International Journal of Environmental Science and Technology (2019) 16:5059–5064
https://doi.org/10.1007/s13762-018-1874-9

ORIGINAL PAPER

Time–temperature superposition for viscoelastic materials with application to asphalt–aggregate mixes

A. Zbiciak[1] · R. Michalczyk[1] · K. Brzeziński[1]

Received: 21 November 2017 / Revised: 29 May 2018 / Accepted: 13 June 2018 / Published online: 19 June 2018

Abstract
This paper presents a new approach to characterisation of asphalt mixtures constitutive models, with application to pavement structures. It is well-known fact that temperature, frequency and time of loading have a great influence on the mechanical properties of bituminous mixtures. For this reason, the properties are usually presented in the frequency domain as complex numbers having real and imaginary parts. This convenient representation in terms of complex modulus and phase angle leads to the dynamic modulus master curve. The effective use of the theory of linear viscoelasticity to characterise constitutive model of asphaltic material is shown in this paper. Viscoelastic constitutive model is represented by a combination of rheological schemes, and its identification is based on both laboratory tests results and mixture composition. The temperature–frequency or temperature–time superposition principle being applied in order to produce master curves of mechanical properties is illustrated with real experimental data. Further, the process of identification of Huet–Sayegh parameters is carried out using best-fitting methods implemented in MATLAB. Fractional rheological model is used as it needs only a small number of elements to fully characterise the response of asphalt materials.

Keywords Asphalt–aggregate mixtures · Complex moduli · Curve fitting · Master curve · Viscoelasticity

Introduction

Since asphalt mixtures are viscoelastic materials, the operating conditions, such as velocity of the travelling load and pavement temperature, have a great influence on the strain state (Graczyk 2010). These conditions may vary within a large range (Rafalski 2007). Reflecting the stress and strain states of asphalt pavement taking into account both the temperature effect and the load velocity requires applying an appropriate computational model. The most important issue is the choice of the proper constitutive model used in order to describe the material response against mechanical and thermal loadings. Furthermore, the identification of material parameters is essential (Radziszewski et al. 2014). It can be done based on laboratory tests or by using an identification procedure proposed by Zbiciak et al. (2017). Among many proposals of rheological models in the literature, the most popular is the Burgers model. However, analysis of a wide range of time–temperature operating conditions can require the use of more complex models such as a generalised Maxwell model or Huet–Sayegh model (Zbiciak and Michalczyk 2014). One of previous paper (Zbiciak et al. 2017) shows comparison of results obtained for the Burgers and the Huet–Sayegh model in one temperature. On this basis, it was decided to use in this study only Huet–Sayegh, which performs better.

In this study, the identification of viscoelastic constitutive models is based on both laboratory tests results and mixture composition (Sybilski et al. 2010). Thus, the process of identification of Huet–Sayegh parameters has been carried out.

Editorial responsibility: iskender AKKURT.

✉ R. Michalczyk
r.michalczyk@il.pw.edu.pl

[1] Institute of Roads and Bridges, Warsaw University of Technology, 16 Armii Ludowej Ave., Warsaw, Poland

Materials and methods

Viscoelastic models of asphalt–aggregate mixtures

In order to describe the rheological properties of asphalt–aggregate mixture, Huet–Sayegh (HS) model has been used. The HS model contains non-classical linear viscoelastic elements whose constitutive properties are defined by fractional derivatives (Podlubny 1999; Butera and Di Paola 2014; Di Mino et al. 2016; Grzesikiewicz et al. 2013; Zbiciak 2012). The fractional derivatives are used in solution of many physical problems (Rzodkiewicz et al. 2009), including viscoelastic behaviour of certain materials from rheological point of view.

In the literature, the HS rheological model is usually defined via its complex stiffness modulus (frequency response), that is (Kim 2009),

$$E^*(i\omega) = E_o + \frac{E_\infty - E_o}{1 + \delta\,(i\omega\,\tau)^{-k} + (i\omega\,\tau)^{-h}}, \tag{1}$$

where ω denotes the frequency of excitation; E_o is the static modulus (as $\omega \to 0$); E_∞ is the glass transition modulus (as $\omega \to \infty$); and $h,\ k$ are parameters obeying the conditions $0 < k < h < 1$; δ is a dimensionless constant; and τ is the characteristic time.

The complex stiffness modulus $E^*(i\omega)$ constitutes the amplitude of the harmonic steady-state stress response $\sigma^*(i\omega) = E^*(i\omega)\, e^{i\omega t}$ excited by the complex strain $\varepsilon^*(i\omega) = e^{i\omega t}$. The frequency response of any linear system can be characterised by the magnitude $|E^*(i\omega)|$, sometimes called the dynamic stiffness modulus $E_{dyn}(\omega)$, and the phase $\phi(\omega)$. Thus, the following representation of the complex quantity holds

$$E^*(i\omega) = E_{dyn}(\omega)\, e^{i\varphi(\omega)}, \quad E_{dyn}(\omega) := |E^*(i\omega)|. \tag{2}$$

The magnitude and phase of the system frequency response, when using a logarithmic frequency scale, are called Bode plots. Moreover, the magnitude as a function of phase is called a Nichols or Black chart. Another characteristic of linear systems results from decomposition of the frequency response into real and imaginary parts

$$E^*(i\omega) = E'(\omega) + i\,E''(\omega), \tag{3}$$

$$E'(\omega) = \mathrm{Re}\,E^* = E_{dyn}(\omega)\,\cos\varphi\,, \quad E''(\omega) = \mathrm{Im}\,E^* = E_{dyn}(\omega)\,\sin\varphi, \tag{4}$$

where E' denotes the storage modulus representing elastic properties of the material and E'' is the loss modulus used to represent viscous properties. A parametric graph visualising the real part of the frequency response versus its imaginary part is called a Nyquist or Cole–Cole plot.

It can be proved that the fractional-order differential equation (FDE) describing the HS stress–strain ($\sigma - \varepsilon$) relationship is as follows:

$$\begin{aligned} &\dot{\sigma}(t) + \delta\,\tau^{-k}\,\mathrm{D}^{1-k}\sigma(t) + \tau^{-h}\,\mathrm{D}^{1-h}\sigma(t) \\ &\quad = E_\infty\,\dot{\varepsilon}(t) + E_o\left[\delta\,\tau^{-k}\,\mathrm{D}^{1-k}\varepsilon(t) + \tau^{-h}\,\mathrm{D}^{1-h}\varepsilon(t)\right], \end{aligned} \tag{5}$$

where the initial condition $\sigma(0) = \sigma_o$ is applied. The following definition of the fractional derivative of the order $\alpha \in (0,\ 1)$ for a function $z(t)$ can be applied to Eq. (5) (Grzesikiewicz et al. 2013):

$$\mathrm{D}^\alpha z\,(t) := \frac{z(0)}{\Gamma(1-\alpha)\,t^\alpha} + \frac{1}{\Gamma(1-\alpha)} \int\limits_0^t \frac{\dot{z}(\xi)}{(t-\xi)^\alpha}\,\mathrm{d}\xi, \tag{6}$$

where $\Gamma(\,\cdot\,)$ denotes the gamma function.

Identification of the asphalt mixture models based only on the stiffness moduli results by using the HS structure is difficult. In order to make this possible, an artificial creep curve was constructed which takes into account the correlation between the secant modulus values obtained in creep tests and the stiffness modulus determined in the cyclic test, where the appropriate relationship between the time and frequency of the load occurs. Thus, the laboratory results of the stiffness modulus tests were converted into points lying on a curve creep, as described in detail in a previous study of the authors (Zbiciak et al. 2017). A similar approach may also be used in analysing the rheological properties of asphalt (Brzezinski and Krainski 2016).

The fundamental problem is that there are no known analytical solutions of the HS creep function. On the other hand, all the characteristics of any linear fractional model (step response, impulse response, etc.) can be obtained by applying numerical algorithms for determining the inverse Laplace transforms.

As it was stated in Zbiciak and Grzesikiewicz (2011), numerical determination of the creep characteristics of the HS model requires the understanding of its transfer function $E^*(s)$, which results directly from Eq. (1) when substituting $i\omega$ by s (see Eq. 7a). The transfer function $E^*(s)$ characterises the response of the system and is defined as the ratio of the Laplace transforms of output and input signals (Eq. 7b)

$$E^*(s) = E^*(i\omega)\,|_{i\omega=s}, \tag{7a}$$

$$E^*(s) = \frac{\sigma^*(s)}{\varepsilon^*(s)}, \tag{7b}$$

where $\sigma^*(s) = \mathrm{L}\,\{\,\sigma(t)\}$ and $\varepsilon^*(s) = \mathrm{L}\,\{\,\varepsilon(t)\}$ are the Laplace transforms of the stress (output) and the strain (input) states, respectively.

Moving back to differential formulation of the HS model, it should be emphasised that the transfer function $E^*(s)$ results directly from Eq. (5) by applying the Laplace transform and using Eq. (7b).

The creep response of the Huet–Sayegh model can be obtained by applying an inverse Laplace transform L^{-1} as it is schematically visualised in Fig. 1, where $H(t)$ denotes a Heaviside step function modelling the excitation and $J(t)$ is the creep function defined as the strain response to a stress step excitation. In the case of the Huet–Sayegh model, obtaining an analytical form of the inverse Laplace

Time domain $\sigma(t) = H(t)$ $\quad\quad$ $\varepsilon(t) = J(t)$

$$\downarrow \; \mathcal{L} \; \downarrow \qquad\qquad \uparrow \; \mathcal{L}^{-1} \; \uparrow$$

Frequency domain $H^*(s) = \frac{1}{s} \rightarrow \left[E^*(s)\right]^{-1} \rightarrow \varepsilon^*(s) = \frac{1}{s\,E^*(s)}$

Fig. 1 Input–output relationship to obtain a creep response

transform is problematic. Thus, the process should be carried out numerically by using the algorithms described in the literature (Valsa and Brancik 1998; Liu 2001).

Creep master curve development

The temperature influences the viscosity of the material (Barzinjy and Zankana 2016). The frequency range of the measured components of the complex modulus (dynamic modulus and phase angle) covers only a small part of the total frequency range at different temperatures. The reduced variables method can be applied in order to reduce the experimental data to single curves (master curves). Master curves cover a wide range of frequencies at a chosen reference temperature. The same idea can be applied to the creep tests where the reduced variables method allows for the visualisation of all the experiments in a single creep master curve covering all times of loadings (at a particular reference temperature).

The method of reduced variables is also known as the temperature–frequency or the temperature–time superposition principle (TTSP) (Kim 2009; Wang 2010). Application of the TTSP requires for the data, collected at different temperatures, to be shifted relative to the time of loading or frequency. Experimental curves representing components of the complex modulus or creep curves obtained at various temperatures can be aligned to form a single master curve. The required shift at a given temperature is defined by the shift factor.

Results and discussion

In order to model the time–temperature relationships for pavement asphaltic layers, we used various equations known in the literature (Zbiciak and Michalczyk 2014). The most popular is the William–Landel–Ferry (WLF) equation. Assuming the WLF equation, we can derive relationship between the shift factor $a(T)$ and temperature T presented in Eq. (8a). It should be substituted into Eq. (1) via parameter τ, which is a function of temperature (Eq. 8b). A summary description of the procedure in one temperature is presented in Zbiciak et al. (2017). The data for asphaltic mixture were obtained from tests conducted in TN-248 study (Sybilski et al. 2010).

$$\log[a(T)] = -\frac{C_1\left(T - T_o\right)}{C_2 + T - T_o}, \tag{8a}$$

$$\tau = \tau_o\, a(T), \tag{8b}$$

where C_1 and C_2 are fitting parameters, T_o denotes the reference temperature and $\tau_o = \tau\left(T_o\right)$ is a constant to be determined for an arbitrarily chosen reference temperature.

The identification results of the HS rheological model for asphalt mixtures used in analysis of the pavement structure are summarised in Tables 1 and 2. Moreover, Figs. 2, 3 and 4 visualise selected curve fitting results presenting experimental creep points and fitted curves for binder course mixture.

Conclusion

In this paper, the problem of characterisation of asphalt mixtures constitutive models, with application to pavement structures, was investigated. In order to describe the rheological properties of asphalt mixture, Huet–Sayegh model was used, because it suitably describes mixture properties in a wide range of time–temperature loading conditions. Identification was done based on laboratory tests data and mixture composition. Based on the results, the following conclusions were drawn:

Table 1 Coefficients of HS model and WLF shift function—binder course mixture (AC16W 50/70)

Identification method	E_o (MPa)	E_∞ (MPa)	k	h	δ	τ(10°C)	C_1	C_2
4 PB results	67	20,295	0.366	0.592	5.334	2.250	19.182	136.907
Mixture composition	37	25,726	0.433	0.885	6.689	0.977	14.437	102.100

Table 2 Coefficients of HS model and WLF shift function—base course mixture (AC16P 50/70)

Identification method	E_o (MPa)	E_∞ (MPa)	k	h	δ	τ(10°C)	C_1	C_2
4 PB results	82	27,503	0.339	0.646	4.658	2.320	19.998	138.710
Mixture composition	78	27,701	0.438	0.802	5.763	0.822	18.490	146.600

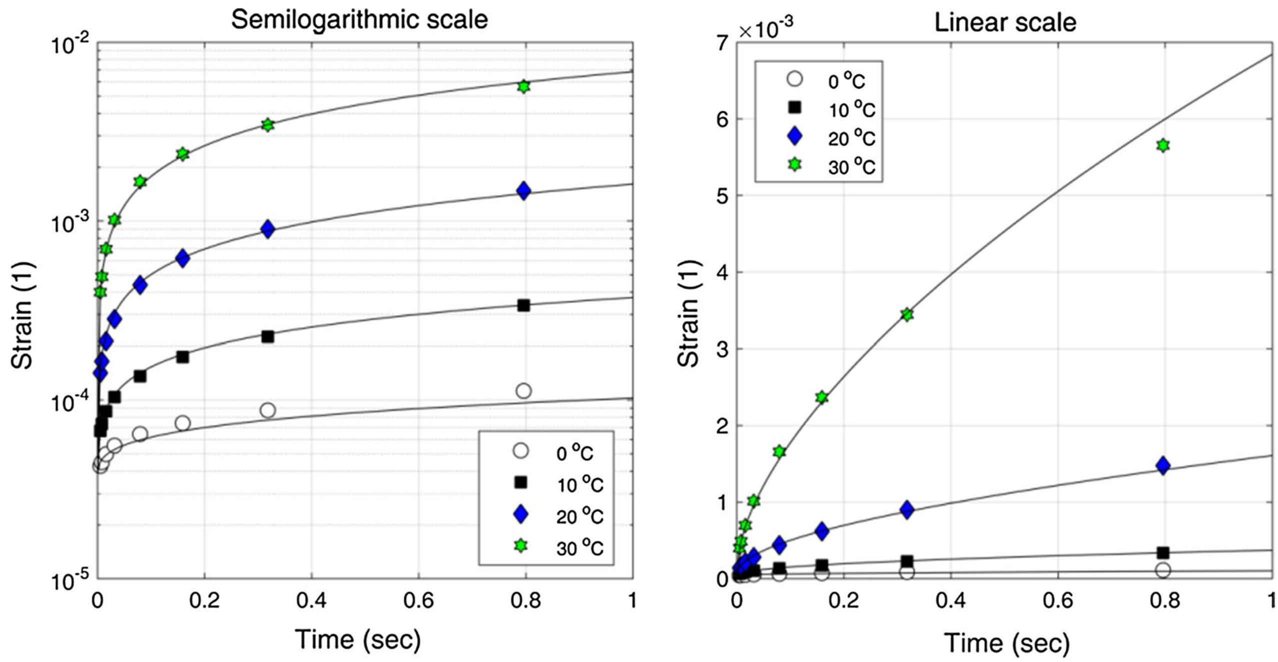

Fig. 2 Curve fitting results (creep curves at different temperatures) based on a mixture composition for AC16W 50/70 (semilogarithmic and linear scales)

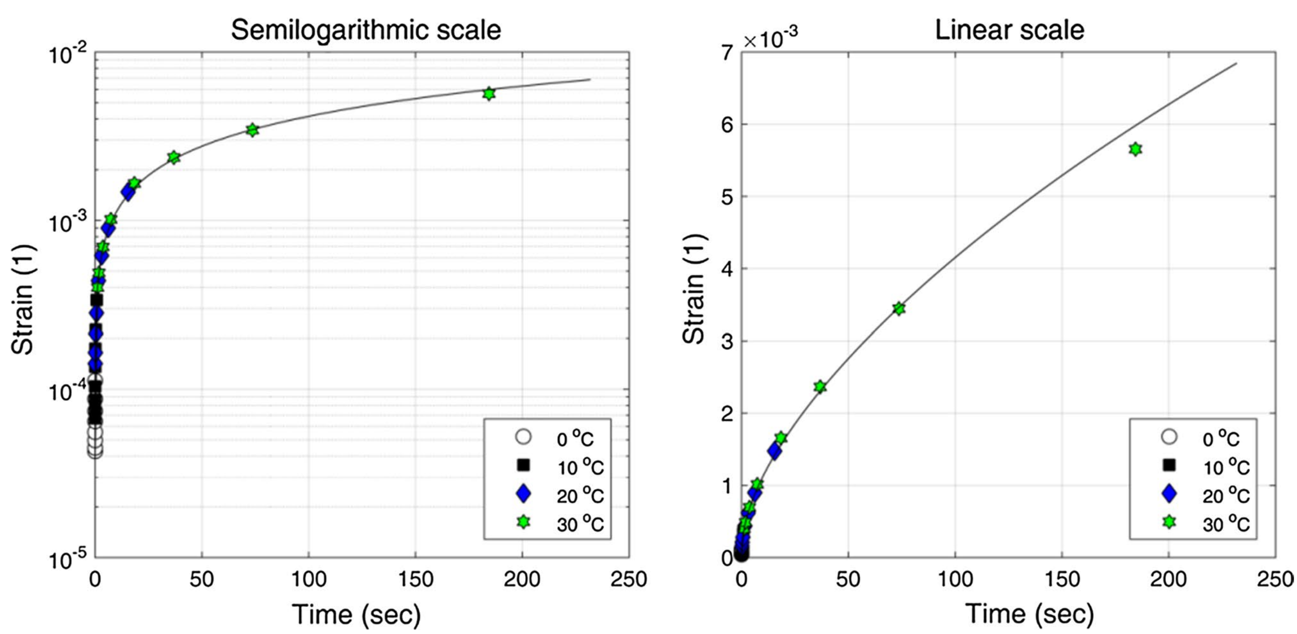

Fig. 3 Creep master curve based on mixture composition for AC16W 50/70 at a reference temperature of 10 °C (semilogarithmic and linear scales)

- The HS model can satisfactorily describe the rheological properties of asphalt mixes. What distinguishes it is the fact that its parameters are relatable to the construction of complex modulus and phase angle master curves, the Black diagram and the Cole–Cole diagram.
- In the time–temperature superposition and the shift factor equations study, the WLF equation generally produced

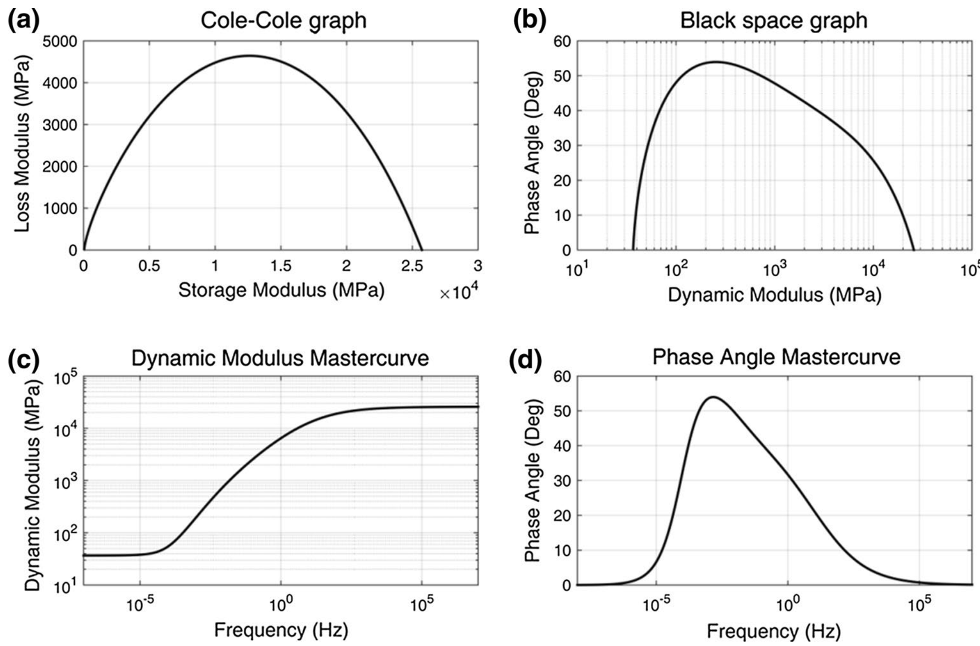

Fig. 4 Dynamic characteristics of AC16W 50/70 rheological model fitted based on mixture composition

very good results compared to the other equations and showed the highest correlation with measured shift factor data.

- The accuracy of the new method based on mixture composition is satisfactory. The main purpose is to avoid labour-intensive and cost-consuming laboratory tests at the early design stages.
- In particular, by applying numerical "extraction" of mixture creep curve from experimental mixture data, the procedure performed much better than classical empirical equations.

Since the analysis was performed only on narrow range of mix designs (Sybilski et al. 2010), it is recommended that the validation be extended to mix designs that include different types of binder and aggregate types. The analysis should also be extended to mixtures containing polymer-modified bitumens.

Acknowledgements This research was conducted by using the funds from Polish Government allocated to statutory aims of the Faculty of Civil Engineering. The data for asphaltic mixtures were obtained from TN-248 study (Sybilski et al. 2010).

References

Barzinjy A, Zankana M (2016) A novel application of the quartz crystal microbalance for determining the rheological properties of the highly viscous liquids. Acta Phys Pol A 130(1):239–244. https://doi.org/10.12693/APhysPolA.130.239

Brzeziński K, Kraiński Ł (2016) Zero shear viscosity estimation using a computer simulation of Van der Poel's nomograph. J Build Chem 1:10–17. https://doi.org/10.17461/j.buildchem.2016.102

Butera S, Di Paola M (2014) Fractional differential equations solved by using Mellin transform. Commun Nonlinear Sci Numer Simul 19(7):2220–2227. https://doi.org/10.1016/j.cnsns.2013.11.022

Di Mino G et al (2016) Linear and nonlinear fractional hereditary constitutive laws of asphalt mixtures. J Civ Eng Manag 22(7):882–889. https://doi.org/10.3846/13923730.2014.914104

Graczyk M (2010) Bearing capacity of the pavement multilayer structures in the national climatic conditions (in Polish). IBDiM, Warsaw

Grzesikiewicz W, Wakulicz A, Zbiciak A (2013) Non-linear problems of fractional calculus in modeling of mechanical systems. Int J Mech Sci 70:90–98

Kim YR (2009) Modeling of asphalt concrete. ASCE Press, McGraw-Hill, New York

Liu Y (2001) A direct method for obtaining discrete relaxation spectra from creep data. Rheol Acta 40:256–260

Podlubny I (1999) Fractional differential equations. Mathematics in science and engineering, vol 198. Academic Press, San Diego

Radziszewski P et al (2014) Quality assessment of bituminous binder based on the viscoelastic properties: Polish experience. J Civ Eng Manag 20(1):111–120

Rafalski L (2007) Road subbases (in Polish). IBDiM, Warsaw

Rzodkiewicz W et al (2009) Optical analyses of Si and GaAs semiconductors by fractional-derivative-spectrum methods. Acta Phys Pol A. https://doi.org/10.12693/aphyspola.116.s-95

Sybilski D et al (2010) Verification of the relationship between the properties of the stiffness modulus results obtained with various laboratory methods according to PN-EN 12697-26 (in Polish). IBDiM, Warsaw

Valsa J, Brancik L (1998) Approximate formulae for numerical inversion of Laplace transforms. Int J Numer Model 11(3):153–166. https://doi.org/10.1002/%28SICI%291099-1204%28199805/06%2911:3%3C153::AID-JNM299%3E3.0.CO;2-C

Wang L (2010) Mechanics of asphalt: microstructure and micromechanics. McGraw-Hill, New York

Zbiciak A (2012) Identification of viscoelastic properties of asphalt–aggregate mixes using classical and fractional rheological models. In: XXI Polish-Russian–Slovak seminar, theoretical foundation of civil engineering, Moscow–Arkhangelsk 2012, Proceedings, Warsaw, pp 187–196

Zbiciak A, Grzesikiewicz W (2011) Characteristics of fractional rheological models of asphalt–aggregate mixtures. Logistyka 6:4119–4124

Zbiciak A, Michalczyk R (2014) Characterization of the complex moduli for asphalt–aggregate mixtures at various temperatures. Proc Eng 91:118–123

Zbiciak A, Brzeziński K, Michalczyk R (2017) Constitutive models of pavement asphaltic layers based on mixture compositions. J Civ Eng Manag. https://doi.org/10.3846/13923730.2015.1128483