



EDITORIAL

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Preface

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The Arabian Journal of Mathematics (AJM) publishes original research articles covering all branches of Pure and Applied Mathematics and Mathematical Physics. More recently, the journal has been receiving articles in fields such as Differential and Riemannian Geometry and Classical Relativity. On the other hand, recent detection of gravitational waves from the coalescence of gravitational objects in close black hole and neutron star binaries by LIGO–VIRGO scientific collaboration, confirmation of black hole Sagittarius A* status at the Milky Way center by ESO’s GRAVITY instrument and first detection of the black hole shadow in the center of elliptic galaxy M87 by the Event Horizon Telescope led to the rapidly growing interest in the study of relativistic astrophysical compact objects. In the light of these recent successes of general theory of relativity, AJM decided to further widen its scope by opening it to researchers working in areas such as Mathematical Physics, General Relativity, and Relativistic Astrophysics. It is in this spirit that AJM is publishing two special issues on Mathematical Physics, General Relativity and Relativistic Astrophysics.

Well-known scientists working in the aforementioned areas were invited to write research or review articles. Of these invited scientists, ten accepted the invitation to contribute. The received papers were classified into two categories, (1) Mathematical Physics & General Relativity and (2) Relativistic Astrophysics. Based on these two categories of papers, the journal decided to publish two special issues with first including papers belonging to Mathematical Physics & General Relativity and the second to include papers on Relativistic Astrophysics. The first volume was published online in May 2019.

A brief overview of the articles appearing in the second issue is as follows:

The first paper by Abdujabbarov, Hakimov, Turimov and Tursunov discusses “Effects of geometric optics in conformal Weyl gravity”. In their article the authors investigate effects of geometric optics as the rotation of polarization vector of light in space–time of gravitationally compact objects in the fourth-order theory of conformal Weyl gravity which is invariant under local conformal transformation. Applying the Pineault–Roeder method to rotation of polarization vector of electromagnetic waves propagating in spacetime of compact object in conformal Weyl gravity, analytic results are obtained in the limit of weak field and slow rotation. For photons traveling parallel to the symmetry axes from the equatorial plane to infinity, it is shown that the rotation of the polarization plane depends on the Weyl parameter γ contrary to the Kerr space–time where there is no rotation of polarization plane for the considered case. Using observational data of the well-known

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blazar 3C 273, located at the distance 749Mpc, the authors estimate that the upper limit on the Weyl parameter is $\gamma \leq 2.10^{-21} \text{cm}^{-1}$.

The second paper entitled “Gravitational collapse of baryonic and dark matter” is contributed by Dey and Joshi. When the pressure inside a collapsing star becomes insufficient to balance the gravitational pull, a massive star undergoes a gravitational collapse whose final fate can be a black hole or a naked singularity depending on the initial conditions of the gravitational collapse. While stars are made up of baryonic matter, there is good indirect evidence that dark matter plays an important role in the formation of large-scale structures in the Universe such as galaxies. It is estimated that around 85% of the total matter in the Universe is dark matter. Knowing that the particle constituent of the dark matter is not yet known, the authors consider the basic properties of the baryonic and dark matter collapse and present their view of the final state of the gravitational collapse in different scenarios. By using the general relativistic techniques of equilibrium, authors present a modification in the Top-Hat collapse model of primordial dark matter, it is argued why the modified collapse process is more likely to take place for the dark matter fields and may describe primordial dark-matter halo formation. It is shown small pressure in the dark-matter field can create stable structures in asymptotic time.

The third paper entitled “Neutron stars in general relativity and scalar-tensor theory of gravity” is contributed by Fattoyev. Neutron stars are considered as astrophysical relativistic laboratories to probe the nature of nuclear matter under extreme conditions of high density and isospin asymmetry as well as to test fundamental theories of gravity in strong-field regime. While there have been significant improvement in understanding properties of dense nuclear matter at and near nuclear saturation density, knowledge of dense matter at super-saturation densities corresponding to the core region of neutron stars remains quite poor. In this paper, the author discusses masses and radii of neutron stars in general relativity and scalar tensor theory of gravity and compares the differences with the current uncertainties in the isovector sector of the nuclear interaction stemming from the nuclear equation of state in the nuclear mean-field framework. The author presents the formalism and results of calculating the structure of neutron stars in the scalar–tensor theory of gravity and demonstrates that the equation of state of nuclear matter can be constrained using canonical- and low-mass neutron-star observations irrespective of the models of gravity used. It is shown that astrophysical and gravitational wave observations of radii of neutron stars with masses less than 1.4 solar masses constrain the nuclear equation of state only, and in particular, the density dependence of the nuclear symmetry energy. It is conjectured that future observations of massive neutron stars may constrain the coupling parameters of scalar–tensor theory provided a general consensus on the dense nuclear matter equation of state is reached.

The fourth article is by Hamoli, Hafiai, De Paolis and Nucita entitled “Free-floating planets (FFPs) in the Milky way”. The discovery of the extrasolar planets in our Galaxy is one of the most discussed issues in the literature. These objects lie outside our solar system and have masses less than about 0.01 solar mass. In order to obtain a useful information about the exoplanets, it is argued that the gravitational microlensing method is the best method to know about the population of FFPs in the Milky Way. In this paper authors first review the bases of microlensing and in particular photometric aspects. Then using microlensing observations the authors have studied the issue of the FFP detection by gravitational microlensing in the Milky Way and estimated the detection efficiency (that is ratio between the number of events for which each second-order effect is detectable and the number of simulated events) of the finite source effect, orbital or satellite parallax and astrometric effect in microlensing events caused by FFPs towards the Galactic bulge. Continuing with astrometric microlensing, authors show how to break the degeneracy in microlensing events caused by FFPs.

The last article in this issue is contributed by Kocherlakota and Joshi on “An approach to stability analyses in general relativity via symplectic geometry”. This is an interesting article which begins with a detailed review of the phase space of classical mechanics, how Hamiltonian flows defined on it and the statements of non-linear, linear and mode stability of autonomous dynamical systems in classical mechanics, using symplectic geometry.

Following a pedagogical and geometric approach, authors claim that their approach considerably simplifies a formal understanding of the statements regarding stability of spacetimes in general relativity. In particular, the governing equations of motion of a Hamiltonian dynamical system are simply the flow equations of the associated symplectic Hamiltonian vector field, defined on phase space, and the non-linear stability analysis of its critical points have simply to do with the divergence of its flow there. Further, the linear stability of a critical point is related to the properties of the tangent flow of the Hamiltonian vector field. Authors also present a study of the of black hole or naked singularity formation from the gravitational collapse of regular matter that is equivalent to a study of the divergence of the Hamiltonian flows of a suitably defined set in the space of all allowed initial data. The authors demonstrate how, reverting back to the usual initial value formulation of general relativity, a typical non-linear stability analysis proceeds in general relativity considering the class



of Lemaître–Tolman–Bondi dust collapse models, of which the Oppenheimer–Snyder–Datt (OSD, collapse model is a member. The OSD collapse process terminates in the formation of a Schwarzschild black hole, and authors are essentially analysing how changes in the initial data from which the collapse evolves (under Einstein’s field equations) affect its formation.

The Arabian Journal of Mathematics thanks all authors who accepted our invitation to submit their research to this second special issue. The editors particularly thank Abdujabbarov, Joshi, Fattoyev, Hamoli, and De-Paolis, for their valuable contributions.

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