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CORRECTION

Correction to: A new concept of smoothness in Orlicz spaces

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The goal of this erratum is to correct a mistake that appears in the proof of Corollary 3. In the second line of the proof we claim that if φ is a superadditive function and

$$M_0(t,\varphi) = \sup_{\epsilon>0} \frac{\varphi(\epsilon^n t)}{\varphi(\epsilon^n)},$$

then there exists $t_0 > 0$ such that $M_0(t,\varphi) \le t^{i_\varphi+1}$ for all $0 < t < t_0$, where

$$i_\varphi = \lim_{t\to 0^+} \frac{\ln(M_0(t,\varphi))}{\ln(t)}.$$

This affirmation is not correct. For example the superadditive function $\varphi(t) = t^{3/2}$ satisfies $M_0(t,\varphi) = t^{3/2}$, $i_\varphi = \frac{3}{2}$, and $M_0(t,\varphi) > t^{i_\varphi+1}$ for all $0 < t < 1$.

However, Corollary 3 is still true. A proof is provided below.

Corollary 3 *Assume $x_0 \in \mathbb{R}$ and $f \in c_n^{\Phi}(x_0)$ such that φ is a superadditive function. Let $\{P_{B(x_0,\epsilon)}(f)\}$ be a net of best Φ-approximation of f from Π^n on $B(x_0,\epsilon)$. Then*

$$\frac{1}{\epsilon\varphi(\epsilon^n)} \int_{B(x_0,\epsilon)} \varphi(|P_{B(x_0,\epsilon)}(f) - C_{x_0,n}(f)|)dx = o(1), \quad as \quad \epsilon \to 0.$$

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Proof If $n = 0$, it is obvious by [1, formula (20)]. Assume $n > 0$ and let

$$M_0(t,\varphi) = \sup_{\epsilon>0} \frac{\varphi(\epsilon^n t)}{\varphi(\epsilon^n)}.$$

Since $M_0(\cdot,\varphi)$ is non-decreasing and non-negative, the limit $i = \lim_{t\to 0^+} M_0(t,\varphi)$ exists. We claim that $i = 0$. In fact, as φ is a superadditive function, it is easy to see that

$$\varphi\left(\frac{u}{2^k}\right) \leq \frac{\varphi(u)}{2^k}, \quad \text{for all} \quad u \geq 0 \quad \text{and} \quad k \in \mathbb{N}.$$

Consequently $0 \leq M_0\left(\frac{1}{2^k},\varphi\right) \leq \frac{1}{2^k}$, for all $k \in \mathbb{N}$, which gives $i = 0$.

Now, let $\beta > 0$ and let $0 < \eta < 1$ be such that

$$M_0(\eta,\varphi) < \frac{\beta}{2}.$$

Then

$$\frac{\varphi(\epsilon^n \eta)}{\varphi(\epsilon^n)} < \frac{\beta}{2} \quad \text{for all} \quad \epsilon > 0. \tag{1}$$

From [1, formula (20)], there exists $\epsilon_0 = \epsilon_0(\eta) > 0$, such that $\epsilon^{-n}|(P_{B(x_0,\epsilon)}(f) - C_{x_0,n}(f))(x)| \leq \eta$, for all $x \in B(x_0,\epsilon)$ and $0 < \epsilon < \epsilon_0$. According to (1) we have

$$\frac{\varphi(|(P_{B(x_0,\epsilon)}(f) - C_{x_0,n}(f))(x)|)}{\varphi(\epsilon^n)} \leq \frac{\varphi(\epsilon^n\eta)}{\varphi(\epsilon^n)} \leq \frac{\beta}{2},$$

for all $x \in B(x_0,\epsilon)$, $0 < \epsilon < \epsilon_0$. Whence by integrating on $B(x_0,\epsilon)$ we can deduce that

$$\frac{1}{\epsilon\varphi(\epsilon^n)} \int_{B(x_0,\epsilon)} \varphi(|P_{B(x_0,\epsilon)}(f) - C_{x_0,n}(f)|)dx < \beta,$$

for all $0 < \epsilon < \epsilon_0$. This completes the proof. □

Finally, we observe that if $\varphi \in \mathcal{F}$ then $\varphi(u) > 0$ for all $u > 0$. Thus, if $\varphi \in \mathcal{F}$ is also superadditive then, for all $0 \leq t < s$, $\varphi(s-t) > 0$, whence $\varphi(t) < \varphi(t) + \varphi(s-t) \leq \varphi(s)$, that is, $\varphi \in \mathcal{F}$ is a strictly increasing function. So, in Theorem 5 and Corollary 4, we replace "φ is a superadditive strictly increasing function" by "φ is a superadditive function".

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Reference

1. Ferreyra, D.E., Levis, F.E., Roldán, M.V.: A new concept of smoothness in Orlicz spaces. Collect. Math. (2021)