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Neuro-Fuzzy Approach for Estimating Energy Dissipation in Skimming Flow over Stepped Spillways

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Abstract In a stepped spillway, the spillway face is provided with a series of steps from near the crest to the toe. The energy dissipation caused by the steps reduces the size of the energy dissipator, generally provided at the toe of the spillway. The hydraulics of stepped spillways is investigated by carrying out laboratory experiments, building models to explain the data, and testing the robustness of the models developed here using a neuro-fuzzy approach. The experiments consist of twenty different stepped spillways tested in a horizontal laboratory flume, a wide range of discharge values, three weir slopes of 15°, 25°, and 45° and different step numbers from 3 to 50 on the ogee surface. The main objective of this paper was to investigate the applicability and accuracy of the neuro-fuzzy approach in estimating the proper values of energy dissipation of skimming flow regime over stepped spillways because of the imprecise, insufficient, ambiguous and uncertain data available. A neuro-fuzzy approach was developed to relate the input and output (energy dissipation) variables. Multiple regression equations based on dimensional analysis theory were developed for computing energy dissipation over stepped spillways. The determination coefficients for the suggested neuro-fuzzy model in training and testing process are 0.974 and 0.966, respectively. It was found that the neuro-fuzzy approach formulation of the problem of solving for the energy dissipation over stepped spillways is more successful than that by regression equations.

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Keywords Energy dissipation · Stepped spillway · Neuro-fuzzy · Skimming regime

الخلاصة

في قناة تصريف المياه الصاعدة يقدم وجه القناة مع سلسلة من الخطوات من قريب القمة إلى أخمص القدم. إن تبديد الطاقة الناجمة عن الدرجات يقلل من حجم مبدد الطاقة ، ويقدم عادة في أخمص قدم القناة. وقد تم التحقيق في هيدروليكية قناة تصريف المياه الصاعدة من خلال تنفيذ التجارب المعملية ، وبناء نماذج لشرح البيانات ، واختبار متانة النماذج المطورة هنا باستخدام النهج العصبي الغامض. وتتكون التجارب من عشرين قناة تصريف مياه صاعدة مختلفة تم اختبارها في مختبر المسايل الأفقي، ومجموعة واسعة من قيم التفريغ، وثلاثة منحدرات سد من 15 و 25 و 45 درجة وأرقام درجة مختلفة من 3 إلى 50 على سطح العقد مستدق الرأس. ويتمثل الهدف الرئيسي من هذه الورقة العلمية في دراسة إمكانية تطبيق ودقة النهج العصبي الغامض في تقدير القيم المناسبة من تبديد الطاقة في نظام تدفق القشط على قناة تصريف مياه صاعدة بسبب البيانات غيرالدقيقة، وغير الكافية ، والغامضة وغير المؤكدة المتاحة. وتم تطوير نهج عصبي غامض لربط متغيرات المدخلات والمخرجات (تبديد الطاقة). وتم كذلك تطوير معادلات الانحدار المتعدد على أساس نظرية تحليل الأبعاد لحساب تبديد الطاقة على قناة تصريف المياه الصاعدة. إن معاملات الحسم للنموذج العصبي الغامض المقترح في عملية التدريب والاختبار هي 0.974 و 0.966 ، على التوالي . وتبين أن صياغة النهج العصبي الغامض لمشكلة معالجة تبديد الطاقة على قناة تصريف المياه الصاعدة هي أكثر نجاحا من تلك التي من معادلات الانحدار.

List of symbols

b	Spillway width
E_1	Energy downstream of spillway before hydraulic jump
E_0	Total energy upstream of spillway
ΔE	The difference between energy upstream and downstream of the spillway ($\Delta E = E_0 - E_1$)
F_r	Supercritical Froude number $= V_1/\sqrt{gy_1}$
g	Acceleration due to gravity

h Step height
H_w Total spillway height from flume bed
l Step length
q Discharge per unit width
Q Discharge
S Weir/spillway slope $(V : H)$
V_a Approach velocity $= (q/y)$
V_1 Velocity at the toe of the spillway
y_0 Depth of flow about 0.60 m upstream of the spillway above the spillway crest
y_1 Depth before hydraulic jump at the spillway toe
y_2 Depth after hydraulic jump

1 Introduction

Stepped spillways have been built in the past, but there is some interest in them because of significant cost savings. Stepped spillways have many applications in dam structures, river engineering, and soil conservation works. Their stilling basins can be constructed at lower costs as energy dissipation in these spillways is high. Stepped spillways have been used for centuries. Recently, new construction materials such as roller compacted concrete (RCC) and design techniques have increased the interest in stepped spillways [1]. The steps produce considerable energy dissipation along the spillway and reduce the size of the required downstream energy dissipation basin. The various flow regimes for stepped spillways are as follows: (i) *nappe or jet flow regime*: the water flows as a succession of free-falling nappes at small discharges [2]; (ii) *skimming flow regime*: Most prototype spillways operate at large discharges per unit width for which the water skims as a coherent stream over the pseudo-bottom (i.e., not solid bottom) formed by step edges, which is characterized by significant form losses and momentum transfer from the main stream to the recirculation zones (e.g., [3,4]); and (iii) *transition flow regime:* For an intermediate range of flow rates, a transition flow regime is observed between the above regimes [5].

Figure 1 shows the schematic classification of nappe, transition and skimming flow regimes.

The hydraulics of flow over stepped spillways is difficult due to the complexity of flow regimes, physical characteristics, and various hydraulic effects such as turbulence. Their implementations in the form of stepped chutes/spillways

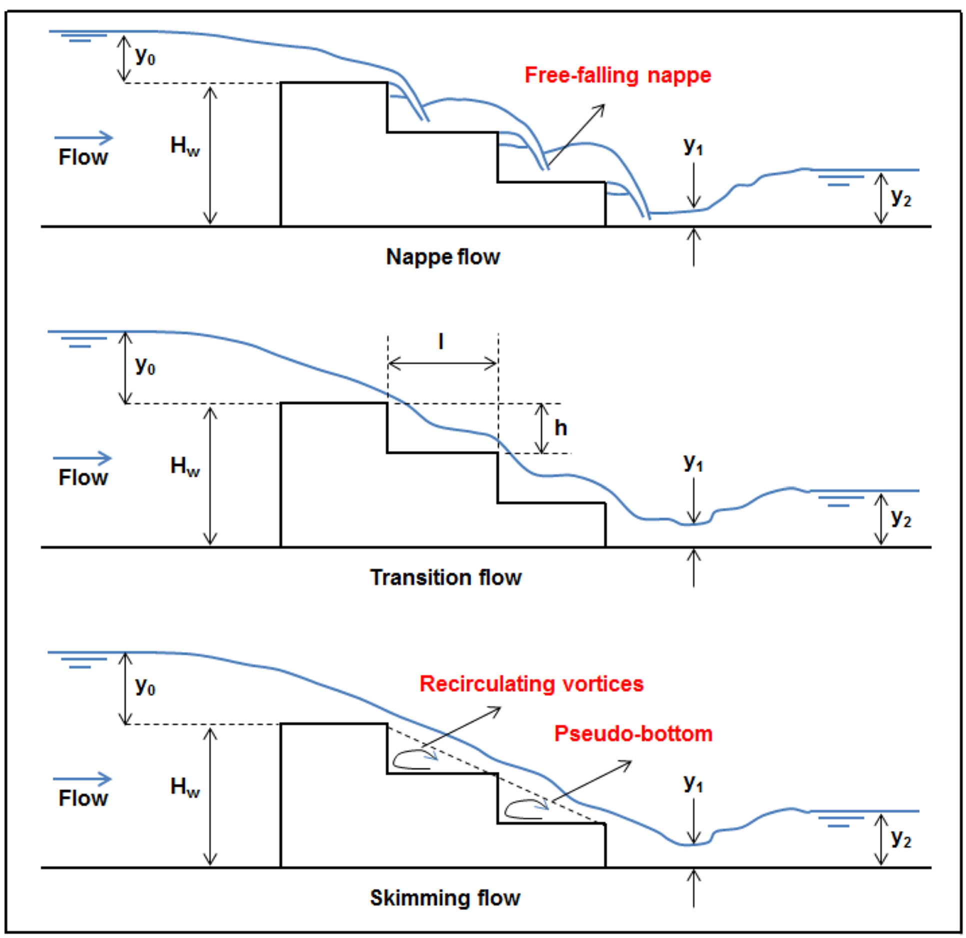

Fig. 1 Schematic classification of three flow regimes: nappe, transition and skimming flow regimes

have become a popular method for handling flood releases. Arguably, a better insight into their hydraulics can enhance their popularity and this is the focus of this paper. Many studies have been carried out to investigate different aspects of overflow in stepped spillways but the hydraulics of stepped spillways by neuro-fuzzy approach has not been fully investigated due to the complexity of flow patterns and resistance.

The primary focus of this research is to investigate the accuracy of a fuzzy rule system approach for estimating the energy dissipation of the skimming flow regime over stepped spillway because of the imprecise, insufficient, ambiguous and uncertain data available. The application of the proposed approach was performed using the measured data for energy dissipation available from the experimental analyses [6]; hence, its performance was tested using some parameters for error estimation.

In recent years, artificial intelligence (AI) techniques such as neuro-fuzzy, artificial neural network (ANN), and genetic programming (GP) models have attracted researchers in many disciplines of science and engineering, since they are capable of correlating large and complex data sets without any prior knowledge of the relationships among them. Notably, ANNs have been applied to other hydraulic problems, e.g., Yuhong and Wenxin [7] to predict the friction factor of open channel flow, Kisi [8] to predict the mean monthly stream flow, Rakhshandehroo et al. [9] for forecasting groundwater level in Shiraz plain and Eslamian et al. [10] to estimate Penman–Monteith reference evapotranspiration. The authors are not aware of the application of neuro-fuzzy to stepped spillways.

Ozger and Yildirim [11] investigated the accuracy of a fuzzy rule system approach to determine the relationship between pipe roughness, Reynolds number, and friction factor. A neuro-fuzzy approach was developed to relate the input (pipe roughness and Reynolds number) and output (friction coefficient) variables. The application of the proposed approach was performed for the data derived from the Moody's diagram. The performance of the proposed model was compared with respect to conventional procedures using some statistical parameters for error estimation. The comparison test results reveal that through fuzzy rules and membership functions, the friction factor can be identified precisely.

Yildirim and Ozger [12] applied neuro-fuzzy approach in estimating Hazen–Williams friction coefficient (C_{HW}) for small-diameter polyethylene pipes. The examination results indicated that through fuzzy rules and membership functions, the proposed model can be successfully used to identify the proper values of the C_{HW} coefficient; hence, accurately estimate friction losses through smooth PE pipes.

Table 1 Geometrical characteristics of physical models of stepped spillways of 50 cm width and average height of 100 cm

Steps number (N)	Steps height h (cm)	Steps length l (cm)	Spillway slope (S)
5	17.22	17.22	45
10	8.5	8.5	45
15	5.5	5.5	45
20	4.3	4.3	45
35	2.46	2.46	45
50	1.72	1.72	45

2 Theoretical Bases and Model Implementation

2.1 Experimental Setup

Experiments on stepped spillways test runs were carried out at the Hydraulic Laboratory of Water Engineering Department, Shahid Chamran University (SCU) in Ahvaz city and Iran. The test runs were installed in two flumes, (i) −0.5 m wide, 8 m in length, and 1.6 m in height; (ii) −0.25 m wide, 10 m in length, and 0.60 m in height. Tables 1 and 2 show some geometrical characteristics of the physical models of stepped spillways with 50 and 25 cm width, respectively.

The flow through the flume was controlled at the end of the laboratory flume by a gate to form a hydraulic jump at the weir toe to enable flow measurements. Thus, discharge values were measured by a calibrated sharp triangle weir (of 53° angle) installed at the downstream of the flume. Discharge water was supplied by a pump (maximum value 50 l/s). Discharge values ranged from 7 to 50 l/s with an accuracy level of ±0.9l/s. Upstream water levels were measured using a point gauge within ±0.1 mm accuracy. All measurements were taken along the centreline of the flume.

Table 2 Geometrical characteristics of physical models of stepped spillways of 25 cm width and average height of 32 cm

Steps number (N)	Steps height h (cm)	Steps length l (cm)	Spillway slope (S)
3	10.5	10.5	45
5	3.9	3.9	45
10	1.96	1.96	45
15	1.3	1.3	45
35	0.8	0.8	45
5	5.6	12	25
10	2.8	6	25
5	5.6	12	25
10	2.8	6	25
15	1.9	4.07	25
5	6.1	22.77	15
10	3	11.2	15
15	2	7.46	15
30	1	3.73	15

In each test run, water depth was measured 0.60 m upstream of the chute, y_0, and after the hydraulic jump, y_2. The thin flow and air entrainment at the spillway toe made it difficult to measure the flow depth (y_1) accurately. Pegram et al. [13] calculated energy dissipation using the conjugate water depths of the hydraulic jump (y_2). In the present study, y_2 was measured with an accuracy level of ±2 mm, where there were few bubbles and less undulation in the tail water.

2.2 Experimental Data

A total of 154 test runs were carried out in the skimming flow regime with three different slopes ($S = 15°, 25°$ and $45°$), seven different step numbers (Tables 1, 2), and varying discharge rates and the measurements for each test run comprised discharge values and two values of water depth (Table 4 in "Appendix"). It should be mentioned that the total number of experiments was 250 including the nappe, transition and skimming flow regimes, but in this study, only skimming data were used for simulation of energy dissipation. Selection of step numbers was based on laboratory flume facilities.

A decision was made to use 68 % of these data points (104 data points) for training and 32 % of the total data points (50 point data) for testing the predictions of the model.

The procedure for selecting training and prediction data was based on plotting relative energy dissipation, drop number, Froude number, slope, number of steps versus discharge and selecting representative data points from high, medium and low ranges. The measurement data points are given in "Appendix"

2.3 Dimensional Analysis

Upstream energy head (E_0), downstream energy head (E_1) and relative energy dissipation ($\Delta E/E_0$) are calculated as follows (Fig. 2):

$$E_0 = H_w + y_0 + \frac{V_0^2}{2g} = H_w + y_0 + \frac{q^2}{2g(H_w + y_0)^2} \tag{1}$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{q^2}{2g y_1^2} \tag{2}$$

$$\frac{\Delta E}{E_0} = \frac{E_0 - E_1}{E_0} = 1 - \frac{E_1}{E_0} \tag{3}$$

where g is acceleration due to gravity, H_w is total spillway height measured with a point gauge after the installation of the spillway at the flume, y_0 is the depth of the flow at a set distance upstream of the spillway and above the spillway crest, q is discharge per unit width, and V_0 is the approach velocity. The depth, y_1, was calculated using the conjugate depth (y_2) expressed as:

$$y_1 = \frac{y_2}{2}\left(\sqrt{1 + 8Fr_2^2} - 1\right) \tag{4}$$

where F_{r2} is Froude number ($F_{r2} = V_2/\sqrt{g y_2}$), V_2 and y_2 are the velocity and water depth at Sect. 2 (after hydraulic jump and the re-establishment of subcritical flow), respectively.

Generally, energy dissipation depends on hydraulic and geometric variables expressed as:

$$\frac{\Delta E}{E_0} = f(q, l, h, H_w, g, N) \tag{5}$$

where l is step length, h is step height, and N is number of steps. In all tests, discharge was regulated in a way to form hydraulic jump at spillway toe, so that supercritical flow at the downstream of the spillway toe may occur (Froude number >1). Although both depth values of y_1 and y_2 were measured, only y_2 values were used to calculate the energy dissipation by means of Eqs. (1)–(4).

The fundamental variables that are important in the hydraulics stepped spillways are geometrical parameters such as total spillway height (H_w), step length (l), step height (h), spillway slope (S), number of steps (N); and hydraulic parameters such as discharge per unit width of canal (q), energy upstream of weir (E_0) and energy downstream of weir (E_1) defined in Eqs. (1) and (2) respectively. Using the Buckingham Π-theorem, relative energy dissipation can be expressed as:

$$\frac{\Delta E}{E_0} = f\left(q^2/gH_w^3, h/l, N, y_c/h, F_{r1}\right) \tag{6}$$

Equation (6) can be rewritten as Eq. (7):

$$\Delta E/E_0 = f(\mathrm{DN}, S, N, y_c/h, F_{r1}) \tag{7}$$

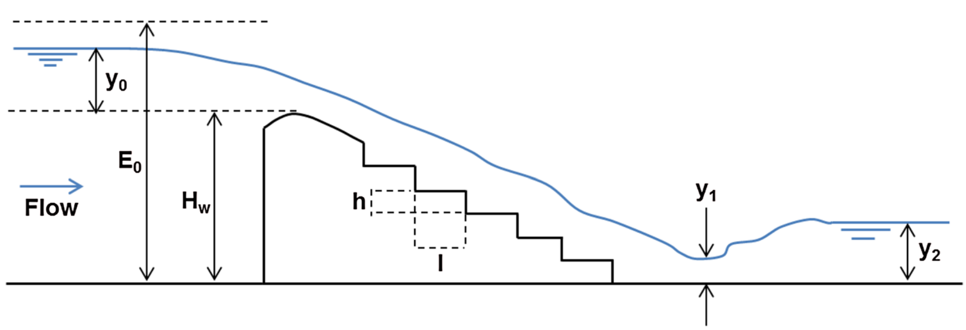

Fig. 2 Location of measured y_1 and y_2 depths downstream of weir

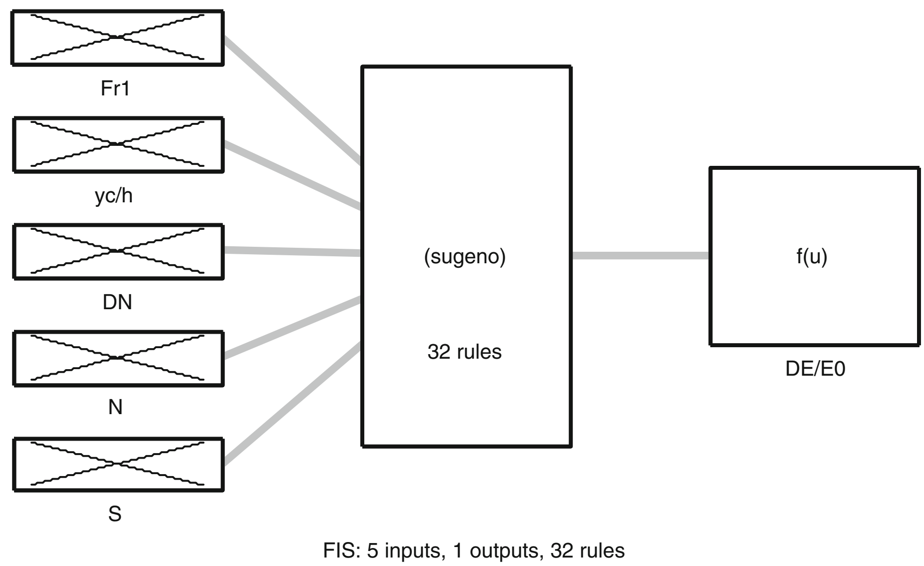

Fig. 3 The model structure of the proposed FIS

where $\mathrm{DN} = q^2/gH_w^3$, y_c is critical depth ($y_c = (q^2/g)^{1/3}$) and $S = h/l$ is the spillway slope. The parameter DN is termed drop number, similar to drop number presented first by Rand [14]. Froude number at Sect. 1 is defined by: $Fr_1 = V_1/\sqrt{gy_1}$. The average flow velocity at any section ($V = q/y$) was calculated as the measured flow rate per unit width ($q = Q/b$) where Q is total discharge, b is the spillway width, and y is depth of water at the appropriate section.

2.4 Implementation of Adaptive Neuro-Fuzzy Inference System (ANFIS)

Adaptive neuro-fuzzy inference system (ANFIS), first introduced by Jang [15], is a universal approximator. The neuro-fuzzy model used in this study implements the Takagi–Sugeno (TS) fuzzy approach [16] to obtain the value of the output variable from input variables. Here, the fuzzy inference system (FIS) has five inputs: drop number (DN), spillway slope (S), number of steps (N), critical depth to step height (y_c/h) and Froude number (F_{r1}); and one output relative energy dissipation ($\Delta E/E_0$).

In this study, ANFIS is selected to train the proposed model. ANFIS Editor GUI of MATLAB is used for implementation. The data (154) is divided into two parts which are the training (104) and testing (50) parts. The testing data is selected randomly, and it is independent from training data. An initial FIS model structure should be specified prior to FIS training. The FIS model consists of grid partitioning techniques which apply grid partition on the data. To train the FIS, backpropagation gradient descent method is employed. The number of training epochs is 1,000, and the training error tolerance is set to 0.01. After the FIS is trained, the model is validated using the testing data that is independent from the data used to train the FIS.

A schematic of the model structure is given in Fig. 3. The FIS model consists of 3 membership functions (MFs) for each input. Gaussian type MF is chosen for input and constant type for output membership function. The MFs for the trained FIS are given in Fig. 4.

2.5 Performance Criteria

The two error measures are used to compare the performance of the various models: determination coefficient (r^2) and root mean square error (RMSE). The study also uses relative error (RE) that is defined as follows:

$$\mathrm{RE} = |T_{\mathrm{True}} - T_{\mathrm{Estimated}}| \,/\, T_{\mathrm{True}} \tag{8}$$

where T_{True} is from experimental tests and $T_{\mathrm{Estimated}}$ is calculated.

3 Results

Figure 5 presents: $\Delta E/E_0$ versus DN and shows that by increasing DN, the relative energy dissipation is reduced. Energy dissipation is reduced with a high gradient until GN reaches a value approximately equal to 0.01 but above this value (DN > 0.01), energy dissipation becomes independent of DN, i.e., it becomes less efficient.

Figure 6 presents: $\Delta E/E_0$ versus y_c/h and shows that by increasing y_c/h, the relative energy dissipation is reduced. But discrepancies among data points are high, and it seems that y_c/h does not affect the variation of $\Delta E/E_0$ solely.

Figure 7 presents: $\Delta E/E_0$ versus F_{r1} and discrepancies among data points are high, and it seems that F_{r1} does not reflect the variation of $\Delta E/E_0$ solely.

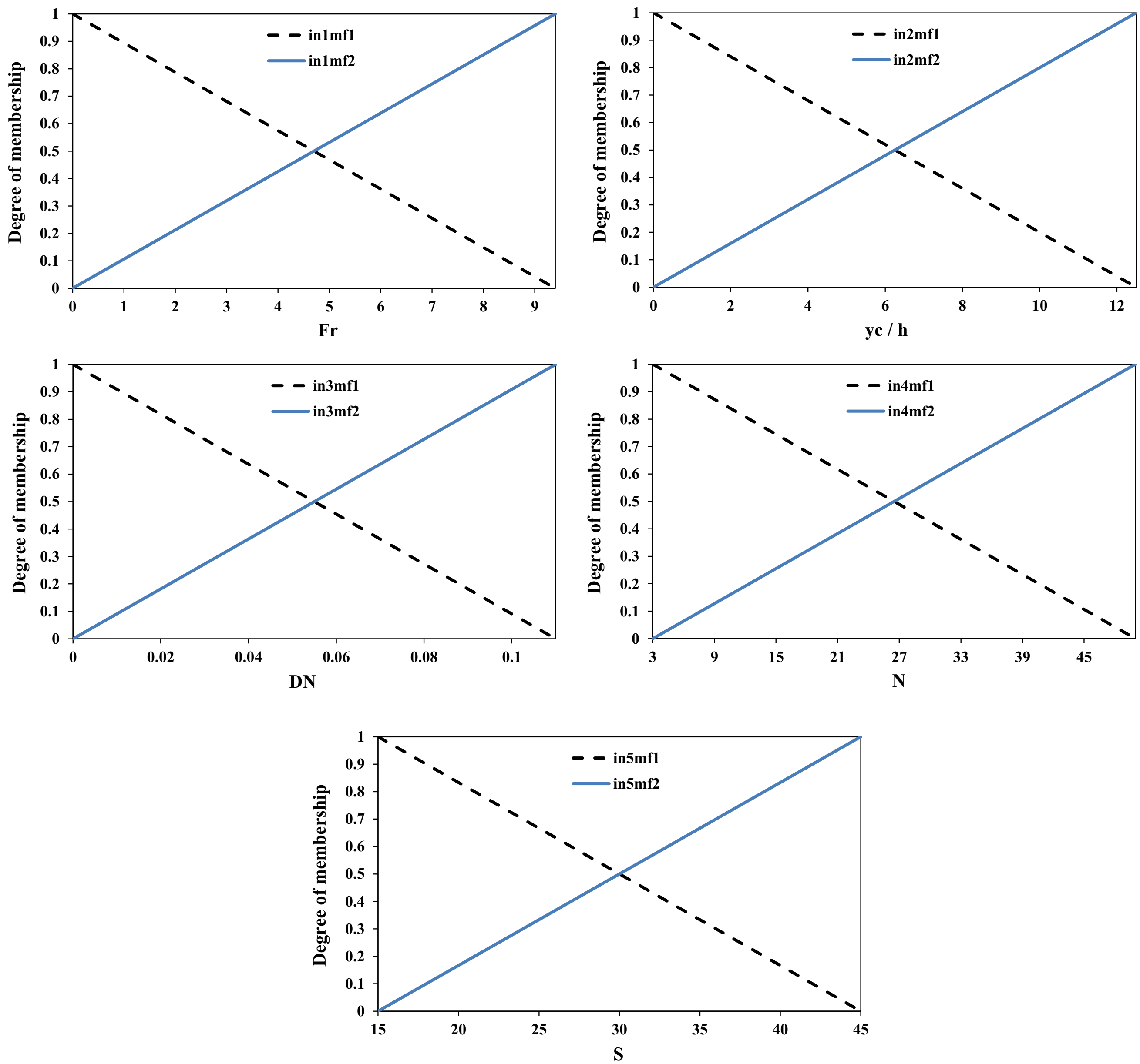

Fig. 4 Membership functions for the trained FIS

Scatter plots of energy dissipation versus to N and S indicate that the correlation is lower than that of the F_{r1}, y_c/h and DN, so their scatter plots are not presented in this study.

The performance of the ANFIS model for calculating energy dissipation of flow over stepped spillways was investigated by plotting a scatter diagram, as shown in Figs. 8 and 9 for training and testing, respectively. The determination coefficients of the proposed neuro-fuzzy model in training and testing process are 0.974 and 0.966, respectively. This suggests high accuracy of implementation of neuro-fuzzy model for estimating the energy dissipation of flows over stepped spillways.

3.1 Regression Analyses

Multiple regression analyses were performed with different combinations of the dimensionless parameters that appear in Eq. (7). Several linear and nonlinear multiple regressions were conducted using the Statistical Package for Social Science (SPSS) software version 17.

The fitted equations for physical models are given by Eqs. (9)–(12):

$$\begin{aligned}\Delta E/E_0 = & -1.063F_{r1}^{-2.119} + 350.365(y_c/h)^{-0.02} \\ & +473.031(\mathrm{DN})^{-0.014} + 61.544N^{0.052} \\ & -858.114S^{0.007}, \quad r^2 = 0.928 \end{aligned} \tag{9}$$

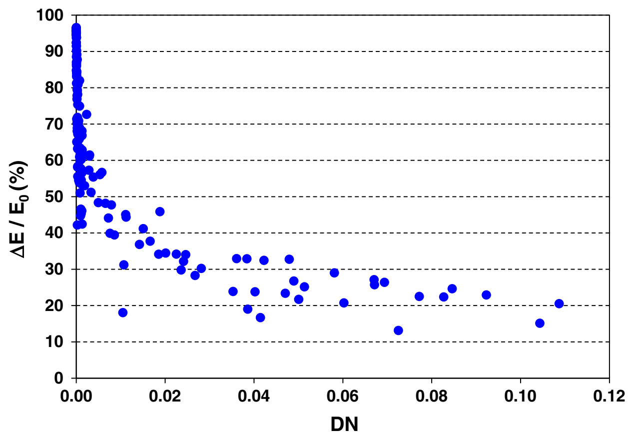

Fig. 5 Variation of $\Delta E/E_0$ versus DN for stepped spillways

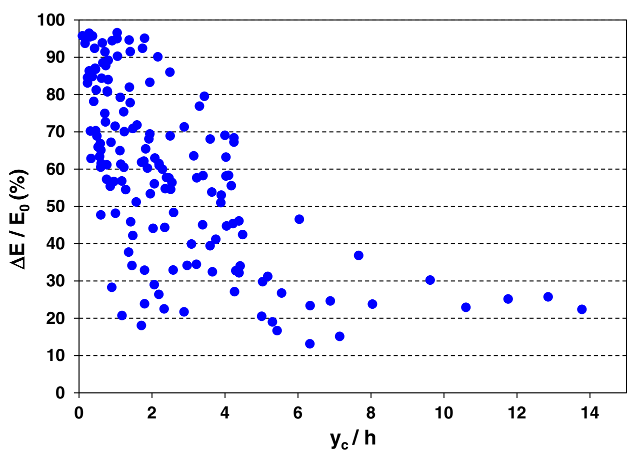

Fig. 6 Variation of $\Delta E/E_0$ versus y_c/h for stepped spillways

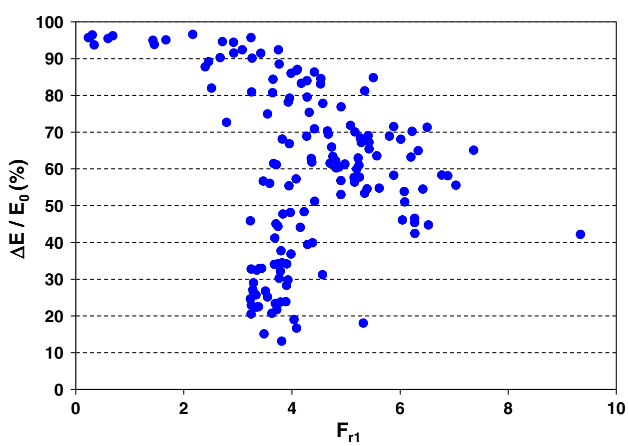

Fig. 7 Variation of $\Delta E/E_0$ versus F_{r1} for stepped spillways

$$\Delta E/E_0 = 26.991(F_{r1})^{0.075}(y_c/h)^{-0.298}(\mathrm{DN})^{-0.048} \times(N)^{0.255}(S)^{-0.063}, \quad r^2 = 0.865 \qquad (10)$$

$$\Delta E/E_0 = -172464.174(F_{r1})^{1.791*10^{-5}} + 172471.549 \times(\mathrm{DN})^{-4.747*10^{-5}}, \quad r^2 = 0.903 \qquad (11)$$

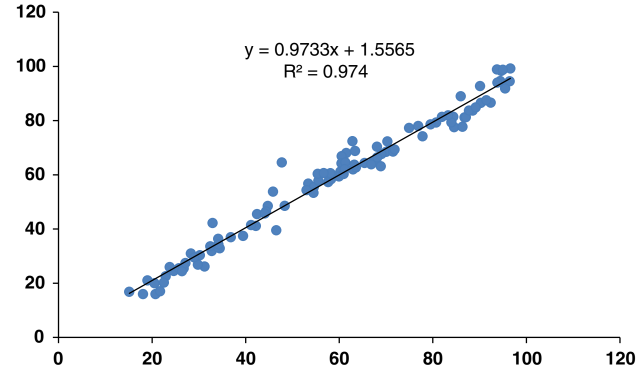

Fig. 8 Predicted and observed values for training data

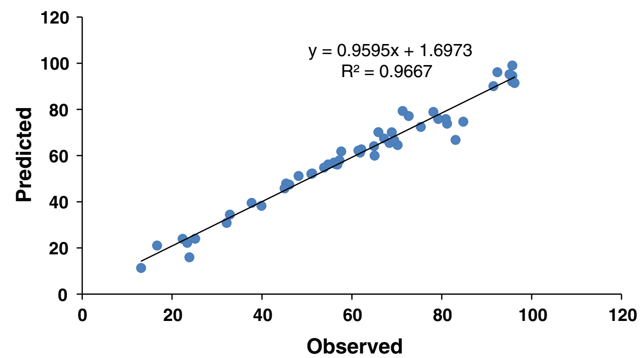

Fig. 9 Predicted and observed values for testing data

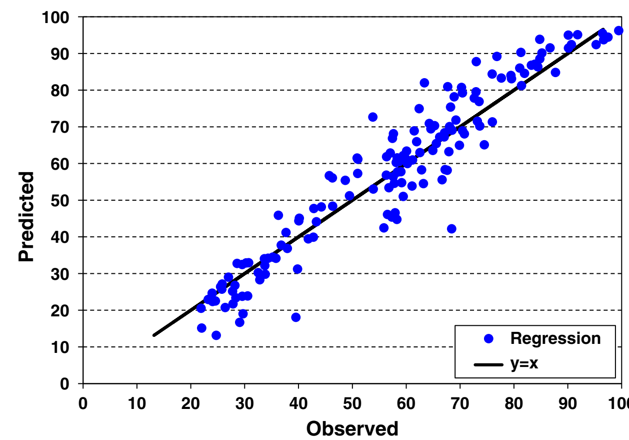

Fig. 10 Performance of regression Eq. (11) and observed data points

$$\Delta E/E_0 = 21.748(F_{r1})^{0.028}(\mathrm{DN})^{-0.133}, \quad r^2 = 0.845 \quad (12)$$

An overview of Eqs. (9)–(12) indicates that the correlation of y_c/h, N and S with energy dissipation is lower (compare Eqs. 10 and 11) than that of the F_{r1} and DN.

In practice, Eq. (11) can be helpful because it includes simple correlations among $\Delta E/E_0$, F_{r1} and DN with $r^2 = 0.903$. With more accuracy, Eq. (9) with $r^2 = 0.928$ can be used.

The performance of regression analysis using Eq. (11) versus the observed data points is shown in Fig. 10, according to

Table 3 Error measurements in regression equations and ANFIS for testing data

Determination coefficient (r^2)	RE (average)	RE (minimum)	RE (maximum)	Equation
0.908	0.0962	0.0022	1.0158	9
0.892	0.1133	0.00097	1.1879	11
0.966	0.0545	0.00056	0.3349	ANFIS

which regression Eq. (11) is capable of predicting the values of energy dissipation.

A comparison among Eqs. 9, 11 and the ANFIS approximation approach in Table 3 demonstrates that ANFIS approach is more accurate than regression analysis. From Table 3, RE (Average) = 0.054 and $r^2 = 0.966$ for the ANFIS approach.

4 Conclusions

The paper applies neuro-fuzzy technique for predicting the energy dissipation in stepped spillways. The data is obtained from laboratory experimental tests on physical stepped spillways. In the proposed fuzzy rule system approach, drop number (DN), Froude number (F_{r1}), slope (S), number of steps (N) and (y_c/h) are the input variables, whereas the relative energy dissipation is employed as the output variable.

Preliminary test runs identified the optimum ANFIS model. The trained network of ANFIS model was able to predict the response with r^2 and RE equal to 0.966 and 0.054, respectively (Table 3). The application of the proposed approach was performed using the measured data for the relative energy dissipation available from the experimental analysis.

The performance of the proposed model was tested using some parameters for error estimation. The results indicate that through fuzzy rules and membership functions, the proposed model can be used with the highest degree of accuracy to identify the proper values of energy dissipation; hence, accurately estimate relative energy dissipation.

Energy dissipation with regression equations had some edge over the fuzzy rules model, both visually and quantitatively. Although the performance of the regression equations in terms of r^2 and RE was good, these values were slightly lower than those predicted by means of the neuro-fuzzy approach, i.e., the latter was found to be more accurate than regression analysis.

Appendix: Experimental Results

Note-Total data points are 154. A decision was made to use 104 data points for training and 50 point data for testing the model.
See Table 4.

Table 4 Experimental results of stepped spillway

Row	$\Delta E/E_0$	F_{r1}	y_c/h	q^2/gH_w^3	N	S	Row	$\Delta E/E_0$	F_{r1}	y_c/h	q^2/gH_w^3	N	S
1	61.523	4.706	0.608	0.001105	5	45	78	21.720	3.719	2.880	0.050072	5	45
2	60.530	4.877	0.596	0.001044	5	45	79	32.943	3.440	2.582	0.036092	5	45
3	63.390	4.758	0.565	0.000890	5	45	80	18.065	5.320	1.710	0.010492	5	45
4	65.931	4.734	0.524	0.000710	5	45	81	74.946	3.549	0.709	0.000747	5	45
5	70.308	4.658	0.459	0.000476	5	45	82	86.357	4.415	0.281	0.000047	5	45
6	78.157	3.931	0.404	0.000325	5	45	83	24.643	3.232	6.885	0.084616	10	45
7	70.203	6.223	0.314	0.000153	5	45	84	16.689	4.086	5.428	0.041449	10	45
8	84.571	4.534	0.233	0.000063	5	45	85	19.022	4.039	5.300	0.038602	10	45
9	60.486	4.807	1.231	0.001111	10	45	86	39.911	4.382	3.083	0.007594	10	45
10	61.312	4.979	1.145	0.000896	10	45	87	54.504	6.423	1.280	0.000543	10	45
11	67.193	5.283	0.878	0.000404	10	45	88	93.836	1.455	0.640	0.000068	10	45
12	80.723	3.642	0.780	0.000283	10	45	89	22.915	3.249	10.603	0.092298	15	45
13	65.084	7.361	0.601	0.000130	10	45	90	23.783	3.791	8.040	0.040247	15	45
14	84.384	3.650	0.619	0.000141	10	45	91	31.232	4.566	5.172	0.010711	15	45
15	81.226	5.346	0.473	0.000063	10	45	92	42.199	9.339	1.475	0.000249	15	45
16	86.788	4.085	0.454	0.000056	10	45	93	23.388	3.692	6.333	0.047045	15	25

Table 4 continued

Row	$\Delta E/E_0$	F_{r1}	y_c/h	q^2/gH_w^3	N	S	Row	$\Delta E/E_0$	F_{r1}	y_c/h	q^2/gH_w^3	N	S
17	87.066	4.101	0.442	0.000051	10	45	94	29.799	3.928	5.032	0.023607	15	25
18	84.818	5.505	0.364	0.000029	10	45	95	39.447	4.290	3.594	0.008597	15	25
19	53.397	5.346	1.956	0.001271	15	45	96	68.119	3.822	1.916	0.001304	15	25
20	60.260	4.820	1.875	0.001121	15	45	97	75.376	4.321	1.225	0.000340	15	25
21	62.144	4.810	1.779	0.000956	15	45	98	79.242	3.956	1.132	0.000269	15	25
22	70.890	4.417	1.477	0.000547	15	45	99	95.727	0.234	0.284	0.000004	15	25
23	70.062	5.166	1.242	0.000326	15	45	100	96.441	0.307	0.278	0.000004	15	25
24	64.960	6.336	1.124	0.000241	15	45	101	22.516	3.382	2.334	0.077210	5	25
25	71.521	5.883	0.991	0.000165	15	45	102	23.893	3.889	1.798	0.035287	5	25
26	83.994	4.279	0.800	0.000087	15	45	103	34.138	3.903	1.451	0.018549	5	25
27	88.529	3.764	0.654	0.000047	15	45	104	55.410	3.945	0.858	0.003837	5	25
28	92.395	3.750	0.427	0.000013	15	45	105	72.669	2.791	0.728	0.002338	5	25
29	56.414	5.158	2.548	0.001169	20	45	106	93.735	0.341	0.166	0.000028	5	25
30	54.585	5.391	2.513	0.001122	20	45	107	95.734	0.258	0.094	0.000005	5	25
31	57.646	5.155	2.467	0.001061	20	45	108	32.761	3.246	4.297	0.047945	10	25
32	57.767	5.249	2.399	0.000976	20	45	109	34.477	3.817	3.217	0.020120	10	25
33	54.751	5.615	2.365	0.000935	20	45	110	56.071	3.590	2.065	0.005324	10	25
34	59.966	5.199	2.286	0.000844	20	45	111	87.756	2.395	0.733	0.000237	10	25
35	60.962	5.239	2.198	0.000750	20	45	112	15.136	3.484	7.144	0.104340	15	15
36	62.960	5.225	2.079	0.000635	20	45	113	13.145	3.811	6.327	0.072493	15	15
37	69.447	4.674	1.941	0.000517	20	45	114	26.775	3.510	5.552	0.048963	15	15
38	65.435	5.427	1.828	0.000431	20	45	115	34.037	3.676	4.416	0.024638	15	15
39	71.836	5.084	1.589	0.000284	20	45	116	32.178	3.787	4.387	0.024160	15	15
40	77.825	4.573	1.402	0.000195	20	45	117	41.184	3.682	3.750	0.015085	15	15
41	90.285	2.673	1.055	0.000083	20	45	118	45.071	3.705	3.388	0.011127	15	15
42	91.466	3.424	0.714	0.000026	20	45	119	48.367	4.225	2.592	0.004981	15	15
43	95.710	3.243	0.372	0.000004	20	45	120	61.490	3.662	2.192	0.003016	15	15
44	42.443	6.275	4.484	0.001343	35	45	121	61.848	4.369	1.716	0.001447	15	15
45	46.099	6.045	4.383	0.001253	35	45	122	81.976	2.512	1.382	0.000756	15	15
46	45.403	6.271	4.222	0.001120	35	45	123	89.186	2.460	0.802	0.000147	15	15
47	44.749	6.526	4.043	0.000984	35	45	124	22.376	3.342	13.781	0.082713	30	15
48	51.015	6.087	3.887	0.000874	35	45	125	25.744	3.337	12.855	0.067132	30	15
49	53.859	6.078	3.639	0.000718	35	45	126	25.166	3.548	11.758	0.051369	30	15
50	58.269	5.884	3.398	0.000584	35	45	127	30.232	3.763	9.622	0.028150	30	15
51	63.573	5.571	3.144	0.000463	35	45	128	36.844	3.982	7.663	0.014220	30	15
52	68.903	5.805	2.499	0.000232	35	45	129	53.027	4.906	3.900	0.001874	30	15
53	83.291	4.175	1.943	0.000109	35	45	130	57.673	5.151	3.227	0.001062	30	15
54	91.516	2.925	1.406	0.000041	35	45	131	94.996	1.428	1.043	0.000036	30	15
55	94.435	2.919	0.908	0.000011	35	45	132	26.388	3.287	2.189	0.069367	5	15
56	46.559	6.271	6.038	0.001040	50	45	133	29.013	3.289	2.064	0.058102	5	15
57	68.387	5.267	4.245	0.000361	50	45	134	32.891	3.397	1.798	0.038402	5	15
58	67.240	5.425	4.245	0.000361	50	45	135	45.866	3.232	1.418	0.018834	5	15
59	55.561	7.034	4.174	0.000344	50	45	136	37.728	3.798	1.360	0.016632	5	15
60	58.334	6.770	4.098	0.000325	50	45	137	48.168	3.971	0.998	0.006582	5	15
61	63.225	6.202	4.027	0.000309	50	45	138	56.691	3.470	0.956	0.005780	5	15
62	58.139	6.883	4.027	0.000309	50	45	139	61.166	3.711	0.764	0.002950	5	15
63	69.063	5.410	4.000	0.000302	50	45	140	57.286	4.078	0.756	0.002861	5	15

Table 4 continued

Row	$\Delta E/E_0$	F_{r1}	y_c/h	q^2/gH_w^3	N	S	Row	$\Delta E/E_0$	F_{r1}	y_c/h	q^2/gH_w^3	N	S
64	68.069	6.013	3.596	0.000220	50	45	141	66.842	3.952	0.584	0.001315	5	15
65	79.533	4.282	3.437	0.000192	50	45	142	68.882	4.275	0.490	0.000779	5	15
66	76.888	4.910	3.301	0.000170	50	45	143	83.096	4.530	0.233	0.000084	5	15
67	71.326	6.503	2.884	0.000113	50	45	144	20.522	3.244	5.008	0.108685	10	15
68	85.997	3.985	2.493	0.000073	50	45	145	27.132	3.277	4.263	0.067043	10	15
69	90.110	3.264	2.162	0.000048	50	45	146	32.461	3.349	3.655	0.042257	10	15
70	95.096	1.669	1.796	0.000027	50	45	147	34.180	3.740	2.964	0.022532	10	15
71	92.397	3.082	1.742	0.000025	50	45	148	44.370	3.742	2.349	0.011221	10	15
72	94.628	2.715	1.376	0.000012	50	45	149	44.105	4.156	2.030	0.007243	10	15
73	96.580	2.165	1.047	0.000005	50	45	150	51.193	4.420	1.568	0.003338	10	15
74	20.735	3.628	1.179	0.060239	3	45	151	56.819	4.907	1.172	0.001392	10	15
75	28.297	3.900	0.899	0.026757	3	45	152	80.929	3.253	0.776	0.000404	10	15
76	47.728	3.832	0.599	0.007919	3	45	153	95.475	0.598	0.298	0.000023	10	15
77	62.845	4.358	0.331	0.001332	3	45	154	96.217	0.686	0.259	0.000015	10	15

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