Correction to: Degenerate Bernstein polynomials

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Correction to: RACSAM

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Unfortunately, errata appear in the statement corresponding Theorems 2.6 and 2.10 in the original paper. In the statements of the theorems, the identities must be corrected as follows. After this modification, the amended proofs are also provided.

Theorem 2.6 For $n, i \in \mathbb{N}$, with $i \leq n$, and $x \in [0, 1]$, we have

$$(x)_{i,\lambda} = \frac{1}{(1-i\lambda)_{n-i,\lambda}} \sum_{k=i}^n \frac{\binom{k}{i}}{\binom{n}{i}} B_{k,n}(x|\lambda).$$

Theorem 2.10 For $n, i \in \mathbb{N}$, with $i \leq n$, and $x \in [0, 1]$, we have

$$\sum_{l=0}^i (x)_l S_{2,\lambda}(i, l) = \frac{1}{(1-i\lambda)_{n-i,\lambda}} \sum_{k=i}^n \frac{\binom{k}{i}}{\binom{n}{i}} B_{k,n}(x|\lambda),$$

where $(x)_l = x(x-1)(x-2) \cdots (x-l+1)$.

Proof of Theorem 2.6

$$\begin{aligned} \sum_{k=1}^n \frac{\binom{k}{1}}{\binom{n}{1}} B_{k,n}(x|\lambda) &= \sum_{k=1}^n \frac{k}{n} \binom{n}{k} (x)_{k,\lambda} (1-x)_{n-k,\lambda} \\ &= \sum_{k=1}^n \binom{n-1}{k-1} (x)_{k,\lambda} (1-x)_{n-k,\lambda} \\ &= \sum_{k=0}^{n-1} \binom{n-1}{k} (x)_{k+1,\lambda} (1-x)_{n-k-1,\lambda} \end{aligned}$$

The original article can be found online at <https://doi.org/10.1007/s13398-018-0594-9>.

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$$\begin{aligned}
&= (x)_{1,\lambda} \sum_{k=0}^{n-1} \binom{n-1}{k} (x-\lambda)_{k,\lambda} (1-x)_{n-k-1,\lambda} \\
&= (x)_{1,\lambda} (1-\lambda)_{n-1,\lambda}.
\end{aligned}$$

Now, we observe that

$$\begin{aligned}
\sum_{k=2}^n \frac{\binom{k}{2}}{\binom{n}{2}} B_{k,n}(x|\lambda) &= \sum_{k=2}^n \frac{k(k-1)}{n(n-1)} \binom{n}{k} (x)_{k,\lambda} (1-x)_{n-k,\lambda} \\
&= \sum_{k=2}^n \binom{n-2}{k-2} (x)_{k,\lambda} (1-x)_{n-k,\lambda} \\
&= \sum_{k=0}^{n-2} \binom{n-2}{k} (x)_{k+2,\lambda} (1-x)_{n-k-2,\lambda} \\
&= (x)_{2,\lambda} \sum_{k=0}^{n-2} \binom{n-2}{k} (x-2\lambda)_{k,\lambda} (1-x)_{n-k-2,\lambda} \\
&= (x)_{2,\lambda} (1-2\lambda)_{n-2,\lambda}.
\end{aligned}$$

Similarly, we have

$$\sum_{k=i}^n \frac{\binom{k}{i}}{\binom{n}{i}} B_{k,n}(x|\lambda) = (x)_{i,\lambda} (1-i\lambda)_{n-i,\lambda}.$$

□

Proof of Theorem 2.10 For $n \geq 0$, the degenerate Stirling number of the second kind is given by

$$(x)_{n,\lambda} = \sum_{k=0}^n S_{2,\lambda}(n, k) (x)_k.$$

Thus, by Theorem 2.6, we get

$$\sum_{l=0}^i (x)_l S_{2,\lambda}(i, l) = \frac{1}{(1-i\lambda)_{n-i,\lambda}} \sum_{k=i}^n \frac{\binom{k}{i}}{\binom{n}{i}} B_{k,n}(x|\lambda).$$

□

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