Arab. J. Math. (2015) 4:77–79
DOI 10.1007/s40065-014-0117-2

Arabian Journal of Mathematics

ERRATUM

A. Bahri

Erratum to: Morse relations and Fredholm deformations of v-convex contact forms

Received: 16 August 2014 / Accepted: 19 August 2014 / Published online: 4 September 2014

Erratum to: Arab J Math (2014) 3:93–187
DOI 10.1007/s40065-014-0098-1

0 Introduction

Unfortunately, the arguments of original article for the conclusions (iii) and (iv) of Theorem 3 are not complete. Our arguments in the original article, p. 181 assert in fact that c_{2k-1} is a non-zero cycle in $H_{2k-1}(C_\beta, L^+ \cup L^- \cup J_\infty^-(\epsilon) \cup \partial_\infty c_{2k-1})$. This does not necessarily imply that c_{2k-1} is non-zero in $H_{2k-1}(C_\beta, L^+ \cup L^- \cup J_\infty^-(\epsilon) \cup \partial_\infty c_{2k-1}) \cup W_u(h_{2k-1,\infty}))$, which is the conclusion that would be required for (iii) and (iv).

It is still possible that the addition of $W_u(h_{2k-1,\infty})$ would yield a factor such as $\mathbb{PC}^{k-1} \times [-1, 1]$ in the second factor after use of the classifying map for the S^1-action, destroying the argument.

Under the assumption of (i) and (ii), this does not happen since the Fadell–Rabinowitz index [3] of $\gamma_{FR}(L^+)$ and $\gamma_{FR}(L^-)$ are at most $(k-2)$, so that the second factor maps at most into $\mathbb{PC}^{k-2} \times [-1, 1] \cup \mathbb{PC}^{k-1} \times \{0\}$. No need to use, as in original article, p. 176 lines -21 to -16, the argument about $\pi_{2k}(S^{2k-1})$ (written $\pi_{2k-2}(S^{2k-1})$ in original article, line -18, p. 176, a misprint) , which is an interesting observation; but we do not in fact need it.

1 Techniques to evaluate γ_{FR} of "sections" to $W_u(h_{2k-1,\infty})$

We now briefly study the Fadell–Rabinowitz index of "sections" to $W_u(h_{2k-1,\infty})$ in order to decide whether this space has a classifying map, given its intersections with $L^\pm$ and its boundary trace $\partial_\infty c_{2k-1}$, valued into $\mathbb{PC}^{k-1} \times [-1, 1]$ or in a lower dimensional complex projective space:

Starting from $W_u(h_{2k-1,\infty})$, we seek a section Σ to the flow-lines of this subset that enter eg L^+ (L^-). We would like to prove that $\gamma_{FR}(\Sigma) \lneqq (k-1)$. We can take this section to be defined by the equations $inf(x_1, \ldots, x_{2k}) = 0$, where the x_ls are the sizes of the various $\pm v$-jumps of $h_{2k-1,\infty}$. Σ is invariant through the flow and it is a set of dimension of dimension $(2k-2)$.

All the non-zero $\pm v$-jumps of Σ are positive.

We observe that, starting in fact from y_{2k}, see original article, section 2.5, p. 125—which dominates all the $h_{2k-1,\infty}$-the $\pm v$-jumps that remain zero all along a flow-line separating two critical points with a difference of Morse indexes equal to 1, cannot disappear; they survive as zero-$H_0^1 \pm v$-jumps in the closure of the flow-line,

The online version of the original article can be found under doi:10.1007/s40065-014-0098-1.

A. Bahri (✉)
Department of Mathematics, Rutgers, The State University of New Jersey,
110 Frelinghuysen Road, Piscataway, NJ 08854-8019, USA
E-mail: abahri@math.rutgers.edu

with their time-parametrization. Indeed, the flow-line occurs in $\cup\Gamma_{2s}$, at infinity. It therefore involves, without taking into account the zero $H_0^1 \pm v$-jump, a difference of Morse indexes at least equal to 1. Any additional loss would yield a difference of Morse indexes equal to 2 to the least.

On the other hand, a surviving $\pm v$-jump induces an S^1-equivariant map valued into S^1 through the map $e^{it(x)}$, $t(x)$ being the time at which this $\pm v$-jump occurs.

Thus, starting from Σ, which is invariant by the flow, and trying to compute its Fadell–Rabinowitz index γ_{FR}, we are led to consider flow-lines from a top critical point at Σ. We observe that we may assume that Σ is made of flow-lines starting at top critical points of $h_{2k-1,\infty}$ because of the properties of our flow. Two distinct smooth pieces of Σ intersect only at a critical point of the flow, this follows from the fact that if a $\pm v$-jump is zero on a curve, it remains zero on all the flow-line through the curve. As observed above, γ_{FR} does not change as long as the difference of Morse indexes is equal to 1 between top and bottom critical points.

γ_{FR} changes therefore only with s difference of Morse indexes equal to 2, without any intermediate critical point involved, between a top z_{2k-1}^{∞} of Σ (of index $(2k-2)$ in Σ because of the constraint involved in Σ) and another $z_{2k-3}^{(\infty)}$ of Σ (of index $(2k-4)$ in Σ for the same reason).

At $z_{2k-3}^{(\infty)}$, there is no surviving zero $H_0^1 \pm v$-jump, only one can be "lost" and the ξ-pieces of $z_{2k-3}^{(\infty)}$ must all be of H_0^1-index zero, also well-oriented as defined in [1, pp. 111–112, Propositions 19 and 21]. Since at least one $\pm v$-jump is zero on a flow-line of Σ, it must not survive at $z_{2k-3}^{(\infty)}$ and it cannot remain "jailed" between two non-zero $\pm v$-jumps of $z_{2k-3}^{(\infty)}$. It must "rotate" and this is only possible if it has a small non-zero (positive) $\pm v$-jump companion on each characteristic ξ-piece allowing it to "travel", that is the zero $\pm v$-jump $\bar{x}$ travels along a ξ-piece with the tiny $\bar{y}$ with it from one edge to the other one. Then, $\bar{y}$ becomes larger, the edge splits into its original $\pm v$-jump $\bar{z}$, $\bar{x}$, which is zero, and $\bar{y}$. $\bar{z}$ decreases in size, $\bar{y}$ increases in size, until $\bar{z}$ becomes tiny on the next ξ-piece, $\bar{x}$ moves also on the next ξ-piece, $\bar{y}$ becomes the edge. Then $\bar{y}$ can rotate around $z_{2k-3}^{(\infty)}$ until $\bar{y}$ is replaced by another $\pm v$-jump and $\bar{y}$ becomes tiny in front of the zero $\bar{x}$ on this next ξ-piece and the process can resume, along $\bar{x}$ to complete a full turn around $z_{2k-3}^{(\infty)}$.

Because of the presence of this extra non-zero $\bar{y}$, all the ξ-pieces of $z_{2k-3}^{(\infty)}$ must be of H_0^1-index zero, well-oriented [1] if characteristic, with no possibility to introduce decreasing normals [1, pp. 111–112, Propositions 19 and 21], including after introducing small ξ-pieces along the large $\pm v$-jumps of $z_{2k-3}^{(\infty)}$.

$z_{2k-3}^{(\infty)}$ is therefore very special and its unstable manifold is entirely in L^+. If $\gamma_{FR}(W_u(z_{2k-3}^{(\infty)}))$ is $(k-2)$, then assuming that there are no intermediate critical points between $z_{2k-1}^{(\infty)}$ and $z_{2k-3}^{(\infty)}$, $\gamma_{FR}(\Sigma)$ is $(k-1)$ since $\overline{W_u(z_{2k-1}^{(\infty)}) \cap \Sigma \cap W_s(z_{2k-3}^{(\infty)})}$ contains a sphere S^2, copy of $\mathbb{PC}^1$.

Observe that $W_u(z_{2k-3}^{(\infty)})$ is described with the help of $(2k-2)$ positive v-jumps at most (two have been lost: $\bar{x}$ and $\bar{y}$). $z_{2k-3}^{(\infty)}$ is made itself of ξ-pieces of H_0^1-index zero, strict H_0^1-index zero if characteristic, well-oriented, with no decreasing normals along its large $\pm v$-jumps.

With the introduction of possible companions (additional positive $\pm v$-jumps), we may assume that $W_u(z_{2k-3}^{(\infty)})$ lies into the subset of L^+ made of curves having the same behavior, namely having ξ-pieces of (strict) H_0^1-index zero well-oriented, with no decreasing normals along its large $\pm v$-jumps. Indeed, if a curve violating the above conditions should arise along a flow-line of $W_u(z_{2k-1}^{(\infty)})$, we can introduce tiny positive v-jumps at positions of decrease and continue the flow in this way.

2 Related directions of research

This reasoning indicates the directions which are natural to continue further the research started in our paper. There are three such directions:

1. Estimate the Fadell–Rabinowitz index of the sets of curves described above, that is the index of the set of curves made of ξ-pieces of H_0^1-index zero, strict H_0^1-index zero if characteristic, well-oriented, with no decreasing normals along its large $\pm v$-jumps.
2. Explore, for the first exotic contact form of Gonzalo and Varela [4] α_1, which we studied in [2], the relation:

$$\partial c_{2k}^{\infty} = c_{2k-1} + h_{2k-1,\infty}$$

see original article, with $h_{2k-1,\infty}$ dominated by a y_{2k} as in original article, p. 125. It is enough to complete this for α_1. Results of existence would follow for all the contact forms of this contact structure, see original article, p. 126, Lemma 2.14. Any cycle c_{2k-1} that does not enter into such a Morse relation survives for all contact forms of the family.
3. Explore the "point to circle" Morse relations with infinity described in section 2.5, pp. 125–131 of original article.

This is a narrow, but maybe fruitful direction of research to further extend results, including results of existence of periodic orbits "in the large", in Contact Form Geometry.

References

1. Bahri, A.: Flow-lines and Algebraic Invariants in Contact Form Geometry PNLDE, vol. 53. Birkhauser, Boston (2003)
2. Bahri, A.: On the contact homology of the first exotic contact form/structure of J. Gonzalo and F. Varela. Arab. J. Math. **3**(2), 211–289 (2014)
3. Fadell, E.R., Rabinowitz, P.H.: Generalized cohomological index theories for Lie group actions with an application to bifurcation questions for Hamiltonian systems. Invent. Math. **45**, 139–174 (1978)
4. Gonzalo, J., Varela, F.: Modeles globaux des varietes de contact. In: Third Schnepfenried Geometry Conference, Asterisque No. 107–108, vol. 1, pp. 163–168. Soc. Math. France, Paris (1983)