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RESEARCH ARTICLE

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Impulsive stabilization of fuzzy neural networks with time-varying delays

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Abstract This paper is concerned with stabilization for a class of Takagi-Sugeno fuzzy neural networks (TSFNNs) with time-varying delays. An impulsive control scheme is employed to stabilize a TSFNN. We firstly establish the model of TSFNNs by using fuzzy sets and fuzzy reasoning and propose the problem of impulsive stabilization for this model. Then, we present several stabilization conditions based on Lyapunov function, inequality techniques and linear matrix inequality approach. Two numerical examples are provided to illustrate the efficiency of impulsive stabilization for TSFNNs by using fixed impulsive interval and variable impulsive intervals, respectively.

Mathematics Subject Classification 93C10 · 93D15

الملخص

تهتم هذه الورقة بقضية الاستقرار لفئة تكاجي- سوجينو (Takagi-Sugeno) لشبكات عصبية غامضة (TSFNNs) مصحوبة بتأخير بزمن متغير. وقد استعمل مخطط تحكم نبضي لاستقرارTSFNN. نقوم أولا بوضع نموذج لِـ TSFNNs وذلك باستخدام المجموعات الغامضة والمنطق الغامض ونقترح مسألة الاستقرار النبضي لهذا النموذج. ثمّ نقدّم عدة شروط للاستقرار معتمدين على دالة ليابونوف، وتقنية المتراجحات، ومقاربة خطية لمتراجحات المصفوفات. نعرض مثالين عددين للتدليل على كفاءة الاستقرار النبضي لِـ TSFNNs وذلك باستعمال فترات نبضية ثابتة وفترات نبضية متغيرة، على التوالي.

1 Introduction

Fuzzy logic theory has been efficiently developed to many applications and shown to be an effective approach to deal with analysis and synthesis problems for complex nonlinear systems. Since Takagi and Sugeno first introduced fuzzy models in [24], the dynamic fuzzy model has become a popular tool and has been successfully and effectively employed in most model-based fuzzy analysis approaches [2,15,23]. In addition, the ordinary Takagi-Sugeno fuzzy model (TSFM) has been further extended to time-delay systems and many interesting results have been reported including stability analysis and synthesis, robust H_∞ control, and stabilization [1,3,9,10,20,27,29]. The main features of the TSFM are as follows: local dynamics in different state space regions are represented by linear models and the overall fuzzy model of the system is achieved by fuzzy blending of these local fuzzy models.

Recently, the fuzzy modeling method has also been used to represent neural network models [11,13,17]. In Huang et al. [13], utilized the TSFM to describe a set of stochastic Hopfield neural networks with time-varying delays and proposed a sufficient condition to guarantee global exponential stability in the mean square.

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More recently, based on the Lyapunov-Krasovskii functional theory, linear matrix inequality (LMI) approach and Leibniz-Newton formula, Hou et al. [11] derived a delay-dependent stability criterion to ensure the global exponential stability of Takagi-Sugeno (TS) fuzzy cellular neural networks with time-varying delays and this criterion removed the assumption that the time derivative of time-varying delays must be smaller than one. Although stability analysis of TS fuzzy neural networks (TSFNNs) has been gradually carried out, it is worth noting that neural networks can exhibit complicated dynamics or chaotic behaviors if the network's parameters and time delays are appropriately specified (see Refs. [7,18]).

Accordingly, some effective controllers for a variety of neural networks have been designed to stabilize states of these neural networks [4,5,8,16,21]. Samidurai et al. [21] and Rakkiyappan et al. [19] established several sufficient criteria for the global exponential stability of neutral-type impulsive neural networks with different kinds of delays via the Lyapunov-Krasovskii functional combining with the LMI approach. In [4, 5], Cheng proposed a decentralized feedback control method to stabilize a class of neural networks with uncertainties and time-varying delays. Despite these fruitful achievements, according to the best of the authors' knowledge, the proposed work in this paper on TSFNNs is new in the literature. The related stabilization problems are interesting and challenging. The reason of using impulsive control to stabilize the TSFNNs in our work is that impulsive control can give better performance than continuous control in some practical cases [6,22,28].

In this paper, we investigate a class of TSFNNs and obtain stabilization conditions under which the TSFNNs can be forced to converge by means of impulsive control approach. A model transformation method used in [30] is adopted to simplify TSFNN model and no differentiability restriction on time-varying delays is required in our results. We also give two numerical examples to illustrate the efficiency of impulsive stabilization for TSFNNs using fixed impulsive interval and variable impulsive intervals, respectively. The proposed approach is also possible to be applied to other complex TSFNNs, such as TSFNNs with environmental noise, TSFNNs with uncertain parameters, TSFNNs with distributed delays and so on.

Throughout this paper, we use the following notations: $\mathbb{R}^n$ denotes the n-dimensional real space; $\mathbb{R}^{n\times m}$ is the set of real $n \times m$ matrices; $\mathbb{R}^+$ is the set of non-negative real numbers; $\mathbb{Z}$ represents the set of positive integer numbers; A^{T} and A^{-1} denote the transpose and inverse of matrix A, respectively; $P > 0$ ($P < 0$, respectively) represents P is a positive (negative, respectively) definite symmetric matrix; $\lambda_{\max}(P)$ ($\lambda_{\min}(P)$, respectively) denotes the maximum (minimum, respectively) eigenvalue of the real symmetric matrix, P. I represents the identity matrix of appropriate dimension; diag$(\cdot)$ is used for represent a block diagonal matrix; $\mathbb{N}(1, n) =: \{1, 2, \ldots, n\}$; $\mathscr{PC}([-\tau_m, 0], \mathbb{R}^n)$ denotes the set of piecewise left continuous functions $\varphi : [-\tau_m, 0] \to \mathbb{R}^n$, $\tau_m > 0$; $\|\cdot\|$ denotes the Euclidean vector norm or spectral norm as appropriate.

2 Model description and preliminaries

Consider the following neural network with time-varying delays

$$\begin{cases} \dot{u}(t) = -Cu(t) + Ag(u(t)) + Bg(u(t-\tau(t))) + J, \\ u(t_0 + s) = \phi(s), \;\; s \in [-\tau_m, 0], \end{cases} \tag{1}$$

where $t > t_0$, $u(t) = (u_1(t), \ldots, u_n(t))^{\mathrm{T}} \in \mathbb{R}^n$ denotes the state vector associated with the neurons; $g(u(t)) = (g_1(u_1(t)), \ldots, g_n(u_n(t)))^{\mathrm{T}}$ and $g(u(t-\tau(t))) = (g_1(u_1(t-\tau_1(t))), \ldots, g_n(u_n(t-\tau_n(t))))^{\mathrm{T}}$ represent the neuron activation functions; $J = (J_1, \ldots, J_n)^{\mathrm{T}}$ is the constant external input vector; $C = \mathrm{diag}(c_1, c_2, \ldots, c_n)$ denotes the state feedback coefficient matrix with positive entries; $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ are connection weight matrix and delayed connection weight matrix, respectively; $\tau(t)$ is a vector of time-varying delays corresponding to the finite speed of axonal signal transmission. A natural assumption on $\tau(t)$ is made as $0 \le \tau_i(t) \le \tau_m$, $i \in \mathbb{N}(1, n)$.$\phi(s) = (\phi_1(s), \ldots, \phi_n(s))^{\mathrm{T}} \in \mathbb{R}^n$ denotes the initial value which is a continuous function defined on $[t_0 - \tau_m, t_0]$.

Throughout the paper, we make the following assumption.

Assumption 2.1 The activation function $g(\cdot)$ satisfies

$$0 \le \frac{g_i(s_1) - g_i(s_2)}{s_1 - s_2} \le l_i, \quad i \in \mathbb{N}(1, n), \tag{2}$$

for any $s_1, s_2 \in \mathbb{R}$ and $s_1 \neq s_2$.

As we know, system dynamics can be captured by a set of fuzzy rules which characterize local correlations in the state space. Each local dynamic described by the fuzzy IF-THEN rules has the property of linear input-output relation. By using the TSFM concept, the authors in [11,13,17] constructed stochastic TS fuzzy Hopfield neural networks, TS fuzzy cellular neural networks, and TS fuzzy bidirectional associative memory (BAM) neural networks, respectively, and derived some interesting results on stability criteria for these neural network models.

Now, we recall the TSFM concept and propose a TSFNNs represented by a TSFM composed of a set of fuzzy implications. The rth rule of this TSFM is of the following form:

Model Rule r:
IF $\theta_1(t)$ is η_1^r and $\cdots$ and $\theta_p(t)$ is η_p^r
THEN

$$\begin{cases} \dot{u}(t) = -C_r(t)u(t) + A_r g(u(t)) + B_r g(u(t-\tau(t))) + J_r, \\ u(t_0+s) = \phi(s), \;\; s \in [-\tau_m, 0], \end{cases} \tag{3}$$

where $\theta(t) = (\theta_1(t), \theta_2(t), \ldots, \theta_p(t))^{\mathrm{T}}$ is the premise variable vector, $\eta_j^k (j \in \mathbb{N}(1,p), r \in \mathbb{N}(1,m))$ is the fuzzy set and r is the number of model rules.

Suppose System (3) has an equilibrium $u^* = [u_1^*, u_2^*, \ldots, u_n^*]$. Then we shift the equilibrium u^* of System (3) to the origin by using the transformation $x(t) = u(t) - u^*$ and System (3) can be transformed into the following form:

Model Rule r:
IF $\theta_1(t)$ is η_1^r and $\cdots$ and $\theta_p(t)$ is η_p^r
THEN

$$\begin{cases} \dot{x}(t) = -C_r(t)x(t) + A_r f(x(t)) + B_r f(x(t-\tau(t))), \\ x(t_0+s) = \varphi(s), \;\; s \in [-\tau_m, 0], \end{cases} \tag{4}$$

where $\varphi \in \mathscr{PC}([-\tau_m, 0], \mathbb{R}^n)$, $f(x(t)) = g(u(t)) - g(u^*)$, $f(x(t-\tau(t))) = g(u(t-\tau(t))) - g(u^*)$,

Next, in order to ensure the origin of System (4) to be stable, we employ an impulsive control scheme which has been utilized to stabilize high-order Hopfield-Type neural networks (see [16]), that is, consider the following system

Model Rule r:
IF $\theta_1(t)$ is η_1^r and $\cdots$ and $\theta_p(t)$ is η_p^r
THEN

$$\begin{cases} \dot{x}(t) = -C_r(t)x(t) + A_r f(x(t)) \\ \qquad\quad +B_r f(x(t-\tau(t))), \;\; t \geq t_0, t \neq t_k, \\ \Delta x(t) = E_{kr}x(t_k), \;\; t = t_k, \;\; k \in \mathbb{Z}, \\ x(\sigma+s) = \varphi(s), \;\; t \in [-\tau_m, 0], \sigma \geq t_0, \end{cases} \tag{5}$$

where $\Delta x(t)|_{t=t_k} = x(t_k^+) - x(t_k^-)$. Here, $x(t_k^+) = \lim_{h\to 0^+} x(t_k+h), x(t_k^-) = \lim_{h\to 0^+} x(t_k-h)$ with discontinuity instants $t_1 < t_2 < \cdots < t_k < \cdots$, $\lim_{k\to\infty} t_k = \infty$, where $t_1 > t_0$. For convenience, let $t_0 = 0$ and $h > 0$ be sufficiently small. Without loss of generality, it is assumed that $x(t_k) = x(t_k^-) = \lim_{h\to 0^+} x(t_k - h)$. $E_{kr} (r \in \mathbb{N}(1,m))$ are the impulsive control coefficient matrices.

Let $h_r(\theta(t))$ be the normalized membership function of the inferred fuzzy set $M_r(\theta(t))$, i.e.,

$$h_r(\theta(t)) = \frac{M_r(\theta(t))}{\sum_{r=1}^m M_r(\theta(t))},$$

where

$$M_r(\theta(t)) = \prod_{j=1}^p \eta_j^r(\theta_j(t)),$$

where $\eta_j^r(\theta_j(t))$ is the grade of membership of $\theta_j(t)$ in η_j^r. According to the theory of fuzzy sets, it is obvious that

$$M_r(\theta(t)) \geq 0, \;\; r \in \mathbb{N}(1,m), \;\; \sum_{r=1}^m M_r(\theta(t)) > 0,$$

for all t. And this implies

$$h_r(\theta(t)) \geq 0,\ r \in \mathbb{N}(1,m),\ \sum_{r=1}^{m} h_r(\theta(t)) = 1,$$

for all t.

To simplify the analysis, let the impulsive coefficient $E_{kr} = E_k r \in \mathbb{N}(1,m)$. Hence, the fuzzy system (5) can be expressed as

$$\begin{cases} \dot{x}(t) = \sum_{r=1}^{m} h_r(\theta(t))[-C_r x(t) + A_r f(x(t)) \\ \qquad\quad +B_r f(x(t-\tau(t)))],\ t \geq t_0, t \neq t_k, \\ \Delta x(t) = E_k x(t_k),\ t = t_k,\ k \in \mathbb{Z}, \\ x(s+\sigma) = \varphi(s),\ t \in [-\tau_m, 0], \sigma \geq t_0. \end{cases} \tag{6}$$

From the definitions of $f(x(t))$ and $f(x(t-\tau(t)))$, we can define, for $i \in \mathbb{N}(1,n)$,

$$F_i(x_i(t)) = \begin{cases} \frac{f_i(x_i(t))}{x_i(t)}, & x_i(t) \neq 0, \\ 0, & x_i(t) = 0, \end{cases} \tag{7}$$

$$F_i(x_i(t-\tau_i(t))) = \begin{cases} \frac{f_i(x_i(t-\tau_i(t)))}{x_i(t-\tau_i(t))}, & x_i(t-\tau_i(t)) \neq 0, \\ 0, & x_i(t-\tau_i(t)) = 0. \end{cases} \tag{8}$$

Thus System (6) can be transformed as follows:

$$\begin{cases} \dot{x}(t) = \sum_{r=1}^{m} h_r(\theta(t))[-C_r x(t) + A_r \hat{F} x(t) \\ \qquad\quad +B_r \hat{F}_\tau x(t-\tau(t))],\ t \geq t_0, t \neq t_k, \\ \Delta x(t) = E_k x(t_k),\ t = t_k,\ k \in \mathbb{Z}, \\ x(s+\sigma) = \varphi(s),\ t \in [-\tau_m, 0], \sigma \geq t_0, \end{cases} \tag{9}$$

where

$$\begin{aligned} \hat{F} &= \mathrm{diag}\Big(F_1(x_1(t)), F_2(x_2(t)), \ldots, F_n(x_n(t))\Big), \\ \hat{F}_\tau &= \mathrm{diag}\Big(F_1(x_1(t-\tau_1(t))), F_2(x_2(t-\tau_2(t))), \\ &\qquad \ldots, F_n(x_n(t-\tau_n(t)))\Big). \end{aligned}$$

Remark 2.2 Recalling the *Assumption 2.1* on the activation functions, it is obvious that $0 \leq F_i(x_i(t)) \leq l_i$ and $0 \leq F_i(x_i(t-\tau_i(t))) \leq l_i$, $i \in \mathbb{N}(1,n)$.

It is worth mentioning that the aim of ensuring stability of System (3) will be accomplished through the transformation $x(t) = u(t) - v^*$ if we design a proper impulsive controller $\Delta x(t_k) = E_k x(t_k)$ to stabilize System (9). Thereby, we only need consider stability of System (9).

In what follows, we need the following definitions and lemmas.

Definition 2.3 (*stable,* [16]). The origin of System (9) with impulses is said to be stable, if for any $\sigma \geq t_0$ and $\varepsilon > 0$, there is $\delta = \delta(\gamma, \varepsilon) > 0$ such that $\varphi \in \mathscr{PC}(\delta)t \geq \sigma$ implies that $\|x(t;\sigma,\varphi)\| < \varepsilon$ for $t \geq \sigma$. If δ is independent of σ, then the origin of System (9) with impulses is said to be uniformly stable.

Definition 2.4 (*Asymptotically stable*, [16]). The origin of System (9) with impulses is said to be (uniformly) asymptotically stable, if it is (uniformly) stable, and there exists $\gamma > 0$ and $T = T(\varepsilon, \gamma)$ for any $\varepsilon > 0$, such that for each $\sigma \geq t_0$ and $\varphi \in \mathscr{PC}([-\tau_m, 0], \mathbb{R}^n)$ with $\sup_{s\in[-\tau_m,0]} \|\varphi(s)\| < \gamma$, $\|x(t;\sigma,\varphi)\| < \varepsilon$ holds for $t > \sigma + T$.

Definition 2.5 (*Exponentially stable*). The origin of System (9) with impulses is said to be exponentially stable, if there exist $\Gamma > 0$, $\mu > 0$ such that for every $\varphi \in \mathscr{PC}([-\tau_m, 0], \mathbb{R}^n)$ satisfy

$$\|x(t;\varphi)\| \leq \Gamma \|x_0\| e^{-\mu(t-t_0)}, \tag{10}$$

where $x_0 = x(t_0^+) \in \mathbb{R}^n$, $t_0 \in \mathbb{R}^+$.

Definition 2.6 [14]. The function $V : \mathbb{R}^n \times [t_0, \infty) \to \mathbb{R}^+$ belongs to class $\mathscr{V}_0$ if it satisfies the following conditions:

(1) the function V is continuous on each of the sets $\mathbb{R}^n \times (t_{k-1}, t_k]$ and for all $t \geq t_0 V(0, t) \equiv 0$;
(2) $V(x, t)$ is locally Lipschitzian in $x \in \mathbb{R}^n$;
(3) for each $k \in \mathbb{Z}$, there exist finite limits

$$\begin{cases} \lim\limits_{(y,t)\to(x,\tau_k^-)} V(y, t) = V(x, \tau_k^-) = V(x, \tau_k), \\ \lim\limits_{(y,t)\to(x,\tau_k^+)} V(y, t) = V(x, \tau_k^+). \end{cases} \tag{11}$$

Definition 2.7 [14]. Let $V \in \mathscr{V}_0$, for $t \in (t_{k-1}, t_k]$, we define

$$D^+V(x(t), t) = \lim_{h\to 0^+} \sup \frac{1}{h}[V(x(t+h), t+h) - V(x(t), t)].$$

Lemma 2.8 [26]. *Let $\Sigma_1, \Sigma_2, \Sigma_3$ be real matrices of appropriate dimensions with $\Sigma_3 > 0$. Then for any vectors x and y with appropriate dimensions,*

$$2x^{\mathrm{T}}\Sigma_1^{\mathrm{T}}\Sigma_2 y \leq x^{\mathrm{T}}\Sigma_1^{\mathrm{T}}\Sigma_3\Sigma_1 x + y^{\mathrm{T}}\Sigma_2^{\mathrm{T}}\Sigma_3^{-1}\Sigma_2 y.$$

Lemma 2.9 [12]. *For any $x \in \mathbb{R}^n$, if $P \in \mathbb{R}^{n\times n}$ is a positive definite matrix, $Q \in \mathbb{R}^{n\times n}$ is a symmetric matrix, then*

$$\lambda_{\min}(P^{-1}Q)x^{\mathrm{T}}Px \leq x^{\mathrm{T}}Qx \leq \lambda_{\max}(P^{-1}Q)x^{\mathrm{T}}Px.$$

Lemma 2.10 (Halanay inequality [25]) *Let $a > b > 0$ and $v(t)$ be a non-negative continuous function on $[t_0 - \tau, t_0]$, and satisfy the following inequality:*

$$D^+v(t) \leq -av(t) + b\widetilde{v}(t), t \geq t_0,$$

where $\widetilde{v}(t) = \sup_{t-\tau\leq s\leq t} v(s)\tau$ is a non-negative constant, then there exists a constant $\lambda > 0$ satisfying

$$v(t) \leq \widetilde{v}(t_0)e^{-\lambda(t-t_0)}, t \geq t_0,$$

where λ is unique positive solution of the following equation:

$$\lambda = a - be^{\lambda\tau}.$$

3 Main results

In this section, we shall state and prove two theorems on asymptotical stability and exponential stability, respectively, for TSFNN (9) with impulses based on two different analysis techniques. Our results show that the impulses play an important role in stabilizing the TSFNN (9).

Theorem 3.1 *The origin of System (9) is uniformly stable if there exists a positive definite matrix P and two positive numbers $\epsilon_1 > 0$ and $\epsilon_2 > 0$ such that*

($\mathcal{C}1$) *the impulsive intervals satisfy*

$$t_k - t_{k-1} < -\ln\alpha_3 / \left(\alpha_1 + \frac{\alpha_2}{\alpha_3}\right), \ k \in \mathbb{Z}$$

hold, where $0 < \alpha_3 < 1$,

$$\alpha_1 = \max_{1\le r\le m} \lambda_{\max}\Big(P^{-1}(-2PC_r + \epsilon_1 PA_rA_r^{\mathrm{T}}P$$
$$+\epsilon_1^{-1}L^2 + \epsilon_2 PB_rB_r^{\mathrm{T}}P)\Big),$$
$$\alpha_2 = \lambda_{\max}\Big(\epsilon_2^{-1}P^{-1}L^2\Big),$$
$$\alpha_3 = \lambda_{\max}\Big(P^{-1}(I+E_k)^{\mathrm{T}}P(I+E_k)\Big),\ k\in\mathbb{Z}.$$

Moreover, if condition $(\mathcal{C}1)$ *holds and*

$(\mathcal{C}2)$ *there exists* $\kappa > 1$ *such that* $x^{\mathrm{T}}(t+s)Px(t+s) < \kappa x^{\mathrm{T}}(t)Px(t) s \in [-\tau_m, 0]$ *and* $\rho = \alpha_1 + \kappa|\alpha_2| < 0$, *then the origin of System* (9) *is uniformly asymptotically stable.*

Proof Consider a Lyapunov function

$$V(x(t),t) = x^{\mathrm{T}}(t)Px(t).$$

Clearly, $V(x(t),t)\in\mathscr{V}_0$ and $\lambda_{\min}(P)\|x(t)\|^2 \le V(x(t),t) \le \lambda_{\max}(P)\|x(t)\|^2$. Hence, there is $\delta=\delta(\varepsilon)>0$ such that $\delta < \varepsilon\sqrt{\frac{\alpha_3\lambda_{\min}(P)}{\lambda_{\max}(P)}}$.

When $t\ne t_k,\ k\in\mathbb{Z}$, we have

$$D^+V(x(t),t) \le \sum_{r=1}^{m} h_r(\theta(t))\Big[2x^{\mathrm{T}}(t)P\Big(-C_rx(t)$$
$$+A_r\hat{F}x(t) + B_r\hat{F}_\tau x(t-\tau(t))\Big)\Big]. \tag{12}$$

Making use of Lemma 2.8, one can obtain

$$2x^{\mathrm{T}}(t)PA_r\hat{F}x(t) \le \epsilon_1 x^{\mathrm{T}}(t)PA_rA_r^{\mathrm{T}}Px + \epsilon_1^{-1}x^{\mathrm{T}}L^2x, \tag{13}$$

and

$$2x^{\mathrm{T}}(t)PB_r\hat{F}_\tau x(t-\tau(t)) \le \epsilon_2 x^{\mathrm{T}}(t)PB_rB_r^{\mathrm{T}}Px + \epsilon_2^{-1}x^{\mathrm{T}}(t-\tau(t))L^2x(t-\tau(t)). \tag{14}$$

Substituting (13) and (14) into (12) together with Lemma 2.9 gives

$$D^+V(x(t),t) \le \alpha_1 V(x(t),t) + \alpha_2 V(x(t-\tau(t)),t). \tag{15}$$

For any $\sigma\ge t_0$ and $\varphi\in\mathscr{PC}(\delta)$, let $x(t)=x(t;\sigma,\varphi)$ be the solution of (9) through (σ,φ).

First, we shall prove that for $t\in[\sigma,t_k]$

$$V(x(t),t) \le \frac{\lambda_{\max}(P)}{\alpha_3}\delta^2, \tag{16}$$

where $\sigma\in(t_{k-1},t_k]$ for some $k\in\mathbb{Z}$.

Obviously, since $\alpha_3<1$ and for any $t\in[\sigma-\tau_m,\sigma]$, there exists $\varpi\in[-\tau_m,0]$ such that $t=\varpi+\sigma$. Therefore, for $\sigma-\tau_m\le t<\sigma$, we have

$$V(x(t),t) = V(x(\varpi+\sigma),\varpi+\sigma)$$
$$= V(\varphi(\varpi),\varpi+\sigma) \le \lambda_{\max}(P)\|\varphi(\varpi)\|^2$$
$$\le \lambda_{\max}(P)\delta^2 < \frac{\lambda_{\max}(P)}{\alpha_3}\delta^2. \tag{17}$$

Suppose that (16) does not hold. Then there is $s \in (\sigma, t_k]$, such that

$$V(x(s), s) > \frac{\lambda_{\max}(P)}{\alpha_3}\delta^2 > \lambda_{\max}(P)\delta^2.$$

By $\sigma \in (t_{k-1}, t_k]$ and the continuity of $V(x(t), t)$ on $[\sigma, t_k]$, then there exists $t_1 \in (\sigma, s]$ such that

$$\begin{cases} V(x(t_1), t_1) = \frac{\lambda_{\max}(P)}{\alpha_3}\delta^2, \\ V(x(t), t) \leq \frac{\lambda_{\max}(P)}{\alpha_3}\delta^2, \ \ \sigma \leq t \leq t_1, \\ D^+(x(t_1), t_1) \geq 0. \end{cases} \tag{18}$$

Note that for $t \in [\sigma - \tau_m, \sigma]$,

$$\begin{cases} V(x(t), t) \leq \lambda_{\max}(P)\delta^2, \\ (\lambda_{\max}(P)/\alpha_3)\delta^2 > \lambda_{\max}(P)\delta^2. \end{cases} \tag{19}$$

Hence, there exists $t_2 \in (\sigma, t_1)$ such that

$$\begin{cases} V(x(t_2), t_2) = \lambda_{\max}(P)\delta^2, \\ V(x(t), t) \geq \lambda_{\max}(P)\delta^2, \ \ t_2 \leq t \leq t_1, \\ D^+(x(t_2), t_2) \geq 0. \end{cases} \tag{20}$$

From (18–20), we have

$$\begin{aligned} V(x(t+s), t+s) &\leq \tfrac{\lambda_{\max}(P)}{\alpha_3}\delta^2 \\ &\leq \tfrac{1}{\alpha_3}V(x(t), t), \ \ s \in [-\tau_m, 0], \end{aligned}$$

for $t \in [t_2, t_1]$, i.e.,

$$x^{\mathrm{T}}(t - \tau(t))Px(t - \tau(t)) \leq (1/\alpha_3)x^{\mathrm{T}}(t)Px(t).$$

Then for $t \in [t_2, t_1]$

$$\begin{aligned} D^+V(x(t), t) &\leq \left(\alpha_1 + \frac{\alpha_2}{\alpha_3}\right)x^{\mathrm{T}}(t)Px(t) \\ &= \left(\alpha_1 + \frac{\alpha_2}{\alpha_3}\right)V(x(t), t). \end{aligned} \tag{21}$$

Integrating Inequality (21) on $t \in [t_2, t_1]$, we get

$$\begin{aligned} &\int_{t_2}^{t_1} \frac{D^+V(x(t), t)}{V(x(t), t)}\mathrm{d}t \leq \int_{t_2}^{t_1} \left(\alpha_1 + \frac{\alpha_2}{\alpha_3}\right)\mathrm{d}t \\ &\leq \int_{t_{k-1}}^{t_k} \left(\alpha_1 + \frac{\alpha_2}{\alpha_3}\right)\mathrm{d}t = \left(\alpha_1 + \frac{\alpha_2}{\alpha_3}\right)(t_k - t_{k-1}) \\ &\leq -\ln\alpha_3. \end{aligned} \tag{22}$$

On the other hand,

$$\begin{aligned} &\int_{t_2}^{t_1} \frac{D^+V(x(t), t)}{V(x(t), t)}\mathrm{d}t \\ &= \int_{V(x(t_2), t_2)}^{V(x(t_1), t_1)} \frac{\mathrm{d}v}{v} \\ &= \int_{\lambda_{\max}(P)\delta^2}^{\frac{\lambda_{\max}(P)}{\alpha_3}\delta^2} \frac{\mathrm{d}v}{v} = -\ln\alpha_3 \end{aligned}$$

which contradicts (22). Hence, Inequality (16) holds.

From (16) and the condition ($\mathcal{C}1$) of Theorem 3.1, we derive

$$\begin{aligned} V(x(t_k^+), t_k^+) &= x^{\mathrm{T}}(t_k)(I + E_k)^{\mathrm{T}} P(I + E_k)x(t_k) \\ &\le \alpha_3 x^{\mathrm{T}}(t_k) P x(t_k) = \alpha_3 V(x(t_k), t_k) \\ &\le \lambda_{\max}(P)\delta^2. \end{aligned} \tag{23}$$

Similarly, we can obtain that

$$V(x(t), t) \le \frac{\lambda_{\max}(P)}{\alpha_3}\delta^2, \quad t_k \le t \le t_{k+1}$$

and $V(x(t_{k+1}), t_{k+1}) \le \lambda_{\max}(P)\delta^2$.

Through simple induction, we can prove that

$$V(x(t), t) \le \frac{\lambda_{\max}(P)}{\alpha_3}\delta^2, \quad t_{k+q} \le t \le t_{k+q+1}$$

and

$$V(x(t_{k+q+1}), t_{k+q+1}) \le \lambda_{\max}(P)\delta^2,$$

for $q = 0, 1, 2, \ldots$.

By virtue of (16) and $\alpha_3 < 1$, we have

$$V(x(t), t) \le \frac{\lambda_{\max}(P)}{\alpha_3}\delta^2, \ t \ge \sigma. \tag{24}$$

Hence, for $t \ge \sigma$,

$$\lambda_{\min}(P)\|x(t)\|^2 \le V(x(t), t) \le \frac{\lambda_{\max}(P)}{\alpha_3}\delta^2, \tag{25}$$

which implies

$$\|x(t)\| \le \delta\sqrt{\frac{\lambda_{\max}(P)}{\alpha_3\lambda_{\min}(P)}} < \varepsilon.$$

Thereby, according to Definition 2.3, the zero solution of TSFNN (9) with impulses is uniformly stable. Using this conclusion together with (17) and (24), for given $\hat{q} > 0$, we can choose a $\delta > 0$ such that $\varphi \in \mathscr{PC}([-\tau_m, 0], \mathbb{R}^n)$ with $\sup_{s\in[-\tau_m,0]} \|\varphi(s)\| < \delta$, which implies that for $t \ge \sigma - \tau_m$, $\|x(t; \sigma, \varphi)\| < \hat{q}$ and

$$V(x(t), t) \le \frac{\lambda_{\max}(P)}{\alpha_3}\delta^2.$$

Now, for given $0 < \varepsilon < \hat{q}$, there exists a number $d = d(\varepsilon) > 0$ such that for $\lambda_{\min}(P)\varepsilon^2 \le s \le \frac{\lambda_{\max}(P)}{\alpha_3}\delta^2$, $d < (\kappa - 1)s$. Let $N = N(\varepsilon) > 0$ be the smallest integer such that $\frac{\lambda_{\max}(P)}{\alpha_3}\delta^2 \le \lambda_{\min}(P)\varepsilon^2 + Nd$. Set $x(t) = x(t; \sigma, \varphi)$, $\hat{\delta} = \max\left\{-\frac{\lambda_{\max}(P)\delta^2}{\rho\alpha_3}, \tau_m\right\}$ and let $T = T(\varepsilon) = (2N - 1)\hat{\delta}$. We shall prove that

$$V(x(t), t) \le \lambda_{\min}(P)\varepsilon^2, \ \text{for } t \ge \sigma + T. \tag{26}$$

Using contradiction approach and mathematical induction, we can prove, in general, that

$$V(x(t), t) \le \lambda_{\min}(P)\varepsilon^2 + (N - i)d,$$

for $t \ge \sigma + (2i - 1)\hat{\delta}i \in \mathbb{N}(1, N)$. Due to the limitation of space, we omit it here.

Therefore, when choosing $i = N$, we obtain

$$V(x(t), t) \leq \lambda_{\min}(P)\varepsilon^2,$$

for $t \geq \sigma + (2N - 1)\hat{\delta}$.

Therefore, Inequality (26) holds, which implies

$$\|x(t)\| < \varepsilon,$$

for $t \geq \sigma + T$. The proof of Theorem 3.1 is completed. □

Based on Lyapunov function together with LMI approach and Halanay inequality, we give the following theorem on exponential stability.

Theorem 3.2 *If there exists positive definite matrices P, Q_1, Q_2 and two positive numbers $a > 0, b > 0$ such that the following LMI*

$$\begin{bmatrix} aI - 2PC_r + LQ_1L & PA_r & PB_r & \frac{1}{2}PB_rL \\ A_r^{\mathrm{T}}P & -Q_1 & 0 & 0 \\ B_r^{\mathrm{T}}P & 0 & -Q_2 & 0 \\ \frac{1}{2}LB_r^{\mathrm{T}}P & 0 & 0 & -bI + \frac{1}{4}LQ_2L \end{bmatrix} < 0 \tag{27}$$

and

$$\gamma_k = \lambda_{\max}\Big(P^{-1}(I + E_k)^{\mathrm{T}}P(I + E_k)\Big) < 1$$

hold, then the zero solution of TSFNN (9) with impulses is exponentially stable with the decay degree μ which is a positive solution of the equation $\mu = \mathring{a} - \mathring{b}e^{\mu\tau_m}$ where $\mathring{a} = \frac{a}{\lambda_{\max}(P)}$, $\mathring{b} = \frac{b}{\lambda_{\min}(P)}$.

Proof Consider the Lyapunov function

$$V(x(t), t) = x^{\mathrm{T}}(t)Px(t) \tag{28}$$

and calculate $D^+V(x(t), t)$ along with (9)

$$D^+V(x(t), t) \leq \sum_{r=1}^{m} h_r(\theta(t))\Big[2x^{\mathrm{T}}(t)P\Big(-C_rx(t) + A_r\hat{F}x(t) + B_r\hat{F}_\tau x(t - \tau(t))\Big)\Big]. \tag{29}$$

From Remark 2.2, we know that

$$-\frac{1}{2}l_i \leq F_i(x_i) - \frac{1}{2}l_i \leq \frac{1}{2}l_i, \ i \in \mathbb{N}(1, n),$$

that is,

$$-\frac{1}{2}L \leq \hat{F} - \frac{1}{2}L \leq \frac{1}{2}L .$$

Therefore, by utilizing Lemma 2.8, we can derive

$$\begin{aligned} &2x^{\mathrm{T}}(t)PA_r\hat{F}x(t)\\ &\quad\le x^{\mathrm{T}}(t)PA_rQ_1^{-1}A_r^{\mathrm{T}}Px + x^{\mathrm{T}}LQ_1Lx, \end{aligned} \tag{30}$$

$$\begin{aligned} &2x^{\mathrm{T}}(t)PB_r\hat{F}_\tau x(t-\tau(t))\\ &\quad= 2x^{\mathrm{T}}(t)PB_r\Big(\hat{F}_\tau - \frac{1}{2}L\Big)x(t-\tau(t))\\ &\qquad -2x^{\mathrm{T}}(t)PB_r\Big(-\frac{1}{2}L\Big)x(t-\tau(t))\\ &\quad\le x^{\mathrm{T}}(t)PB_rQ_2^{-1}B_r^{\mathrm{T}}Px\\ &\qquad +x^{\mathrm{T}}(t-\tau(t))\Big(\hat{F}_\tau - \frac{1}{2}L\Big)^{\mathrm{T}}Q_2\\ &\qquad \times\Big(\hat{F}_\tau - \frac{1}{2}L\Big)x(t-\tau(t))\\ &\qquad +x^{\mathrm{T}}(t)PB_rLx(t-\tau(t))\\ &\quad\le x^{\mathrm{T}}(t)PB_rQ_2^{-1}B_r^{\mathrm{T}}Px\\ &\qquad +\frac{1}{4}x^{\mathrm{T}}(t-\tau(t))LQ_2Lx(t-\tau(t))\\ &\qquad +x^{\mathrm{T}}(t)PB_rLx(t-\tau(t)). \end{aligned} \tag{31}$$

Combining with (29–31) and using Lemma 2.9, it follows that

$$D^{+}V(x(t),t) \le \sum_{r=1}^{m} h_r(\theta(t))\xi^{\mathrm{T}}(t)\Theta\xi(t) - \mathring{a}V(t) + \mathring{b}V(t-\tau(t)) \tag{32}$$

where

$$\begin{aligned} \xi^{\mathrm{T}}(t) &= [x^{\mathrm{T}}(t) \quad x^{\mathrm{T}}(t-\tau(t))],\\ \mathring{a} &= \frac{a}{\lambda_{\max}(P)}, \mathring{b} = \frac{b}{\lambda_{\min}(P)},\\ \Theta &= \begin{bmatrix} \Upsilon_1 & \frac{1}{2}PB_rL\\ \frac{1}{2}LB_r^{\mathrm{T}}P & -bI + \frac{1}{4}LQ_2L \end{bmatrix}\\ \Upsilon_1 &= aI - 2PC_r + PA_rQ_1^{-1}A_r^{\mathrm{T}}P\\ &\quad +LQ_1L + PB_rQ_2^{-1}B_r^{\mathrm{T}}P. \end{aligned}$$

It results from the Schur Complement Lemma that, LMI (27) is equivalent to $\Theta < 0$. Hence, we obtain

$$D^{+}V(x(t),t) \le -\mathring{a}V(t) + \mathring{b}V(t-\tau(t)). \tag{33}$$

Accordingly, by Lemma 2.10, it follows from (33) that for $t \in (t_{k-1}, t_k]k \in \mathbb{Z}$,

$$V(x(t),t) \le V(x(t_{k-1}^{+}), t_{k-1}^{+})e^{-\mu(t-t_{k-1})}, \tag{34}$$

where μ is the unique positive root of $\mu = \mathring{a} - \mathring{b}e^{\mu\tau_m}$.

Meanwhile, from Lemma 2.9, one has

$$\begin{aligned} V(x(t_k^{+}), t_k^{+}) &= x^{\mathrm{T}}(t_k^{+})Px(t_k^{+})\\ &= [(I+E_k)x(t_k)]^{\mathrm{T}}P[(I+E_k)x(t_k)]\\ &\le \gamma_k x^{\mathrm{T}}(t_k)Px(t_k). \end{aligned} \tag{35}$$

Therefore, for $t \in (t_0, t_1]$,

$$V(x(t),t) \le V(x_0, t_0^{+})e^{-\mu(t-t_0)},$$

which leads to

$$V(x(t_1), t_1) \leq V(x_0, t_0^+)e^{-\mu(t_1-t_0)},$$

and

$$V(x(t_1^+), t_1^+) \leq \gamma_1 V(x(t_1), t_1) \leq \gamma_1 V(x_0, t_0^+)e^{-\mu(t_1-t_0)}.$$

Similarly, for $t \in (t_1, t_2]$,

$$\begin{aligned} V(x(t), t) &\leq V(x(t_1^+), t_1^+)e^{-\mu(t-t_1)} \\ &\leq \gamma_1 V(x_0, t_0^+)e^{-\mu(t-t_0)}, \end{aligned}$$

then

$$V(x(t_2), t_2) \leq \gamma_1 V(x_0, t_0^+)e^{-\mu(t_2-t_0)}$$

and

$$V(x(t_2^+), t_2^+) \leq \gamma_2 V(x(t_2), t_2) \leq \gamma_1\gamma_2 V(x_0, t_0^+)e^{-\mu(t_2-t_0)}.$$

The rest may be deduced by analogy. In general, for $t \in (t_k, t_{k+1}]$,

$$V(x(t), t) \leq \gamma_1\gamma_2 \cdots \gamma_k V(x_0, t_0^+)e^{-\mu(t-t_0)}, \tag{36}$$

which yields

$$\|x(t)\| \leq \Gamma\|x_0\|e^{-\frac{\mu}{2}(t-t_0)}, \tag{37}$$

where $\Gamma = \sqrt{(\gamma_1\gamma_2 \ldots \gamma_k)\lambda_{\max}(P)/\lambda_{\min}(P)}$.

Therefore, according to Definition 2.5, the zero solution of TSFNN (9) with impulses is exponentially stable.

□

Remark 3.3 Both Theorems 3.1 and 3.2 do not need the differential assumption on time-varying delays, which makes our results more applicable. Moreover, when the number of fuzzy model rules is set as one, i.e., $m = 1$, system (9) degenerates into a general impulsive neural networks with time-varying delays. Therefore, one can easily extend our results to stability of general impulsive neural networks with time-varying delays.

4 Numerical simulations

In this section, we will apply the proposed results to impulsive stabilization of TSFNNs by using two impulsive strategies.

Simulation 1–Stabilization of almost periodic orbits using fixed impulsive intervals

Consider a TSFNN with time-varying delays and impulses:

$$\begin{cases} \dot{x}(t) = \sum_{r=1}^{2} h_r(\theta(t))[-C_r x(t) + A_r f(x(t)) \\ \qquad\quad +B_r f(x(t-\tau(t)))], \ t \geq t_0, t \neq t_k, \\ \Delta x(t) = E_k x(t_k), \ t = t_k, \ k \in \mathbb{Z}, \\ x(s) = \varphi(s), \ t \in [-1.6, 0], \end{cases} \tag{38}$$

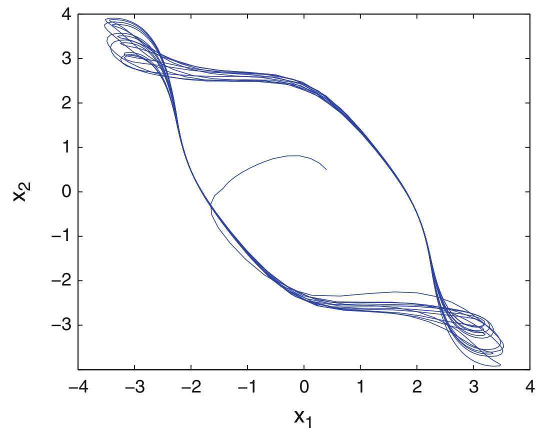

Fig. 1 The phase plot of state variables of (38) without impulsive control

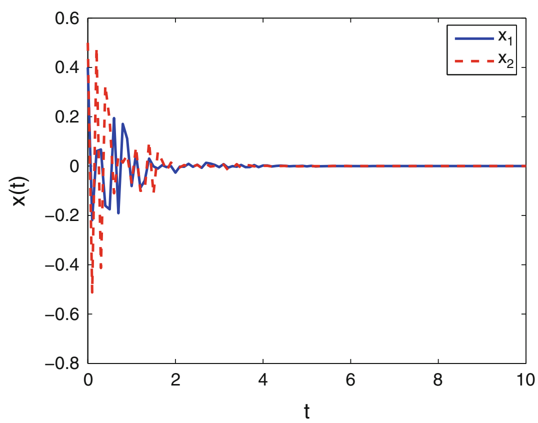

Fig. 2 The time responses of state variables of (38) with impulsive control

where the model parameters are given as follows:

$$C_1 = C_2 = \mathrm{diag}(1, 1),$$

$$A_1 = \begin{bmatrix} 2 & -0.1 \\ -5 & 2 \end{bmatrix}, A_2 = \begin{bmatrix} 2 & -3 \\ 3 & 2 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -1.5 & -0.1 \\ -0.2 & -1.5 \end{bmatrix}, B_2 = \begin{bmatrix} -2.5 & -1 \\ 1.2 & -2 \end{bmatrix}$$

$$E_k = -\begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}, k \in \mathbb{Z},$$

$$f_i(x) = \tanh(x), \tau_i(t) = 1 + 0.6\cos^2(t), i = 1, 2,$$

Here, the membership functions are given as $h_1(\theta(t)) = \sin^2(\frac{x_1(t)}{3})$, $h_2(\theta(t)) = \cos^2(\frac{x_1(t)}{3})$.

Without impulsive effects, i.e., $E_k = 0$, $k \in \mathbb{Z}$, the TSFNN (38) exhibits almost periodic behaviors. The phase plot of state variables with initial values $\varphi_1(s) = 0.4$ and $\varphi_2(s) = 0.5(s \in [-1.6, 0])$ is shown in Fig. 1. Now, let us impose impulses on the model and take $t_k - t_{k-1} = 0.08$ for every $k \in \mathbb{Z}$. In Theorem 3.1, choose $P = I$, $\epsilon_1 = 0.1$, $\epsilon_2 = 2$, it is straightforward to verify that the impulsive intervals satisfy $t_k - t_{k-1} < 0.0866$. Therefore, according to Theorem 3.1, the origin of TSFNN (38) is uniformly stable. The time responses of state variables are depicted in Fig. 2.

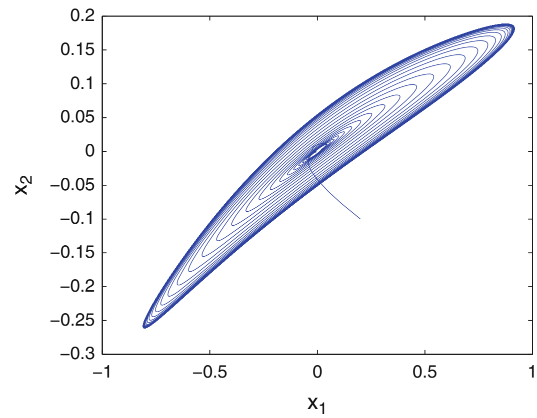

Fig. 3 The phase plot of state variables of (39) without impulsive control

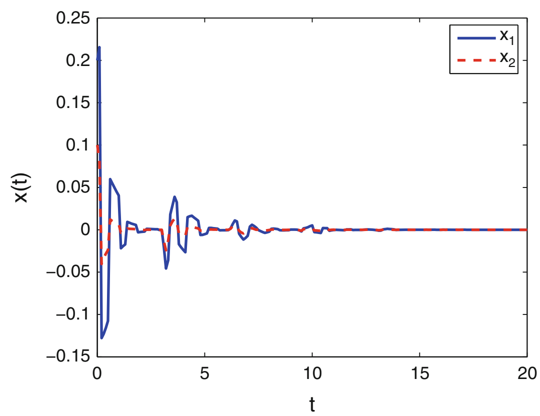

Fig. 4 The time responses of state variables of (39) with impulsive control

Simulation 2–Stabilization of periodic orbits using variable impulsive intervals
Consider another TSFNN with time-varying delays and impulses:

$$\begin{cases} \dot{x}(t) = \sum_{r=1}^{2} h_r(\theta(t))[-C_r x(t) + A_r f(x(t)) \\ \qquad\qquad +B_r f(x(t-\tau(t)))],\ t \geq t_0, t \neq t_k, \\ \Delta x(t) = E_k x(t_k),\ \ t = t_k,\ \ k \in \mathbb{Z}, \\ x(s) = \varphi(s),\ \ t \in [-1, 0], \end{cases} \tag{39}$$

where the model parameters are given as follows:

$$C_1 = C_2 = \text{diag}(3.5, 3.5),$$
$$A_1 = \begin{bmatrix} 3 & -0.1 \\ 0.4 & 2 \end{bmatrix}, A_2 = \begin{bmatrix} 1.8 & 20 \\ 0.1 & 1.8 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} -1.5 & -0.1 \\ -1.2 & -1.3 \end{bmatrix}, B_2 = \begin{bmatrix} -1.63 & 0.1 \\ 0.1 & -1.63 \end{bmatrix},$$
$$E_k = -\begin{bmatrix} 0.6 & 0 \\ 0 & 0.6 \end{bmatrix}, k \in \mathbb{Z},$$
$$f_i(x) = \tanh(0.6x), \tau_i(t) = 3, i = 1, 2,$$

Here, the membership functions are given as $h_1(\theta(t)) = \frac{1}{1+e^{sin(x_1)}}$, $h_2(\theta(t)) = 1 - h_1(\theta(t))$.

Without impulsive effects, i.e., $E_k = 0, k \in \mathbb{Z}$, the TSFNN (55) exhibits periodic orbits The phase plot of state variables with initial values $\varphi_1(s) = 0.2$ and $\varphi_2(s) = -0.1$ $(s \in [-3, 0])$ is shown in Fig. 3. Now, let

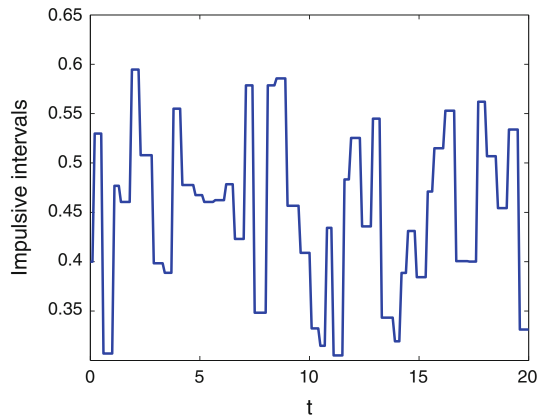

Fig. 5 The variable impulsive intervals in Simulation 2

us impose impulses on the model and by using the Matlab LMI Control Toolbox to solve the LMI (27), we obtain the solution as follows:

$$P = \begin{bmatrix} 46.6676 & -0.3143 \\ -0.3143 & 42.7611 \end{bmatrix} > 0,$$
$$a = 88.6283, b = 66.9761,$$
$$Q_1 = \begin{bmatrix} 208.1018 & 6.0348 \\ 6.0348 & 137.1213 \end{bmatrix} > 0,$$
$$Q_2 = \begin{bmatrix} 202.8380 & 29.8833 \\ 29.8833 & 140.6959 \end{bmatrix} > 0.$$

Thus it can be directly verified that

$$\gamma_k = 0.1600 < 1,\ k \in \mathbb{Z},\ \mu = 0.124 > 0.$$

Thereby, all the conditions in Theorem 3.2 are satisfied, which implies that the origin of the TSFNN (39) has been stabilized. Figure 4 shows the stabilized state trajectories of TSFNN (39) under initial values $\varphi_1(s) = 0.2$ and $\varphi_2(s) = 0.1 (s \in [-3, 0])$ after imposing impulsive control on the model with random impulsive intervals $t_k - t_{k-1} \in [0.3, 0.6]$. The varieties of the random impulsive intervals are shown in Fig. 5.

5 Conclusions

Although a few efforts on stability analysis of TSFNNs have been made, little study has been performed for stabilization of TSFNNs. In this paper, we discuss stabilization of TSFNNs with time-varying delays and present two stabilization criteria to guarantee the stability of the networks by employing an impulsive control scheme. One of the proposed conditions is in terms of LMI which can be readily solved by using standard numerical software. Two periodic TSFNNs have been provided to demonstrate the applicability and validity of the proposed control scheme.

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