Simplified Model to Predict Deflection and Natural Frequency of Steel Pole Structures

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Abstract Steel pole structures are suitable alternate to transmission line towers, due to difficulty encountered in finding land for the new right of way for installation of new lattice towers. The steel poles have tapered cross section and they are generally used for communication, power transmission and lighting purposes. Determination of deflection of steel pole is important to decide its functionality requirement. The excessive deflection of pole may affect the signal attenuation and short circuiting problems in communication/transmission poles. In this paper, a simplified method is proposed to determine both primary and secondary deflection based on dummy unit load/moment method. The predicted deflection from proposed method is validated with full scale experimental investigation conducted on 8 m and 30 m high lighting mast, 132 and 400 kV transmission pole and found to be in close agreement with each other. Determination of natural frequency is an important criterion to examine its dynamic sensitivity. A simplified semi-empirical method using the static deflection from the proposed method is formulated to determine its natural frequency. The natural frequency predicted from proposed method is validated with FE analysis results. Further the predicted results are validated with experimental results available in literature.

Keywords Steel monopole · Simplified model · Transmission / Communication monopole · Natural frequency · Unit load method · Semi-empirical method

Introduction

The steel poles are preferred over lattice towers due to aesthetic appearance. The steel poles are used in urban areas, where it is difficult to get land for installation of lattice towers. The steel poles have smaller base dimension compared to steel lattice towers and they are constructed in segments, which is easy for transportation and construction [1]. The steel poles have smaller plan dimension compared to lattice towers designed for similar functionality requirement in transmission and distribution field. The steel tubular poles are generally used for power transmission, communication, lighting and for advertisement purposes. A computer aided optimum design approach is proposed for the design of steel tubular pole structures [2]. The corrosion of steel pole pose serious threat to structural stability since it is a continuum structure and they are subjected to lesser wind load as compared to lattice towers. A tapered modeling technique considering geometric non-linearity was proposed to estimate the displacement of steel pole structures [3]. A simplified formula is proposed for the analysis of tapered poles based on equivalent pole concept [4]. Tapered steel poles are used for mounting CCTV cameras to monitor the traffic control system. Finite element analysis of tapered steel poles are carried out using elastoplastic solid element with eight nodes considering all geometric, material and contact non linearity along with surface contact algorithm. The load deformation results from FE analysis are in good agreement with experimental results [5]. A simplified formula is proposed to find the bending and local buckling of polygonal steel tubes subjected to both axial and bending load [6]. Three dimensional modeling of steel pole along with loading and boundary conditions is used for strength and deflection calculation [7]. The amplitude reduction technique through a plausible

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mechanism to reduce the vibration of tapered aluminium highway lighting pole was proposed to reduce its excessive oscillation [8]. However, a simplified approach to determine the deflection and natural frequency of steel pole to determine the preliminary design parameters is the need of the hour.

Present Study

In present study, a simplified method is proposed to determine the deflection of steel pole structures based on dummy unit load/moment method. The proposed method is validated with full scale experimental investigation conducted on 8 m and 30 m high lighting mast, 132 and 400 kV transmission poles. Since these structures are dynamically sensitive, a simplified semi-empirical method is proposed to determine the natural frequency from static deflection predicted using the proposed method. The predicted natural frequency is validated FE analysis results.

Proposed Method for Deflection Determination of Steel Pole

Deflection Due to Lateral Load

Consider the pole as shown in Fig. 1 with xyz co-ordinate system. The unit dummy load method is used to determine the displacement of steel pole. According to this method, the lateral displacement δ_{lat} at a distance z_d from the top of the pole is expressed as;

$$\delta_{lat} = \int_{z_d}^{H^p} \frac{Mm}{EI} dz \quad (1)$$

where ‘ M ’ and ‘ m ’ are the moments due to the applied and unit dummy loads applied in the transverse direction along the height of the pole, E is the modulus of elasticity and H^p is the pole height.

Assuming that a point load ‘ P ’ is acting at the top of the pole. The cross-sectional properties of the pole are calculated as per ASCE 48-05 [9] Manual recommendations. The moment of inertia ‘ I ’ is expressed as;

$$I = CD^3t$$

C is constant, D is diameter, and t is the thickness of the pole.

The bottom and top diameter of the pole are designated as D_b and D_t respectively. Then the slope of the pole is expressed as;

$$s_p = \left[\frac{D_b - D_t}{H^p} \right]$$

The lateral displacement δ_{lat} at a distance z_d from the top of the pole due to the application of lateral load P at a distance z_p from the top of the pole is expressed as;

$$\delta_{lat} = \int_{z_d \text{ or } z_p}^{H^p} \frac{P(z - z_p)(z - z_d)}{E[Ct(D_t + s_p z)^3]} dz \quad (2)$$

The lower limit of the integral has to be changed as z_d or z_p depending upon the location of deflection to be calculated. If the deflection measuring point is below the load point application, then the lower limit of the integral is z_d or vice versa. Solving the integral equation by method of partial fractions, the above integral equation is expressed as;

$$\int_{z_d \text{ or } z_p}^{H^p} \frac{P(z - z_p)(z - z_d)}{E[Ct(D_t + s_p z)^3]} dz = \frac{P}{CEt} \left[\int_{z_d \text{ or } z_p}^{H^p} \frac{A}{(D_t + s_p z)} + \int_{z_d \text{ or } z_p}^{H^p} \frac{B}{(D_t + s_p z)^2} + \int_{z_d \text{ or } z_p}^{H^p} \frac{C}{(D_t + s_p z)^3} \right] dz \quad (3)$$

The constants A , B , C are obtained as;

$$A = \frac{1}{s_p^2}$$

$$B = \frac{-1}{s_p^2} [(z_p + z_d)s_p + 2D_t]$$

$$C = \frac{D_t^2 + (z_d + z_p)D_t + z_d z_p s_p^2}{s_p^2}$$

Substituting the constants in the above integral equation, the deflection of the pole δ_{lat} is obtained as;

$$\delta_{lat} = \frac{P}{CEt} \left\{ A \left[\frac{\ln(D_t + s_p z)}{s_p} \right]_{z=z_d \text{ or } z_p}^{z=H^p} + B \left[\frac{-1}{s_p(D_t + s_p z)} \right]_{z=z_d \text{ or } z_p}^{z=H^p} + C \left[\frac{-1}{2s_p(D_t + s_p z)^2} \right]_{z=z_d \text{ or } z_p}^{z=H^p} \right\} \quad (4)$$

Deflection Due to Vertical Load

The deflection due to primary moment also contributes to the deflection of the pole. The moment due to vertical load is calculated as

$$M = V\delta_{lat} \quad (5)$$

where V is the vertical load applied and δ_{lat} is the deflection due to application of lateral load as shown in Fig. 2. Thus

Fig. 1 Pole dimension details **Fig. 2** Pole deflection

the lateral displacement due to vertical load δ_{vert} through moment M at a distance z_d from the top of the pole is expressed as;

$$\delta_{\text{vert}} = \int_{z_d}^{\text{HP}} \frac{M(z - z_d)}{\text{CEt}(D_t + s_p z)^3} dz \quad (6)$$

Solving the above equation through integration by parts, the deflection due to vertical load through moment is obtained as;

$$\delta_{\text{vert}} = \frac{M}{\text{CEt}} \left[\int_{z_d}^{\text{HP}} \frac{A}{(D_t + s_p z)} + \int_{z_d}^{\text{HP}} \frac{B}{(D_t + s_p z)^2} + \int_{z_d}^{\text{HP}} \frac{C}{(D_t + s_p z)^3} \right]$$

Substituting the value of constants in the above integral equation, the deflection due to vertical load δ_{vert} is obtained as;

$$\delta_{\text{vert}} = \frac{M}{\text{CEt}} \left\{ \left[\frac{-1}{s_p^2 + (D_t + s_p z)} \right]_{z=z_d \text{ or } z_p}^{z=\text{HP}} + \left(\frac{D_t}{s_p} + z_d \right) \left[\frac{-1}{2s_p(D_t + s_p z)^2} \right]_{z=z_d \text{ or } z_p}^{z=\text{HP}} \right\} \quad (7)$$

Thus the total deflection of the tapered steel pole is calculated as;

$$\delta_{\text{tot}} = \delta_{\text{lat}} + \delta_{\text{vert}} \quad (8)$$

Analytical and Experimental Investigation on Deflection of Steel Pole

The full scale experimental investigation is conducted to determine the deflection of different types of steel pole structures such as 8 m & 30 m lighting pole and 132 & 400 kV transmission pole. The analytical investigation is carried out using FE software NE-NASTRAN [10] to determine its natural frequency and deflection. The wind load on pole is calculated based on IS:875 (Part 3) 1987 [11]. The wind load on conductor, earthwire and insulators are determined as per IS:802 (Part 1/Section 1)1995 [12] codal recommendations.

8 m Street Lighting Pole

This type of pole is used for street lighting in rural and urban areas. The main shaft of the pole is tapered hollow with octagonal cross-section and is of 4 mm thick with 350 MPa as yield stress. The base plate of 20 mm thick and

250 MPa yield stress is welded to the bottom of the shaft. The schematic view of the pole with geometric configuration, dimension and the load tree is shown in Fig. 3. The entire pole is assembled and erected in the test bed. The load is applied at discrete nodal points along the height of the pole. The load is applied in steps of 50, 75, 90, 95 and 100% of the design load through load cells fixed at each load point. The application of load is controlled through centralized servo-controlled system through hydraulic actuators available in Tower Testing and Research Station, CSIR-SERC, Chennai, India. The deflection is measured at the top of the pole through total station instrument. The experimental set-up of the pole is shown in Fig. 4.

The pole is modeled in NE-NASTRAN, a non-linear FE analysis software, which accounts for both geometric and material non-linearity. The entire shaft including base plate is modeled using 4 noded plate shell elements, which can resist membrane bending and shear forces. The elastic and plastic material property of steel is represented by an

Fig. 3 Schematic view of 8 m lighting pole

Fig. 4 Experimental set-up

elasto-plastic bi-linear model, with a modulus of elasticity as 2E5 MPa up to yield and 2000 MPa above yield stress. The boundary condition is assumed such that the base of the pole is fixed and the load is applied at designated height of the pole similar to testing. The incremental load and predictor–corrector iteration for each increment of load is followed for non-linear analysis. For post yielding, isotropic hardening model is assumed and the yielding was modeled as Von Mises criterion. For the numerical solution convergence, arc-length method in conjunction with modified Newton–Raphson method is used. Non-linear static-analysis carried out and the deformed view of the pole is shown in Fig. 5a. The natural frequency of the pole is determined from eigen value analysis available in NE-NASTRAN. The deformed first mode view of FE model is shown in Fig. 5b. The deflection is computed using the proposed formula from Eq. 4 for lateral load and Eq. 7 for the deflection due to self-weight of lighting mass at the top of the pole and both the loads are added. The comparison of load vs deflection from experimental, FE analysis and from proposed method is shown in Fig. 6.

Fig. 5 FE models of 8 m pole. **a** Deformed FE model, **b** first mode frequency (2.4 Hz)

30 m Lighting Pole

The 30 m lighting pole is used for mass lighting in transportation hubs such as bus shelters, railway stations, airports and ports. The main shaft consists of 20 sided regular polygon with tapered cross-section. The shaft is made of three segments joined together by telescopic slip splice connection. The schematic view of the lighting mast along with loading tree is shown in Fig. 7. The erected pole in test bed is shown in Fig. 8.

The pole is modeled in NE-NASTRAN with similar assumptions as explained in “8 m Street Lighting Pole” section. In the region of slip splice connection, the combined thickness of shaft is assumed for modeling. The deformed view and first mode deformed model are shown in Fig. 9a, b respectively. The comparison of the deflection from proposed method with experiment and FE analysis is shown in Fig. 10.

132 kV Transmission Pole

The description of this pole is already explained in Ref. [1]. The configuration, geometric and load application details are shown in Fig. 11. The pole is assembled and erected in the test bed as shown in Fig. 12. The pole is modeled in

Fig. 6 Comparison of deflection for 8 m lighting pole

Fig. 7 Schematic view of 30 m lighting pole

Fig. 8 Experimental setup of 30 m lighting pole

Fig. 9 FE models of 30 m lighting pole. **a** Deformed FE model, **b** first mode frequency (0.9 Hz)

NE-NASTRAN assuming the material properties and non-linear assumptions similar to 30 m lighting pole as explained in “30 m Lighting Pole” section. The loads are applied at discrete nodal points and the deformed FE model

of the pole is shown in Fig. 13a. The deformed FE model showing the first mode of vibration of the pole is shown in Fig. 13b.

The combined thickness of the shaft is assumed in the region of slip splice connection for the computation of deflection using the proposed model. The deflection due to vertical load is computed based on the moment due to application of vertical load on the pole. The comparison of deflection from experiment, FE analysis and proposed method is shown in Fig. 14.

400 kV Transmission Pole

The description of 400 kV transmission pole is already explained in Ref. [1]. The schematic view of the pole showing the geometric configuration, cross arm and load application details are shown in Fig. 15. The pole is assembled and erected on the circular test bed through rock anchor system to resist the uplift force as shown in Fig. 16.

The combined thickness of the pole in the region of attachment of cross-arm and ground wire collar to the main shaft is considered in the FE modeling. All other non-linear material properties are assumed similar to 132 kV transmission pole. The deformed FE model of the pole due to the application of loads in reliability condition is shown in Fig. 17a. The deformed first mode of the pole from eigenvalue analysis is shown in Fig. 17b. The deflection from the proposed method is computed based on deflection due to lateral load on the mast. The deflection due to vertical load at the tip of cross-arm is computed from its moment due to vertical load and lever arm length which is equal to length of cross arm from the center of the main shaft. The average weighed thickness of the cross arm shaft including the collar thickness is considered for secondary deflection

Fig. 10 Comparison of deflection for 30 m lighting pole

Fig. 11 132 kV transmission pole

computation using the proposed method. The comparison of deflection from experiment, FE analysis and proposed semi-empirical method is shown in Fig. 18.

Proposed Semi-empirical Method for Determination of Natural Frequency of Steel Pole

Literature exists to find the natural frequency of tapered hollow steel poles considering all geometric, material and contact non-linearity based on regression analysis [13]. The natural frequency and mode shape of tower supporting utility was investigated based on model order reduction technique namely the lumped mass method [14]. The vibration and buckling problem of tapered cantilever beam was solved through Rayleigh and Timoshenko quotient method. It was found that Timoshenko approach was more reliable than Rayleigh's quotient method [15]. Rigid multibody system was adopted to determine the natural

Fig. 12 Experimental set-up of 132 kV transmission pole [1]

frequency of tapered cantilever beam in free bending employing Euler–Bernoulli Beam theory [16]. The natural frequency of stepped cantilever beam is compared using Rayleigh model, modified Rayleigh's model and FE analysis using ANSYS. The stiffness of beam is updated in each step of modified Rayleigh's model and the predicted natural frequency was found to be in close agreement with FE analysis results [17]. In the present study, natural frequency of pole is determined from the unit load deflection δ_{unit} predicted using the proposed formula by applying unit load at the top of the pole as shown in Fig. 2. The stiffness of the pole is obtained as;

$$k = \frac{1}{\delta_{unit}} \quad (9)$$

The cross sectional area of the pole is calculated based on ASCE 48-05 [9] manual recommendations. For tapered pole, the cross sectional area is calculated as;

$$A = C \cdot D \cdot t$$

where C is constant, D is diameter and t is the thickness of the pole. The average cross sectional area is considered for

Fig. 13 FE models of 132 kV transmission pole. **a** Deformed FE model, **b** first mode frequency (1.2 Hz)

the calculation of mass m of the pole. Thus the undamped natural frequency of the pole is obtained as;

$$\omega = \sqrt{\frac{k}{m}} \quad (10)$$

Based on the experimental investigation conducted on 30 m lighting mast, the damping ratio was found to be 4.5% in Ref. [18]. Hence 4.5% damping is assumed in the present study and the damped natural frequency is computed for all the poles and compared with FE analysis results. The same 4.5% damping is assumed in FE analysis. The proposed natural frequency is validated with experimentally determined natural frequency of 400 kV transmission pole available in Ref. [1] and the comparison with FE analysis results for all other poles are tabulated in Table 1.

Results and Discussion

It can be observed from Figs. 6, 10 and 14 that the experimental results closely matches with deflection from the proposed formula for 8 m and 30 m high lighting mast and 132 kV transmission pole. The FE analysis results varies up to 5 to 10% with experimental results at 100% load for these poles. This is due to the reason that the second order deflection due to application of vertical load is not captured in FE analysis, whereas in the proposed model the deflection due to vertical load near the mast is taken into account through moment deflection formulation.

In case of deflection of 400 kV transmission pole as seen from Fig. 18, the experimental and FE analysis deflection value varies by 30%. This is due to the reason that the moment exerted on mast due to application of

Fig. 14 Comparison of deflection for 132 kV transmission pole

Fig. 16 Experimental set-up of 400 kV transmission pole [1]. **a** Experimental set-up, **b** cross-arm, **c** ground wire

(a)

(b)

(c)

Fig. 17 FE model of 400 kV transmission pole. **a** FE model, **b** FE model first mode frequency (1 Hz)

(a)

(b)

Fig. 18 Comparison of deflection for 400 kV transmission pole

Table 1 Comparison of natural frequency of poles

S. no.	Type of pole	Natural frequency of pole in (Hz) based on		
		FE model	Proposed method	Experiment
1	8 m lighting pole	2.4	2.4	—
2	30 m lighting pole	0.9	0.9	—
3	132 kV transmission pole	1.2	1.2	—
4	400 kV transmission pole	1.0	0.9	0.9 ^a

^aReference [1]

Further the natural frequency predicted from proposed method is in close agreement with experimental and FE analysis results. Thus the dynamic sensitivity of these poles can be estimated as quick reference without the involvement of complex FE modeling. The wind load calculation method i.e., force co-efficient or gust factor method can be easily determined. Hence the preliminary design check can be carried out through this quick estimate of deflection and natural frequency from the proposed method, which is very much useful for the design engineers.

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