RACSAM (2015) 109:549–550
DOI 10.1007/s13398-014-0208-0

ERRATUM

Erratum to: The number of twists with large torsion of an elliptic curve

Filip Najman

Published online: 11 December 2014

Erratum to: RACSAM DOI 10.1007/s13398-014-0199-x

The statements in the proofs of Proposition 7 and Proposition 8 that all elliptic curves with j-invariants 0 and 1728 belong to 3 and 2 families of quadratic twists is false. There are infinitely many families of quadratic twists with j-invariant 0 and infinitely many with j-invariant 1728.

Accordingly, Proposition 7 of the original paper should be as follows:

Proposition 1 *Among all the elliptic curves with j-invariant 0, there exists infinitely many with trivial torsion, infinitely many with torsion C_2, infinitely many with torsion C_3, and 1 with torsion C_6.*

Proof An elliptic curve with j-invariant 0 is of the form

$$E_D : y^2 = x^3 + D, \quad \text{where } D \in \mathbb{Q}^*/(\mathbb{Q}^*)^6.$$

It follows that $E_D(\mathbb{Q})[2] \simeq \mathbb{Z}/2\mathbb{Z}$ if D is a cube and and $E_D(\mathbb{Q})[2]$ is trivial otherwise. A computation using division polynomials proves that there is no 4-torsion in $E_D(\mathbb{Q})$.

By [1, Theorem 3] it follows that $E_D(\mathbb{Q})[3] \simeq \mathbb{Z}/3\mathbb{Z}$ if D is a square and $E_D(\mathbb{Q})[3]$ is trivial otherwise.

There can be no other torsion in $E_D(\mathbb{Q})$ by [2, Proposition 1].

In Remark 2, it should say "infinitely many large torsion twists" instead of "4 large torsion twists" and the sentence "For example E_2 has 4 large torsion twists" should be deleted.

Also, Proposition 8 of the original paper should be as follows:

The online version of the original article can be found under doi:10.1007/s13398-014-0199-x.

F. Najman (✉)
Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia
e-mail: fnajman@math.hr

Proposition 2 *Among all the elliptic curves with* j*-invariant* 1728, *there exists infinitely many with torsion* C_2, *infinitely many with torsion* $C_2 \oplus C_2$ *and* 1 *with torsion* C_4.

Proof An elliptic curve with j-invariant 1728 is of the form

$$E_D : y^2 = x^3 + Dx, \quad \text{where } D \in \mathbb{Q}^*/(\mathbb{Q}^*)^4.$$

It follows that $E_D(\mathbb{Q})[2] \simeq \mathbb{Z}/2\mathbb{Z}$ if D is not a square and $E_D(\mathbb{Q})[2] \simeq \mathbb{Z}/2\mathbb{Z} \oplus \mathbb{Z}/2\mathbb{Z}$ if D is a square. A computation using division polynomials shows that the only elliptic curve with 4-torsion in this family is $E_4 : y^2 = x^3 + 4x$ with $E_4(\mathbb{Q})_{tors} \simeq \mathbb{Z}/4\mathbb{Z}$. By [1, Theorem 3], there is no p-torsion in $E_D(\mathbb{Q})$ for any odd prime p.

References

1. Dieulefait, L., González-Jiménez, E., Jiménez Urroz, J.: On fields of definition of torsion points of elliptic curves with complex multiplication. Proc. Am. Math. Soc. **139**, 1961–1969 (2011)
2. Parish, J.L.: Rational torsion in complex-multiplication elliptic curves. J. Number Theory **33**, 257–265 (1989)