Hom-left-symmetric color dialgebras, Hom-tridendriform color algebras and Yau's twisting generalizations

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Abstract

The goal of this paper is to introduce and give some constructions and study properties of Hom-left-symmetric color dialgebras and Hom-tridendriform color algebras. Next, we study their connection with Hom-associative color algebras, Hom-post-Lie color algebras and Hom-Poisson color dialgebras. Finally, we generalize Yau's twisting to a class of color Hom-algebras and use endomorphisms or elements of centroids to produce other color Hom-algebras from given one.

Keywords Hom-tridendriform color algebra · Hom-associative color algebra · Hom-left-symmetric color dialgebra · Hom-Poisson color dialgebra

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1 Introduction

The investigations of various q -deformations (quantum deformations) of Lie algebras began a period of rapid expansion in 1980's stimulated by introduction of quantum groups motivated by applications to the quantum Yang-Baxter equation, quantum inverse scattering methods and constructions of the quantum deformations of universal enveloping algebras of semi-simple Lie algebras. In [2,19–25,31,32,39–41] various versions of q -deformed Lie algebras appeared in physical contexts such as string theory, vertex models in conformal field theory, quantum mechanics and quantum field theory in the context of q -deformations of infinite-dimensional algebras, primarily the q -deformed Heisenberg algebras, q -deformed oscillator algebras and q -deformed Witt and q -deformed Virasoro algebras, and some interesting q -

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deformations of the Jacobi identity for Lie algebras in these q -deformed algebras were observed.

Hom–Lie algebras and more general quasi-Hom–Lie algebras were introduced first by Larsson, Hartwig and Silvestrov [30], where the general quasi-deformations and discretizations of Lie algebras of vector fields using more general σ -derivations (twisted derivations) and a general method for construction of deformations of Witt and Virasoro type algebras based on twisted derivations have been developed, initially motivated by the q -deformed Jacobi identities observed for the q -deformed algebras in physics, along with q -deformed versions of homological algebra and discrete modifications of differential calculi. The general abstract quasi-Lie algebras and the subclasses of quasi-Hom–Lie algebras and Hom–Lie algebras as well as their color algebras generalizations have been introduced [30,35–38,56]. Subsequently, various classes of Hom–Lie admissible algebras have been considered in [49]. In particular, in [49], the Hom-associative algebras have been introduced and shown to be Hom–Lie admissible, that is leading to Hom–Lie algebras using commutator map as new product, and in this sense constituting a natural generalization of associative algebras, as Lie admissible algebras leading to Lie algebras via commutator map as new product. In [49], moreover several other interesting classes of Hom–Lie admissible algebras generalising some classes of non-associative algebras, as well as examples of finite-dimensional Hom–Lie algebras have been described. Since these pioneering works [30,35–38,49], Hom-algebra structures have developed in a popular broad area with increasing number of publications in various directions. In Hom-algebra structures, defining algebra identities are twisted by linear maps. Hom-algebras structures are very useful since Hom-algebra structures of a given type include their classical counterparts and open more possibilities for deformations, extensions of homology and cohomology structures and representations, Hom-coalgebra, Hom-bialgebras and Hom–Hopf algebras (see for example [1,3–11,18,26,27,29,33–35,50,53–55,58,60,63] and references therein).

Dendriform algebras were introduced by Loday as algebras with two operations which dichotomize the notion of associative algebras [42]. They are connected to K-theory, Hopf algebras, homotopy Gerstenhaber algebra, operads, homology, combinatorics, and quantum field theory where they occur in the theory of renormalization of Connes and Kreimer. While tridendriform algebra were introduced later by Loday and Ronco in their study of polytopes and Koszul duality [43].

Hom-tridendriform algebras were introduced in [48] as a twisted generalization of tridendriform algebras. More precisely, a Hom-tridendriform algebra is a linear space A , together with three bilinear maps $\dashv, \vdash, \cdot : A \otimes A \rightarrow A$ and a linear map $\alpha : A \rightarrow A$ satisfying, for $x, y, z \in A$,

$$\begin{aligned}(x \dashv y) \dashv \alpha(z) &= \alpha(x) \dashv (y \dashv z + y \vdash z + y \cdot z), \\ \alpha(x) \vdash (y \vdash z) &= (x \dashv y + x \vdash y + x \cdot y) \vdash \alpha(z), \\ (x \dashv y) \cdot \alpha(z) &= \alpha(x) \cdot (y \vdash z), (x \vdash y) \cdot \alpha(z) = \alpha(x) \vdash (y \cdot z), \\ (x \cdot y) \dashv \alpha(z) &= \alpha(x) \cdot (y \dashv z), (x \cdot y) \cdot \alpha(z) = \alpha(x) \cdot (y \cdot z), \\ (x \vdash y) \dashv \alpha(z) &= \alpha(x) \vdash (y \dashv z).\end{aligned}$$

When $\alpha = id$, we recover tridendriform algebras. The constructions of these algebras and their relationships with Hom-pre-Lie algebras are provided.

Connections between Hom-bialgebras and Hom-coalgebras, Hom-dendriform and tridendriform systems and Rota–Baxter and Hom–Rota–Baxter Hom-algebra structures are considered in [44–48,51].

As for Hom-Post-Lie algebras, they have been studied in [13] as a twisted generalization of post-Lie algebra. Post-Lie algebras first arise from the work of Bruno Vallette [57] in 2007 through the purely operadic technique of Koszul dualization. In [52], it is shown that they also arise naturally from differential geometry of homogeneous spaces and Klein geometries, topics that are closely related to Cartan's method of moving frames. The universal enveloping algebras of post-Lie algebras and the free post-Lie algebra are studied. Some examples and related structures are given. Further, they were extended to graded case in [16].

It has been noticed by D. Yau that one can deform associative and Lie algebra structures into Hom-associative and Hom-Lie algebras by composition of associative algebra multiplication with algebra endomorphism. This procedure, known as Yau's twisting or a composition method has been intensively used for many other Hom-algebraic structures such as Hom-Akivis algebras [17], Hom-Lie Yamaguti algebras [28], Hom-Poisson algebras [61], Hom-Post-Lie algebras [13], Hom-pre-Lie (or left Hom-symmetric) algebras [48, 49], G-Hom-associative algebras [49, 59], Hom-dendriform algebras [48, 59], modules over Hom-bialgebras [62]. Left-Hom-symmetric dialgebras and Hom-Poisson dialgebras, their connection to Hom-Leibniz algebras using Loday commutator and their constructions using composition method have been considered in [15].

Here we prove that this technique works for a big class of Hom-algebraic structures and for other special linear maps as averaging operator or element of centroid. In the literature, the graded version, including super and color case, of many of these structures were studied.

The paper is organized as follows. In Sect. 2, we show on the one hand that the tensor product of two Hom-associative color algebras is also a Hom-associative color algebra. On the other hand, that the tensor product of a commutative Hom-associative color algebra and a Hom-left-symmetric color algebra is a Hom-left-symmetric color algebra. We give a construction of a Hom-associative color dialgebra from an associative color algebra and an averaging operator. Next, we introduce Hom-left-symmetric color dialgebras and study their connection with Hom-Poisson color dialgebras. In Sect. 3, we show that Hom-Post-Lie and Hom-associative color algebras may come from Hom-tridendriform color algebras. Next, we pointed out that the opposite of any Hom-tridendriform color algebra is also one, and any Hom-tridendriform color algebra carries a structure of Hom-dendriform algebra. Section 4 is devoted to a Yau's twisting generalization to Hom-algebraic structures with a finite number of bilinear products, and use averaging operators or centroids to produce other color Hom-algebras from given one.

Throughout this paper, all graded vector spaces are assumed to be over a field $\mathbb{K}$ of characteristic different from 2.

2 Hom-left-symmetric color dialgebras

For an abelian group G and G -graded linear space $V = \bigoplus_{g \in G} V_g$, an element $x \in V$ is said to be homogeneous of the degree (color) $g \in G$ if $x \in V_g$. The set of all homogeneous elements of V is denoted by $\mathcal{H}(V)$. A linear mapping $f : V \rightarrow W$ between two G -graded linear spaces $V = \bigoplus_{g \in G} V_g$ and $W = \bigoplus_{g \in G} W_g$ is said to be homogeneous of the degree $d \in G$ if $f(V_g) \subseteq W_{g+d}$ for all $g \in G$. Homogeneous mapping is said to be even if it is of degree 0_G (degree zero), that is $f(V_g) \subseteq W_g$ for any $g \in G$. A bilinear mapping $f : V \times V \rightarrow V$ is said to be even if $f(V_a, V_b) \subseteq V_{a+b}$ for all $a, b \in G$. An algebra $\mathcal{A}$ is said to be G -graded if its underlying linear space is G -graded, that is $\mathcal{A} = \bigoplus_{g \in G} A_g$ and

furthermore $A_a A_b \subseteq A_{a+b}$, for all $a, b \in G$. A homomorphism $f : A \rightarrow B$ of G -graded algebras is a homomorphism of algebras which is also an even mapping.

Given an abelian group G , a map $\varepsilon : G \times G \rightarrow \mathbb{K}^*$ is called a skew-symmetric bicharacter or a commutation factor on G if for all $a, b, c \in G$:

- (i) $\varepsilon(a, b)\varepsilon(b, a) = 1$;
- (ii) $\varepsilon(a, b + c) = \varepsilon(a, b)\varepsilon(a, c)$;
- (iii) $\varepsilon(a + b, c) = \varepsilon(a, c)\varepsilon(b, c)$.

Remark that $\varepsilon(a, 0) = \varepsilon(0, a) = 1$, $\varepsilon(a, a) = \pm 1$ for all $a \in G$, where 0 is the identity of G . If x and y are two homogeneous elements of degree a and b respectively and ε is a skew-symmetric bicharacter, then we shorten the notation by writing $\varepsilon(x, y)$ instead of $\varepsilon(a, b)$.

Definition 1 [64] A Hom-associative color algebra is a triple (A, μ, α) consisting of a G -graded linear space A , an even bilinear map $\mu : A \times A \rightarrow A$ and an even homomorphism $\alpha : A \rightarrow A$ satisfying

$$\mu(\alpha(x), \mu(y, z)) = \mu(\mu(x, y), \alpha(z)), \quad (1)$$

for any $x, y, z \in \mathcal{H}(A)$ (homogeneous elements of A).

If in addition $\mu = \varepsilon(\cdot, \cdot)\mu^{op}$ i.e. $\mu(x, y) = \varepsilon(x, y)\mu(y, x)$, for any $x, y \in \mathcal{H}(A)$, the color Hom-associative algebra $(A, \mu, \varepsilon, \alpha)$ is said to be a commutative Hom-associative color algebra.

Theorem 1 is proved by a direct calculation.

Theorem 1 Let $(A_1, \cdot_1, \varepsilon, \alpha_1)$ and $(A_2, \cdot_2, \varepsilon, \alpha_2)$ be two Hom-associative color algebras. Then $A = A_1 \otimes A_2$ is a color Hom-associative algebra for twisting map $\alpha := \alpha_1 \otimes \alpha_2 : A \rightarrow A$ and the product $*$: $A \otimes A \rightarrow A$ defined by

$$\alpha(a_1 \otimes a_2) := \alpha_1(a_1) \otimes \alpha_2(a_2), \quad (a_1 \otimes a_2) * (b_1 \otimes b_2) := \varepsilon(a_2, b_1)(a_1 \cdot_1 b_1) \otimes (a_2 \cdot_2 b_2).$$

Corollary 1 The tensor product of two associative color algebras is also an associative color algebra.

Definition 2 A Hom-color algebra or a color Hom-algebra $(S, \cdot, \varepsilon, \alpha)$ is an G -graded linear space S equipped with even bilinear multiplication $\cdot$, even twisting map α and commutation factor ε . A Hom-color algebra is called a Hom-left-symmetric color algebra if the following Hom-left-symmetric color identity (or ε -Hom-left-symmetric identity)

$$(x \cdot y) \cdot \alpha(z) - \alpha(x) \cdot (y \cdot z) = \varepsilon(x, y)((y \cdot x) \cdot \alpha(z) - \alpha(y) \cdot (x \cdot z)) \quad (2)$$

is satisfied for all $x, y, z \in \mathcal{H}(S)$.

Example 1 Let $A = A_{(-1)} \oplus A_{(1)} = \langle e_2, e_3 \rangle \oplus \langle e_1 \rangle$ be a $G = \{-1, +1\}$ -graded linear space. Then A is a Hom-left-symmetric color algebra together with the bicharacter

$$\varepsilon(i, j) = (-1)^{(i-1)(j-1)/4},$$

the multiplication $e_1 e_1 = -e_1$, $e_2 e_1 = -ae_2$, $e_3 e_1 = e_3$, $e_1 e_2 = -ae_2$, and the even linear map $\alpha : A \rightarrow A$ defined by

$$\alpha(e_1) = e_1, \quad \alpha(e_2) = ae_2, \quad \alpha(e_3) = -e_3, \quad a \in \mathbb{R}.$$

Theorem 2 is proved by a straightforward computation.

Theorem 2 If $(S, *, \varepsilon, \alpha_S)$ is a Hom-left-symmetric color algebra and $(A, \cdot, \varepsilon, \alpha_A)$ a commutative Hom-associative color algebra, then $(S \otimes A, \circ, \varepsilon, \alpha_{S \otimes A})$ is a Hom-left-symmetric color algebra, with

$$\alpha_{S \otimes A} = \alpha_S \otimes \alpha_A,$$

$$(x \otimes a) \circ (y \otimes b) = \varepsilon(a, y)(x * y) \otimes (a \cdot b)$$

for all $x, y \in \mathcal{H}(S)$, $a, b \in \mathcal{H}(A)$.

Recall that an ideal of a color Hom-algebra $(A, \cdot, \varepsilon, \alpha)$ is a graded subspace I such that $\alpha(I) \subseteq I$ and $I \cdot A = A \cdot I \subseteq I$.

Proposition 1 Let $(S, \cdot, \varepsilon, \alpha)$ be a Hom-left-symmetric color algebra and I an ideal of S such that for any $i, j \in \mathcal{H}(I)$ and $a, b, c, d \in \mathcal{H}(S)$,

$$\alpha(i) \cdot (a \cdot b) - (i \cdot a) \cdot \alpha(b) = \varepsilon(i, a)(\alpha(a) \cdot (i \cdot b) - (a \cdot i) \cdot \alpha(b)),$$

$$\alpha(c) \cdot (d \cdot j) - (c \cdot d) \cdot \alpha(j) = \varepsilon(c, d)(\alpha(d) \cdot (c \cdot j) - (d \cdot c) \cdot \alpha(j)).$$

Then $(I \oplus S, \dashv, \vdash, \alpha_{I \oplus S})$ is a Hom-left-symmetric color dialgebra with

$$\alpha_{I \oplus S} = \alpha_I \oplus \alpha_S,$$

$$(i_1 + a_1) \dashv (i_2 + a_2) = i_1 a_2 + a_1 a_2,$$

$$(i_1 + a_1) \vdash (i_2 + a_2) = a_1 i_2 + a_1 a_2$$

for $i_1, a_1 \in S_p$, $i_1, a_2 \in S_q$.

Proof It is straightforward by calculation. $\square$

Definition 3 A Hom-left-symmetric color dialgebra is a G -graded linear space S equipped with a bicharacter $\varepsilon : G \otimes G \rightarrow \mathbb{K}^*$ on G and two even bilinear products $\dashv : S \times S \rightarrow S$ and $\vdash : S \times S \rightarrow S$ satisfying the identities

$$\alpha(x) \dashv (y \dashv z) = \alpha(x) \dashv (y \vdash z), \quad (3)$$

$$(x \vdash y) \vdash \alpha(z) = (x \dashv y) \vdash \alpha(z), \quad (4)$$

$$\alpha(x) \dashv (y \dashv z) - (x \dashv y) \dashv \alpha(z) = \varepsilon(x, y)(\alpha(y) \vdash (x \dashv z) - (y \vdash x) \dashv \alpha(z)), \quad (5)$$

$$\alpha(x) \vdash (y \vdash z) - (x \vdash y) \vdash \alpha(z) = \varepsilon(x, y)(\alpha(y) \dashv (x \vdash z) - (y \vdash x) \vdash \alpha(z)) \quad (6)$$

for all $x, y, z \in \mathcal{H}(S)$.

Remark 1 Relation (6) means that $(S, \vdash, \varepsilon, \alpha)$ is a Hom-left-symmetric color algebra. So, any Hom-left-symmetric color algebra is a Hom-left-symmetric color dialgebra.

Definition 4 [14] A Hom-associative color dialgebra is defined as a quintuple $(D, \dashv, \vdash, \varepsilon, \alpha)$ where D is a G -graded linear space, $\dashv, \vdash : D \otimes D \rightarrow D$ are even bilinear maps, $\varepsilon : G \otimes G \rightarrow \mathbb{K}^*$ is a bicharacter and $\alpha : D \rightarrow D$ is an even linear map such that the following five axioms

$$(x \vdash y) \dashv \alpha(z) = \alpha(x) \vdash (y \dashv z), \quad (7)$$

$$\alpha(x) \dashv (y \dashv z) = (x \dashv y) \dashv \alpha(z), \quad (8)$$

$$(x \dashv y) \dashv \alpha(z) = \alpha(x) \dashv (y \vdash z), \quad (9)$$

$$(x \vdash y) \vdash \alpha(z) = \alpha(x) \vdash (y \vdash z), \quad (10)$$

$$\alpha(x) \vdash (y \vdash z) = (x \dashv y) \vdash \alpha(z) \quad (11)$$

are satisfied for $x, y, z \in \mathcal{H}(D)$.

Remark 2 If $(A, \dashv, \vdash, \varepsilon, \alpha)$ is a Hom-associative color dialgebra in which $\dashv = \vdash =: \mu$, then $(A, \mu, \varepsilon, \alpha)$ is a Hom-associative color algebra. Hence, any Hom-associative color algebra $(A, \mu, \varepsilon, \alpha)$ is a Hom-associative color dialgebra with $\dashv := \mu =: \vdash$.

Lemma 1 A Hom-left-symmetric color dialgebra S is a Hom-associative color dialgebra if and only if both products of S are color Hom-associative.

Proof If a Hom-left-symmetric color dialgebra S is a Hom-associative color dialgebra, then both products $\dashv$ and $\vdash$ defined over S are Hom-associative according to Definition 4. Conversely, if each product of a Hom-left-symmetric color dialgebra is Hom-associative, then from (5), we get axiom (3). $\square$

We need the below definition for the next theorem.

Definition 5 A Nijenhuis operator on Hom-associative color algebra $(A, \mu, \varepsilon, \alpha)$ is an even linear map $N : A \rightarrow A$ such that $\alpha \circ N = N \circ \alpha$ and

$$\mu(N(x), N(y)) = N(\mu(N(x), y) + \mu(x, N(y)) - N(\mu(x, y))) \quad (12)$$

for all $x, y \in \mathcal{H}(A)$.

Theorem 3 Let $(A, \mu, \varepsilon, \alpha)$ be a Hom-associative color algebra and $N : A \rightarrow A$ a Nijenhuis operator. Then the new multiplication $\mu^N : A \rightarrow A$ given by

$$\mu^N(x, y) = \mu(N(x), y) + \mu(x, N(y)) - N(\mu(x, y))$$

makes A into a Hom-associative color algebra.

Corollary 2 Let $(A, \mu, \varepsilon, \alpha)$ be an associative color algebra, $\alpha : A \rightarrow A$ be an even endomorphism and $N : A \rightarrow A$ be a Nijenhuis operator commuting with α . Then $\mu_\alpha^N = \alpha \circ \mu^N$ is a Hom-associative color algebra.

Now we introduce averaging operator to produce Hom-associative color dialgebras from Hom-associative color algebras.

Definition 6 An averaging operator on Hom-associative color algebra $(A, \mu, \varepsilon, \alpha)$ is an even linear map $\beta : A \rightarrow A$ such that $\alpha \circ \beta = \beta \circ \alpha$ and

$$\beta(\mu(\beta(x), y) = \mu(\beta(x), \beta(y)) = \beta(\mu(x, \beta(y))) \quad (13)$$

for all $x, y \in \mathcal{H}(A)$.

Theorem 4 Let $(A, \cdot, \varepsilon)$ be an associative color algebra and $\alpha : A \rightarrow A$ an averaging operator such that $(A, \cdot, \varepsilon, \alpha)$ be a Hom-associative color algebra. For any $x, y \in \mathcal{H}(A)$, define new operations on A by $x \vdash y = \alpha(x) \cdot y$ and $x \dashv y := x \cdot \alpha(y)$. Then $(A, \dashv, \vdash, \varepsilon, \alpha)$ is a Hom-associative color dialgebra.

Proof We prove one axiom. The others are proved similarly. For all $x, y, z \in \mathcal{H}(A)$,

$$\begin{aligned} \alpha(x) \dashv (y \dashv z) - (x \dashv y) \dashv \alpha(z) &= \alpha(x) \cdot \alpha(y \cdot \alpha(z)) - (x \cdot \alpha(y)) \cdot \alpha^2(z) \\ &\stackrel{(13)}{=} \alpha(x) \cdot (\alpha(y) \cdot \alpha(z)) - (x \cdot \alpha(y)) \cdot \alpha^2(z) \\ &\stackrel{(1)}{=} \alpha(x) \cdot (\alpha(y) \cdot \alpha(z)) - \alpha(x) \cdot (\alpha(y) \cdot \alpha(z)) = 0. \end{aligned}$$

This proves the axiom. $\square$

Definition 7 [14] A Hom-Poisson color dialgebra $(P, \dashv, \vdash, \{-, -\}, \varepsilon, \alpha)$ is a sextuple in which P is a G -graded linear space, $\dashv, \vdash, \{-, -\} : P \otimes P \rightarrow P$ are three even bilinear maps, $\varepsilon : G \otimes G \rightarrow \mathbb{K}^*$ is a bicharacter and $\alpha : P \rightarrow P$ is an even linear map such that

$$\{x \dashv y, \alpha(z)\} = \alpha(x) \dashv \{y, z\} + \varepsilon(y, z)\{x, z\} \dashv \alpha(y), \quad (14)$$

$$\{x \vdash y, \alpha(z)\} = \alpha(x) \vdash \{y, z\} + \varepsilon(y, z)\{x, z\} \vdash \alpha(y), \quad (15)$$

$$\{\alpha(x), y \dashv z\} = \varepsilon(x, y)\alpha(y) \vdash \{x, z\} + \{x, y\} \dashv \alpha(z) = \{\alpha(x), y \vdash z\} \quad (16)$$

for all $x, y, z \in \mathcal{H}(P)$.

Lemma 2 [14] Let $(D, \dashv, \vdash, \varepsilon, \alpha)$ be a Hom-associative color dialgebra, and $\{x, y\} = x \dashv y - \varepsilon(x, y)y \vdash x$ for any $x, y \in \mathcal{H}(D)$. Then $(D, \dashv, \vdash, \{-, -\}, \varepsilon, \alpha)$ is a Hom-Poisson color dialgebra.

Theorem 5 Let $(D, \dashv, \vdash, \varepsilon, \alpha)$ be a Hom-left-symmetric color dialgebra with Hom-associative products. Then $(D, \dashv, \vdash, \{-, -\}, \varepsilon, \alpha)$ is a Hom-Poisson color dialgebra, where $\{x, y\} = x \dashv y - \varepsilon(x, y)y \vdash x$, for any $x, y \in \mathcal{H}(D)$.

Proof It follows from Lemmas 1 and 2. $\square$

3 Hom-tridendriform color algebras

We introduce Hom-tridendriform color algebras, give some properties and study their connection with Hom-post-Lie color algebras and color Hom-associative algebras.

Definition 8 A Hom-tridendriform color algebra is a sextuple $(T, \dashv, \vdash, \cdot, \varepsilon, \alpha)$ consisting of a G -graded linear space T , three even bilinear maps $\dashv, \vdash, \cdot : T \otimes T \rightarrow T$, a bicharacter $\varepsilon : G \times G \rightarrow \mathbb{K}^*$ and an even linear map $\alpha : T \rightarrow T$ satisfying

$$(x \dashv y) \dashv \alpha(z) = \alpha(x) \dashv (y \vdash z + \varepsilon(z, y)y \vdash z + \varepsilon(z, y)y \cdot z), \quad (17)$$

$$(x \vdash y) \dashv \alpha(z) = \varepsilon(z, x)\alpha(x) \vdash (y \dashv z), \quad (18)$$

$$\alpha(x) \vdash (y \vdash z) = (\varepsilon(x, y)x \dashv y + x \vdash y + x \cdot y) \vdash \alpha(z), \quad (19)$$

$$(x \dashv y) \cdot \alpha(z) = \varepsilon(y, x)\alpha(x) \cdot (y \vdash z), \quad (20)$$

$$(x \vdash y) \cdot \alpha(z) = \alpha(x) \vdash (y \cdot z), \quad (21)$$

$$(x \cdot y) \dashv \alpha(z) = \varepsilon(z, x)\alpha(x) \cdot (y \dashv z), \quad (22)$$

$$(x \cdot y) \cdot \alpha(z) = \alpha(x) \cdot (y \cdot z) \quad (23)$$

for $x, y, z \in \mathcal{H}(T)$.

Remark 3 When the color Hom-associative product is identically null, we get a Hom-tridendriform color algebra [14].

Example 2 Let $A = A_0 \oplus A_1 = \langle e_1, e_2 \rangle \oplus \langle e_3 \rangle$ be a three-dimensional superspace. The multiplications

$$e_2 \dashv e_2 = e_2 \vdash e_2 = ae_1, \quad e_2 \cdot e_2 = -ae_1,$$

$$e_3 \dashv e_3 = e_3 \vdash e_3 = be_1, \quad e_3 \cdot e_3 = be_1,$$

and the even linear map $\alpha : A \rightarrow A$ defined by

$$\alpha(e_1) = e_1, \quad \alpha(e_2) = e_1 + e_2, \quad \alpha(e_3) = -e_3$$

make A into a Hom-tridendriform color algebra, for any $a, b \in \mathbb{R}$.

Proposition 2 Let $(T, \dashv, \vdash, \cdot, \varepsilon, \alpha)$ be a Hom-tridendriform color algebra. Then $(T, \dashv^{op}, \vdash^{op}, \cdot^{op}, \varepsilon, \alpha)$ is also a Hom-tridendriform color algebra, with

$$x \dashv^{op} y := y \vdash x, \quad x \vdash^{op} y := y \dashv x, \quad x \cdot^{op} y := y \cdot x$$

for any $x, y \in \mathcal{H}(T)$.

Proof The proof is straightforward by calculation by using axioms in Definition 8. $\square$

Proposition 3 Let $(A, \dashv, \vdash, \cdot, \varepsilon, \alpha)$ be a Hom-tridendriform color algebra. Then $(A, \dashv, \vdash', \varepsilon, \alpha)$ is a Hom-dendriform color algebra, where $x \vdash' y := x \vdash y + x \cdot y$ for any $x, y \in \mathcal{H}(T)$.

Proof It comes immediately from Definition 8. $\square$

We need the following definition for the next proposition.

Definition 9 Let $(A, \cdot, \varepsilon, \alpha)$ be a Hom-associative color algebra and $\lambda \in \mathbb{K}$. An even linear map $R : A \rightarrow A$ is called a Rota–Baxter operator of weight λ on A if it satisfies the identities

$$R \circ \alpha = \alpha \circ R, \quad (24)$$

$$R(x) \cdot R(y) = R(R(x) \cdot y + x \cdot R(y) + \lambda x \cdot y), \quad (25)$$

for any $x, y \in \mathcal{H}(A)$.

Theorem 6 Let $(A, \cdot, \varepsilon, \alpha, R)$ be a Rota–Baxter Hom-associative color algebra of weight λ . Let us define three new operations $\dashv, \vdash$ and $*$ on A by

$$x \dashv y := x \cdot R(y), \quad x \vdash y := \varepsilon(x, y)R(x) \cdot y \quad \text{and} \quad x * y := \lambda \varepsilon(x, y)x \cdot y.$$

Then $(A, \dashv, \vdash, *, \varepsilon, \alpha)$ is a Hom-tridendriform color algebra.

Example 3 Let $G = \{-1, +1\}$ be an abelian group and $A = A_{(-1)} \oplus A_{(1)} = \langle e_2 \rangle \oplus \langle e_1 \rangle$ a G -graded two-dimensional linear space. The quintuple $(A, \cdot, \varepsilon, \alpha, R)$ is a Rota–Baxter Hom-associative color algebra of weight λ with

- the multiplication, $e_1 \cdot e_1 = -e_1$, $e_1 \cdot e_2 = e_2$, $e_2 \cdot e_1 = e_2$, $e_2 \cdot e_2 = e_1$,
- the bicharacter, $\varepsilon(i, j) = (-1)^{(i-1)(j-1)/4}$,
- the even linear map $\alpha : A \rightarrow A$ defined by : $\alpha(e_1) = e_1$, $\alpha(e_2) = -e_2$,
- the Rota–Baxter operator $R : A \rightarrow A$ given by : $R(e_1) = -\lambda e_1$, $R(e_2) = -\lambda e_2$.

Therefore, $(A, \dashv, \vdash, *, \varepsilon, \alpha)$ is a Hom-tridendriform color algebra with

$$\begin{aligned} e_1 \dashv e_1 &= \lambda e_1, & e_1 \vdash e_1 &= \lambda e_1 & e_1 * e_1 &= -\lambda e_1, \\ e_1 \dashv e_2 &= -\lambda e_2, & e_1 \vdash e_2 &= -\lambda e_2, & e_1 * e_2 &= \lambda e_2, \\ e_2 \dashv e_1 &= -\lambda e_2, & e_2 \vdash e_1 &= -\lambda e_2 & e_2 * e_1 &= \lambda e_2, \\ e_2 \dashv e_2 &= -\lambda e_1, & e_2 \vdash e_2 &= \lambda e_1, & e_2 * e_2 &= -\lambda e_1. \end{aligned}$$

Proposition 4 Let $(A, \cdot, \varepsilon, R, \alpha)$ be a Rota–Baxter Hom-associative color algebra of weight λ . Define the operations $\dashv$ and $\vdash$ by

$$x \dashv y := x \cdot R(y) + \lambda x \cdot y \quad \text{and} \quad x \vdash y := \varepsilon(x, y)R(x) \cdot y.$$

Then $(A, \dashv, \vdash, \varepsilon)$ is a Hom-dendriform color algebra.

Corollary 3 *Let $(A, \cdot, \varepsilon, R)$ be a Rota–Baxter Hom-associative color algebra of weight 0. We define the even bilinear operations $\dashv: A \times A \rightarrow A$ and $\vdash: A \times A \rightarrow A$ on A by $x \dashv y := x \cdot R(y)$ and $x \vdash y := \varepsilon(x, y)R(x) \cdot y$. Then $(A, \dashv, \vdash, \varepsilon)$ is a Hom-dendriform color algebra.*

In Theorem 7, we associate a Hom-associative color algebra to any Hom-tridendriform color algebra.

Theorem 7 *Let $(T, \dashv, \vdash, \cdot, \varepsilon, \alpha)$ be a Hom-tridendriform color algebra and define operation $*$ by $x * y = x \vdash y + \varepsilon(x, y)x \dashv y + x \cdot y$. Then $(T, *, \varepsilon, \alpha)$ is a Hom-associative color algebra.*

Proof For any $x, y, z \in \mathcal{H}(T)$

$$\begin{aligned} as_{\alpha}(x, y, z) &= (x \vdash y) \vdash \alpha(z) + \varepsilon(x, y)(x \dashv y) \vdash \alpha(z) + (x \cdot y) \vdash \alpha(z) \\ &\quad + \varepsilon(x, z)\varepsilon(y, z)(x \vdash y) \dashv \alpha(z) + \varepsilon(x, z)\varepsilon(y, z)\varepsilon(x, y)(x \dashv y) \dashv \alpha(z) \\ &\quad + \varepsilon(x, z)\varepsilon(y, z)(x \cdot y) \dashv \alpha(z) + (x \vdash y) \cdot \alpha(z) \\ &\quad + \varepsilon(x, y)(x \dashv y) \cdot \alpha(z) + (x \cdot y) \cdot \alpha(z) - \alpha(x) \vdash (y \vdash z) \\ &\quad - \varepsilon(y, z)\alpha(x) \vdash (y \dashv z) - \alpha(x) \vdash (y \cdot z) \\ &\quad - \varepsilon(x, y)\varepsilon(x, z)\alpha(x) \dashv (y \vdash z) - \varepsilon(x, y)\varepsilon(x, z)\varepsilon(y, z)\alpha(x) \dashv (y \dashv z) \\ &\quad - \varepsilon(x, y)\varepsilon(x, z)\alpha(x) \dashv (y \cdot z) - \alpha(x) \cdot (y \vdash z) \\ &\quad - \varepsilon(y, z)\alpha(x) \cdot (y \dashv z) - \alpha(x) \cdot (y \cdot z). \end{aligned}$$

The left-hand side vanishes by axioms in Definition 8. This prove that $(A, *, \varepsilon, \alpha)$ is a Hom-associative color algebra. This completes the proof. $\square$

Remark 4 Whenever $(T, \dashv, \vdash, \cdot, \varepsilon, \alpha)$ is commutative, we recover [16, Lemma 3.3].

Definition 10 [12] A Hom–Poisson color algebra consists of a G -graded linear space A , a multiplication $\mu: A \times A \rightarrow A$, an even bilinear bracket $\{\cdot, \cdot\}: A \times A \rightarrow A$ and an even linear map $\alpha: A \rightarrow A$ such that

- (i) $(A, \mu, \varepsilon, \alpha)$ is a Hom-associative color algebra,
- (ii) $(A, \{\cdot, \cdot\}, \varepsilon, \alpha)$ is a Hom–Lie color algebra,
- (iii) the color Hom–Leibniz identity is satisfied, i.e.,

$$\{\alpha(x), \mu(y, z)\} = \mu(\{x, y\}, \alpha(z)) + \varepsilon(x, y)\mu(\alpha(y), \{x, z\}) \quad (26)$$

for any $x, y, z \in \mathcal{H}(A)$.

Lemma 3 [12] *Let $(A, \mu, \varepsilon, \alpha)$ be a Hom-associative color algebra.*

Then $(A, \mu, \{\cdot, \cdot\} = \mu - \varepsilon(\cdot, \cdot)\mu^{op}, \varepsilon, \alpha)$ is a Hom–Poisson color algebra.

Theorem 8 *Let $(A, \dashv, \vdash, \cdot, \varepsilon, \alpha)$ be a Hom-tridendriform color algebra.*

Then $(A, *, [-, -], \varepsilon, \alpha)$ is a Hom–Poisson color algebra, where

$$x * y := x \vdash y + \varepsilon(x, y)x \dashv y + x \cdot y \quad \text{and} \quad [x, y] := x * y - \varepsilon(x, y)y * x.$$

Proof The proof follows from Theorem 7 and Lemma 3. $\square$

Definition 11 [16] A Hom-post-Lie color algebra $(L, [-, -], \cdot, \varepsilon, \alpha)$ is a Hom-Lie color algebra $(L, [-, -], \varepsilon, \alpha)$, i.e.

$$[x, y] = -\varepsilon(x, y)[y, x] \quad (\varepsilon\text{-skew-symmetry}) \quad (27)$$

$$\varepsilon(z, x)[\alpha(x), [y, z]] + \varepsilon(x, y)[\alpha(y), [z, x]] + \varepsilon(y, z)[\alpha(z), [x, y]] = 0$$

$$(\varepsilon\text{-Hom-Jacobi identity}) \quad (28)$$

together with an even bilinear map $\cdot : L \otimes L \rightarrow L$ such that

$$\alpha(z) \cdot [x, y] - [z \cdot x, \alpha(y)] - \varepsilon(z, x)[\alpha(x), z \cdot y] = 0, \quad (29)$$

$$\alpha(z) \cdot (y \cdot x) - \varepsilon(z, y)\alpha(y) \cdot (z \cdot x) + \varepsilon(z, y)(y \cdot z) \cdot \alpha(x) - (z \cdot y) \cdot \alpha(x) + \varepsilon(z, y)[y, z] \cdot \alpha(x) = 0 \quad (30)$$

for any $x, y, z \in \mathcal{H}(L)$.

Example 4 Let $G = \mathbb{Z}_2 \times \mathbb{Z}_2$ be an abelian group and L be a three-dimensional G -graded linear space defined by

$$L_{(0,0)} = 0, \quad L_{(0,1)} = \langle e_2 \rangle, \quad L_{(1,0)} = \langle e_1 \rangle, \quad L_{(1,1)} = \langle e_3 \rangle.$$

Then $(L, [-, -], \cdot, \varepsilon, \alpha)$ is a Hom-Post-Lie color algebra with

- the bicharacter: $\varepsilon((i_1, i_2), (j_1, j_2)) = (-1)^{i_1 j_1 + i_2 j_2}$,
- the bracket: $[e_1, e_2] = -e_3, \quad [e_1, e_3] = -e_2, \quad [e_2, e_3] = -e_1$,
- the multiplication: $e_2 e_1 = -e_3, \quad e_2 e_3 = e_1$,
- the even linear map $\alpha : L \rightarrow L$: $\alpha(e_1) = -e_1, \alpha(e_2) = -e_2, \alpha(e_3) = e_3$.

Proposition 5 Let $(L, [-, -], \cdot, \varepsilon, \alpha)$ be a color post-Hom-Lie algebra. With the product $x * y = x \cdot y + \frac{1}{2}[x, y]$, $(L, *, \varepsilon, \alpha)$ is a color Hom-Lie admissible algebra, that is the map $*$: $L \times L \rightarrow L$ satisfies the ε -Hom-Jacobi identity.

Proof It follows from direct computation by using axioms in Definition 11. $\square$

To any Hom-tridendriform color algebra one can associate a Hom-post-Lie color algebra, as stated in the following result.

Theorem 9 Let $(T, \dashv, \vdash, \cdot, \varepsilon, \alpha)$ be a Hom-tridendriform color algebra, and let $x \circ y = x \vdash y - y \dashv x$ and $[x, y] = x \cdot y - \varepsilon(x, y)y \cdot x$ for any $x, y \in \mathcal{H}(T)$. Then $(T, \circ, [-, -], \varepsilon, \alpha)$ is a Hom-post-Lie color algebra.

Proof The ε -skew-symmetry and the ε -Hom-Jacobi identity are trivial, by Lemma 3. Now, for any $x, y, z \in \mathcal{H}(T)$, we have

$$\begin{aligned} & \alpha(x) \circ (y \circ z) - \varepsilon(x, y)\alpha(y) \circ (x \circ z) + \varepsilon(x, y)(y \circ x) \circ \alpha(z) - (x \circ y) \circ \alpha(z) \\ & + \varepsilon(x, y)[y, x] \circ \alpha(z) = \alpha(x) \vdash (y \vdash z) - \alpha(x) \vdash (z \vdash y) - (y \vdash z) \dashv \alpha(x) \\ & + (z \dashv y) \dashv \alpha(x) - \varepsilon(x, y)\alpha(y) \vdash (x \vdash z) \\ & + \varepsilon(x, y)\alpha(y) \vdash (z \dashv x) + \varepsilon(x, y)(x \vdash z) \dashv \alpha(y) \\ & - \varepsilon(x, y)(z \dashv x) \dashv \alpha(y) + \varepsilon(x, y)(y \vdash x) \vdash \alpha(z) \\ & - \varepsilon(x, y)(x \dashv y) \vdash \alpha(z) \\ & - \varepsilon(x, y)\alpha(z) \dashv (y \vdash x) \\ & + \varepsilon(x, y)\alpha(z) \dashv (x \vdash y) - (x \vdash y) \vdash \alpha(z) + (y \dashv x) \vdash \alpha(z) \end{aligned}$$

$$\begin{aligned}
& +\alpha(z) \dashv (x \vdash y) - \alpha(z) \dashv (y \dashv x) \\
& +\varepsilon(x, y)(y \cdot x) \vdash \alpha(z) - (x \cdot y) \vdash \alpha(z) - \varepsilon(x, y)\alpha(z) \dashv (y \cdot x) + \alpha(z) \dashv (x \cdot y)
\end{aligned}$$

which vanishes by axioms in Definition 8. Next,

$$\begin{aligned}
& \alpha(z) \circ [x, y] - [z \circ x, \alpha(y)] - \varepsilon(z, x)[\alpha(x), z \circ y] \\
& = \alpha(z) \vdash (x \cdot y) - \varepsilon(x, y)\alpha(z) \vdash (y \cdot x) - (x \cdot y) \dashv \alpha(z) + \varepsilon(x, y)(y \cdot x) \dashv \alpha(z) \\
& \quad - (z \vdash x) \cdot \alpha(y) + (x \dashv z) \cdot \alpha(y) + \varepsilon(x + z, y)\alpha(y) \cdot (z \vdash x) \\
& \quad - \varepsilon(x + z, y)\alpha(y) \cdot (x \dashv z) - \varepsilon(z, x)\alpha(x) \cdot (z \vdash y) + \varepsilon(z, x)\alpha(x) \cdot (y \dashv z) \\
& \quad + \varepsilon(x, y)(z \vdash y) \cdot \alpha(x) - \varepsilon(x, y)(y \dashv z) \cdot \alpha(x).
\end{aligned}$$

The left-hand side vanishes by axioms in Definition 8. $\square$

4 Generalization of Yau's twisting composition method

In this section, we generalize Yau's twisting to a large class of color Hom-algebras and use the centroids to produce other color Hom-algebras from given one. To state the fundamental result, Theorem 10, of this section, we give the following definitions.

Definition 12 By a color Hom-algebra we mean a $(n + 3)$ -uple $(A, \mu_1, \dots, \mu_n, \varepsilon, \alpha)$ in which A is a G -graded linear space, $\mu_i : A \otimes A \rightarrow A$ ($i = 1, \dots, n$) are even bilinear maps, $\varepsilon : G \times G \rightarrow \mathbb{K}^*$ is a bicharacter and $\alpha : A \rightarrow A$ is an even linear map, called the twisting map. If in addition, $\alpha \circ \mu_i = \mu_i \circ (\alpha \otimes \alpha)$ ($i = 1, \dots, n$), the color Hom-algebra $(A, \mu_1, \dots, \mu_n, \varepsilon, \alpha)$ is said to be multiplicative. We call a Hom- X color algebra a color Hom-algebra for which the axioms defining the structure of X are linear combination of the terms of the form $\mu_j \circ (\mu_i \otimes \alpha)$ or $\mu_j \circ (\alpha \otimes \mu_i)$.

Example 5 (1) A Hom- X color algebra $(A, \mu, \varepsilon, \alpha)$ for which

$$\mu(\mu(x, y), \alpha(z)) - \mu(\alpha(x), \mu(y, z)) = 0 \quad (31)$$

is called a Hom-associative color algebra.

(2) A Hom- X color algebra $(A, \mu, \varepsilon, \alpha)$ for which

$$\mu(\alpha(x), \mu(y, z)) + \varepsilon(y, z)\mu(\mu(x, z), \alpha(y)) - \mu(\mu(x, y), \alpha(z)) = 0 \quad (32)$$

is called a Hom-Leibniz color algebra.

(3) A Hom- X color algebra $(A, \mu_1, \mu_2, \varepsilon, \alpha)$ for which (31) and (32) hold for μ_1 and μ_2 respectively and

$$\mu_2(\alpha(x), \mu_1(y, z)) + \varepsilon(y, z)\mu_2(\mu_1(x, z), \alpha(y)) - \mu_1(\mu_2(x, y), \alpha(z)) = 0 \quad (33)$$

is called a Hom-Leibniz-Poisson color algebra.

Remark 5 The following color Hom-algebras enter in these categories of algebras: Hom-Lie color algebras, Hom-pre-Lie (or left Hom-symmetric) color algebras, Hom-post-Lie color algebras, Hom-left symmetric color dialgebras, Hom-Poisson color dialgebras, Hom-pre-Poisson color algebras, Hom-post-Poisson color algebras, Hom-Leibniz-Poisson color algebras, Hom-tridendriform color algebras and so on.

Definition 13 Let $(A, \mu_1, \dots, \mu_n, \varepsilon, \alpha)$ and $(A', \mu'_1, \dots, \mu'_n, \varepsilon, \alpha')$ be two color Hom-algebras. An even linear map $f : A \rightarrow A'$ is said to be a morphism of color Hom-algebras if $f \circ \alpha = \alpha' \circ f$ and $f(\mu_i(x, y)) = \mu'_i(f(x), f(y))$ for all $x, y \in \mathcal{H}(A)$ and $i = 1, \dots, n$.

Definition 14 Let $(A, \mu_1, \dots, \mu_n, \varepsilon, \alpha)$ be a multiplicative color Hom-algebra.

(1) For $k \in \mathbb{N}^* = \{1, 2, 3, \dots\}$, the k th derived color Hom-algebra of type 1 of A is defined by

$$A_1^k = \left(A, \mu_1^{(k)} = \alpha^k \circ \mu_1, \dots, \mu_n^{(k)} = \alpha^k \circ \mu_n, \varepsilon, \alpha^{k+1} \right). \quad (34)$$

(2) For $k \in \mathbb{N}^* = \{1, 2, 3, \dots\}$, the k th derived color Hom-algebra of type 2 of A is defined by

$$A_2^k = \left(A, \mu_1^{(2^k-1)} = \alpha^{2^k-1} \circ \mu_1, \dots, \mu_n^{(2^k-1)} = \alpha^{2^k-1} \circ \mu_n, \varepsilon, \alpha^{2^k} \right); \quad (35)$$

and $A_1^0 = A_2^0 = (A, \mu_1, \dots, \mu_n, \varepsilon, \alpha)$ and $A_1^1 = A_2^1 = (A, \alpha \circ \mu_1, \dots, \alpha \circ \mu_n, \varepsilon, \alpha^2)$.

Definition 15 A color Hom-algebra $(A, \mu_1, \dots, \mu_n, \varepsilon, \alpha)$ endowed with an even linear map $R : A \rightarrow A$ such that

$$\mu_i(R(x), R(y)) = R(\mu_i(R(x), y) + \mu_i(x, R(y)) + \lambda \mu_i(x, y)), \quad i = 1, \dots, n \quad (36)$$

with $\lambda \in \mathbb{K}$ and $x, y \in \mathcal{H}(A)$, is called a Rota–Baxter color Hom-algebra, and R is called a Rota–Baxter operator on A .

The below result allows to get Hom- X color algebras from either an X -color algebra on the one hand or another Hom- X color algebra on the other hand.

Theorem 10 Let $(A, \mu_1, \dots, \mu_n, \varepsilon, \alpha)$ be a Rota–Baxter Hom- X color algebra and $\beta : A \rightarrow A$ be an endomorphism of A . Then, for any nonnegative integer n ,

$$A_\beta = \left(A, \mu_\beta^1 = \beta^n \circ \mu_1, \dots, \mu_\beta^n = \beta^n \circ \mu_n, \beta^n \circ \alpha \right)$$

is a Rota–Baxter Hom- X color algebra, where $\beta^n = \beta \circ \beta^{n-1}$.

Moreover, suppose that $(A', \mu'_1, \dots, \mu'_n, \varepsilon, \alpha')$ is another Hom- X color algebra and $\beta' : A' \rightarrow A'$ be an algebra endomorphism. If $f : A \rightarrow A'$ is a morphism of Hom- X color algebras that satisfies $f \circ \beta = \beta' \circ f$, then $f : A_\beta \rightarrow A_{\beta'}$ is also a morphism of Hom- X color algebras.

Proof The proof of the first part follows from the following facts.

For any $x, y, z \in \mathcal{H}(X)$, $1 \leq i, j \leq n$,

$$\begin{aligned} \mu_\beta^i(\mu_\beta^j(x, y), (\beta^n \circ \alpha)(z)) &= \mu_\beta^i(\mu_\beta^j(x, y), \beta^n(\alpha(z))) \\ &= \beta^n \mu_i(\beta^n \mu_j(x, y), \beta^n(\alpha(z))) \\ &= (\beta^n \circ \beta^n)(\mu_i(\mu_j(x, y), \alpha(z))) \\ &= \beta^{2n}(\mu_i(\mu_j(x, y), \alpha(z))), \\ \mu_\beta^i((\beta^n \circ \alpha)(x), \mu_\beta^j(y, z)) &= \mu_\beta^i(\beta^n(\alpha(x)), \mu_\beta^j(y, z)) \\ &= \beta^n(\mu_i(\beta^n(\alpha(x)), \beta^n(\mu_j(y, z)))) \\ &= (\beta^n \circ \beta^n)(\mu_i(\alpha(x), \mu_j(y, z))) \end{aligned}$$

$$= \beta^{2n} (\mu_i(\alpha(x), \mu_j(y, z))) .$$

To prove the Rota–Baxter identity (36) for μ_β^i , we have

$$\begin{aligned} \mu_\beta^i(x, y) &= \beta^n(\mu_i(x, y)) = \beta^n(\mu_i(R(x), y) + \mu_i(x, R(y)) + \lambda\mu_i(x, y)) \\ &= \beta^n\mu_i(R(x), y) + \beta^n\mu_i(x, R(y)) + \lambda\beta^n\mu_i(x, y) \\ &= \mu_\beta^i(R(x), y) + \mu_\beta^i(x, R(y)) + \lambda\mu_\beta^i(x, y). \end{aligned}$$

For the second assertion, we have

$$\begin{aligned} f(\mu_\beta^i(x, y)) &= f(\beta^n(\mu_i(x, y))) = f(\mu_i(\beta^n(x), \beta^n(y))) = \mu_i'(f(\beta^n(x)), f(\beta^n(y))) \\ &= \mu_i'(\beta^n(f(x)), \beta^n(f(y))) = \beta^n(\mu_i'(f(x), f(y))) = \mu_\beta^i(f(x), f(y)). \end{aligned}$$

This ends the proof. $\square$

Remark 6 Whenever $\alpha = Id$ in the first part of the previous theorem, we obtain a Hom- X color algebra from an X -color algebra.

Example 6 If $(T, \dashv, \vdash, \cdot, \varepsilon, \alpha)$ is a Hom-tridendriform color algebra and $\beta : T \rightarrow T$ an endomorphism of T , then $T_\beta = (T, \dashv_\beta = \beta^n \circ \dashv, \vdash_\beta = \beta^n \circ \vdash, \cdot_\beta = \beta^n \circ \cdot, \beta^n \circ \varepsilon)$ is a Hom-tridendriform color algebra, for any nonnegative integer n .

Moreover, suppose that $(T', \dashv', \vdash', \cdot', \varepsilon', \alpha')$ is another Hom-tridendriform color algebra and $\beta' : T' \rightarrow T'$ a Hom-tridendriform color algebra endomorphism. If $f : T \rightarrow T'$ is a morphism of Hom-tridendriform color algebra that satisfies $f \circ \beta = \beta' \circ f$, then $f : T_\beta \rightarrow T'_{\beta'}$ is a morphism of Hom-tridendriform color algebras.

Proof We shall only prove relation (17), the others being proved analogously. Then, for any $x, y, z \in \mathcal{H}(T)$,

$$\begin{aligned} (x \dashv_\beta y) \dashv_\beta (\beta^n \circ \alpha)(z) &= \beta^n(\beta^n(x \dashv y) \dashv (\beta^n \circ \alpha)(x)) = \beta^{2n}((x \dashv y) \dashv \alpha(x)) \\ &= \beta^{2n}(\alpha(x) \dashv (y \dashv z + \varepsilon(z, y)y \vdash z + \varepsilon(z, y)y \cdot z)) \\ &= (\beta^n \circ \alpha)(x) \dashv_\beta \beta^n(y \dashv z + \varepsilon(z, y)y \vdash z + \varepsilon(z, y)y \cdot z) \\ &= (\beta^n \circ \alpha)(x) \dashv_\beta (y \dashv_\beta z + \varepsilon(z, y)y \vdash_\beta z + \varepsilon(z, y)y \cdot_\beta z). \end{aligned}$$

For the second assertion, we have

$$\begin{aligned} f(x \dashv_\beta y) &= f(\beta^n(x) \dashv \beta^n(y)) = f(\beta^n(x)) \dashv' f(\beta^n(y)) \\ &= \beta^n(f(x)) \dashv' \beta^n(f(y)) = f(x) \dashv'_{\beta'} f(y). \end{aligned}$$

This completes the proof. $\square$

We have the following series of consequence of Theorem 10.

Corollary 4 Let $(A, \mu_1, \dots, \mu_n, \varepsilon)$ be an X -color algebra and $\beta : A \rightarrow A$ be an endomorphism of A . Then $A_\beta = (A, \beta \circ \mu_1, \dots, \beta \circ \mu_n, \varepsilon, \beta)$ is a multiplicative Hom- X color algebra.

Proof Take $\alpha = Id$ in Theorem 10. $\square$

Corollary 5 Let $(A, \mu_1, \dots, \mu_n, \varepsilon, \alpha)$ be a Hom- X color algebra such that α be invertible. Then $(A, \mu_{\alpha^{-1}} = \alpha^{-1} \circ \mu_1, \dots, \mu_{\alpha^{-1}} = \alpha^{-1} \circ \mu_n, \varepsilon)$ is an X -color algebra.

Proof Take $n = 1$ and $\beta = \alpha^{-1}$ in Theorem 10. $\square$

Corollary 6 Let $(A, \mu_1, \dots, \mu_n, \varepsilon, \alpha)$ be a Hom- X color algebra. Then the k th derived color Hom-algebra of type 1 and the k th derived color Hom-algebra of type 2 are Hom- X color algebras.

Proof It is sufficient to take $\beta = \alpha$, and $n = k$ and $n = 2^k - 1$ respectively in Theorem 10. $\square$

Now we introduce the notion of centroids for Hom- X color algebras. They allow to provide Hom- X color algebras from a given one, and thus they play here the role of twisting.

Definition 16 Let $(A, \mu_1, \dots, \mu_n, \varepsilon, \alpha)$ be a color Hom-algebra. An even linear map $\beta : A \rightarrow A$ is said to be an element of the centroid if

$$\beta(\mu_i(x, y)) = \mu_i(\beta(x), y) = \mu_i(x, \beta(y)) \quad \text{for any } x, y \in \mathcal{H}(A).$$

The centroid of A is defined by

$$\begin{aligned} \text{Cent}(A) &= \{\beta : A \rightarrow A \text{ even linear map} \mid \forall x, y \in \mathcal{H}(A) : \\ &\quad \beta(\mu_i(x, y)) = \mu_i(\beta(x), y) = \mu_i(x, \beta(y))\} \end{aligned}$$

Theorem 11 Let $(A, \mu_1, \dots, \mu_n, \varepsilon, R, \alpha)$ be a Rota–Baxter Hom- X color algebra and $\beta_1, \beta_2 : A \rightarrow A$ be a pair of commuting elements of the centroid such that $\beta_i \circ R = R \circ \beta_i, i = 1, 2$.

(1) Define bilinear maps $\mu_\beta^i : A \times A \rightarrow A, (i = 1, \dots, n)$, by

$$\mu_\beta^i(x, y) := \mu_i((\beta_2\beta_1)(x), y).$$

Then $A_{\beta_1, \beta_2}^1 = (A, \mu_\beta^1, \dots, \mu_\beta^n, \varepsilon, R, \beta_2\beta_1\alpha)$ is also a Rota–Baxter Hom- X color algebra.

(2) Define bilinear maps $\mu_\beta^i : A \times A \rightarrow A, (i = 1, \dots, n)$, by

$$\mu_\beta^i(x, y) := \mu_i(\beta_1(x), \beta_2(y)).$$

Then $A_{\beta_1, \beta_2}^2 = (A, \mu_\beta^1, \dots, \mu_\beta^n, \varepsilon, R, \beta_2\beta_1\alpha)$ is also a Rota–Baxter Hom- X color algebra.

Proof (1) For any $x, y \in \mathcal{H}(A)$, we have

$$\begin{aligned} \mu_\beta^i(\mu_\beta^j(x, y), (\beta_2\beta_1\alpha)(z)) &= \mu_\beta^i\left(\mu_\beta^j(x, y), \beta_2(\beta_1(\alpha(z)))\right) \\ &= \mu_i(\beta_2\beta_1(\mu_j(\beta_2\beta_1(x), y)), \beta_2\beta_1(\alpha(z))) \\ &= \beta_2\mu_i(\beta_1(\beta_2\mu_j(\beta_1(x), y)), \beta_2\beta_1(\alpha(z))) \\ &= \beta_2^2\mu_i(\beta_1(\beta_2\mu_j(\beta_1(x), y)), \beta_1(\alpha(z))) \\ &= \beta_2^2\beta_1\mu_i(\beta_2\mu_j(\beta_1(x), y), \beta_1(\alpha(z))) \\ &= \beta_2^3\beta_1\mu_i(\beta_1\mu_j(x, y), \beta_1(\alpha(z))) \\ &= \beta_2^3\beta_1^2\mu_i(\mu_j(x, y), \beta_1(\alpha(z))) \\ &= \beta_2^3\beta_1^3\mu_i(\mu_j(x, y), \alpha(z)). \end{aligned}$$

We have a similar proof for $\mu_\beta^i((\beta_2\beta_1\alpha)(x), \mu_\beta^j(y, z))$. Next,

$$\mu_\beta^i(R(x), R(y)) = \mu_i(\beta_2\beta_1(R(x)), R(y)) = \beta_2\beta_1\mu_i(R(x), R(y))$$

$$\begin{aligned}
&= \beta_2 \beta_1 R(\mu_i(R(x), y) + \mu_i(x, R(y)) + \lambda \mu_i(x, y)) \\
&= R(\beta_2 \beta_1 \mu_i(R(x), y) + \beta_2 \beta_1 \mu_i(x, R(y)) + \lambda \beta_2 \beta_1 \mu_i(x, y)) \\
&= R(\mu_i(\beta_2 \beta_1 R(x), y) + \mu_i(\beta_2 \beta_1(x), R(y)) + \lambda \mu_i(\beta_2 \beta_1(x), y)) \\
&= R\left(\mu_\beta^i(R(x), y) + \mu_\beta^i(x, R(y)) + \lambda \mu_\beta^i(x, y)\right).
\end{aligned}$$

(2) For any $x, y \in \mathcal{H}(A)$, we have

$$\begin{aligned}
\mu_\beta^i(\mu_\beta^j(x, y), (\beta_2 \beta_1 \alpha)(z)) &= \mu_\beta^i\left(\mu_\beta^j(x, y), \beta_2(\beta_1(\alpha(z)))\right) \\
&= \mu_i(\beta_1(\mu_j(\beta_1(x), \beta_2(y))), \beta_2(\beta_2 \beta_1(\alpha(z)))) \\
&= \beta_1 \beta_2 \mu_i(\beta_1 \beta_2(\mu_j(x, y)), \beta_2 \beta_1(\alpha(z))) \\
&= \beta_1^2 \beta_2^2 \mu_i(\mu_j(x, y), \beta_2 \beta_1(\alpha(z))), \\
&= \beta_1^3 \beta_2^3 \mu_i(\mu_j(x, y), \alpha(z)).
\end{aligned}$$

Similarly, one proves $\mu_\alpha^i((\beta_2 \beta_1 \alpha)(x), \mu_\alpha^j(y, z)) = \beta_1^3 \beta_2^3 \mu_i(\alpha(x), \mu_j(y, z))$. Next,

$$\begin{aligned}
\mu_\beta^i(R(x), R(y)) &= \mu_i(\beta_1(R(x)), \beta_2 R(y)) = \beta_2 \beta_1 \mu_i(R(x), R(y)) \\
&\quad \beta_2 \beta_1 R(\mu_i(R(x), y) + \mu_i(x, R(y)) + \lambda \mu_i(x, y)) \\
&= R(\beta_2 \beta_1 \mu_i(R(x), y) + \beta_2 \beta_1 \mu_i(x, R(y)) + \lambda \beta_2 \beta_1 \mu_i(x, y)) \\
&= R(\mu_i(\beta_1 R(x), \beta_2(y)) + \mu_i(\beta_1 x, \beta_2(R(y))) + \lambda \mu_i(\beta_1(x), \beta_2(y))) \\
&= R\left(\mu_\beta^i(R(x), y) + \mu_\beta^i(x, R(y)) + \lambda \mu_\beta^i(x, y)\right).
\end{aligned}$$

This completes the proof. $\square$

Let us observe that $A_{\beta_1, Id}^1 = A_{\beta_1, Id}^2$.

Corollary 7 Let $(A, \mu_1, \dots, \mu_n, \varepsilon, R, \alpha)$ be a Rota–Baxter Hom- X color algebra and $\beta \in \text{Cent}(A)$. Define bilinear maps $\mu_\beta^i : A \times A \rightarrow A$, $(i = 1, \dots, n)$, by

$$\mu_\beta^i(x, y) := \mu_i(\beta(x), y).$$

Then $(A, \mu_\beta^1, \dots, \mu_\beta^n, \varepsilon, R, \beta \circ \alpha)$ is also a Rota–Baxter Hom- X color algebra.

Note that $(A, \mu_\beta^1, \dots, \mu_\beta^n, \varepsilon, \beta)$ is also a Hom- X color algebra whenever α is the identity map in Corollary 7.

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