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v_c -Noetherian domains and Krull domains

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Abstract Let D be an integrally closed domain, $\{V_\alpha\}$ be the set of t -linked valuation overrings of D , and v_c be the star operation on D defined by $I^{v_c} = \bigcap_\alpha IV_\alpha$ for all nonzero fractional ideals I of D . In this paper, among other things, we prove that D is a v_c -Noetherian domain if and only if D is a Krull domain, if and only if $v_c = v$ and every prime t -ideal of D is a maximal t -ideal. As a corollary, we have that if D is one-dimensional, then $v_c = v$ if and only if D is a Dedekind domain.

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1 Introduction

Let D be an integral domain. It is well known that there are at least four star operations on D , say, d , v , t , and w . (Definitions related to star operations will be reviewed in Sect. 2). It has been studied by many researchers when $*_1 = *_2$ for $*_i = d, v, t$, or w . It is known that if D is integrally closed, then $d = t$ if and only if D is a Prüfer domain [8, Proposition 34.12] and $w = t$ if and only if D is a Prüfer v -multiplication domain (PvMD) [12, Theorem 3.5]. In [9], Heinzer studied when $d = v$; in particular, he showed that if D is integrally closed, then $d = v$ if and only if D is an h -local Prüfer domain in which each maximal ideal is invertible [9, Theorem 5.1]. Houston–Zafrullah [10] studied an integral domain in which each nonzero t -ideal is a v -ideal. Among other things, they showed that D is a PvMD with $t = v$ if and only if D is an independent ring of Krull type whose maximal t -ideals are t -invertible [10, Theorem 3.1]. In [11], Hwang–Chang studied PvMDs on which $t = v$. Mimouni studied integral domains on which $w = t$ [13] and $d = w$ [14]. Also, Picozza and Tartarone studied integral domains on which $d = w$ [15]. El Bagdadi and Gabelli studied integral domains with $w = v$; in particular, they showed that if D is integrally closed, then $w = v$ if and only if D is an independent ring of Krull type in which each maximal t -ideal is t -invertible [4, Theorem 3.3].

If D is integrally closed, then there are two more star operations b and v_c that are *e.a.b.* star operations of finite character. Fontana–Picozza [6] studied some classes of integral domains defined by the b -operation (in the more general setting of semistar operations). For example, they showed that if D is integrally closed, then (i) $b = w$ if and only if $b = d$, if and only if D is a Prüfer domain [6, Proposition 34]; (ii) $b = t$ if and only if D is a v -domain on which b is a unique *e.a.b.* star operation of finite character [6, Proposition 35]; (iii) $b = v$ if and only if D is an h -local Prüfer domain whose maximal ideals are finitely generated (hence invertible) [6, Proposition 32]; (iv) if D is b -Noetherian, then D is a Krull domain [6, Theorem 21]; and (v) D is a one-dimensional b -Noetherian domain if and only if D is a Dedekind domain [6, Theorem 11]. These results are the motivation of this paper.

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This paper consists of four sections including introduction. In Sect. 2, we review the definitions and preliminary results related to star operations. Then, in Sect. 3, we study the integrally closed domains on which $v_c = d, b, v, t$, or w . We also give an example of integrally closed domains on which $v_c \neq d, b, w, t$, and v . Finally, in Sect. 4, we prove that if D is integrally closed, then D is a v_c -Noetherian domain if and only if D is a Krull domain, if and only if $v_c = v$ and each prime t -ideal of D is a maximal t -ideal. As a corollary, we have that if D is one-dimensional, then $v_c = v$ if and only if D is a Dedekind domain.

2 Definitions related to star operations

Let D be an integral domain with quotient field K . An *overring* of D means a subring of K containing D . Let $\mathbf{F}(D)$ be the set of nonzero fractional ideals of D and $\mathbf{f}(D)$ be the set of nonzero finitely generated fractional ideals of D ; so $\mathbf{f}(D) \subseteq \mathbf{F}(D)$, and $\mathbf{f}(D) = \mathbf{F}(D)$ if and only if D is Noetherian. A mapping $I \mapsto I^*$ of $\mathbf{F}(D)$ into $\mathbf{F}(D)$ is called a *star operation* on D if for all $0 \neq a \in K$ and $I, J \in \mathbf{F}(D)$, (i) $(aD)^* = aD$ and $(aI)^* = aI^*$, (ii) $I \subseteq I^*$; $I \subseteq J$ implies $I^* \subseteq J^*$, and (iii) $(I^*)^* = I^*$. Given a star operation $*$ on D , the $*$ -operation is defined by $I^{*f} = \bigcup \{J^* \mid J \subseteq I \text{ and } J \in \mathbf{f}(D)\}$; and the $*$ -operation is given by setting $I^{*w} = \{x \in K \mid xJ \subseteq I \text{ for some } J \in \mathbf{f}(D) \text{ with } J^* = D\}$ for all $I \in \mathbf{F}(D)$. A star operation $*$ is said to be of *finite character* if $*_f = *$. Clearly, $(*)_f = *_f$ and $(*)_w = *_w$; so, $*$ and $*$ are of finite character. An $I \in \mathbf{F}(D)$ is called a $*$ -ideal if $I^* = I$, while a $*$ -ideal is a *maximal $*$ -ideal* if it is maximal among proper integral $*$ -ideals of D . Let $*\text{-Max}(D)$ denote the set of maximal $*$ -ideals of D . We know that if D is not a field, then $*_f\text{-Max}(D) \neq \emptyset$ and $*_f\text{-Max}(D) = *_w\text{-Max}(D)$. For any two star operations $*_1$ and $*_2$ on D , we mean by $*_1 \leq *_2$ that $I^{*1} \subseteq I^{*2}$ for all $I \in \mathbf{F}(D)$. It is easy to see that if $*_1 \leq *_2$, then $(*_1)_f \leq (*_2)_f$ and $(*_1)_w \leq (*_2)_w$. Also, $*_w \leq *_f \leq *$ for any star operation $*$ on D .

The most well-known examples of star operations are the d -, v -, t -, and w -operations. The d -operation is just the identity function on $\mathbf{F}(D)$, i.e., $I^d = I$ for all $I \in \mathbf{F}(D)$; so, $d = d_f = d_w$. The v -operation is defined by $I^v = (I^{-1})^{-1}$, where $I^{-1} = \{x \in K \mid xI \subseteq D\}$, and the t -operation (resp., w -operation) is defined by $t = v_f$ (resp., $w = v_w$). Let $*$ be a star operation on D . It is known that $I^v = \bigcap \{xD \mid x \in K \text{ and } I \subseteq xD\}$ [8, Theorem 34.1], and hence, $I \subseteq I^* \subseteq I^v$ for any $I \in \mathbf{F}(D)$. Thus, $d \leq * \leq v$, $d \leq *_f \leq t$, and $d \leq *_w \leq w$. Clearly, each t -ideal is a $*$ -ideal, and thus, each maximal $*$ -ideal is a t -ideal if and only if $*_w = w$. As in [2, page 224], we say that an overring R of D is $*$ -linked over D if $I^* = D$ implies $(IR)^v = R$ for all $I \in \mathbf{f}(D)$. Note that if $I \in \mathbf{f}(D)$, then $I^* = D \Leftrightarrow I^{*f} = D \Leftrightarrow I^{*w} = D$; hence, $*$ -linkedness $= *_f$ -linkedness $= *_w$ -linkedness. Moreover, if $*_1 \leq *_2$ are star operations on D , then the $*_2$ -linked overrings of D are $*_1$ -linked over D .

A star operation $*$ on D is called an *endlich arithmetisch brauchbar* (*e.a.b.*) star operation if $(AB)^* \subseteq (AC)^*$ for all $A, B, C \in \mathbf{f}(D)$ implies $B^* \subseteq C^*$. Obviously, $*$ is *e.a.b.* if and only if $*_f$ is an *e.a.b.* star operation. It is known that if D admits an *e.a.b.* star operation, then D is integrally closed [8, Corollary 32.8]. For the converse, note that D is integrally closed if and only if $D = \bigcap V$, where V ranges over all valuation overrings of D . Therefore, if D is integrally closed, then the mapping $I \mapsto I^b = \bigcap IV$ of $\mathbf{F}(D)$ into $\mathbf{F}(D)$ is an *e.a.b.* star operation of finite character [8, pp. 396–398]. More generally, we have the following.

Lemma 2.1 ([3, Lemma 3.1]). *Let D be an integrally closed domain, $*$ be a star operation on D , and $\{V_\alpha\}$ be the set of $*$ -linked valuation overrings of D . Then, the map $*_c : \mathbf{F}(D) \rightarrow \mathbf{F}(D)$, given by $I \mapsto I^{*c} = \bigcap_\alpha IV_\alpha$, is an *e.a.b.* star operation of finite character on D , such that $*_w = (*_c)_w \leq *_c$ and $*_f\text{-Max}(D) = *_c\text{-Max}(D)$. In particular, $d_c = b$.*

An $I \in \mathbf{F}(D)$ is said to be $*$ -invertible if $(II^{-1})^* = D$. Clearly, $I \in \mathbf{F}(D)$ is $*_f$ -invertible if and only if $II^{-1} \not\subseteq P$ for all $P \in *_f\text{-Max}(D)$. We say that D is a *Prüfer $*$ -multiplication domain* (P^*MD) if each nonzero finitely generated ideal of D is $*_f$ -invertible. The next result is a very nice characterization of P^*MD s, which is essential to the subsequent arguments of this paper.

Lemma 2.2 *The following statements are equivalent for an integral domain D .*

- (1) D is a P^*MD .
- (2) $*_w$ is an *e.a.b.* star operation.
- (3) D is integrally closed and $*_w = *_c$.

Proof (1) $\Leftrightarrow$ (2) [5, Theorem 3.1].

(1) $\Leftrightarrow$ (3) This appears in [3, Theorem 3.7]. □

Let $*$ be a star operation on D . We say that D is $*$ -Noetherian if D satisfies the ascending chain condition on integral $*$ -ideals of D . Hence, d -Noetherian domains are just the Noetherian domains, v -Noetherian domains are Mori domains, and w -Noetherian domains are strong Mori domains. Clearly, if D is $*$ -Noetherian, then $*_f = *$. Also, if $*_1 \leq *_2$ are star operations on D , then $*_1$ -Noetherian domains are $*_2$ -Noetherian. Hence, Noetherian domain $\Rightarrow$ strong Mori domain $\Rightarrow$ Mori domain.

An integral domain D is h -local if each nonzero ideal of D is contained in only finitely many maximal ideals and each nonzero prime ideal is contained in a unique maximal ideal. We say that D is an *independent ring of Krull type* if D is a PvMD in which each nonzero ideal of D is contained in only finitely many maximal t -ideals and each nonzero prime t -ideal is contained in a unique maximal t -ideal. It is easy to see that an h -local Prüfer domain is an independent ring of Krull type, and the converse holds if each maximal ideal is a t -ideal.

3 The v_c -operation on integrally closed domains

Let D be an integrally closed domain. As we noted in Sect. 2, there are at least six star operations on D , say, d, b, v, t, w , and v_c , such that:

where $*_1 \longrightarrow *_2$ means $*_1 \leq *_2$ for $*_i = d, b, v, t, w$, or v_c . In this section, we study when $d = v_c$ (Theorem 3.1), $b = v_c$ (Theorem 3.2), $w = v_c$ (Theorem 3.3), $v_c = t$ (Corollary 3.4), and $v_c = v$ (Corollary 3.5).

Our first result is the v_c -operation analogue of [6, Proposition 34] that $b = d$ if and only if D is a Prüfer domain.

Theorem 3.1 (cf. [6, Proposition 34]) *The following statements are equivalent for an integrally closed domain D .*

- (1) $d = b$.
- (2) $b = w$.
- (3) $d = v_c$.
- (4) $d = t$.
- (5) D is a Prüfer domain.

Proof (1) $\Rightarrow$ (5) Since $d_w = d$, by Lemma 2.2, D is a PdMD, which is exactly a Prüfer domain.

(5) $\Rightarrow$ (4) It is well known and easy to see that an invertible ideal is a t -ideal. Thus, the result follows.

(4) $\Rightarrow$ (3) $\Rightarrow$ (2) These follow from the fact that $d \leq w \leq v_c \leq t$ and $d \leq b \leq v_c$.

(2) $\Rightarrow$ (1) By Lemma 2.1, $d = b_w$, so if $b = w$, then $d = b_w = w_w = w$. Thus, $d = b$. $\square$

Theorem 3.2 (cf. [14, Proposition 2.2]) *The following statements are equivalent for an integrally closed domain D .*

- (1) $d = w$.
- (2) $b = v_c$.
- (3) Every maximal ideal of D is a t -ideal.

Proof This follows directly from the fact that $*_c\text{-Max}(D) = *_f\text{-Max}(D)$ for any star operation $*$ on D , $b_w = d$, $(v_c)_w = w$, $d_c = b$, and $w_c = v_c$ by Lemma 2.1. $\square$

It is known that D is a PvMD if and only if D is integrally closed and $t = w$ [12, Theorem 3.5]. We next give another proof of this result.

Theorem 3.3 *The following statements are equivalent for an integrally closed domain D .*

- (1) $w = t$.
- (2) $w = v_c$.
- (3) D is a PvMD.

Proof (1) $\Rightarrow$ (2) This follows, because $w \leq v_c \leq t$.

(2) $\Rightarrow$ (3) This is an immediate consequence of Lemma 2.2.

(3) $\Rightarrow$ (1) Note that w and t are of finite character; so, it suffices to show that $I^w = I^t$ for all nonzero finitely generated ideal I of D . Let I be a nonzero finitely generated ideal of D . Then, I is t -invertible, and hence, I is w -invertible, because $t\text{-Max}(D) = w\text{-Max}(D)$. Therefore, $I^t = (II^{-1})^w I^t \subseteq ((II^{-1})^w I^t)^w = (II^{-1} I^t)^w = (I(I^{-1} I^t)^w)^w = I^w \subseteq I^t$, and thus, $I^w = I^t$. $\square$

Recall that D is a v -domain if the v -operation (equivalently, t -operation) on D is an $e.a.b.$ star operation. It is known that D is a v -domain if and only if each nonzero finitely generated ideal of D is v -invertible [8, Theorem 34.6]. Clearly, PvMDs are v -domains, and if $v = t$, then v -domains are PvMDs.

Corollary 3.4 *Let D be an integrally closed domain.*

- (1) *If $v_c = t$, then D is a v -domain.*
- (2) *If every maximal t -ideal of D is a v -ideal, then $v_c = t$ if and only if D is a PvMD.*
- (3) *[6, Proposition 35]) $b = t$ if and only if D is a v -domain and D has a unique $e.a.b.$ star operation of finite character.*

Proof (1) If $v_c = t$, then t is an $e.a.b.$ star operation, and since $t = v_f$, v is also an $e.a.b.$ star operation. Thus, D is a v -domain.

(2) If $v_c = t$, then by (1), D is a v -domain, and hence, each nonzero finitely generated ideal of D is v -invertible. However, since each maximal t -ideal is a v -ideal, v -invertible ideals must be t -invertible. Thus, D is a PvMD. The converse follows directly from Theorem 3.3.

(3) If $b = t$, then $v_c = t$, and hence, D is a v -domain. Also, recall that if $*$ is an $e.a.b.$ star operation of finite character, then $b \leq * \leq t$. Hence, b is a unique $e.a.b.$ star operation of finite character. Conversely, if D is a v -domain, then t is an $e.a.b.$ star operation of finite character. Thus, $b = t$ by the uniqueness of an $e.a.b.$ star operation of finite character. $\square$

It is known that if D is integrally closed, then $d = v$ if and only if D is an h -local Prüfer domain in which each maximal ideal is invertible [9, Theorem 5.1], if and only if $b = v$ [6, Proposition 32].

Corollary 3.5 *The following statements are equivalent for an integrally closed domain D .*

- (1) $w = v$.
- (2) $v_c = v$.
- (3) D is a PvMD and $t = v$.
- (4) D is an independent ring of Krull type in which each maximal t -ideal is t -invertible.

Proof (1) $\Rightarrow$ (2) This follows, because $w \leq v_c \leq v$.

(2) $\Rightarrow$ (3) Clearly, $t = v$, because $v_c \leq t \leq v$. Also, by Corollary 3.4, D is a v -domain, and since $t = v$, we have that D is a PvMD.

(3) $\Rightarrow$ (1) By Theorem 3.3, $w = t$, and thus, $w = v$.

(3) $\Leftrightarrow$ (4) [10, Theorem 3.1]. $\square$

We next give an example of integral domains on which $v_c \neq d, b, w, t$, and v .

Example 3.6 Let R be an integrally closed domain which is not a v -domain, X be an indeterminate over R , and $D = R[X]$ be the polynomial ring over R .

- (1) D is integrally closed but not a v -domain (cf. [7, Theorem 4.1]).
- (2) D is not a Prüfer domain, and so, $v_c \neq d$ by Theorem 3.1.
- (3) There is a maximal ideal of D that is not a t -ideal (e.g., $M + XR[X]$ for a maximal ideal M of R). Hence, $v_c \neq b$ by Theorem 3.2.
- (4) Since D is not a v -domain (hence not a PvMD), $v_c \neq w, t$, and v by Theorem 3.3, Corollaries 3.4 and 3.5.

For an explicit example, let $\mathbb{R}$ be the field of real numbers, $\overline{\mathbb{Q}}$ be the algebraic closure of $\mathbb{Q}$ in $\mathbb{R}$, y be an indeterminate over $\mathbb{R}$, and $R = \overline{\mathbb{Q}} + y\mathbb{R}[y]$. Then, R is an integrally closed domain which is not a v -domain [7, page 161].

4 v_c -Noetherian domains

In [6], Fontana-Picozza studied b -Noetherian domains in a more general setting of semistar operations. They showed that if $\dim(D) = 1$, then D is a b -Noetherian domain if and only if D is a Dedekind domain [6, Theorem 18]. They also proved that if D is an integrally closed b -Noetherian domain, then D is a Krull domain [6, Theorem 21]. This is a generalization of the well-known result that an integrally closed Noetherian domain is a Krull domain [8, Theorem 43.4]. We next give a v_c -Noetherian domain analogue.

Theorem 4.1 *The following statements are equivalent for an integrally closed domain D .*

- (1) D is a v_c -Noetherian domain.
- (2) D is a Krull domain.
- (3) $v_c = v$ and every prime t -ideal of D is a maximal t -ideal.
- (4) D is a strong Mori domain.

Proof (1) $\Rightarrow$ (2) Recall that D is a Krull domain if and only if D is a completely integrally closed Mori domain [10, Theorem 2.3]. Note also that $v_c \leq t$; so, each t -ideal is a v_c -ideal, and thus, v_c -Noetherian domains are Mori domains. Hence, it suffices to show that D is completely integrally closed.

Let x be a nonzero element in the quotient field of D , such that x is almost integral over D . Then, there is a nonzero ideal I of D , such that $xI \subseteq I$; so, $xI^{v_c} \subseteq I^{v_c}$. Since D is v_c -Noetherian, there is a nonzero finitely generated ideal J of D , such that $J^{v_c} = I^{v_c}$. Now, let $\{V_\alpha \mid \alpha \in \Lambda\}$ be the set of t -linked valuation overrings of D . Then, $\bigcap_{\alpha \in \Lambda} V_\alpha = D$ and $J^{v_c} V_\alpha = J V_\alpha$ for all $\alpha \in \Lambda$. Hence, $xJ V_\alpha = xJ^{v_c} V_\alpha \subseteq J^{v_c} V_\alpha = J V_\alpha$, and since J is finitely generated, we have $x \in xJ V_\alpha \subseteq V_\alpha$. Therefore, $x \in \bigcap_{\alpha \in \Lambda} V_\alpha = D$. Thus, D is completely integrally closed.

(2) $\Rightarrow$ (1) Recall that Krull domains are PvMDs with $t = v$. Hence, $v_c = v$ by Corollary 3.5. Note also that Krull domains are v -Noetherian. Thus, D is v_c -Noetherian.

(2) $\Rightarrow$ (3) It is well known that each prime t -ideal of a Krull domain is a maximal t -ideal. Thus, the result follows from Corollary 3.5.

(3) $\Rightarrow$ (2) Since each prime t -ideal is a maximal t -ideal, by Corollary 3.5, each prime t -ideal is t -invertible. Thus, D is a Krull domain [10, Theorem 2.3].

(2) $\Leftrightarrow$ (4) This appears in [16, Theorem 2.8]. □

Corollary 4.2 (cf. [6, Theorem 18]) *The following statements are equivalent for a one-dimensional integrally closed domain D .*

- (1) D is a b -Noetherian domain.
- (2) D is a v_c -Noetherian domain.
- (3) $v_c = v$.
- (4) D is a Krull domain.
- (5) D is a Dedekind domain.
- (6) D is a Noetherian domain.

Proof (1) $\Rightarrow$ (2) Since $b \leq v_c$, b -Noetherian domains are v_c -Noetherian. Thus, D is a v_c -Noetherian domain.

(2) $\Leftrightarrow$ (3) $\Leftrightarrow$ (4) These follow directly from Theorem 4.1.

(4) $\Rightarrow$ (5) This is well known [8, Theorem 43.16].

(5) $\Rightarrow$ (1) It is known that Dedekind domains are Noetherian and Noetherian domains are d -Noetherian. Thus, $d \leq b$ implies that D is b -Noetherian.

(5) $\Leftrightarrow$ (6) [8, Theorem 37.8]. □

Clearly, Noetherian $\Rightarrow b$ -Noetherian $\Rightarrow v_c$ -Noetherian $\Rightarrow v$ -Noetherian. We close this paper with two examples of integral domains that (i) v -Noetherian domains that are not v_c -Noetherian and (ii) v_c -Noetherian domains that are not b -Noetherian.

Example 4.3 (1) Let D be an integrally closed Mori domain that is not a Krull domain. Then, D is a v -Noetherian domain but not a v_c -Noetherian domain by Theorem 4.1. For a concrete example, let F be a field, y, z be indeterminates over F , $F(y, z)$ be the quotient field of the polynomial ring $F[y, z]$, X be an indeterminate over $F(y, z)$, $F(y, z)[[X]]$ be the power series ring over $F(y, z)$ and $D = F + XF(y, z)[[X]]$. Then, D is an integrally closed Mori domain [1, Theorem 3.2], but, clearly, D is not a Krull domain.

(2) Let $\{X_\alpha\}$ be an infinite set of indeterminates over a field F and $D = F[[X_\alpha]]$ be the polynomial ring over D . Then, D is a Krull domain, and hence, D is a v_c -Noetherian domain. However, note that each prime ideal of D is a b -ideal and D has an infinite chain of prime ideals. Thus, D is not a b -Noetherian domain.

Obviously, b -Noetherian domains are integrally closed, and hence, Noetherian domains that are not integrally closed are not b -Noetherian. However, we do not know any example of b -Noetherian domains that are not Noetherian.

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