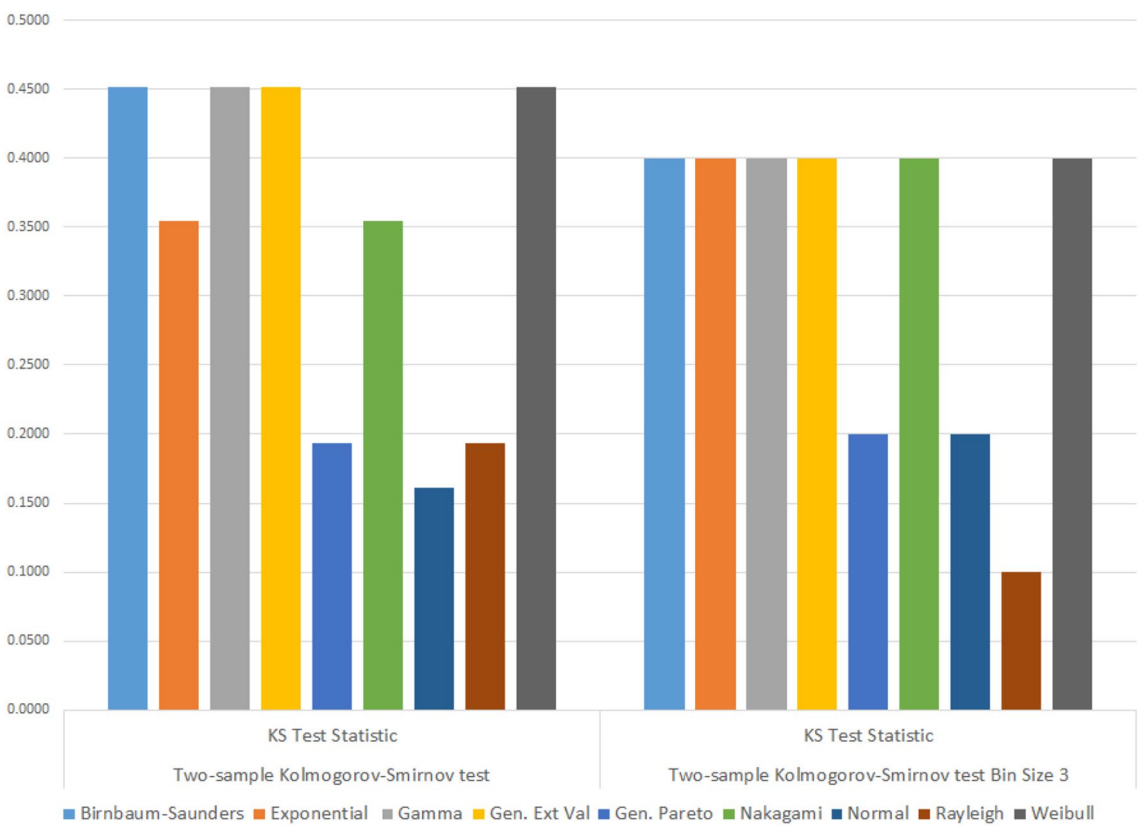


Fig. 15 Piarco—ranked by KS

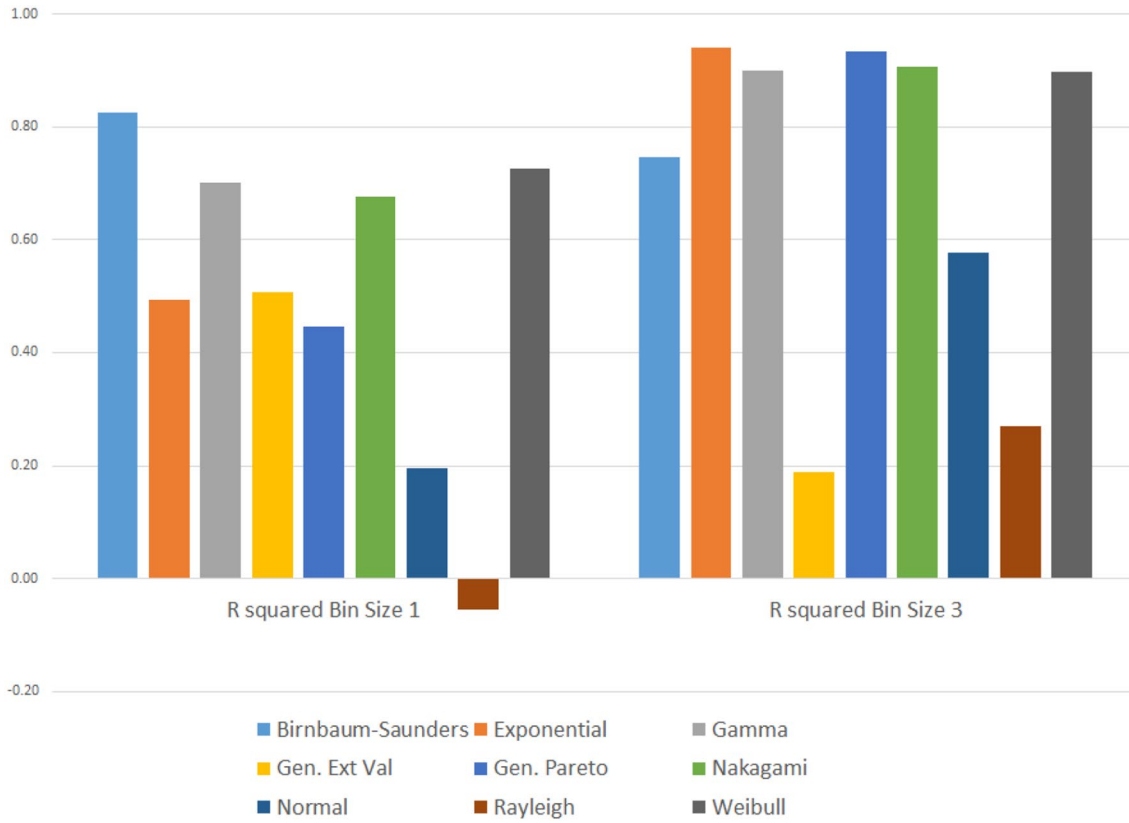


Fig. 16 Piarco—Chi Squared

Results and discussion

The estimated parameters for each distribution are shown in the Appendix. The performance of these distributions were compared using the goodness of fit metrics described in Sect. 3.4 (Tables 6, 7, 8, 9).

As evident, rankings varied depending on the goodness of fit metric used. Although in some other studies goodness of fit metrics corroborated each other [32, 38, 39], similar variability was observed in [55].

Figures 8, 9, 10, 11, 12, 13, 14, 15, 16 and 17 show the details for each goodness of fit metric. This is particularly evident in Figs. 8 and 9 which show rankings by NMSE and Chi-squared metrics, where the Birnbaum–Sanders distribution was particularly ill fitted as compared to Figs. 11 and 12 in which it is comparable when evaluated using the R^2, and AIC and BIC criteria, respectively.

The variability in rankings raises the question of suitability of any given metric to the application. Consequently, some method of determining which goodness-of-fit criterion is best suited to the application has to be found or a new application-specific information criterion (ASIC) has to be formulated.

Application-specific information criterion

Wind models are used to calculate the expected energy generated by wind turbines. In this case, expected energy output over a particular time would be an important consideration in design and investment decisions. The ability of the distribution to accurately estimate this value is crucial.

Consider a wind turbine modeled as a 3MW unit using a piecewise linear model with a cut-in speed (cis) of 3.5 ms^{-1}, rated speed (*rs*) of 14 ms^{-1} and cut-out speed (cos) of 25 ms^{-1} as shown in Fig. 18.

The expected energy output of the turbine over a given period of time is calculated according to Eq. 19.

$$E = \mathrm{hrs} \times \int_{\mathrm{cis}}^{\mathrm{cos}} P(v) \times f(v)\,\mathrm{d}v \tag{19}$$

where $P(v)$ is the turbine power vs speed characteristic (Fig. 18) and $f(v)$ is the distribution function used to model the data.

For this work, the proposed ASIC is defined as a normalized weighted error function (in this case normalized error in expected energy is used), with the weightings defined by the turbine characteristic.

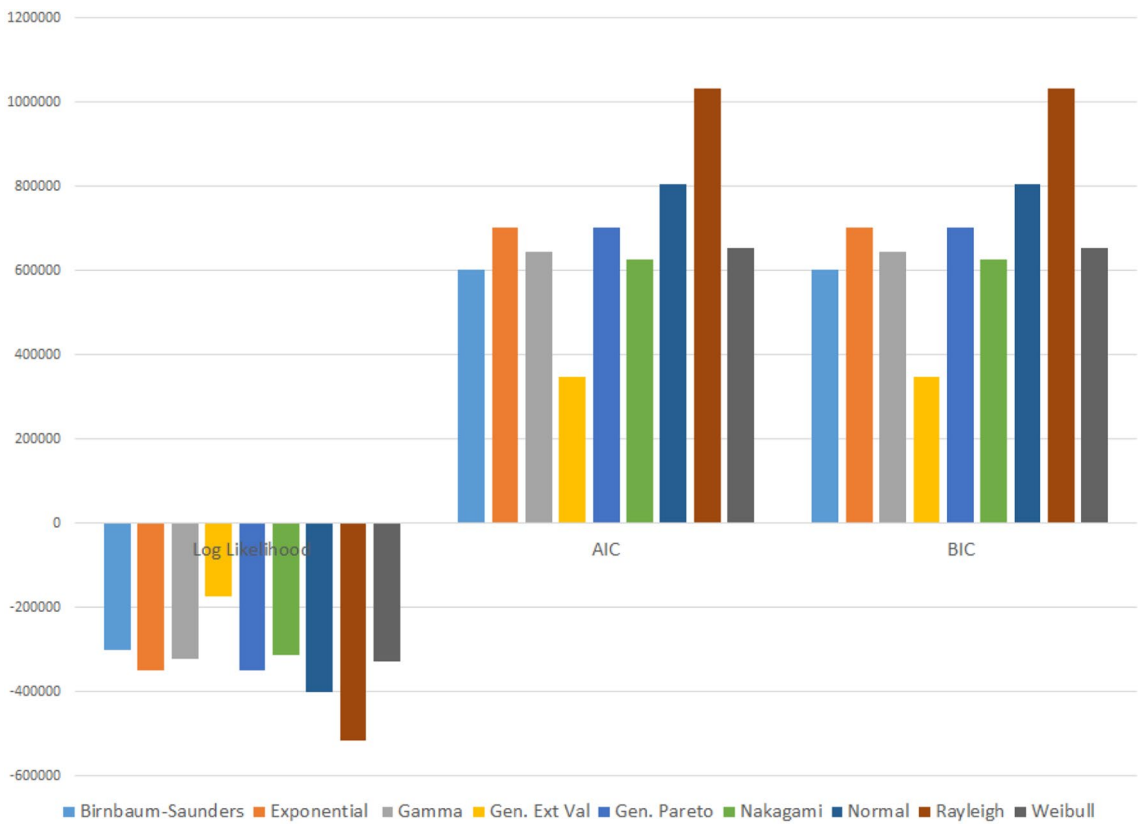
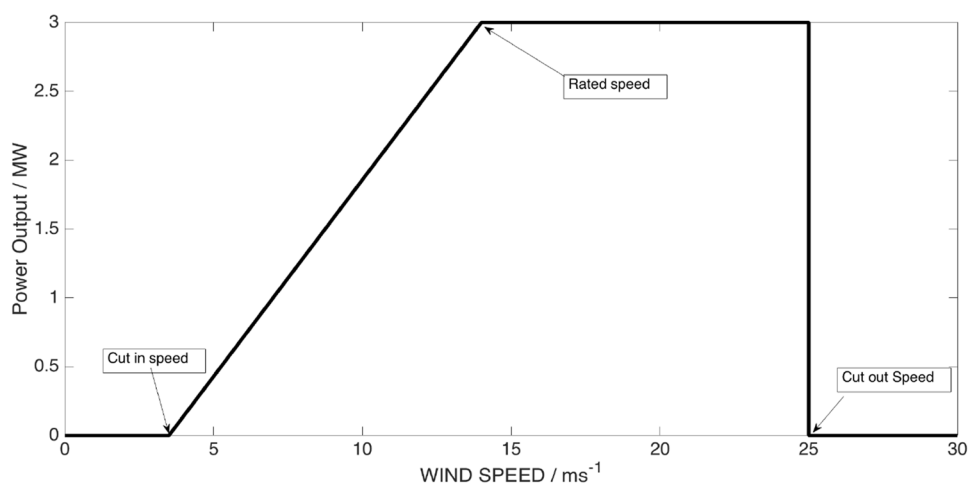


Fig. 17 Piarco—Log-Likelihood, AIC and BIC

Fig. 18 Wind turbine power characteristic



$$\mathrm{ASIC} = \frac{\mathrm{hrs} \times \left[\int_{\mathrm{cis}}^{\mathrm{cos}} P(v) \times \hat{f}(v)\,\mathrm{d}v - \int_{\mathrm{cis}}^{\mathrm{cos}} P(v) \times f(v)\,\mathrm{d}v\right]}{\mathrm{hrs} \times \int_{\mathrm{cis}}^{\mathrm{cos}} P(v) \times f(v)\,\mathrm{d}v},$$
$$= \frac{\int_{\mathrm{cis}}^{\mathrm{cos}} P(v) \times \left(\hat{f}(v) - f(v)\right)}{\int_{\mathrm{cis}}^{\mathrm{cos}} P(v) \times f(v)\,\mathrm{d}v}\,\mathrm{d}v \tag{20}$$

where $\hat{f}(v)$ is the estimated distribution function.

Using this approach, sections of the distribution which contribute more to the application are more heavily weighted than those that do not. In this case, the fit of the distributions below wind speeds of 3.5 ms^{-1} or above 25 ms^{-1} are not as important since the wind turbine does not output any power for those conditions. Using the chosen ASIC, the fit of the

Table 10 Expected energy at Crown Point by distribution

Distribution	Expected energy/MWh	%Difference from actual
Actual	155,270	0
Generalized extreme value	163,810	5.50
Generalized Pareto	142,140	− 8.46
Normal	139,560	− 10.12
Nakagami	139,560	− 14.59
Rayleigh	179,520	15.62
Gamma	81,110	− 47.76
Weibull	72,843	− 53.09
Birnbaum–Saunders	295,810	90.51
Exponential	4.0429E−08	− 100

Table 11 Expected energy predicted at Piarco by distribution

Distribution	Expected energy/MWh	%Difference from actual
Actual	111,560	0
Normal	113,930	2.12
Nakagami	121,920	9.28
Rayleigh	123,060	10.30
Generalized Pareto	97,240	− 12.84
Gamma	77,874	− 30.20
Generalized extreme Value	71,804	− 35.64
Weibull	71,187	− 36.19
Exponential	0.00012435	− 100.00
Birnbaum–Saunders	403,780	261.94

data over the range of power producing speeds of the turbine is assessed. This marks a departure from the philosophy behind other goodness-of-fit tests which equally weight all sections of a distribution or weight them based on probability and do not consider any external information in the determination of goodness of fit.

The actual energy output for Piarco was calculated as approximately 112 GWh, while the value for Crown Point was 155 GWh. Tables 10 and 11 show the percentage difference in energy predicted by the models as compared to the energy derived directly from the wind data. As evident, the results did not match any ranking derived from the conventional goodness-of-fit metrics.

Among the traditional goodness-of-fit tests, the Chi-squared and Kolmogorov–Smirnov tests produced similar results to the ASIC in that they placed similar candidate distributions within the top four ranked distributions, albeit with a different order. This indicates that they may be better suited as goodness-of-fit tests for the purpose of wind energy studies than the other traditional goodness of fit metrics

Table 12 Parameter Estimates for distributions

Distribution	Parameter	Location	
		Crown Point	Piarco
Birnbaum–Saunders	beta	0.1009	1.0274
	gamma	9.636	2.6596
Exponential	lambda	7.3244	5.1314
Gamma	*k*	0.3606	0.4944
	theta	20.314	10.4568
Generalized extreme value	xi	− 0.15	4.2971
	sigma	4.9517	0.437
	mu	5.0824	0.1016
Generalized Pareto	xi	− 0.2299	− 0.2013
	Sigma	8.7412	6.2781
	Theta	0.0001	0.0001
Nakagami	Mu	0.1807	0.2244
	omega	82.1115	51.0448
Normal	sigma	7.3244	4.971
	mu	5.3352	5.1314
Rayleigh	sigma	6.4075	5.052
Weibull	lamda	5.3442	3.9135
	k	0.4825	0.6195

utilised in this paper. Given that the application space is known, however, using an ASIC would still be preferrable since rankings are made according to a parameter (energy in this case) which is meaningful to users of the data.

Finally, it is also noteworthy that the Weibull distribution, which is traditionally used in wind modelling in the Caribbean, performed poorly for both datasets using all the metrics investigated. This is likely due to the large amount of low to zero wind speed measurements. Castellanos [37] has also noted that the Weibull distribution performs poorly when the data contains a large proportion of low wind speeds (Figs. 19, 20; Table 12).

Conclusions

The Weibull distribution was found to perform relatively poorly as a wind probability model for both sites. The Rayleigh distribution performed consistently better than the Weibull but was still ill suited as a model for the data.

The inconsistency in results for the goodness of fit led to the conceptualization of application-specific information criteria (ASIC) as a more meaningful approach for assessment of goodness of fit in cases where the secondary, application-specific features must be calculated from the primary data.

For the application in question, the normalized error in expected energy is used as a goodness-of-fit metric to rank candidate distributions. The advantage of this technique

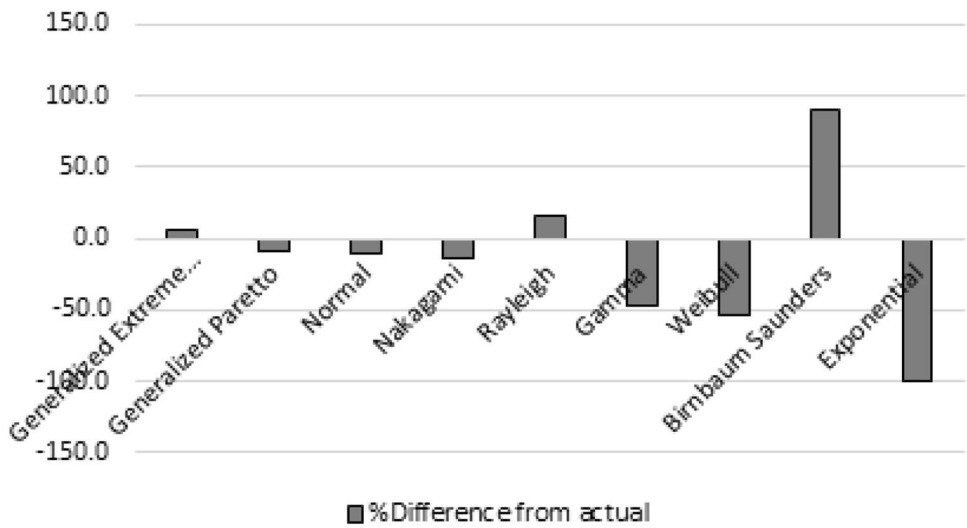


Fig. 19 Crown Point ranked by expected energy

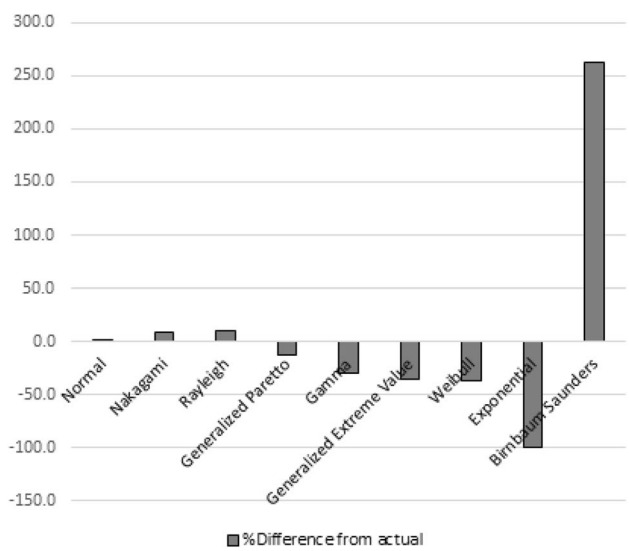


Fig. 20 Piarco ranked by expected energy

is that the distributions can be examined in terms of over-estimation or under-estimation of expected energy as well as the magnitude of deviation while using a metric that is meaningful in the context of the intended application space.

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