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QED plasma in the early universe

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Abstract Renormalization scheme of quantum electrodynamics (QED) at high temperatures is used to calculate the effective parameters of relativistic plasma in the early universe. Renormalization constants of QED play the role of effective parameters of the theory and can be used to determine the collective behavior of the medium. We explicitly show that the dielectric constant, magnetic reluctivity, Debye length and the plasma frequency depend on temperature in the early universe. Propagation speed (v_{prop}), refractive index (i_r), plasma frequency (ω) and Debye shielding length (λ_D) of a QED plasma are computed at extremely high temperatures in the early universe. We also found the favorable conditions for the existence of relativistic plasma from these calculations.

Mathematics Subject Classification 85A40

1 Introduction

Renormalization is a process of removal of singularities in gauge theories. Perturbation theory is needed to calculate the radiative corrections in terms of the higher-order processes in a medium. KLN (Kinoshita–Lee and Nauenberg) theorem [9, 10, 12, 28, 29] requires an order by order cancellation of singularities to assure the finiteness of a theory at all orders of perturbative expansion. It works very well for QED in vacuum. The coupling constant of the theory, as an expansion parameter of QED perturbative series, is sufficiently small to suppress the higher-order effects in vacuum and ensures the finiteness of QED parameters. However, this coupling constant starts to increase at temperatures larger than the electron mass and a relativistic QED plasma can locally exist.

Renormalization of gauge theories at extremely high temperatures needs thermal modification of particle propagators of the theory. This modified propagators may be identified as thermally dressed propagators and each one of them has an additional term in real-time formalism [10, 11] to incorporate the probability of interaction of propagating particles in the medium. The interaction with the medium induces temperature dependence to the physically measurable parameters [1–3, 8, 13, 14, 18, 19, 26] of the theory such as electron mass, charge and wavefunction in the system and these parameters quadratically grow with temperature. The impact of this temperature dependence on astrophysical systems [4, 7, 25] is already known.

It has been previously shown that the temperature affects the electromagnetic properties of a medium itself which determines the behavior of the system under extreme conditions [1–3, 8, 10, 11, 13, 14, 18, 19, 26]. Renormalization constants of QED in a medium refer to physically measurable parameters of the theory which affect the electromagnetic properties of the medium and are associated with the dynamically generated plasma screening mass of photons and the propagation speed of photons [2, 10, 19, 30, 32]. Debye shielding

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length is also calculated as a function of temperature which corresponds to the relativistic plasma in the early universe.

The mechanism of propagation of light in a medium helps us to understand the inflation and generation of anisotropy [4–7, 23, 24, 30, 32] in the early universe. A similar type of effect has already been seen in QCD using the perturbative calculations of QCD in the real-time formalism [16]. In this paper, we consider the effect of dynamically generated mass of photon. However, this analysis can be expanded by incorporating the effects of interaction of neutrino with the electromagnetic field as well [15, 17, 20–22, 27, 31]. However, the thermal effects on the electromagnetic coupling are not included.

This paper is organized as follows: Sect. 2 briefly describes the calculational scheme which is used to compute the electromagnetic properties of a QED medium and the parameters of the relativistic plasma in Sect. 3. Section 4 reports the results of Sect. 3 and discuss them in detail.

2 Calculational scheme

The standard Lagrangian and the Feynman rules of QED remain unchanged in this scheme of calculations. Only the mass, wavefunction and charge of electron as QED parameters are simply replaced by the corresponding renormalization parameters. Renormalization techniques are not unique. More than one renormalization schemes are available in quantum statistical (hot and dense) media and each one of them has its own benefits and limitations. Mainly the real-time formalism [10] and the imaginary time formalism [11] are well established. We use the covariant real-time formalism [10, 11] in Minkowski space which can easily check the KLN theorem at finite temperatures. Interaction of propagating particles with the medium is incorporated by adding a statistical term to the photon propagator as:

$$\frac{1}{K^2} \longrightarrow \frac{1}{K^2} - 2\pi i \delta(K^2) n_B(K), \quad (1)$$

and the photon distribution function is given as:

$$n_B(K) = \frac{1}{e^{\beta K} - 1}.$$

K is the four momentum of photon and β is the inverse temperature. The corresponding fermion propagators are given as:

$$\frac{(\not{P} + m_e)}{P^2 + m_e^2} \longrightarrow (\not{P} + m) \left[\frac{1}{P^2 + m_e^2} + 2\pi n_F(P) \right] = S_\beta(P) \quad (2)$$

and the corresponding fermion distribution function is expressed as:

$$n_F(P) = \frac{1}{e^{\beta P} + 1}.$$

All the above relations are expressed in usual notation of QED. P corresponds to the four momentum of fermion and K to the photon momentum. p corresponds to the magnitude of three momentum of electron and k is the magnitude of three momentum of photon or its wavenumber. The temperature dependence of the renormalization constants has already been calculated in literature [4, 5] as a function of temperature T . The dominant thermal contribution is quadratically dependent on temperature at the one-loop level. These renormalization constants (if finite) are multiplied with the bare value of the corresponding parameter to give the renormalized values of parameters which correspond to the physically measurable values of those parameters in the given background.

Real-time formalism is used to calculate QED renormalization constants in a hot and dense medium by changing the mass renormalization constant Z_1 , which gives an additional contribution $\frac{\delta m}{m}$ to m , the bare mass of the electron. The wavefunction renormalization constant Z_2 is calculated from a derivative of Z_1 [9–11, 18, 28]. The calculation of Z_3 , the charge renormalization constant, is related to the vacuum polarization tensor of QED and the coupling of the electromagnetic field in a medium of charged particles. Vacuum polarization tensor $\Pi_{\mu\nu}$ gives the polarization of electromagnetic waves in four-dimensional space and explains the longitudinal component of the polarization tensor corresponding to the nonzero longitudinal component. $\Pi_{\mu\nu}$ plays a

key role in computing the electromagnetic properties of the medium itself and leads to the calculation of a plasma-generating mass and then the Debye shielding facilitates the phase change into the relativistic plasmas.

First-order thermal contributions to the renormalization constant of QED are calculated for generalized temperatures up to the one loop level already [1–3, 14, 18, 26] at generalized temperatures and different ranges of chemical potentials corresponding to the density of the medium. In this paper, we ignore the density effects as they are not relevant in the early universe. It has been calculated that renormalization of QED in a hot medium of the early universe yields the self-mass (or self-energy) of the electron [1] as

$$\frac{\delta m}{m} \simeq \frac{\alpha \pi T^2}{3m^2} \left[1 - \frac{6}{\pi^2} c(m\beta) \right] + \frac{2\alpha}{\pi} \frac{T}{m} a(m\beta) - \frac{3\alpha}{\pi} b(m\beta), \quad (3)$$

and the wavefunction renormalization constant can be written as

$$Z_2^{-1}(\beta) = Z_2^{-1}(T=0) - \frac{2\alpha}{\pi} \int_0^\infty \frac{dk}{k} n_B(k) - \frac{5\alpha}{\pi} b(m\beta) + \frac{\alpha T^2}{\pi v E^2} \ln \frac{1+v}{1-v} \left[\frac{\pi^2}{6} + m\beta a(m\beta) - c(m\beta) \right]. \quad (4)$$

The electron charge renormalization constant [3] is:

$$Z_3 \simeq 1 + \frac{2e^2}{\pi^2} \left[\frac{ma(m\beta)}{\beta} - \frac{c(m\beta)}{\beta^2} + \frac{1}{4} \left(m^2 + \frac{1}{3} \omega^2 \right) b(m\beta) \right], \quad (5)$$

which corresponds to QED coupling constant because QED coupling α is related to the charge e through the relation $\alpha = \frac{e^2}{4\pi}$, the well-known Masood's abc functions $a_i(m\beta)$ for different values of 'i' corresponding to different letters and are given as:

$$a(m\beta) = \ln(1 + e^{-m\beta}), \quad (6)$$

$$b(m\beta) = \sum_{n=1}^{\infty} (-1)^n Ei(-nm\beta), \quad (7)$$

$$c(m\beta) = \sum_{n=1}^{\infty} (-1)^n \frac{e^{-nm\beta}}{n^2}. \quad (8)$$

An interesting limit for the early universe corresponds to temperatures which are high enough to take $m\beta \rightarrow 0$, the functions $a(m\beta)$ and $b(m\beta)$ that tend to give vanishing contribution, and $c(m\beta)$ that approaches to $-\frac{\pi^2}{12}$.

The calculation of Z_3 , the charge renormalization constant, is related to the vacuum polarization tensor due to the coupling of electromagnetic field with the medium of electromagnetically charged particles. Vacuum polarization tensor $\Pi_{\mu\nu}$ gives the polarization of electromagnetic waves in four-dimensional space and explains the longitudinal component of polarization corresponding to the nonzero longitudinal component. $\Pi_{\mu\nu}$ plays a key role in computing the electromagnetic properties of the medium itself and leads to the calculation of plasma-generating mass and then the Debye shielding length, indicating the phase change into the relativistic plasmas.

3 Vacuum polarization tensor in QED

Vacuum polarization tensor of photon at the two-loop level gives the second-order hot corrections to charge the renormalization constant of QED at low temperature. This contribution basically comes from the self-mass and vertex-type electron loop corrections inside the vacuum polarization tensor, as in vacuum the counter term has to be included to cancel the singularities.

The longitudinal and the transverse components of the vacuum polarization tensor are calculated [3] in terms of abc functions and the leading contributions to Π_L and Π_T are calculated using Eqs. (5–7). The leading order thermal contributions are evaluated as:

$$\begin{aligned}\Pi_L(p, T) &= -\frac{p^2}{|p|^2} u^\mu u^\nu \Pi_{\mu\nu}(p, T) \\ &= \frac{2\alpha^2 T^2 p^2}{3|p|^2} \left(1 + \frac{p_0^2}{2m^2}\right),\end{aligned}\quad (9)$$

and

$$\begin{aligned}\Pi_T(p, T) &= -\frac{1}{2}[\Pi_L(p, T) - g^{\mu\nu} \Pi_{\mu\nu}^b(p, T)] \\ &= \frac{\alpha^2 T^2}{3} \left[\frac{1}{2} - \frac{p^2}{|p|^2} \left(1 + \frac{p_0^2}{2m^2}\right) \right],\end{aligned}\quad (10)$$

whereas the corresponding detailed expressions for Π_L and Π_T up to the one-loop level are given [3] as:

$$\begin{aligned}\Pi_L \simeq \frac{4e^2}{\pi^2} \left(1 - \frac{\omega^2}{k^2}\right) &\left[\left(1 - \frac{\omega}{2k} \ln \frac{\omega+k}{\omega-k}\right) \left(\frac{ma(m\beta)}{\beta} - \frac{c(m\beta)}{\beta^2}\right) \right. \\ &\left. + \frac{1}{4} \left(2m^2 - \omega^2 + \frac{11k^2 + 37\omega^2}{72}\right) b(m\beta) \right].\end{aligned}\quad (11)$$

It is also worth mentioning at this point that $\Pi_L = 0$ in vacuum due to the absence of self-interaction of photons and is directly related to the mass of photon. Thermal contributions, given in Eq. (10), vanish at temperatures below the nucleosynthesis temperatures and we do not have to worry about any longitudinal component below $10^{10} K$. The corresponding thermal contribution to Π_T for generalized temperature is however an additive contribution to vacuum value and is given as:

$$\begin{aligned}\Pi_T \simeq \frac{2e^2}{\pi^2} &\left[\frac{\omega^2}{k^2} + \left(1 - \frac{\omega^2}{k^2}\right) \frac{\omega}{2k} \ln \frac{\omega+k}{\omega-k} \times \left(\frac{ma(m\beta)}{\beta} - \frac{c(m\beta)}{\beta^2}\right) \right. \\ &\left. + \frac{1}{8} \left(2m^2 + \omega^2 + \frac{107\omega^2 + 131k^2}{72}\right) b(m\beta) \right].\end{aligned}\quad (12)$$

These components of the vacuum polarization tensor are used to determine the electromagnetic properties of a medium with hot photons and will be discussed in the next section in a little more detail.

4 Electromagnetic properties and plasma screening

It has been shown that the photons in this medium develop a plasma screening mass which can be obtained from the longitudinal and transverse component of the vacuum polarization tensor $\Pi_L(0, k)$ and $\Pi_T(k, k)$ where $K^2 = \omega^2 - k^2 = 0$ in vacuum.

In this scheme of calculations, longitudinal and transverse components (Π_L and Π_T , respectively) of vacuum polarization tensor $\Pi_{\mu\nu}$ play a crucial role in the calculation of the electromagnetic properties of a medium. The electromagnetic properties of a medium are determined in terms of the measurable values of parameters such as electric permittivity $\epsilon(K)$, magnetic permeability $\mu(K)$, refractive index i_R , propagation speed v_{prop} and the magnetic moment μ_a of charged particles in a medium. Electric permittivity $\epsilon(K)$ and the magnetic permeability $\mu(K)$ are related to Π_L and Π_T , as [3, 11, 32]:

$$\epsilon(K) = 1 - \frac{\Pi_L}{K^2} \quad (13)$$

and

$$\frac{1}{\mu(K)} = 1 + \frac{K^2 \Pi_T - \omega^2 \Pi_L}{k^2 K^2}.$$

The electric permittivity and magnetic permeability of the medium satisfy the relations [4],

$$\epsilon(K) = 1 - \chi_e \quad (14)$$

and

$$\mu(K) = 1 + \chi_m, \quad (15)$$

whereas χ_e and χ_m give the dielectric constant and magnetization of the medium at a given temperature, respectively. The number density of different particles in such a medium are evaluated in terms of the electromagnetic properties of the medium. The refractive index of such a medium can be expressed as

$$n(K) = \sqrt{\mu\epsilon}, \quad (16)$$

and the inverse of magnetic permeability corresponds to the magnetic reluctance of the medium. The longitudinal and transverse components can be evaluated from the vacuum polarization tensor directly by using appropriate limits of photon frequency ω and wavenumber k :

$$\begin{aligned} \epsilon(K) \cong & 1 - \frac{4e^2}{\pi^2 K^2} \left(1 - \frac{\omega^2}{k^2}\right) \left(1 - \frac{\omega}{2k} \ln \frac{\omega+k}{\omega-k}\right) \left(\frac{ma(m\beta)}{\beta} - \frac{c(m\beta)}{\beta^2}\right) \\ & + \frac{1}{4} \left(2m^2 - \omega^2 + \frac{11k^2 + 37\omega^2}{72}\right) b(m\beta) \end{aligned} \quad (17)$$

and

$$\begin{aligned} \frac{1}{\mu(K)} \cong & 1 - \frac{2e^2}{\pi^2 k^2 K^2} \left[\omega^2 \left\{ \left(1 - \frac{\omega^2}{k^4} - \left(1 + \frac{k^2}{\omega^2}\right) \left(1 - \frac{\omega^2}{k^2}\right) \frac{\omega}{2k} \ln \frac{\omega+k}{\omega-k}\right\} \right. \right. \\ & \left. \left. \times \left(\frac{ma(m\beta)}{\beta} - \frac{c(m\beta)}{\beta^2}\right) - \frac{1}{8} \left(6m^2 - \omega^2 + \frac{129\omega^2 - 109k^2}{72}\right) b(m\beta) \right\} \right]. \end{aligned} \quad (18)$$

Thermal contributions to electrical permittivity (Eq. 17) and magnetic permeability $\mu(K)$ in (Eq. 18) are used to evaluate the thermal contribution to the propagation speed of light v_{prop} and other relevant parameters in the early universe.

$$v_{\text{prop}} = \sqrt{\frac{1}{\mu(K)\epsilon(K)}}, \quad (19)$$

where refractive index r_i can be expressed in the medium as:

$$r_i = \frac{c}{v_{\text{prop}}} = \sqrt{\frac{\mu(K)\epsilon(K)}{\mu_0(K)\epsilon_0(K)}}. \quad (20)$$

The vacuum polarization tensor at finite temperature can be used to determine the phase of the medium indicating the overall properties of the medium. The electromagnetic properties of such a medium change with temperature due to modified electromagnetic couplings α because of thermal corrections to electric charge. K_L is evaluated by taking $\omega = k_0 = 0$ and p very small. Debye shielding length λ_D of such a medium is then given by the inverse of K_L :

$$\lambda_D = \frac{1}{K_L}, \quad (21)$$

and the corresponding frequency ω_D can be expressed as:

$$\omega_D = \frac{2\pi v_{\text{prop}}}{\lambda_D} = 2\pi K_L v_{\text{prop}}, \quad (22)$$

which satisfy the relation

$$f_D \lambda_D = v_D, \quad (23)$$

The ω_p is also related to ω_T from $\Pi_T(\omega = |k|)$ as

$$\omega_p = \frac{2\pi v_D}{\lambda_D}.$$

The longitudinal and transverse components of the photon frequency ω and the momentum k are evaluated in two different limits of magnitudes of ω and k and can be studied in two important limits

Case 1: $\omega = k_0 = 0$ and the magnitude of three momentum p is very small.

In this limit, the longitudinal and transverse components of the propagation vector K read as

$$K_L^2 = \Pi_L(0, k) \simeq \frac{4e^2}{\pi 2} \left[\left(\frac{ma(m\beta)}{\beta} - \frac{c(m\beta)}{\beta^2} \right) + \frac{1}{4} \left(2m^2 - \omega^2 + \frac{11k^2 + 37\omega^2}{72} \right) b(m\beta) \right] \quad (24)$$

and

$$K_T^2 = \Pi_T(0, k) \simeq \frac{2e^2}{\pi 2} \left[\frac{1}{8} \left(2m^2 + \frac{131k^2}{72} \right) b(m\beta) \right]. \quad (25)$$

and the magnitude of vector K is calculated as

$$|K| = \sqrt{(K_L^2 + K_T^2)}. \quad (26)$$

Case 2: $\omega = |p|$ and the magnitude of three momentum p is still very small.

$$\omega_L^2 = \Pi_L(|k|, k) \simeq 0, \quad (27)$$

$$\omega_T^2 = \Pi_T(|k|, k) \simeq \frac{2e^2}{\pi 2} \left[\frac{ma(m\beta)}{\beta} - \frac{c(m\beta)}{\beta^2} + \frac{1}{8} \left(2m^2 - k^2 + \frac{238k^2}{72} \right) b(m\beta) \right] \quad (28)$$

and

$$\omega = \sqrt{(\omega_L^2 + \omega_T^2)}. \quad (29)$$

The plasma frequency is defined as $\omega_p^2 = \omega_T^2$ as $\omega_L^2 = 0$ and the Debye shielding length is obtained by equation (10b). These results can easily be generalized to different situations using the initial values of Π_L and Π_T from equations (11–12).

5 Results and discussions

The physically measurable parameters of QED in certain ranges of photon frequency ω and momentum k are discussed in the last section. We have studied the temperature dependence of QED parameters as functions of temperature, measured in units of electron mass. Natural system of units is used such that $\hbar = c = 1$ and mass, energy and momentum are all expressed in units of MeV. All of the QED parameters such as electric permittivity, magnetic permeability, refractive index and the propagation speed, are also normalized to unity in vacuum to compare with the corresponding thermal corrections up to the one-loop level. Plots of these functions indicate the behavior of QED parameters with the increase in temperature. It also shows how the plasma screening was dissolved near the nucleosynthesis temperature ($T \simeq m$) when the dynamically generated mass vanishes due to the low energy of photons.

Figure 1 shows the plot of K_L^2 and ω_T^2 as a function of temperature. These two functions have quadratic dependence on temperature, though the change in frequency is slower than the momentum. Therefore, the change in the longitudinal component of the propagation vector is faster than the transverse frequency, showing the fast bending but slower increase in transverse oscillation.

On the other hand, it is obvious from Fig. 2 that the transverse component of propagation vector is not affected significantly, as it just depends on the $b(m\beta)$ and gives ignorable contribution as compared to quadratic function $c(m\beta)$. Moreover, we know that $K_L^2 = 0$ at low temperature and it just appears to contribute at $T \geq 10^9 K$. Negligible change in K_T^2 gives the assurance that the speed remains almost the same. At $T > 10^{10} K$,

Fig. 1 Temperature dependence of the longitudinal component of the propagation vector (square) and the transverse frequency (square) is plotted showing that the Debye shielding length is greater than the plasma frequency

Fig. 2 On comparing the plots of K_L^2 and K_T^2 , it can be clearly seen that the propagation vector in the transverse direction is independent of temperature. The inverse of Debye length ($1/k_L$) increases with temperature, but the transverse motion will not be affected

K_L^2 has a quadratic dependence on T which indirectly affects the propagation velocity by mainly the bending angle and then slowing it down inside the plasma.

When the plasma screening is large enough, it traps the light and the speed of light is not a measurable speed any more. Propagation with the transverse frequency makes its phase velocity, which is responsible for oscillation of light waves within the Debye sphere, of length proportional to $\frac{1}{k_L}$. To clearly identify the unique behavior of light in this range by comparison, we plot the magnitude of wave vector k along with K_L^2 and ω_T^2 in Fig. 3.

Fig. 3 Comparison of the plots of k ; the magnitude of the propagation vector, longitudinal propagation K_L^2 and transverse frequency ω_T^2

Fig. 4 Debye length and the plasma frequency are plotted as a function of temperature showing that the screening length increases with T . When temperature rises around 16 times the mass of the electron which is around 8 MeV, a phase transition takes place and the plasma frequency becomes greater than the shielding length

It is obvious that k is directly proportional to T and is contributed both by the longitudinal and transverse components. When the temperature reaches around 12 MeV or higher, the contribution of the transverse frequency dominates over the momentum. On the other hand, the longitudinal component of the wavenumber is an inverse of the Debye length; increase in K_L indicates the decrease in the Debye shielding length with temperature. Quadratic increase in K_L^2 corresponds to the inverse proportionality of the Debye length with temperature.

It is also clearly seen in the graph that the transverse component of k is significant at $T \leq 6$ MeV. Then the transverse contribution is reduced as compared to the longitudinal component.

Fig. 5 All of the plasma parameters are plotted as functions of temperature such that the reluctance, propagation speed, and dielectric constant all increase with temperature

A plot of plasma parameters in Fig. 4 shows that the Debye length decreases with increase in plasma frequency at rising temperature. It means that the system behaves as plasma for the region of temperature $1 \leq T/m \leq 16$. Therefore, the proper QED plasma phase in the universe was there only when $T \leq 16$ MeV.

A phase transition occurred at around 16 MeV and an unusual behavior of trapping of light was shown above that temperature. It also explains the extremely large coupling constant of QED. This looks a little unnatural. We have plotted some of the important QED parameters as functions of temperature in Fig. 5. It can be easily seen that the electric charge and the propagation speed are slowly changing functions of temperature and are almost independent of temperature below 5 MeV. However, the mass of electron, dielectric constant and magnetic reluctance of the medium change significantly and their quadratic behavior is obvious.

Thus, the electromagnetic properties of such a medium are tremendously changed with temperature. Light propagation is not much affected because the transverse properties are almost same, whereas the medium mainly affects the longitudinal behavior of light and plasma screening is present in the longitudinal plane only.

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