Stoch PDE: Anal Comp (2023) 11:1732–1739
https://doi.org/10.1007/s40072-022-00267-5

CORRECTION

Correction to: Convergent numerical approximation of the stochastic total variation flow

Ĺubomír Baňas[1] · Michael Röckner[1] · André Wilke[1]

Received: 6 July 2022 / Revised: 7 July 2022 / Accepted: 8 July 2022 /
Published online: 4 August 2022

Abstract

We correct two errors in our paper [4]. First error concerns the definition of the SVI solution, where a boundary term which arises due to the Dirichlet boundary condition, was not included. The second error concerns the discrete estimate [4, Lemma 4.4], which involves the discrete Laplace operator. We provide an alternative proof of the estimate in spatial dimension $d = 1$ by using a mass lumped version of the discrete Laplacian. Hence, after a minor modification of the fully discrete numerical scheme the convergence in $d = 1$ follows along the lines of the original proof. The convergence proof of the time semi-discrete scheme, which relies on the continuous counterpart of the estimate [4, Lemma 4.4], remains valid in higher spatial dimension. The convergence of the fully discrete finite element scheme from [4] in any spatial dimension is shown in [3] by using a different approach.

Correction to: Stoch PDE: Anal Comp (2021) 9:437–471 https://doi.org/10.1007/s40072-020-00169-4

Funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) - SFB 1283/2 2021 - 317210226.

The original article can be found online at https://doi.org/10.1007/s40072-020-00169-4.

✉ Ĺubomír Baňas
banas@math.uni-bielefeld.de

Michael Röckner
roeckner@math.uni-bielefeld.de

André Wilke
awilke@math.uni-bielefeld.de

1 Department of Mathematics, Bielefeld University, 33501 Bielefeld, Germany

1 Introduction

Let $\mathcal{O} \subset \mathbb{R}^d$ be an open convex domain with piecewise smooth boundary. We consider numerical approximation of the stochastic total variation flow

$$\begin{aligned} \mathrm{d}X &= \operatorname{div}\left(\frac{\nabla X}{|\nabla X|}\right)\mathrm{d}t - \lambda(X-g)\,\mathrm{d}t + X\,\mathrm{d}W, && \text{in } (0,T)\times\mathcal{O},\\ X &= 0 && \text{on } (0,T)\times\partial\mathcal{O},\\ X(0) &= x_0 && \text{in } \mathcal{O}, \end{aligned} \tag{1}$$

which is constructed via the discretization of the regularized problem

$$\begin{aligned} \mathrm{d}X^\varepsilon &= \operatorname{div}\left(\frac{\nabla X^\varepsilon}{\sqrt{|\nabla X^\varepsilon|^2+\varepsilon^2}}\right)\mathrm{d}t - \lambda(X^\varepsilon-g)\,\mathrm{d}t + X^\varepsilon\,\mathrm{d}W && \text{in } (0,T)\times\mathcal{O},\\ X^\varepsilon &= 0 && \text{on } (0,T)\times\partial\mathcal{O},\\ X^\varepsilon(0) &= x_0 && \text{in } \mathcal{O}. \end{aligned} \tag{2}$$

Throughout the paper we employ the notation from [4]. The first error is corrected in Sect. 2 and the correction of the second error is provided in Sect. 3.

2 Definition of the SVI solution and the uniqueness proof

In the proof of [4, Theorem 3.1] the term IV in (29) is wrongly rewritten as

$$IV = -\left(X_1^\varepsilon - X_{2,n}^{\varepsilon,\delta}, \operatorname{div}\frac{\nabla X_{2,n}^{\varepsilon,\delta}}{\sqrt{|\nabla X_{2,n}^{\varepsilon,\delta}|^2+\varepsilon^2}}\right) = \left(\nabla X_1^\varepsilon - \nabla X_{2,n}^{\varepsilon,\delta}, \frac{\nabla X_{2,n}^{\varepsilon,\delta}}{\sqrt{|\nabla X_{2,n}^{\varepsilon,\delta}|^2+\varepsilon^2}}\right)$$

since X_1^ε is only in $BV(\mathcal{O})$ and possibly non-zero at the boundary. Hence, to show the uniqueness of the SVI solutions for $\varepsilon > 0$ requires a modification of the definition which takes into account the value at the solution at the boundary. This definition is consistent with the one from [2], which also shows uniqueness in case $\varepsilon = 0$.

We define the following functionals which include the corresponding boundary terms

$$\bar{\mathcal{J}}_{\varepsilon,\lambda}(u) = \begin{cases} \mathcal{J}_{\varepsilon,\lambda}(u) + \int_{\partial\mathcal{O}} |\gamma_0(u)|\,\mathrm{d}\mathcal{H}^{n-1} & \text{for } u \in BV(\mathcal{O}) \cap L^2(\mathcal{O}),\\ +\infty & \text{for } u \in BV(\mathcal{O}) \setminus L^2(\mathcal{O}), \end{cases}$$

and

$$\bar{\mathcal{J}}_{\lambda}(u) = \begin{cases} \mathcal{J}_{\lambda}(u) + \int_{\partial\mathcal{O}} |\gamma_0(u)|\,\mathrm{d}\mathcal{H}^{n-1} & \text{for } u \in BV(\mathcal{O}) \cap L^2(\mathcal{O}),\\ +\infty & \text{for } u \in BV(\mathcal{O}) \setminus L^2(\mathcal{O}), \end{cases}$$

where $\gamma_0(u)$ is the trace of $u \in BV(\mathcal{O})$ on the boundary and $\mathrm{d}\mathcal{H}^{n-1}$ is the Hausdorff measure on $\partial\mathcal{O}$. $\bar{\mathcal{J}}_{\varepsilon,\lambda}$ and $\bar{\mathcal{J}}_{\lambda}$ are both convex and lower semicontinuous on $L^2(\mathcal{O})$ and the lower semicontinuous hulls of $\bar{\mathcal{J}}_{\varepsilon,\lambda}|_{\mathbb{H}_0^1}$ or $\bar{\mathcal{J}}_{\lambda}|_{\mathbb{H}_0^1}$ respectively, cf. [1, Proposition 11.3.2]. We define the SVI solution as follows.

Definition 2.1 Let $0 < T < \infty$, $\varepsilon \in [0, 1]$ and $x_0 \in L^2(\Omega, \mathcal{F}_0; \mathbb{L}^2)$ and $g \in \mathbb{L}^2$. Then a $(\mathcal{F}_t)$-adapted map $X^\varepsilon \in L^2(\Omega; C([0,T]; \mathbb{L}^2)) \cap L^1(\Omega; L^1((0,T); BV(\mathcal{O})))$ (denoted by $X \in L^2(\Omega; C([0,T]; \mathbb{L}^2)) \cap L^1(\Omega; L^1((0,T); BV(\mathcal{O})))$ for $\varepsilon = 0$) is called an SVI solution of (2) (or (1) if $\varepsilon = 0$) if $X^\varepsilon(0) = x_0$ ($X(0) = x_0$), and for each $(\mathcal{F}_t)$-progressively measurable process $G \in L^2(\Omega \times (0,T), \mathbb{L}^2)$ and for each $(\mathcal{F}_t)$-adapted $\mathbb{L}^2$-valued process Z with $\mathbb{P}$-a.s. continuous sample paths, s.t. $Z \in L^2(\Omega \times (0,T); \mathbb{H}_0^1)$, which together satisfy the equation

$$\mathrm{d}Z(t) = -G(t)\,\mathrm{d}t + Z(t)\,\mathrm{d}W(t),\ \ t \in [0,T],$$

it holds for $\varepsilon \in (0, 1]$ that

$$\begin{aligned}
&\frac{1}{2}\mathbb{E}\left[\|X^\varepsilon(t) - Z(t)\|^2\right] + \mathbb{E}\left[\int_0^t \bar{\mathcal{J}}_{\varepsilon,\lambda}(X^\varepsilon(s))\,\mathrm{d}s\right]\\
&\quad\le \frac{1}{2}\mathbb{E}\left[\|x_0 - Z(0)\|^2\right] + \mathbb{E}\left[\int_0^t \bar{\mathcal{J}}_{\varepsilon,\lambda}(Z(s))\,\mathrm{d}s\right]\\
&\quad+ \frac{1}{2}\mathbb{E}\left[\int_0^t \|X^\varepsilon(s) - Z(s)\|^2\,\mathrm{d}s\right] + \mathbb{E}\left[\int_0^t \big(X^\varepsilon(s) - Z(s), G\big)\,\mathrm{d}s\right],
\end{aligned}\tag{3}$$

and analogously for $\varepsilon = 0$ it holds that

$$\begin{aligned}
&\frac{1}{2}\mathbb{E}\left[\|X(t) - Z(t)\|^2\right] + \mathbb{E}\left[\int_0^t \bar{\mathcal{J}}_{\lambda}(X(s))\,\mathrm{d}s\right]\\
&\quad\le \frac{1}{2}\mathbb{E}\left[\|x_0 - Z(0)\|^2\right] + \mathbb{E}\left[\int_0^t \bar{\mathcal{J}}_{\lambda}(Z(s))\,\mathrm{d}s\right]\\
&\quad+ \frac{1}{2}\mathbb{E}\left[\int_0^t \|X(s) - Z(s)\|^2\,\mathrm{d}s\right] + \mathbb{E}\left[\int_0^t (X(s) - Z(s), G)\,\mathrm{d}s\right].
\end{aligned}\tag{4}$$

The existence of SVI solutions (3), (4) follows as in [4, Theorem 3.1] by the lower semicontinuity of $\bar{\mathcal{J}}_{\varepsilon,\lambda}$, $\bar{\mathcal{J}}_{\lambda}$, respectively. The uniqueness of SVI solution (4) follows from [2, Theorem 3.2].

To show uniqueness of the SVI solution (3) we proceed as in [4, Theorem 3.1] with exception that the term IV in (29) takes a different form. In particular, to obtain uniqueness we have to show the following estimate:

$$IV := \left(X_1^\varepsilon - X_{2,n}^{\varepsilon,\delta}, -\mathrm{div}\frac{\nabla X_{2,n}^{\varepsilon,\delta}}{\sqrt{|\nabla X_{2,n}^{\varepsilon,\delta}|^2 + \varepsilon^2}}\right) \le \bar{\mathcal{J}}_{\varepsilon,0}(X_1^\varepsilon) - \bar{\mathcal{J}}_{\varepsilon,0}(X_{2,n}^{\varepsilon,\delta}).\tag{5}$$

We note that term IV is well defined since $\operatorname{div}\frac{\nabla X_{2,n}^{\varepsilon,\delta}}{\sqrt{|\nabla X_{2,n}^{\varepsilon,\delta}|^2+\varepsilon^2}} \in \mathbb{L}^2$ for a.a. $(\omega, t) \in \Omega \times (0, T)$. Indeed, from [4, Lemma 3.2] for $\delta > 0$, $n < \infty$ we deduce by parabolic regularity theory that $X_{2,n}^{\varepsilon,\delta}(\omega, t) \in \mathbb{H}^2$ for a.a. $(\omega, t) \in \Omega \times (0, T)$ and a direct calculation yields that

$$\left|\operatorname{div}\frac{\nabla X_{2,n}^{\varepsilon,\delta}}{\sqrt{|\nabla X_{2,n}^{\varepsilon,\delta}|^2+\varepsilon^2}}\right| \leq 2\frac{|\nabla X_{2,n}^{\varepsilon,\delta}||\nabla^2 X_{2,n}^{\varepsilon,\delta}|}{\left(|\nabla X_{2,n}^{\varepsilon,\delta}|^2+\varepsilon^2\right)^{\frac{3}{2}}} + \frac{|\Delta X_{2,n}^{\varepsilon,\delta}|}{\sqrt{|\nabla X_{2,n}^{\varepsilon,\delta}|^2+\varepsilon^2}}.$$

We show the inequality in (5) by the integration by parts formula using a density argument. We fix $(\omega, t) \in \Omega \times (0, T)$ and proceed below with $X_1^\varepsilon \equiv X_1^\varepsilon(\omega, t)$, $X_{2,n}^{\varepsilon,\delta} \equiv X_{2,n}^{\varepsilon,\delta}(\omega, t)$. We consider an approximating sequence $x_k \in C^\infty(\mathcal{O}) \cap BV(\mathcal{O})$, s.t. $x_k \to X_1^\varepsilon$ strongly in $\mathbb{L}^1$ and

$$\mathcal{J}_{\varepsilon,0}(x_k) \to \mathcal{J}_{\varepsilon,0}(X_1^\varepsilon) \quad \text{for } k \to \infty, \tag{6}$$

cf., [1, Theorems 10.1.2, 13.4.1 and Remark 10.2.1] or [6, Theorem 5.2].

Note that, since $X_1^\varepsilon(\omega, t) \in \mathbb{L}^2$ it is straightforward to modify the proof of [1, Theorems 10.1.2] (see for instance [1, Proposition 2.2.4] for the $\mathbb{L}^p$ properties of the mollifiers) such that the sequence x_k converges strongly in $\mathbb{L}^2$:

$$\|x_k - X_1^\varepsilon\|_{\mathbb{L}^2} \to 0 \quad \text{for } k \to \infty. \tag{7}$$

Using the integration by parts formula [1, Theorem 10.2.1] we obtain that

$$\begin{aligned}\left(x_k - X_{2,n}^{\varepsilon,\delta}, -\operatorname{div}\frac{\nabla X_{2,n}^{\varepsilon,\delta}}{\sqrt{|\nabla X_{2,n}^{\varepsilon,\delta}|^2+\varepsilon^2}}\right) &= \left(\nabla(x_k - X_{2,n}^{\varepsilon,\delta}), \frac{\nabla X_{2,n}^{\varepsilon,\delta}}{\sqrt{|\nabla X_{2,n}^{\varepsilon,\delta}|^2+\varepsilon^2}}\right)\\ &+ \int_{\partial\mathcal{O}} \gamma_0(x_k)\frac{\nabla X_{2,n}^{\varepsilon,\delta}}{\sqrt{|\nabla X_{2,n}^{\varepsilon,\delta}|^2+\varepsilon^2}} \cdot \nu \, \mathrm{d}\mathcal{H}^{n-1} - \int_{\partial\mathcal{O}} \gamma_0(X_{2,n}^{\varepsilon,\delta})\frac{\nabla X_{2,n}^{\varepsilon,\delta}}{\sqrt{|\nabla X_{2,n}^{\varepsilon,\delta}|^2+\varepsilon^2}} \cdot \nu \, \mathrm{d}\mathcal{H}^{n-1},\end{aligned} \tag{8}$$

where ν is the outer unit normal vector to $\partial\mathcal{O}$ and $\mathcal{H}^{n-1}$ is the Hausdorff measure on $\partial\mathcal{O}$.

Since $X^{\varepsilon,\delta}_{2,n} \in \mathbb{H}^1_0$ it holds that $\gamma_0(X^{\varepsilon,\delta}_{2,n}) = 0$ and the second boundary integral vanishes. The first boundary integral can be estimated as

$$\int_{\partial\mathcal{O}} \gamma_0(x_k) \frac{\nabla X^{\varepsilon,\delta}_{2,n}}{\sqrt{|\nabla X^{\varepsilon,\delta}_{2,n}|^2 + \varepsilon^2}} \cdot \nu \, \mathrm{d}\mathcal{H}^{n-1} \leq \int_{\partial\mathcal{O}} |\gamma_0(x_k)| \left| \frac{\nabla X^{\varepsilon,\delta}_{2,n}}{\sqrt{|\nabla X^{\varepsilon,\delta}_{2,n}|^2 + \varepsilon^2}} \cdot \nu \right| d\mathcal{H}^{n-1}$$
$$\leq \int_{\partial\mathcal{O}} |\gamma_0(x_k)| \, \mathrm{d}\mathcal{H}^{n-1} = \int_{\partial\mathcal{O}} \left|\gamma_0(X^{\varepsilon}_1)\right| \, \mathrm{d}\mathcal{H}^{n-1}, \tag{9}$$

where the last equality follows from the fact that the trace of $x_k \in C^\infty(\mathcal{O}) \cap BV(\mathcal{O})$ coincides with the trace of X^{ε}_1, cf. [1, Remark 10.2.1].

By the convexity of $\mathcal{J}_{\varepsilon,0}$ we deduce that

$$\left(\nabla(x_k - X^{\varepsilon,\delta}_{2,n}), \frac{\nabla X^{\varepsilon,\delta}_{2,n}}{\sqrt{|\nabla X^{\varepsilon,\delta}_{2,n}|^2 + \varepsilon^2}} \right) \leq \mathcal{J}_{\varepsilon,0}(x_k) - \mathcal{J}_{\varepsilon,0}(X^{\varepsilon,\delta}_{2,n}). \tag{10}$$

Hence, (5) follows after substituting (10), (9) into (8) and taking the limit for $k \to \infty$ and noting (6), (7).

The rest of the proof follows analogously to the original proof of [4, Theorem 3.1].

3 Convergence of the full discretization

In the proof of [4, Lemma 4.4] it is concluded that

$$\frac{1}{2} \sum_{K,K' \in \mathcal{T}_h} \bar{v}_h^T A_K^T M^{-1} A_{K'} \bar{v}_h \left((|\nabla v_h|^2 + \varepsilon^2)_K^{-\frac{1}{2}} + (|\nabla v_h|^2 + \varepsilon^2)_{K'}^{-\frac{1}{2}} \right)$$
$$\geq \frac{1}{2} \sum_{K,K' \in \mathcal{T}_h} \sqrt{(|\nabla v_h|^2 + \varepsilon^2)_{K'}^{-\frac{1}{2}}} \, \bar{v}_h^T A_K^T M^{-1} A_{K'} \bar{v}_h \sqrt{(|\nabla v_h|^2 + \varepsilon^2)_K^{-\frac{1}{2}}} \geq 0,$$

which is not justified. Lemma 4.4 is required to obtain the estimate (48) in [4, Lemma 4.5] (note that the continuos counterpart of the estimate in Lemma 3.2 is obtained using Proposition 2.1), which is in turn required to show [4, Theorem 4.1].

In this section we show an analogue of the estimate in [4, Lemma 4.4] for a slightly modified numerical scheme in dimension $d = 1$. Given $J \in \mathbb{N}$ and a mesh size $h = 1/J$ we consider a uniform partiton $\mathcal{T}_h = \cup_{j=1}^J T_j$ of the spatial domain $\mathcal{O} = (0, 1)$ into subintervals $T_j = (x_{i-1}, x_i)$ with nodes $x_j = jh$, $j = 0, \ldots, J$. As in [4] we consider a finite element space $\mathbb{V}_h \subset \mathbb{H}^1_0$ of piecewise linear globally continuous functions on subordinated to $\mathcal{T}_h$. The standard nodal interpolation operator $\mathcal{I}_h : C(\bar{\mathcal{O}}) \to \mathbb{V}_h$ is defined as

$$\mathcal{I}_h \Phi(x_j) = \Phi(x_j) \quad \forall \; j = 0, \ldots, L.$$

We define the discrete (mass-lumped) $\mathbb{L}^2$-inner product $(\cdot, \cdot)_h$ on $\mathbb{V}_h$ as

$$(\varphi, \psi)_h = \int_{\mathcal{O}} \mathcal{I}_h(\langle \varphi, \psi \rangle)(x)\, \mathrm{d}x = h \sum_{j=1}^{J-1} \varphi(x_j), \psi(x_j) \text{ for } \varphi, \psi \in \mathbb{V}_h, \tag{11}$$

with the corresponding discrete norm $\|\psi\|_h^2 = (\psi, \psi)_h$.

It is well known that the above discrete inner product and the norm satisfy (cf. [5]):

$$\|v_h\|_{\mathbb{L}^2} \le \|v_h\|_h \le C\|v_h\|_{\mathbb{L}^2} \qquad \forall\, v_h \in \mathbb{V}_h, \tag{12}$$

$$\big|(v_h, w_h)_h - (v_h, w_h)\big| \le Ch\|v_h\|_{\mathbb{L}^2}\|w_h\|_{\mathbb{H}^1} \qquad \forall\, v_h, w_h \in \mathbb{V}_h. \tag{13}$$

We define the mass-lumped Discrete Laplace operator $\Delta_h : \mathbb{V}_h \to \mathbb{V}_h$ through the identity

$$(\Delta_h v_h, w_h)_h = -(\nabla v_h, \nabla w_h). \tag{14}$$

The next lemma is the counterpart of [4, Lemma 4.4] for the $1d$ discrete Laplace operator (14). Numerical experiments (not stated in this paper) indicate that the result also holds for $d > 1$ (possibly under some additional assumptions on the shape of the mesh). Nevertheless, the proof of the result for $d > 1$ remains open, so far.

Lemma 3.1 *Let Δ_h be the discrete Laplacian defined by* (14). *Then for any $v_h \in \mathbb{V}_h$, $\varepsilon, h > 0$ the following inequality holds:*

$$\left(\frac{\nabla v_h}{\sqrt{|\nabla v_h|^2 + \varepsilon^2}}, \nabla(-\Delta_h v_h)\right) \ge 0.$$

Proof Since $\mathbb{V}_h$ is the space of piecewise linear functions over $\mathcal{T}_h$, it holds for $v_h \in \mathbb{V}_h$ that

$$\delta_x v_h^j := \partial_x v_h(x)\big|_{T_j} = \frac{v_h(x_j) - v_h(x_j)}{h}.$$

By definition (11) and (14) we deduce that

$$\Delta_h v_h^j := \Delta_h v_h(x_j) = \frac{v_h(x_{j+1}) - 2v_h(x_j) + v_h(x_{j-1})}{h^2} = \frac{\delta_x v_h^{j+1} - \delta_x v_h^j}{h},$$

and $\Delta_h v_h^0 = \Delta_h v_h^J = 0$.

By the above properties we deduce that

$$
\begin{aligned}
&\left(\frac{\nabla v_h}{\sqrt{|\nabla v_h|^2+\varepsilon^2}}, \nabla(-\Delta_h v_h)\right) = -\left(\frac{\partial_x v_h}{\sqrt{|\partial_x v_h|^2+\varepsilon^2}}, \partial_x \Delta_h v_h\right) \\
&= -\sum_{j=1}^{J}\int_{T_j} \frac{\partial_x v_h}{\sqrt{|\partial_x v_h|^2+\varepsilon^2}} \partial_x \Delta_h v_h \,\mathrm{d}x \\
&= -h\sum_{j=1}^{J} \frac{\delta_x v_h^j}{\sqrt{|\delta_x v_h^j|^2+\varepsilon^2}} \delta_x \Delta_h v_h^j = -\sum_{j=1}^{J} \frac{\delta_x v_h^j}{\sqrt{|\delta_x v_h^j|^2+\varepsilon^2}} \left(\Delta_h v_h^j - \Delta_h v_h^{j-1}\right) \\
&= -\sum_{j=1}^{J} \frac{\delta_x v_h^j}{\sqrt{|\delta_x v_h^j|^2+\varepsilon^2}} \Delta_h v_h^j + \sum_{j=1}^{J} \frac{\delta_x v_h^j}{\sqrt{|\delta_x v_h^j|^2+\varepsilon^2}} \Delta_h v_h^{j-1} \\
&= -\sum_{j=1}^{J-1} \frac{\delta_x v_h^j}{\sqrt{|\delta_x v_h^j|^2+\varepsilon^2}} \Delta_h v_h^j + \sum_{j=1}^{J-1} \frac{\delta_x v_h^{j+1}}{\sqrt{|\delta_x v_h^{j+1}|^2+\varepsilon^2}} \Delta_h v_h^j \\
&= \sum_{j=1}^{J-1} \left(\frac{\delta_x v_h^{j+1}}{\sqrt{|\delta_x v_h^{j+1}|^2+\varepsilon^2}} - \frac{\delta_x v_h^j}{\sqrt{|\delta_x v_h^j|^2+\varepsilon^2}}\right) \Delta_h v_h^j \\
&= \frac{1}{h}\sum_{j=1}^{J-1} \left(\frac{\delta_x v_h^{j+1}}{\sqrt{|\delta_x v_h^{j+1}|^2+\varepsilon^2}} - \frac{\delta_x v_h^j}{\sqrt{|\delta_x v_h^j|^2+\varepsilon^2}}\right) (\delta_x v_h^{j+1} - \delta_x v_h^j) \\
&\geq 0
\end{aligned}
$$

where we used the convexity of $\sqrt{|\cdot|^2+\varepsilon^2}$ to deduce the last inequality. □

Using the above lemma one can show the convergence for a slight modification of the fully discrete numerical scheme of [4] where the standard $\mathbb{L}^2$-inner product is replaced by the discrete inner product (11) as follows: given $x_0,\ g \in \mathbb{L}^2$ we set $X_{\varepsilon,h}^0 = \mathcal{P}_h x_0$, $g^h := \mathcal{P}_h g$ and obtain $X_{\varepsilon,h}^i$ for $i = 1, \dots, N$ as the solution of the following system:

$$
\begin{aligned}
\left(X_{\varepsilon,h}^i, v_h\right)_h &= \left(X_{\varepsilon,h}^{i-1}, v\right)_h - \tau\left(\frac{\nabla X_{\varepsilon,h}^i}{\sqrt{|\nabla X_{\varepsilon,h}^i|^2+\varepsilon^2}}, \nabla v_h\right) \\
&\quad - \tau\lambda\left(X_{\varepsilon,h}^i - g^h, v_h\right)_h + \left(X_{\varepsilon,h}^{i-1}, v_h\right)_h \Delta_i W \quad \forall\, v_h \in \mathbb{V}_h. \qquad (15)
\end{aligned}
$$

By the equivalence of the norms $\|\cdot\|$ and $\|\cdot\|_h$ (cf. (12)) the convergence of the above numerical approximation for $d = 1$ follows as in [4] with [4, Lemma 4.4] replaced by Lemma 3.1.

We note that the convergence proof remains valid for $d \geq 1$ in the case of the time-semi discrete variant of the original numerical scheme from [4]:

$$\left(X_\varepsilon^i, \varphi\right) = \left(X_\varepsilon^{i-1}, \varphi\right) - \tau \left(\frac{\nabla X_\varepsilon^i}{\sqrt{|\nabla X_\varepsilon^i|^2 + \varepsilon^2}}, \nabla\varphi\right) - \tau\lambda \left(X_\varepsilon^i - g, \varphi\right) + \left(X_\varepsilon^{i-1}, \varphi\right) \Delta_i W \qquad \forall\, \varphi \in \mathbb{H}_0^1.$$

In the semi-discrete setting one employs the continuous counterpart of Lemma 3.1 and proceeds as in the proof of [4, Lemma 3.2] to obtain the space-continuous version of the stronger estimate (48) in Lemma 4.5 from [4]. Then the convergence proof of the above semi-discrete numerical scheme follows analogically as in the case of the fully discrete numerical approximation; we skip the detailed exposition for brevity and instead refer to [3, Section 4], from where the necessary components of the proof can be deduced.

Finally, we conclude that a convergence proof of the fully discrete numerical approximation for $d \geq 1$, which avoids the use of [4, Lemma 4.4], is provided in the upcoming paper [3].

Acknowledgements The authors would like to thank Martin Ondreját for pointing the two mistakes in the original paper to us.

References

1. Attouch, H., Buttazzo, G., Michaille, G.: Variational analysis in Sobolev and BV spaces, volume 6 of MPS/SIAM Series on Optimization. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA; Mathematical Programming Society (MPS), Philadelphia, PA (2006)
2. Barbu, V., Röckner, M.: Stochastic variational inequalities and applications to the total variation flow perturbed by linear multiplicative noise. Arch. Ration. Mech. Anal. **209**(3), 797–834 (2013)
3. Baňas, Ĺ., Röckner, M., Wilke, A.: Convergent numerical approximation of the stochastic total variation flow: the higher dimensional case. preprint
4. Baňas, Ĺ., Röckner, M., Wilke, A.: Convergent numerical approximation of the stochastic total variation flow. Stoch. Partial Differ. Equ. Anal. Comput. **9**(2), 437–471 (2021)
5. Brenner, S.C., Scott, L.R.: The Mathematical Theory of Finite Element Methods, 2nd edn. Springer, New York (2002)
6. Temam, R.: Problèmes mathématiques en plasticité. Méthodes Mathématiques de l'Informatique [Mathematical Methods of Information Science], vol. 12. Gauthier-Villars, Montrouge (1983)

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.