Correction to Notes on reduced Rickart rings, I. Representation and equational axiomatizations

Jānis Cīrulis¹ · Insa Cremer²

Received: 4 November 2019 / Accepted: 8 November 2019 / Published online: 9 December 2019
© The Managing Editors 2019

Correction to: Beitr Algebra Geom (2018) 59:375–389
<https://doi.org/10.1007/s13366-017-0373-3>

The aim of this note is to correct two misprints and a fault in a theorem in the original paper by Cīrulis and Cremer (2018). The authors of the paper are grateful to P. Nielsen for evoking a discussion about the proof of the theorem.

1. Theorem 2.4(f) should read as

$$(f) \quad (a, b)'(a''b)' = (a''b)'.$$

In its proof, the reference (e) should be replaced by (f).

2. In a private communication, Pace Nielsen turned authors' attention to a gap in the proof of Theorem 6.5: it is asserted on line 3 of its proof that an identity $x'' = x^\circ$ is immediate from the definition $x' := 1 - x^\circ$ (x'' stands for $(x')'$), but this is the case only if another identity $(1 - x^\circ)^\circ = 1 - x^\circ$ holds. Even a more elementary additional assumption does the job, but in any case the axiom system presented in the theorem is incomplete and must be supplemented. Below, an improved version of the theorem is given together with a revised proof; the new axiom is (iii).

Theorem 6.5 *A ring is a reduced Rickart ring iff it has unity and admits an operation $^\circ$ such that*

$$(i) \quad xx^\circ = x = x^\circ x, \quad (ii) \quad (xy)^\circ = x^\circ y^\circ, \quad (iii) \quad 0^\circ = 0.$$

If this is the case, then the focal operation of the ring is defined by $x' := 1 - x^\circ$.

The original article can be found online at <https://doi.org/10.1007/s13366-017-0373-3>.

✉ Jānis Cīrulis
janis.cirulis@lu.lv
Insa Cremer
insa.kremere@gmail.com

¹ Institute of Mathematics and Informatics, University of Latvia, Raiņa b. 29, Rīga 1459, Latvia

² Department of Mathematics, University of Latvia, Jelgavas 3, Rīga 1004, Latvia

Proof Let R be any ring. First assume that R is reduced and Rickart. For all $x \in R$, put $x^\circ := x''$; then (i)–(iii) follow by Propositions 3.8(b, g, e) and 2.4(a). Conversely, suppose that an operation $^\circ$ on R satisfies (i)–(iii) and put, for all x , $x' := 1 - x^\circ$. Then

$$\begin{aligned} x'' &= (1 - x^\circ)' = 1 - (1 - x^\circ)^\circ = 1 - ((1 - x^\circ)^\circ - 0^\circ) \\ &= 1 - ((1 - x^\circ)^\circ - (x - xx^\circ)^\circ) = 1 - ((1 - x^\circ)^\circ - (x(1 - x^\circ))^\circ) \\ &= 1 - ((1 - x^\circ)^\circ - x^\circ(1 - x^\circ)^\circ) = 1 - (1 - x^\circ)(1 - x^\circ)^\circ = 1 - (1 - x^\circ) = x^\circ. \end{aligned}$$

Easy calculations by help of (i)–(iii) show that then axioms of Theorem 6.3 are fulfilled; so, R is reduced and Rickart. $\square$

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.