Correction to: A $p(\cdot)$ -Poincaré-type inequality for variable exponent Sobolev spaces with zero boundary values in Carnot groups

Thomas Bieske¹ · Robert D. Freeman¹

Published online: 3 January 2019
© Springer Nature Switzerland AG 2019

Correction to: Anal.Math.Phys. (2018) 8:289–308
<https://doi.org/10.1007/s13324-018-0235-7>

In this short note, we remark on the main theorem of [1, Theorem 5.1]. First, the authors would like to thank the referee for pointing out that a more general result related to [1, Theorem 5.1], was proved for vector fields satisfying Hörmander’s condition by Li et al. [3, Theorem 1.10]. In [3, Theorem 3.1], the authors first show a more general result for boundedness of fractional integral operators in variable Lebesgue spaces on spaces of homogeneous type and combine this with a representation formula proved in [2] to establish a higher order Poincaré inequality for vector fields satisfying Hörmander’s condition on Boman chains (and therefore metric balls). Here they took the assumptions that $p(\cdot)$ is log-Hölder continuous and $1 < p^- \leq p^+ < Q/m$, where Q is the homogenous dimension and m is the dimension of the variable exponent space (in our case $m = 1$).

However, the main theorem [1, Theorem 5.1] does not follow directly from [3, Theorem 1.10]. The difference is in the assumption that $p(\cdot)$ is log-Hölder continuous in [3, Theorem 1.10] while [1, Theorem 5.1] requires that the variable exponent $p(\cdot)$ satisfies the jump condition. This allows for the case where $p^- > Q$ in the gauge balls discussed in Section 5 of [1]. Thus, the condition $1 < p^- \leq p^+ < Q$ in [3, Theorem 1.10] is violated. Consequently, when $p^- > Q$, then the hypotheses of [3, Theorem 1.10] do not hold but those of [1, Theorem 5.1] do hold.

Furthermore, the main theorem [1, Theorem 5.1] does not follow from combining [3, Theorem 3.1] and the representation formula of functions with boundary value

The original article can be found online at <https://doi.org/10.1007/s13324-018-0235-7>.

✉ Thomas Bieske
tbieske@usf.edu

Robert D. Freeman
rfreeman1@mail.usf.edu

¹ Department of Mathematics and Statistics, University of South Florida, Tampa, FL 33620, USA

zero for vector fields established in [4]. Again, we have the possibility that $p^- > Q$ in gauge balls but this fails the conditions in [3, Theorem 3.1].

In both cases, it is obvious that if $p^+ \leq \frac{Qp^-}{Q-p^-}$ in gauge balls, then we have the jump condition satisfied and this also agrees with the conditions in [3, Theorem 1.10] and thus [1, Theorem 5.1] does indeed follow directly from [3, Theorem 1.10]. Similarly, this is also true for [3, Theorem 3.1] and we could use representation formulas in [4] to easily obtain [1, Theorem 5.1].

References

1. Bieske, T., Freeman, R.D.: A $p(x)$ -Poincaré-type inequality for variable exponent Sobolev spaces with zero boundary values in Carnot groups. *Anal. Math. Phys.* **8**, 289–308 (2018)
2. Franchi, B., Lu, G., Wheeden, R.L.: Representation formulas and weighted Poincaré inequalities for Hörmander vector fields. *Ann. Inst. Fourier (Grenoble)* **45**, 577–604 (1995)
3. Li, X., Lu, G.Z., Tang, H.L.: Poincaré and Sobolev inequalities for vector fields satisfying Hörmander's condition in variable exponent Sobolev spaces. *Acta Math. Sin. Engl. Ser.* **31**(7), 1067–1085 (2015)
4. Lu, G.: Weighted Poincaré and Sobolev inequalities for vector fields satisfying Hörmander's condition and applications. *Rev. Mat. Iberoam.* **8**, 367–439 (1992)

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.