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ORIGINAL ARTICLE

Learners' mathematical identities: exploring relationships between high school learners and significant others

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Abstract

Many researchers have tried to understand why some learners engage in and others disengage from mathematics by exploring learners' mathematical identities. Significant others (i.e., teachers, peers and family members) offer learners' different opportunities for mathematical identity support, but no study has explored their collective role in supporting or constraining learners' identities. In this paper, we explore how relationships between high school learners and different groups of significant others shaped learners' narrated mathematical identities. Fifty Grade 10 learners responded to a mathematical identity questionnaire. After analysing the results of the questionnaire, six learners were purposively selected for a semi-structured interview, and the interview data were analysed thematically. Identities are described as robust-leaning, mixed or fragile-leaning, depending on learners' narrated confidence, persistence, beliefs and sense of belonging to or exclusion from mathematics communities. Relationships with teachers, peers and family members are described as robust-enhancing or fragile-enhancing, depending on the kind of identity encouraged by the relationships. The findings show that many learners narrated fragile-leaning identities, and all the learners showed some robustness and fragility in their identities. The more groups of significant others a learner developed robust-enhancing relationships with, the more likely the learner narrated a robust-leaning identity. We argue that understanding learners' identities requires understanding their relationships with the three different groups of significant others as a collective.

Keywords Mathematical identity · Significant others

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Introduction

Repeated negative mathematical experiences have been linked to a decline in participation and interest in mathematics, with many learners choosing not to pursue mathematics as they move to higher grades (Darragh, 2015; Grootenboer & Jorgensen, 2009; Spaull, 2013). Some researchers have argued that the abstract nature of mathematics supports feelings among learners that mathematics is boring and is only useful for career opportunities (Andersson et al., 2015; Boaler et al., 2000). Learners' disengagement from mathematics has led researchers to investigate learners' mathematical identities and their relationships with mathematics, as a way of trying to understand why some learners enjoy mathematics and others hate it (e.g., Darragh, 2015; Hall et al., 2018; Sfard & Prusak, 2005). Understanding how learners construct their mathematical identities may suggest strategies likely to encourage more learners to enjoy and choose mathematics.

Mathematical identities are constructed in relation to others (Hall et al., 2018). Significant others in learners' lives (i.e., their teachers, peers and family members) offer learners' identity support which the learners can accept or reject (Gardee, 2019). Learners' mathematical identities are related to how they think their significant others position them and how they are positioned by their significant others (Hall et al., 2018).

A number of researchers have investigated the role of teachers in supporting learners' identities and found that how teachers communicate with learners and how they present mathematics in class are linked to learners' identities (e.g., Aydeniz & Hodge, 2011; Gardee, 2019). Some researchers have explored the role of peers and family members in how learners enact their identities and found that learners' views of the expectations of their peers and family members support or constrain their identities (e.g., Darragh, 2015; Hall et al., 2018; Zhang & Wang, 2021). Previous studies have tended to focus on the role of each group of significant others separately, and the extent of the collective role played by significant others in affording learners' identities has not been investigated. We therefore explored relationships between learners and three different groups of significant others: teachers, peers and family members, and how these relationships afforded learners' narrated identities. The following research questions guided the study:

1. What kinds of mathematical identities do high school learners narrate?
2. What kinds of relationships, with different groups of significant others, do high school learners narrate and how do these relationships collectively afford learners' mathematical identities?

In the next section, we outline our perspective on mathematical identities, followed by a discussion of research focusing on how significant others support learners' mathematical identities.

Learners' mathematical identities and relationships between learners and their significant others

Identity is a social construct strongly linked to a person's interactional experiences with others. Specifically, learners' mathematical identities are shaped by their interactions with their teachers, peers and families in mathematics classrooms and after-school and home communities.

Learners' mathematical identities

Many researchers do not precisely state what identity is and how it can be empirically studied. Identity has been difficult to define and operationalise (Fellus, 2019; Graven & Heyd-Metzuyanim, 2019; Ntow & Adler, 2019). Wenger's (1998) social practice theory places identity as central to practice and learning in communities of practice. Identity is therefore enacted in community with others (see also Darragh, 2016). Sfard and Prusak (2005) argue that identity is discursive and thus is produced as we narrate ourselves to ourselves and others (see also Bishop, 2012). These two seemingly contrasting positions, that identity can be enacted or narrated, are intimately interrelated because our actions are never separate from our discourse. Marks and O'Mahoney (2014) argue that identity is composed of personal identity, which is a view of the self, and social identity, which is how a person relates to the collective. Investigating both personal and social identities contributes to understanding identity since personal identity is supported or constrained by the social identities made available by society (Marks & O'Mahoney, 2014), and social identities are supported or constrained by the personal identity that a person enacts or narrates.

McGee (2015) argues that a learner's identity can be robust or fragile, depending on how the learner deals with persistent negative mathematical experiences. A learner with a robust identity is self-motivated to persist with mathematics despite persistent negative mathematical experiences, which means that their persistence with mathematics is intrinsically motivated, showing a strong relationship with mathematics. A learner with a fragile identity may not like mathematics but may persist with the subject to please others, which means that their persistence with mathematics is extrinsically motivated, showing a delicate relationship with mathematics (McGee, 2015). Intrinsically motivated persistence is motivated by a personal desire to conquer challenging activities, while extrinsically motivated persistence is motivated by external factors such as career opportunities, rewards or punishment or trying to please other people (McGee, 2015; Woolfolk, 2010).

We define mathematical identities, be they enacted or narrated, as composed of personal and social identities, which can be robust or fragile. Four different possible combinations of personal and social identities are suggested: robust personal and social identities, fragile personal and social identities, robust personal identities and fragile social identities and fragile personal identities and robust social identities.

Different researchers focus on different indicators of identities. Sabbah and Heyd-Metzuyanim (2019) focused on ethnicity and religion, Bishop (2012) focused on beliefs and persistence, Darragh (2013) focused on confidence and a sense of

belonging to mathematics communities and McGee (2015) focused on race and persistence. Researchers who want to understand the identities of populations focus on indicators specific to groups of people (e.g., Sabbah & Heyd-Metzuyanim, 2019), while those who want to understand identities at a personal level focus on indicators specific to individuals (e.g., Bishop, 2012). Some researchers focus on both sets of indicators (e.g., McGee, 2015).

Similarly to McGee (2015), we wanted to understand identities at both a social and personal level so we focused on confidence, persistence, beliefs about mathematics and a sense of belonging to or exclusion from mathematics communities. Confidence is a person's trust in their mathematical responses (Sander & Sanders, 2009). Persistence is not giving up when solving mathematical problems and pursuing mathematics despite facing challenges (Sutherland et al., 2013). Beliefs are personal views about mathematics formed by past experiences (Eleftherios & Theodosios, 2007). A sense of belonging is a feeling of being a member of a community of mathematics practice (Wenger, 1998), while a sense of exclusion is a feeling of not being a member of a community of mathematics practice. Confidence, persistence and beliefs are personal characteristics and are therefore indicators of personal identities, while a sense of belonging or exclusion shows how a person relates to the collective and is an indicator of social identities. Since personal and social identities are mutually constitutive (Gardee, 2019; Marks & O'Mahoney, 2014), and no indicator of identity operates in isolation from the other indicators (Fellus, 2019), all of the indicators influence each other.

McGee (2015) focused on a single indicator of personal identity, persistence, and a single indicator of social identity, race, and distinguished learners' overall identities as robust or fragile. For a deeper understanding of identity, we focus on three indicators of personal identity and a single but more encompassing indicator of social identity, a sense of belonging. We also make further distinctions in relation to robust and fragile identities, which will be discussed below.

Learners' mathematical identities and significant others

Goos and Bennison (2019) argue that a person's identity is constructed across different mathematics communities, suggesting that understanding how these communities help produce identities will help us understand learners' identities. While some studies have focused on how relationships between learners and teachers afford learners' identities (e.g., Aydeniz & Hodge, 2011; Boaler et al., 2000; Gardee, 2019; Gardee & Brodie, 2022; Grootenboer, 2013), others have focused on how relationships between learners and peers or learners and family members afford learners' identities (e.g., Heyd-Metzuyanim & Sfard, 2012; Sfard & Prusak, 2005; Turner et al., 2013; Zhang & Wang, 2021). We could not find any study that focused on how relationships between learners and the three different groups of significant others: teachers, peers and family members, support learners' identities.

Many learners find mathematics unappealing because they view the subject as dealing only with numbers rather than with ways of thinking (Epstein et al., 2010), suggesting fragilities in identities related to how mathematics is presented

by teachers. Some learners pursue mathematics for career opportunities and not for enjoyment, attributing this fragile position to the abstract nature of mathematics (Andersson et al., 2015; Aydeniz & Hodge, 2011; Boaler et al., 2000). If teachers teach mathematics for deep understanding, for example, by encouraging classroom discussion and showing learners that mathematics is about making connections between ideas, then learners' mathematical experiences may shift positively (Anderson et al., 2018; Andersson et al., 2015; Boaler et al., 2018; Cribbs et al., 2020), leading to the creation of more robust identities. Grootenboer (2013) found that teachers who exposed learners to tasks with multiple solutions or solution pathways supported robust identities. He also found that some teachers were quick to assist learners, which he argued robs learners of the opportunity to develop robust identities. He argued that teachers need to be skilled in finding a balance between protecting learners' identities and allowing learners to struggle with challenging problems if learners are to develop more robust identities.

Teachers' social relationships with learners can also support learners' mathematical identities (Forster, 2000; Gardee, 2019, 2021; Gardee & Brodie, 2022; Noren, 2011). Teachers who relate to their learners with care, concern and thoughtfulness, attend to learners' emotions and treat learners with respect and courtesy support learners' identities (Grootenboer, 2013). A teacher's explanation to a learner that it is normal and necessary to struggle to understand mathematics can reduce the learner's fears of mathematics (Braathe & Solomon, 2015), supporting the learner's identity. Some teachers label learners as good or weak at mathematics and believe that these are characteristics inherent in the learners (Gardee, 2019), contributing to the enactment of robust or fragile identities. In a follow-up paper, Gardee (2021) argued further that if teachers offer learners similar opportunities to participate in class irrespective of their abilities then the learners are more likely to develop more robust identities. Some teachers still publicly compare learners with each other, and this constrains many learners' identities, because of feeling embarrassed and inadequate (Grootenboer, 2013).

Many learners are pressured to pursue mathematics by their parents, and some are discouraged from pursuing the subject by their peers, which constrains the learners' identities (Gweshe & Brodie, 2019; Hall et al., 2018; McGee, 2015). Gardee and Brodie (2023) showed that a learner can be offered an identity of high, equal or low status by a peer and can construct their identity by affiliating with or resisting the offered identity. Gardee and Brodie (2023) argued that the kind of identity that a learner offers and constructs when working with others depends on the learner's personal and social identities.

Heyd-Metzuyanim and Sfard (2012) argued that if learners are not guided by their teacher, they may negatively influence each other's identities showing the importance of the collective role of teachers and peers in affording learners' identities. In another study, teachers facilitated class discussions in which peers attended to the explanations of problem-solving strategies of other learners. The peers were allowed to disagree with a learner's explanation but not in ways that positioned the learner as not capable of contributing to a group's discussion (Turner et al., 2013), a response usually offered to other learners by peers that constrain identities. Instead, the teachers encouraged peers to request clarifications and justifications in ways that

helped each other feel capable of contributing to the discussion (Turner et al., 2013). Langer-Osuna et al. (2020) found that learners who work on a task without guidance from the teacher can dominate the discussion by giving commands and ideas, which affects participation patterns and identity development. They argued that teachers need to teach learners how to give each other opportunities to participate fully in a group. The studies by Heyd-Metzuyanim and Sfard (2012), Langer-Osuna et al. (2020) and Turner et al. (2013) showed that peers usually constrain each other's identities, but if they receive proper guidance from their teachers, they can support each other's identities, strengthening the idea that teachers and peers can collectively support learners' identities.

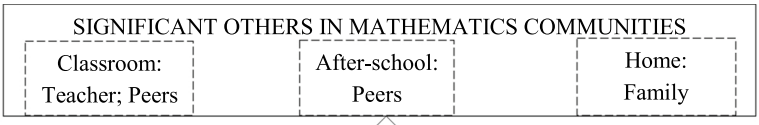
Several researchers have found that family members can support or constrain learners' identities. Sfard and Prusak (2005) found that if parents' educational expectations for their children are accompanied by family support rather than pressure, then learners can develop robust identities. If parents speak positively about mathematics, then their children will believe them, aim to meet the parents' educational expectations and develop more robust identities (McGee & Martin, 2011). Learners with parents who have high educational expectations are likely to develop more positive beliefs about mathematics than learners with parents who have low educational expectations (Zhang & Wang, 2021). Parents who praise their children when successful and encourage them when struggling, support their children's identities (Cunningham, 2021). Showing children the usefulness of mathematics outside of school and telling children stories of people similar to them who pursued mathematics and became successful in life support the children's identities (Cunningham, 2021). The collective role played by parents and teachers in supporting learners' identities was suggested when a learner frequently referenced her parents' and teachers' educational expectations, which she embodied and tried to meet, leading to the development of a robust identity (Hall et al., 2018).

The research reviewed here has shown that teachers, peers and family members support or constrain learners' mathematical identities. Despite the importance of these three groups of significant others for learners' identities, and some investigations into the collective role of two groups, no study has attempted to explore the collective role of all three groups in affording learners' identities.

Conceptual framework

Learners' personal and social identities are constructed through interaction with their teachers, peers and family members in mathematics classes, and after-school and home communities. Significant others in these communities offer learners different opportunities for identity support which the learners accept or reject, leading to the construction of different identities (Aydeniz & Hodge, 2011; Epstein et al., 2010; Gardee, 2019; McGee, 2015).

Identities can be robust or fragile (McGee, 2015) and are fluid (Bishop, 2012), so they can be constituted by different mixes of robustness and fragility. In this study, we consider a range of identities—from robust-leaning identities, those constituted by more robustness than fragility, through mixed identities, those constituted by



MATHEMATICAL IDENTITIES				
		PERSONAL IDENTITY		
		Robust-leaning	Mixed	Fragile-leaning
SOCIAL IDENTITY	Robust-leaning	*Robust-leaning personal and social identity*	*Mixed personal identity and robust-leaning social identity*	*Fragile-leaning personal identity and robust-leaning social identity*
	Mixed	*Robust-leaning personal identity and mixed social identity*	*Mixed personal and social identity*	*Fragile-leaning personal identity and mixed social identity*
	Fragile-leaning	*Robust-leaning personal identity and fragile-leaning social identity*	*Mixed personal identity and fragile-leaning social identity*	*Fragile-leaning personal and social identity*

Fig. 1 A framework for learners' identities

relatively equal mixes of robustness and fragility and to fragile-leaning identities, those constituted by more fragility than robustness. Since a person's identity is constituted by personal and social identities (Marks & O'Mahoney, 2014), their personal identity can be robust-leaning, mixed or fragile-leaning, and so can their social identity. The three kinds of personal identities and three kinds of social identities suggest that learners can construct nine different possible combinations of personal and social identities.

Figure 1 presents a framework showing relationships between learners' significant others and learners' personal and social identities.

Method

As we argued above, identities can be enacted or narrated. In this study, we focus only on narrated identities. The focus of this study was to understand how relationships between learners and their significant others as narrated by learners afforded learners' narrated personal and social identities. A qualitative research design within an interpretive paradigm was adopted (Lodico et al., 2010) because the focus was on learners' socially constructed realities, and data were analysed to show how the learners interpreted their and others' thoughts, actions and feelings (McMillan & Schumacher, 2010).

Permission to conduct the study was granted by the university's ethics committee. The participating high school was selected because of its convenience. The school is a township school similar to many other South African schools that learners from low socio-economic households attend. Parents do not pay school fees and learners

Table 1 Questions assigned to each indicator and total possible scores

Indicator	**No. of questions**	**Total possible score**	**Example** of a question
Confidence	4	20 (4×5)	I am confident in maths
Persistence	4	20 (4×5)	In maths, when faced with a hard problem I continue trying until I get it
Beliefs	5	25 (5×5)	Ability in maths increases when one studies hard
Sense of belonging	3	15 (3×5)	In maths, my contributions are valued

are given free lunch, textbooks and stationery. The provincial department of education and the school principal permitted us to conduct the research at the school. All learners and their parents gave consent for learners' participation in the study.

Sample, instruments and data collection

Fifty out of 328 Grade 10 learners volunteered to take part in the study and were assured of confidentiality and anonymity. For ethical reasons, we could only work with the 50 volunteers, which we did not consider to be a setback since the purpose of the study was to understand relationships rather than to generalise findings. The 50 learners responded to a 16-item mathematical identity questionnaire using a Likert scale, and after analysing the questionnaire responses, we selected some of the learners to participate in a semi-structured interview about their identities and their relationships with significant others. The questionnaire and interview schedules were piloted at a similar school during the previous year, and some changes were made to the instruments, for example, simplifying the language in the questionnaire. Items in the questionnaire and semi-structured interview were adapted from previous studies investigating learners' identities (e.g., Bishop, 2012; Darragh, 2015).

The purpose of the questionnaire was twofold: to gain an overview of the fifty learners' narrated identities and to help us purposively select learners for the semi-structured interview. The questionnaire consisted of 13 questions asking learners about their personal identities (i.e., confidence, persistence and beliefs) and 3 questions about their social identities (i.e., sense of belonging). Questionnaire responses ranged from strongly agree, scored as 5, to strongly disagree, scored as 1. The questionnaire was answered after school, and learners ticked their responses on paper using pencils or pens.

A total personal identity score was calculated by adding together the scores for confidence, persistence and beliefs. A learner's sense of belonging score was their social identity score. Although identities are far more complex than such quantification allows, this was the best way to get an overview from a number of learners, and we found that the questionnaire data were confirmed in the interviews.

Table 1 shows the indicators of learners' narrated identities, the number of questions asked, the total possible score and an example of a question for each indicator.

Table 2 shows the nine possible identity groups (see also Fig. 1). Four out of the 9 identity groups were observed in the data (as will be discussed later in the paper). These are cells 1, 5, 8, and 9 (as in italics).

We intended to select two learners from each of the four identity groups observed in the data to participate in a semi-structured interview. The two learners narrating robust-leaning personal and social identities, Rose and Ben (all names are pseudonyms), agreed to participate in the interview. None of the learners narrating mixed personal and social identities or mixed personal identities and fragile-leaning social identities (cells 5 and 8 in Table 2) agreed to participate in the interview. Since many of the learners had narrated fragile-leaning personal and social identities (cell 9 in Table 2), we felt that it was reasonable to select four learners from cell 9: Kelly, Neo, Tom and Cindy. The purpose of the interview was twofold: to understand the learners' narrated identities more deeply and to understand the relationships between the learners' significant others and the learners' narrated identities, from the learners' perspectives.

Individual interviews were conducted after school in the learners' classrooms. The interviews lasted for about 40 min each, and they were audio-recorded and transcribed. Before conducting the interview, learners were put at ease regarding the purpose of the interview and informed that no one except the researcher would see their responses, and their responses would not affect their school results. They were asked to answer honestly and told that they could choose not to answer any question. Some examples of the interview questions were:

- How do you see yourself as a maths learner?
- Please tell me about how you relate with your teacher in maths.
- How do you relate with other learners in maths in your class?
- What does your family say about you as a maths learner?

Data analysis

The questionnaire

Since we had 13 questions asking learners about their personal identities and each question's score ranged from 5 = strongly agree to 1 = strongly disagree, we considered the minimum total score indicating robustness in personal identities to be $13 \times 4 = 52$ and the maximum to be $13 \times 5 = 65$. So, learners in the personal identity score range of 52–65 were considered having more robustness than fragility in their personal identities and were therefore narrating robust-leaning personal identities. The score range indicating fragile-leaning personal identities was from $13 \times 1 = 13$ to $13 \times 2 = 26$, suggesting more fragility than robustness in personal identities. Total personal identity scores of 27–51 indicated relatively equal mixes of robustness and fragility in personal identities, and learners in this score range were considered narrating mixed personal identities. Learners in the social identity score range of 12–15 were considered narrating robust-leaning social identities. The score range indicating fragile-leaning social identities was 3–6, while social identity scores of 7–11

Table 2 The nine possible identity groups

		Personal identity		
		Robust-leaning	**Mixed**	**Fragile-leaning**
Social identity	**Robust-leaning**	*1. Robust-leaning personal and social identity*	2. Mixed personal identity and robust-leaning social identity	3. Fragile-leaning personal identity and robust-leaning social identity
	Mixed	4. Robust-leaning personal identity and mixed social identity	*5. Mixed personal and social identity*	6. Fragile-leaning personal identity and mixed social identity
	Fragile-leaning	7. Robust-leaning personal identity and fragile-leaning social identity	*8. Mixed personal identity and fragile-leaning social identity*	*9. Fragile-leaning personal and social identity*

Table 3 Score ranges for narrated identities

Identity	Score range
Personal identity	
Robust-leaning	52–65
Mixed	27–51
Fragile-leaning	13–26
Social identity	
Robust-leaning	12–15
Mixed	7–11
Fragile-leaning	3–6

indicated mixed social identities. Table 3 shows the score ranges indicating the different learners' personal and social identities.

The interview

The learners' interview responses were thematically analysed, and the coding process took place in two stages. First, to understand learners' narrated identities, we coded for responses indicating confidence, persistence, beliefs and a sense of belonging to mathematics communities. Second, we coded for responses indicating relationships between learners and significant others. Two kinds of relationships were considered: robust-enhancing relationships, which encouraged the narration of robust-leaning personal and social identities; and fragile-enhancing relationships, which encouraged the narration of fragile-leaning personal and social identities. Table 4 shows the codes for relationships between learners and significant others, code descriptors and examples of learner quotes. The codes in Table 4 are predetermined codes drawn from the literature. The reported codes are not exhaustive since codes considered insignificant in relation to the study were left out.

Findings

All of the learners' narrated identities were constituted by some robustness and fragility. Relationships between learners and their significant others collectively supported or constrained learners' narrated identities.

Learners' narrated identities

The questionnaire

Table 5 shows the nine different possible combinations of learners' personal and social identities and the learners who narrated each identity combination. Four out of the nine possible combinations were seen in our data and five were not.

Table 4 Codes for relationships between learners and significant others

Code	Code descriptor and example of a quote	
	Robust-enhancing actions	**Fragile-enhancing actions**
Teacher pedagogy	Teacher teaches for understanding: *He encourages us to try harder and not leave a problem incomplete*	Teacher does not teach for understanding: *When the teacher teaches it's boring, it's just equations and sums*
Peer beliefs	Positive reaction to negative peer beliefs: *They say maths is difficult. I see that they don't understand what they are saying and what maths is*	Negative reaction to negative peer beliefs: *They say maths is boring and difficult. It makes me have a mind that says maths is difficult*
Peer responses	Positive peer responses: *When we are given homework we stay after school and help each other*	Negative peer responses: *They laugh at me*
Family pressure	Family does not pressure learner to pursue mathematics: *My family says they are very proud of me and will still love me whether I pass or fail*	Family pressures learner to pursue mathematics: *I'm the only one who has Grade 10 in my family and I feel that I have to carry on and make my parents proud*
Family support	Family supports learner: *She encourages me not to stop but to keep trying*	Family does not support learner: *They are busy most of the time. They don't even have time to sign my books*

Table 5 Identity combinations and narrated identities

Personal identities				
		Robust-leaning	**Mixed**	**Fragile-leaning**
Social identities	**Robust-leaning**	1. L26, L50 (2 learners)	2	3
	Mixed	4	5. L1, L19, L24, L30, L44 (5 learners)	6
	Fragile-leaning	7	8. L2, L5, L12, L27, L29, L32, L37, L42 (8 learners)	9. L3, L4, L6, L7, L8, L9, L10, L11, L13, L14, L15, L16, L17, L18, L20, L21, L22, L24, L23, L28, L31, L33, L34, L35, L36, L38, L39, L40, L41, L43, L45, L46, L47, L48, L49 (35 learners)

Table 6 Selected learners' identity scores

Identity	Learner code	Confidence (20)	Persistence (20)	Beliefs (25)	Personal identity (65)	Social identity (15)
Robust-leaning personal and social identities	Rose (L26)	15	17	24	56	14
	Ben (L50)	18	18	23	59	15
Fragile-leaning personal and social identities	Kelly (L7)	5	8	9	22	5
	Neo (L18)	6	7	9	22	5
	Tom (L28)	5	5	6	16	4
	Cindy (L2)	9	7	8	24	4

The majority of learners in the study were in cell 9, with some in cells 5 and 8 and two in cell 1. Each learner's personal and social identity score combination fits into one of the nine cells allowing for the differentiation of the learners' identities (see also in the Appendix). All the learners showed different mixes of robustness and fragility in their narrated identities suggesting that identities are narrated along a continuum. For example, L42 in cell 8 and L22 in cell 9, respectively, had personal identity scores of 27 and 26 and social identity scores of 6 each.

Six learners were selected for the semi-structured interview based on the questionnaire data. They were Rose (L26) and Ben (L50), narrating robust-leaning personal and social identities, and Kelly (L7), Neo (L18), Tom (L28) and Cindy (L2), narrating fragile-leaning personal and social identities (Table 6). Findings from the interview, which we discuss in the next section, confirmed these six learners' narrated identities from the questionnaire.

The interview

Robust-leaning personal and social identities The interviews showed that Ben and Rose narrated robust-leaning personal and social identities. The two learners described their mathematical experiences as follows:

> Ben: I'm very good at maths because there is nothing that I think I'm unable to do. I'm willing to do more work and I put much effort into maths that I never believed I could because it seems like a fun subject. My teacher has been teaching me since last year so he understands how I do things. Maths is useful except for doing classwork.
> Rose: Maths increases my confidence, when I get a sum right I want to do another one. I find it [maths] interesting because sometimes we just say maths is difficult and we find something right and we laugh in class. You can even do it [maths] just for fun. We [peers] help each other because we know each other from primary.

These two learners trusted their mathematical capabilities, and the trust was linked to past experiences with mathematics. Both learners suggested a personal desire to pursue mathematics. Positive descriptions of previous experiences with mathematics by the two learners suggested positive beliefs about mathematics. The learners' positive comments about mathematics, their teacher and peers indicated a sense of belonging to their classroom mathematics community and the narration of robust-leaning social identities. Ben's negative views of classwork suggested that learners narrating robust-leaning personal and social identities can show some fragility in their narrated identities.

Fragile-leaning personal and social identities The interviews showed that Kelly, Tom, Neo and Cindy narrated fragile-leaning personal and social identities. Kelly, Tom and Neo described their mathematical experiences as follows:

> Kelly: Sometimes I feel that I can't do maths but I work hard because I want to make my teacher proud. I feel like my teacher doesn't know me very well, he doesn't talk to me that much. My mind says maths is boring and difficult.
> Tom: I'm not so good at maths. I don't think I will pass maths this term, even if I try it's just hard. My family says that I can't do anything without maths so I have to put in the effort, but maths is a hard and horrible subject that can't be conquered. I don't talk with other learners that much, I believe I'm an introvert.
> Neo: I don't think I'm good at maths because I failed last year. Some [peers] say I won't make it, and it discourages me. I'm the only one with Grade 10 in my family and I feel that I have to carry on doing maths and make my parents proud.

These learners' comments showed that they had low confidence in mathematics. External factors, such as teachers and family members, influenced the learners to pursue mathematics, suggesting the narration of extrinsically motivated persistence. The learners' past experiences with mathematics were unpleasant for them, possibly leading to the development of negative beliefs about mathematics. The learners suggested that they did not feel a connection with other members of their classroom mathematics communities, indicating a sense of exclusion from their classroom mathematics communities. Tom constantly described mathematics as a subject that could not be "conquered", which meant that for him, mathematics could not be understood.

Cindy's narrative did not indicate low confidence but it indicated extrinsically motivated participation:

> Cindy: I participate because I look forward to succeeding and making a fortune. If maths is not needed for jobs I won't focus on it, I will focus on the job. My family says I can't go anywhere without maths so I try by all means to practice and love maths. I can say that maths is difficult, that's why I don't compromise on my time for maths. Other learners are selfish, they don't share what they know with me but I share everything with them.

Table 7 Robust-enhancing relationships related to teacher pedagogy

Learner	Teacher pedagogy
Rose	He encourages us to try harder and not leave a problem incomplete. Sometimes I get it right and that makes me feel that I can be an achiever. When we don't get it he helps us. I would say I'm improving and gaining confidence because of my teacher
Ben	He gives clues and explains things a little bit further and that makes me put in effort. He praises me and that strengthens me to do more challenging problems
Neo	If I don't know a sum, my teacher gives me examples so that I can understand. It helps me to try harder. He says maths is easy if you practice. It makes me think that I have to try harder because when I try I find new ways of solving problems. If he can do it then I can also do it

Cindy's participation was motivated by a desire to get a well-paying job, and her persistence was motivated by family pressure. Since extrinsically motivated persistence contributes to fragility in identity (McGee, 2015), we argue that extrinsically motivated participation contributes to fragility in identity. Believing that mathematics was difficult influenced Cindy to put effort into mathematics, indicating persistence motivated by negative beliefs about mathematics and fragility in her narrated identity. Cindy felt a sense of exclusion from her classroom mathematics community linked to not sharing ideas with her peers, indicating a narrated fragile-leaning social identity.

Relationships between learners and their significant others

We wanted to understand the role of teachers, peers and family members in supporting or constraining learners' narrated identities. In this section, we present learners' narratives about their relationships with their different groups of significant others. We do not have the significant others' narratives about the learners.

Learner-teacher relationships

Three learners, Rose, Ben and Neo, had robust-enhancing relationships with their teachers. Rose and Ben narrated robust-leaning personal and social identities while Neo narrated fragile-leaning personal and social identities. The other three learners, Kelly, Tom and Cindy, had fragile-enhancing relationships with their teachers, and they narrated fragile-leaning personal and social identities. Rose, Ben, Kelly and Cindy were taught by the same teacher, with Rose and Ben in the same class while Kelly and Cindy were in another class.[1] Neo and Tom were in the same class and were taught by a different teacher from the other four learners.

Robust-enhancing relationships with teachers Rose, Ben and Neo felt that their teachers taught mathematics for understanding. The learners appreciated their

[1] Ability grouping was not practiced at the participating school.

Table 8 Fragile-enhancing relationships related to teacher pedagogy

Learner	Teacher pedagogy
Kelly	He tells us the method of solving a problem and we work on the problem. I don't connect with him most of the time…he doesn't say much about me. He says maths is fun and challenging but that makes me scared to face challenging problems
Cindy	I feel like most learners don't understand the teacher. I don't share my methods with him because he says it's not the right method. He tells us that if you practice you will understand maths and if you love maths you will pass it. I try to love it but it's difficult
Tom	When the teacher teaches us it's boring, it's just equations and sums. He says maths is easy if you work on it every day. It makes me think I can do it but most of the time I just think I can't do it

teachers' pedagogies, and their comments indicated that teachers' pedagogies can support or constrain learners' narrated confidence and persistence (Table 7).

The teachers allowed learners to struggle with problems, which encouraged the learners to "do more challenging problems" and to feel successful. The teachers explained and the learners felt supported by their teachers' pedagogies. Because depth in teaching mathematics is related to positive beliefs about mathematics (Boaler et al., 2018), we argue that these learners' experiences of their teachers' pedagogies strengthened their beliefs about mathematics. Neo's comment indicated that teachers can act as learners' role models.

Fragile-enhancing relationships with teachers Kelly and Cindy felt that their teacher did not teach mathematics for understanding (Table 8). Tom felt that his teacher did not assist him to like mathematics.

Kelly and Cindy felt that their teacher insisted on particular methods and did not explain why and did not give learners a chance to ask questions or give their methods, which negatively affected their participation and sense of belonging to their classroom mathematics community. Tom suggested that his teacher did not change his teaching style, leading to boredom. Not teaching mathematics for deep understanding leads to learners not understanding and developing negative beliefs about mathematics (Boaler et al., 2018). The learners' comments suggested that telling learners that mathematics is fun or easy may not be sufficient to support learners' narrated identities.

Learner-peer relationships

Two learners, Rose and Ben, had robust-enhancing relationships with their peers, and they narrated robust-leaning personal and social identities. Four learners, Neo, Kelly, Tom and Cindy had fragile-enhancing relationships with their peers and they narrated fragile-leaning personal and social identities.

Table 9 Peer beliefs and responses

Learner	Peer beliefs	Peer responses
Rose	Other learners say that maths is hard. I find it interesting because sometimes we just say maths is difficult and we find something right and laugh in class	When we are given homework we stay after school and help each other, we don't laugh at each other. Some learners say I'm good at maths and it increases my confidence, I try to work harder and help other kids
Ben	They say maths is difficult. I see that they don't understand what they are saying and what maths is	We work together, sit in groups and share everything related to maths. They praise and encourage me to do more and that encourages me to help more learners

Robust-enhancing relationships with peers Rose and Ben had good relationships with their peers and resisted their peers' negative beliefs about mathematics (Table 9).

Rose and Ben's beliefs about mathematics were different from their peers' beliefs, indicating different experiences with mathematics from their peers. For Rose, struggling with a problem and eventually solving it was an interesting experience. Ben's comment suggests that not understanding what mathematics is can contribute to developing a negative view of mathematics.

Despite Rose and Ben's peers having negative beliefs about mathematics, they worked together on homework. Rose and Ben were encouraged and praised by their peers, which supported their narrated confidence, persistence and sense of belonging to the classroom and after-school mathematics communities.

Fragile-enhancing relationships with peers Peers' negative beliefs and attitudes constrained Neo, Kelly, Cindy and Tom's narrated identities (Table 10). These learners' comments suggested that they did not believe in their mathematical capabilities and they felt that their peers' attitudes discouraged them from participating in mathematical activities.

Peers' negative beliefs about mathematics supported these learners' negative beliefs. The learners felt excluded from their classroom mathematics communities, linked to being laughed at and made fun of when stuck on problems. The learners' narrated confidence in and beliefs about mathematics were constrained.

Learner-family relationships

Two learners, Rose and Kelly, had robust-enhancing relationships with their families. Rose narrated robust-leaning personal and social identities while Kelly narrated fragile-leaning personal and social identities. Four learners, Neo, Tom, Cindy and

Table 10 Peer beliefs and responses

Learner	Peer beliefs	Peer responses
Neo	They say maths is boring, I sometimes feel like yes it's boring	They laugh at me [when stuck] and it makes me think that maths is difficult. Some say I won't make it
Kelly	They say it's boring and difficult. It makes me have a mind that says maths is difficult and I can't solve all the problems all the time	I participate but not all the time because they sometimes embarrass me
Cindy	They say many negative things about maths. It affects me to listen to them. I begin to have that same belief about maths	They don't assist me [when stuck]. All they do is say hahaha! It's wrong here and stuff. It makes me think that I'm not good enough to learn maths
Tom	They say it's difficult. It affects me negatively because if the entire class doesn't think that they can do maths then I too feel as if I can't because no one is stepping up and trying to conquer maths	[If stuck], very few help me… It makes me perceive maths as a hard subject that can't be conquered

Table 11 Supportive families

Learner	Family support
Rose	Sometimes when I get something wrong I cry but my mom tells me that I'm good at maths. She encourages me not to stop but to keep trying. Sometimes I teach her some sums and that makes me feel that I'm growing up in maths and should not quit
Kelly	My family says they are proud of me and will still love me whether I pass or fail. When I get stuck on a problem my brother helps me

Ben had fragile-enhancing relationships with their families. Neo, Tom and Cindy narrated fragile-leaning personal and social identities while Ben narrated robust-leaning personal and social identities.

Robust-enhancing relationships with family members Rose and Kelly felt that their families supported them, which was robust-enhancing (Table 11). There were no indications of family pressure from Rose or Kelly's responses.

Rose and Kelly's comments indicated that they were encouraged and helped when struggling by their family members. Rose's narrated persistence and confidence were supported and related to a robust-enhancing learner-family relationship.

Fragile-enhancing relationships with family members Family members pressured Neo, Tom and Cindy to pursue mathematics but offered little support (Table 12). The sources of pressure were family members' past experiences with mathematics and the assumption that success in mathematics was a gateway to a better socio-economic status. Family members' past experiences with mathematics and their busy schedules limited the amount of support given to the learners. Ben's family pressured him not to pursue mathematics.

Neo, Tom and Cindy's comments indicated that coming from a family that historically did not study mathematics in the higher grades and the assumption that mathematics is a gateway to a better socio-economic status by family members can contribute to a narrative of extrinsically motivated persistence in mathematics. The learners felt that they were not being supported by their families, which was fragile-enhancing.

Ben felt that he was not being supported in dealing with the challenges that come with studying mathematics, which possibly constrained his narrated identity. Ben's family did not support him and they pressured him not to pursue mathematics. Ben resisted this pressure, illustrating robustness in his narrated identity which was supported by his teacher and peers, and indicated the collective role played by

Table 12 Non-supportive families

Learner	Family pressure	Family support
Neo	I'm the only one with Grade 10 in my family, I feel I have to carry on doing maths and make my parents proud	They also find maths difficult because they didn't do Grade 10, so they can't help me
Tom	My family says that I can't do anything without maths so I have to put in the effort. I feel it's compulsory to do maths to succeed in life	I live with my granny. She doesn't know maths so she can't help me. All she says is that maths is difficult but I must do it to succeed in life. It makes me feel as if I can't do maths but it's something that I need to do to achieve my goals
Cindy	My family says I can't go anywhere and I can't make a living without maths. I feel it's a must to love maths and to learn it	They are busy most of the time. They don't even have time to sign my books
Ben	Aah! [Hesitantly]. Honestly, my family doesn't say anything [about him as a maths learner] because they are not used to encouraging me	Last year they said I must do mathematical literacy[a] because maths was difficult for me. They don't give me the support that I expect

[a]Mathematical literacy is a subject that learners who find mathematics challenging can opt to take in South Africa (Spaull, 2013)

Table 13 Learners' relationships with significant others and their narrated identities

Learner	Rose	Ben	Neo	Kelly	Cindy	Tom
Teacher	RE	RE	RE	FE	FE	FE
Peers	RE	RE	FE	FE	FE	FE
Family	RE	FE	FE	RE	FE	FE
Identity	RL	RL	FL	FL	FL	FL

RE robust-enhancing relationship, *FE* fragile-enhancing relationship, *RL* robust-leaning, *FL* fragile-leaning

significant others in supporting or constraining learners' narrated identities. Ben had a fragile-enhancing relationship with his family but narrated robust-leaning personal and social identities, suggesting that having a fragile-enhancing relationship with only one group of significant others does not necessarily support the narration of fragile-leaning personal and social identities.

Summary

Table 13 shows that Rose had robust-enhancing relationships with all three groups of significant others while Ben had robust-enhancing relationships with two groups of significant others, his teacher and peers. Rose and Ben narrated robust-leaning personal and social identities. Neo and Kelly each had a robust-enhancing relationship with one group of significant others, Neo with his teacher and Kelly with her family. Tom and Cindy did not have robust-enhancing relationships with any of the three groups of significant others. Neo, Kelly, Tom and Cindy narrated fragile-leaning personal and social identities.

Discussion and conclusion

In discussing our findings, we address our two research questions. The first question is as follows: What kinds of mathematical identities do high school learners narrate? We found that while all the learners' narratives indicated some robustness and fragility in identities, more than half of the learners narrated fragile-leaning personal and social identities. This study resonates with other studies that have found that many learners enact fragile identities (Aydeniz & Hodge, 2011; Bishop, 2012; Epstein et al., 2010; Hall et al., 2018). The learners' narrated identities arose from their experiences with mathematics and social interactions with their teachers, peers and family members in different communities of mathematics practice.

The second research question is as follows: What kinds of relationships, with different groups of significant others, do high school learners narrate and how do these relationships collectively afford learners' mathematical identities? Many researchers have investigated the relationships between teachers and learners, peers and learners and family members and learners separately, despite learners being in constant interaction with all three groups of significant others (e.g., Boaler et al., 2000; Gardee, 2019; McGee, 2015; McGee & Martin, 2011). We moved forward

by trying to understand how these three groups of significant others afford learners' narrated mathematical identities collectively. The findings showed that the relationships between learners and their significant others, as narrated by the learners, were either robust- or fragile-enhancing. Robust-enhancing relationships with at least two groups of significant others supported the narration of robust-leaning personal and social identities while fragile-enhancing relationships with at least two groups of significant others supported the narration of fragile-leaning personal and social identities.

Two learners, Rose and Ben, reported that they had robust-enhancing relationships with their teacher and peers, and they narrated robust-leaning personal and social identities. Three learners, Kelly, Cindy and Tom, reported that they had fragile-enhancing relationships with their teacher and peers, and they narrated fragile-leaning personal and social identities. This finding supports studies that have shown that teachers can work with peers to support more robust identities (Heyd-Metzuyanim & Sfard, 2012; Turner et al., 2013).

Peers' and family members' relationships with learners were usually fragile-enhancing for the learners. This finding, coupled with the finding that more than half of the learners narrated fragile-leaning personal and social identities, shows the importance of peers and family members in supporting learners' narrated identities. Some peers supported each other by working together on homework and praising each other. Ben was not supported and pressured to pursue mathematics by his family, and he developed a fragile-enhancing relationship with the family.

Ben's case suggests that a learner can resist a fragile-enhancing relationship with one group of significant others because of the support that they get from the other two groups of significant others. Neo and Kelly's cases suggest that having a robust-enhancing relationship with only one group of significant others may not be enough to overcome the other fragile-enhancing relationships.

This study examines learners' narrated identities by analysing learners' narratives about their relationships with their teachers, peers and family members. Confidence, persistence, beliefs and a sense of belonging to mathematics communities are part of learners' narrated identities and are supported or constrained through the kinds of relationships that learners develop with their teachers, peers and family members. Teachers' pedagogies and peers' and family members' experiences with mathematics are key to the kinds of relationships developed between learners and these three groups of significant others and therefore key to the kinds of identities that learners narrate.

We recognise some limitations in our study. Our findings depend on our choice and definitions of the indicators of identities and are based on a limited number of learners at one school and from one grade. The fifty learners who volunteered to participate in the study may have been more mathematically confident than those who did not volunteer. Future research may investigate other indicators or work with a larger sample and several grades, providing a broader range of experiences and allowing for more general results. Separating questionnaire scores indicating robustness and fragility in identity led to learners with score differences of only one or two points located in different identity cells. However, this approach, of quantifying a qualitative construct and creating

groups based on this quantification, did help to gain an overview of learners' identities based on their narrated personal and social identities.

We relied on the learners' narrations of their interactions with their significant others, and we did not observe the actual relationships. We did not interview significant others about the learners, which could have added depth to our findings. Interviewing the significant others about their personal and social mathematical identities might have helped us to understand why they offered learners the social identities they offered.

Our study is the first to try to understand the relationship between learners' identities and learners' relationships with their teachers, peers and family members as a collective. The study shows that a deeper understanding of identity requires analysing learners' collective relationships with the three groups of significant others, indicating the importance of all three groups of significant others to learners' identities. We have broadened McGee's (2015) definition of identity, which considers two identity indicators and two identity groups, by considering four identity indicators and nine identity groups. The collective role of significant others in identity and the broader definition of identity can be used by educators and researchers to better understand the development of learners' mathematical identities.

Appendix

Learners' questionnaire scores and narrated identities

Cell number	Identity	Learner code	Confidence (20)	Persistence (20)	Beliefs (25)	Personal identity (65)	Social identity (15)
1	Robust-leaning personal and social identities	L26	15	17	24	56	14
		L50	18	18	23	59	15
5	Mixed personal and social identities	L1	8	12	18	38	8
		L19	8	10	19	37	8
		L24	12	12	14	38	7
		L30	12	12	15	39	7
		L44	8	12	18	38	9
8	Mixed personal identities and fragile-leaning social identities	L3	13	6	9	28	4
		L5	7	12	9	28	6
		L12	12	10	7	29	5
		L27	9	7	11	27	5
		L29	10	9	8	27	5
		L32	8	7	12	27	5
		L37	9	8	10	27	5
		L42	9	8	10	27	6

Cell number	Identity	Learner code	Confidence (20)	Persistence (20)	Beliefs (25)	Personal identity (65)	Social identity (15)
9	Fragile-leaning personal and social identities	L2	9	7	8	24	4
		L4	6	6	9	21	5
		L6	8	7	7	22	6
		L7	5	8	9	22	5
		L8	6	7	9	22	4
		L9	8	8	7	23	6
		L10	8	6	7	21	6
		L11	7	7	6	20	6
		L13	6	7	5	18	3
		L14	8	6	9	26	3
		L15	6	8	7	21	6
		L16	6	7	5	18	5
		L17	5	9	8	22	5
		L18	6	7	9	22	5
		L20	8	7	6	21	5
		L21	4	9	7	20	6
		L22	9	8	9	26	6
		L24	10	4	11	25	6
		L23	8	11	6	25	6
		L28	5	5	6	16	4
		L31	8	9	9	26	6
		L33	8	9	9	26	6
		L34	8	5	11	24	3
		L35	11	8	7	26	6
		L36	11	8	7	26	5
		L38	9	8	9	26	6
		L39	8	6	11	25	4
		L40	8	6	8	22	4
		L41	9	9	8	26	3
		L43	7	8	10	25	6
		L45	7	7	7	21	5
		L46	9	8	9	26	6
		L47	9	8	8	25	6
		L48	8	6	7	21	3
		L49	8	9	9	26	5

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Declarations

Consent to participate All participants and their parents consented that the participants would participate in the study after having been assured of confidentiality and anonymity. Ethical clearance was granted by the University of the Witwatersrand Ethics Committee. Protocol Number: 2017ECE005D.

Conflict of interest The authors declare no competing interests.

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