ORIGINAL ARTICLE

Fractional-order rumor propagation model with memory effect

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Abstract

The harm of online rumor cannot be ignored, and the research on its spreading mechanism is the core issue of rumor governance. People's judgment on whether information is a rumor is affected by early memory. In order to simulate the spread process of rumors better, so as to timely control rumors, this paper establishes a fractional rumor propagation model with memory effect based on the SIR rumor propagation model. Then, the basic regeneration number of the model is calculated, and the stability analysis of the equilibrium point is performed. Finally, the model is solved and simulated using an efficient and high-precision numerical solution algorithm. It was found that the lower the order, the stronger the memory effect. The simulation results show that the fractional-order rumor propagation model can be used to study the rumor propagation process more realistically. By enhancing the memory effect, the rumor propagation scale can be reduced and the harm of rumor can be reduced. The difference between this study and the past is that the fractional differential equation is used to describe the influence of memory effect on rumor propagation.

1 Introduction

Online social platforms have become the main place for people to express their view and exchange opinions. Mainstream social platforms such as WeChat, Weibo, and Twitter generate billions of messages every day. But there are many artificial online rumors hidden in a huge amount of information. Due to the convenience and speed of online social platforms, a rumor may diffuse the entire network in a short period of time. It has the characteristics of fast spreading, wide spreading, and great influence on society, so as to bring harm to the harmony and stability of the society, stable economic development, and even national security. For example, during COVID-19, a rumor that "Shuanghuanglian can resist the COVID-19" received widespread attention. As a result, people flocked to drugstore to buy Shuanghuanglian oral liquid, which caused a large number of people to gather. This not only disrupted the pharmaceutical market, but also brought a lot of unnecessary troubles and challenges to the

work of epidemic prevention. Therefore, studying the law of the spreading of rumors, then controlling it, and eliminating the harm of rumors has become a key issue that many scholars pay attention to.

For the research on the spreading of rumors, most scholars build rumors propagation models based on infectious disease models and then study the rules of the spread process of rumors. Daley and Kendall (1964) defined a rumor spreading rule similar to infectious diseases. The people in a closed space are divided into three categories: those who have not heard the rumors (ignorants), actively spread rumors (spreaders), and no longer spread rumors (stiflers). Rumors are spread through the "contact" between the spreader and the ignorance. Ignorance has a high chance of becoming a spreader when he comes into contact with a spreader, and a spreader may become stifler after infecting ignorant people around him. This is the earliest research on the rumor propagation using infectious disease model. Later researchers obtained many rumors propagation models such as SIR based on DK model (Zanette 2001; Moreno et al. 2004; Nekovee et al. 2007). Qiu et al. (2016) improved the SIR model in consideration of authoritative behavior and human forgetting rules, and found that rumor spreading behavior is time-dependent. Luo and Ma (2018) discussed

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the ISS_pR model of the impact of positive news on rumor propagation, and simulated rumor propagation in a scale-free network. Simulation results show that the model can well control and suppress the density of rumors. Liu et al. (2019) developed a rumor propagation dynamic model based on compartment method, defined the node as a user and its equipment, and considered the interaction between users and equipment in the process of network rumor propagation. All network nodes are divided into four parts: rumor neutrality, rumor receiving, believing rumor, and denying rumor. Finally, the network rumor is controlled by adjusting the parameters in the model. Huang et al. (2021) established a two-layer network. In the communication layer, simulate the spreading of rumors; in the contact layer, the spreading of infectious diseases is simulated, and the interaction between infectious diseases and rumors is studied. With the increase in attention to rumor governance, more and more scholars are engaged in the research of rumor (Chen and Wang 2020; Giorno and Spina 2016; Chen 2019; Wang et al. 2014). Most scholars mainly study and analyze the spreading of rumors by establishing differential equations (Zanette Nov 2001; Moreno et al. May 2004; Nekovee et al. 2007; Qiu et al. 2016; Luo and Ma 2018; Liu et al. 2019; Huang et al. 2021; Chen and Wang 2020; Giorno and Spina 2016; Chen 2019; Wang et al. 2014). These studies only consider the influence of time on the spreading of rumors. Wang et al. (2012) considered the influence of time and space factors on information diffusion by establishing partial differential equations. Zhu et al. (2021) established a 2SIR partial differential rumor propagation model, discussed the global stability of equilibrium points in homogeneous and heterogeneous networks, and proposed rumor control strategies, respectively. Zhu and Wang (2018) considered the uncertainty of human behavior, established a partial differential rumor propagation model, and found that the network topology and the uncertainty of human behavior determine the final scale of rumors. In the previous research on the spread of rumors, the current state of the system does not depend on the previous state and is a memoryless propagation process. However, when rumors are spread on the Internet, the experience and knowledge of netizens will affect their judgment on whether the news is a rumor. The early memory becomes blurred due to forgetting and has little impact on the present (Saeedian et al. 2017), and under normal circumstances, people rely on the latest memory, that is, the newly acquired knowledge to make judgments of information. So, there is obviously a memory effect in the spreading process of rumors. In the previous studies, the memory effect was achieved through integer-order differential equations (Zhang and Xu 2015).

In this study, the difference from the past is that fractional-order differential equations are used to describe the impact of memory effects on rumor spreading. The current state of the system depends on the past state through the "memory core" in the function. The structure of this paper is as follows: in Sect. 2, the fractional-order rumor propagation differential equation is established. Section 3 explores the equilibrium point of the model and its stability conditions. In the fourth section, the fractional-order differential equation is numerically simulated and compared with SIR model and real data to verify the effectiveness of the model. The fifth section puts forward some suggestions on the governance of rumors according to the research results, and indicates the next research direction.

2 Fractional-order rumor propagation model

Fractional-order calculus is a valuable tool for studying the influence of memory effects on system dynamics (Saeedian et al. Feb 2017; Ebadi et al. 2016). Recently, many scholars have studied the spreading of infectious diseases by establishing fractional-order differential equations (Ali et al. 2021a, b; Ahmad et al. 2020; Atangana 2020; Jahanshahi et al. 2021; Zafar et al. (2021a, b, c; 2022a, b). In view of the similarities between the spreading of rumors and the spreading of infectious diseases, this provides a reference for the construction of a fractional-order rumor propagation model. In general, the evolution of the rumor propagation is described by the first-order differential equations or partial differential equations. By replacing it with fractional-order differentiation, the equation appears as a time-dependent function or memory core, so that the state of the system depends on all past states. The advantage provided by the Caputo form is that when solving such fractional-order differential equations, it is not necessary to define the initial conditions of the fractional order, and only the initial values of the integer order differential equations need to be known. In addition, in the definition of Caputo fractional-order differential, the time correlation function is a power law function, which can reflect the fact that the contribution of the earlier state is significantly lower than the contribution of the current state of the dynamic system (Saeedian et al. Feb 2017). Therefore, this paper establishes a fractional-order differential equation of rumor spreading in Caputo form for related research.

In the SIR rumor propagation model, the population is divided into three categories: ignorant (Ignorant, *I*), rumor spreader (Spreader, *S*), and immune (Stifler, *R*). Everyone belongs to one of these three categories, and with the spread of rumors, the category of people may change, which results in a differential equation:

$$\frac{dI(t)}{dt} = \mu N - \frac{\beta I(t)S(t)}{N} - \nu I(t)$$
$$\frac{dS(t)}{dt} = \frac{\beta I(t)S(t)}{N} - \gamma S(t) - \nu S(t) \qquad (1)$$
$$\frac{dR(t)}{dt} = \gamma S(t) - \nu R(t)$$

N represents the total number of netizens, which tends to stabilize within a certain period of time. $I(t)$, $S(t)$, $R(t)$ represent the number of people in each state at the moment, respectively. Obviously, $I(t) + R(t) + S(t) = N$. The specific meaning of each parameter is detailed in Table 1.

This differential equation describes the changes in the number of people in the spreading process of rumors. In this process, the state at the current moment only depends on the state at the last moment and has nothing to do with the previous state. In order to observe the influence of the memory effect, we first rewrite the equation into a fractional-order differential equation with memory effect. The rewritten equation is as follows:

$$\frac{dI(t)}{dt} = \mu N - \int_{t_0}^{t} k(t - t')\left[\frac{\beta I(t')S(t')}{N} - \nu I(t')\right]dt'$$
$$\frac{dS(t)}{dt} = \int_{t_0}^{t} k(t - t')\left[\frac{\beta I(t')S(t')}{N} - \gamma S(t') - \nu S(t')\right]dt' \qquad (2)$$
$$\frac{dR(t)}{dt} = \int_{t_0}^{t} k(t - t')\left[\gamma S(t') - \nu R(t')\right]dt'$$

$k(t - t')$ is the time-dependent function. In the Caputo form of the fractional-order differential equation, it is a power function with a slow decay, that is to say, the longer the time, the less influence it has on the system. This is in line with the fact that long-term memory is not as powerful as the latest memory on individual judgments of rumors.

Generally speaking, $k(t - t') = \frac{1}{\Gamma(m-\alpha)}(t - t')^{m-\alpha-1}$, Γ is gamma function. α is the order of the fractional-order differential equation. The strength of the memory effect is controlled by α. In this study, only the case where $0 < \alpha < 1$ is considered. $m = \lceil \alpha \rceil$, that is, m is the smallest integer greater than or equal to α. Obviously, $m = 1$. Thus, the complete Caputo form of fractional-order rumor propagation differential equation is obtained:

$${}_{t_0}^{c}D_t^{\alpha}I(t) = \mu N - \frac{\beta I(t)S(t)}{N} - \nu I(t)$$
$${}_{t_0}^{c}D_t^{\alpha}S(t) = \frac{\beta I(t)S(t)}{N} - \gamma S(t) - \nu S(t) \qquad (3)$$
$${}_{t_0}^{c}D_t^{\alpha}R(t) = \gamma S(t) - \nu R(t)$$

${}_{t_0}^{c}D_t^{\alpha}$ represents the Caputo fractional differential of order $\alpha (0 < \alpha < 1)$, ${}_{t_0}^{c}D_t^{\alpha}f(t) = \frac{1}{\Gamma(1-\alpha)}\int_{t_0}^{t}\frac{f'(t)}{(t-t')^{\alpha}}dt'$. In the Caputo fractional-order differential equation, α determines the strength of the memory effect by controlling the decay speed of the time function. The smaller the value of α, the slower the decay speed of the time function, which means the existence of long memory. Conversely, the larger the value of α, the faster the time function decays, and the memory effect will be relatively weakened. When $\alpha = 1$, the fractional-order differential equation becomes an integer order and there is no memory. Therefore, the previous SIR model is a special case of this model, a differential equation degenerated from fractional order to integer order. In this study, for the convenience of research, we assume that all three types of people have the same memory, that is, they have the same α.

3 Equilibrium point and stability of the proposed model

For a system of Caputo fractional-order differential equations, when the order $\alpha \in (0, 1)$,

$${}^{c}D^{\alpha}f_1(x) = f(x, y)$$
$${}^{c}D^{\alpha}f_2(x) = g(x, y) \qquad (4)$$

Let ${}^{c}D^{\alpha}f_1(x) = 0$, ${}^{c}D^{\alpha}f_2(x) = 0$, can get the equilibrium point $(\bar{x}, \bar{y})$. Using their corresponding Jacobian matrix, we can judge whether the equilibrium point is stable. The Jacobian matrix is shown below:

$$\begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{pmatrix}$$

When the eigenvalues of the matrix satisfy $|\arg(\lambda)| > \frac{\alpha\pi}{2}$ or all eigenvalues have negative real parts, the equilibrium point has asymptotic stability. As shown in Fig. 1, in the

Table 1 The meaning of parameter

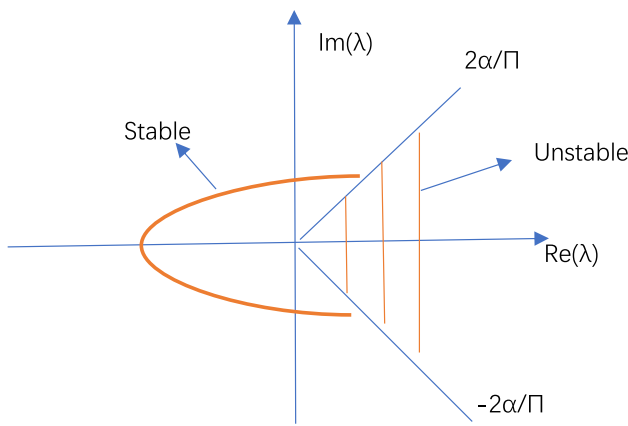| Parameter	Meaning of parameter
μ	The probability of increase in netizen (the online rate)
ν	The probability of a decrease in netizens (the offline rate)
β	The probability that an ignorant person chooses to spread a rumor after contact with a rumor spreader
γ	The probability that the rumor spreader will no longer spread the rumor

Fig. 1 Stable area of equilibrium point

Fig. 2 The relationship between R_0, β, and γ

complex plane, the yellow straight line is the unstable area, and the yellow curve is the stable area.

Since the first two equations have nothing to do with R, so when the current two parameters are determined, R is also determined. Let

$$
{}_{t_0}^{c}D_t^{\alpha}I(t) = 0 \\
{}_{t_0}^{c}D_t^{\alpha}S(t) = 0
\tag{5}
$$

We can figure out the equilibrium point $\left(\frac{\mu N}{v}, 0\right)$ and $(\bar{I}, \bar{S})$. Equation (6) shows the specific value.

$$
\bar{I} = \frac{N(\gamma + v)}{\beta}, \quad \bar{S} = \frac{\mu N \beta - v N(v + \gamma)}{\beta(v + \gamma)}
\tag{6}
$$

When the equilibrium point is $\left(\frac{\mu N}{v}, 0\right)$, the basic regeneration number R_0 of the model can be obtained from the regeneration matrix. According to the differential equation, we can get it as follows:

$$
F = \frac{\beta I}{N}, \quad V = \gamma + v
\tag{7}
$$

R_0 of the equations is the maximum eigenvalue of FV^{-1} that is shown in Eq. (8).

$$
\rho(FV^{-1}) = \frac{\beta I}{N(\gamma + v)}
\tag{8}
$$

Bring in the rumor-free equilibrium point and the basic regeneration number R_0 is $\frac{\beta \mu}{v(\gamma + v)}$. R_0 means the number of people a rumor disseminator spreads to before he decides not to spread a rumor. The larger the R_0, the more difficult it is to stop the spread of rumors. It can be seen that the basic regeneration number is jointly determined by β, γ, μ, v and has a positive correlation with the value of β, μ and negative correlation with the value of γ, v. Because the online time of netizens is relatively stable,

they will not suddenly go online and offline on a large scale under normal circumstances, so the values of online rate μ and offline rate v are relatively stable and have little impact on the basic regeneration number. So this article only studies the impact of β, γ on R_0. The relationship between them is shown in Fig. 2.

As shown in Fig. 2, the basic regeneration number also increases with the increase in β, and the basic regeneration number decreases with the increase in γ, which is consistent with the theoretical analysis.

The Jacobian matrix of the no-rumor equilibrium point $\left(\frac{\mu N}{v}, 0\right)$ is shown below:

$$
\begin{pmatrix}
-v & \frac{-\beta \mu}{v} \\
0 & \frac{\beta \mu}{v} - \gamma - v
\end{pmatrix}
$$

We can get the eigenvalue $\lambda_1 = -v$, $\lambda_2 = \frac{\beta \mu}{v} - \gamma - v$. Obviously, $\lambda_1 < 0$. When $R_0 < 1$, eigenvalue $\lambda_2 < 0$. The equilibrium point has asymptotic stability. This is consistent with the conclusions obtained in integer-order differential equations. The equilibrium point $\left(\frac{\mu N}{v}, 0\right)$ is called the equilibrium point without rumors, that is, the number of people spreading rumors is zero. In real life, this situation generally does not exist, and rumors cannot disappear completely. It is believed that the number of rumors will tend to be stable and will not have a great impact on others and society in the end.

The Jacobian matrix of the equilibrium point $(\bar{I}, \bar{S})$ is given below:

$$
\begin{pmatrix}
\frac{-\mu \beta}{v + \gamma} & -\gamma - v \\
\frac{\mu \beta - v(v + \gamma)}{\gamma + v} & 0
\end{pmatrix}
$$

The eigenvalue is shown as follows:

$$\lambda_1 = \frac{-\mu\beta - \sqrt{\mu^2\beta^2 - 4[\mu\beta - \nu(\nu+\gamma)](\nu+\gamma)^2}}{2(\nu+\gamma)}$$

$$\lambda_2 = \frac{-\mu\beta + \sqrt{\mu^2\beta^2 - 4[\mu\beta - \nu(\nu+\gamma)](\nu+\gamma)^2}}{2(\nu+\gamma)}$$

According to the stability condition of the equilibrium point of the fractional-order differential equation, when $\left|\arg(\lambda_1)\right| > \frac{\alpha\pi}{2}, \left|\arg(\lambda_2)\right| > \frac{\alpha\pi}{2}$, that is,

$$\left|\arg\left(\frac{-\mu\beta + \sqrt{\mu^2\beta^2 - 4[\mu\beta - \nu(\nu+\gamma)](\nu+\gamma)^2}}{2(\nu+\gamma)}\right)\right| > \frac{\alpha\pi}{2} \tag{9}$$

$$\left|\arg\left(\frac{-\mu\beta - \sqrt{\mu^2\beta^2 - 4[\mu\beta - \nu(\nu+\gamma)](\nu+\gamma)^2}}{2(\nu+\gamma)}\right)\right| > \frac{\alpha\pi}{2} \tag{10}$$

the equilibrium point $(\bar{I}, \bar{S})$ has asymptotic stability.

4 Numerical simulation

There are not as many methods to solve nonlinear fractional ordinary differential equations as integer-order equations. Most scholars use the predictive correction algorithm based on Adams–Bashforth–Moulton (Diethelm 2010). However, the efficiency and scope of application of this method are limited. This paper uses the high-efficiency and high-precision numerical solution algorithm proposed by Xue (Xue 2018) to study Caputo-type fractional-order differential equations. We assume that the number of netizens $N = 1000$, of which 980 do not know the rumors at the initial moment, and 15 know and spread the rumors, that is, $I(0) = 980, S(0) = 15$. Some people know but do not believe the rumors, so they will not spread, $R(0) = 5$. The spread probability of rumors is 0.6, $\beta = 0.6$. Without considering the rumor refutation by the official and authoritative institutions, people always tend to believe the first thing they come into contact with. In the short term, the probability that the rumor disseminator will no longer spread the rumor is lower than the probability of spreading, $\gamma = 0.3$. The probability of Internet users suddenly going online and offline is relatively low. In this paper, it is considered that the number of Internet users is constant, that is, the online rate is equal to the offline rate, $\mu = \nu = 0.001$. According to the setting of parameter value in this paper, $R_0 = \frac{\beta\mu}{\nu(\gamma+\nu)} = \frac{0.6}{0.301} > 1$.

Figure 3 shows the relationship between the three types of people in the fractional-order rumor propagation model when $R_0 > 1$. It can be seen intuitively from the figure that among the rumor spreader, the ignorant, and the stifler, there is not a simple linear relationship between the two. As the rumors spread, the number of ignorant people will decrease,
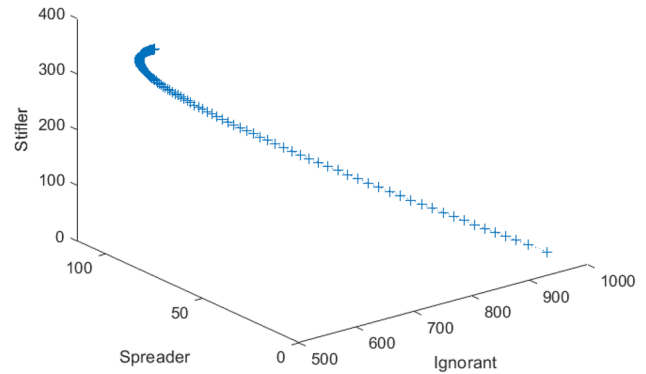

Fig. 3 The changing relationship between the three groups of people
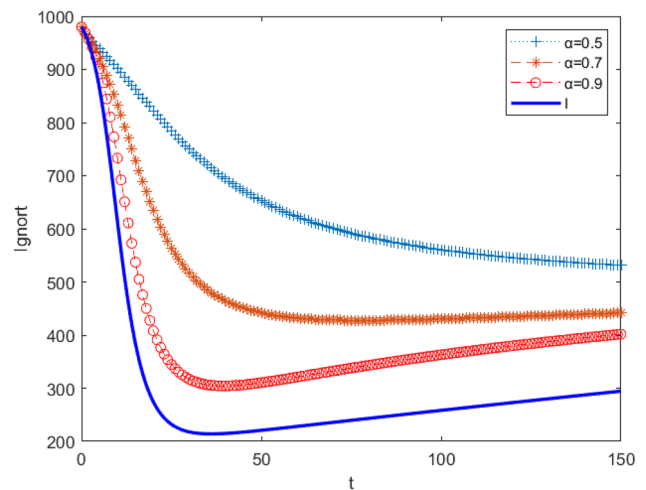

Fig. 4 The change of ignorant over time when $R_0 > 1$ (the blue solid line represents the change of I in the SIR model, and the others are the changes of I under different orders) (Color figure online)

and the number of spreaders and stiflers will increase in a short period of time, but the number of immunized people will increase faster. After a certain period of time, the number of rumors spreaders will also decrease. It will not have a big impact on society and others, and the rumors will eventually fade away.

4.1 Comparison with SIR model

Figure 4 shows the changes in the number of ignorants in the fractional-order rumor propagation model under different orders and the SIR rumor propagation model. It can be seen from the figure that in the initial stage of the spread of rumors, the larger the order, the faster the number of ignorants will decrease. There are great differences in the change trend of the number of ignorants in different orders. When $\alpha = 0.5$ and $\alpha = 0.7$, the number of ignorants decreases with the increase in time and finally tends to be stable. However,

when $\alpha = 0.9$, the number of ignorants first decreases and then increases, which is similar to the trend of ignorants in SIR model. This may be due to the existence of "forgetting" mechanism. With the growth of time, people forget rumors. This is also the memory effect at work. The greater the order, the weaker the memory effect, so the faster the forgetting. But in real life, this kind of thing will not happen in a short time. Even if it happens, the number of people who forget is very small and will not form an increasing trend.

As can be seen from Fig. 5, the number of rumors spread first increases and then decreases with the growth of time, which is in line with the actual trend of rumors spread in real life. The change trend of rumor disseminators is slightly different under different orders. Specifically, the larger the order, the earlier the number of rumor disseminators reaches the peak, the faster the rumor subsides, and the fewer rumor disseminators in the end. It is obvious from the figure that the peak of $\alpha = 0.9$ is higher than $\alpha = 0.7$ and they are higher than $\alpha = 0.5$. This means that the higher the order, the weaker the memory effect, and the rapid spread of rumors at the same time the disappearance quickly, which verifies the theoretical analysis of the memory effect in Sect. 2.

From the analysis in Sect. 3, we can see that when $R_0 > 1$, there is no rumor-free equilibrium, that is, the number of people spreading rumors will not be zero. The blue dotted line in Fig. 5 represents the change in the number of spreaders in the SIR rumor propagation model. It can be seen that when the time is greater than a certain value, the number of spreaders of rumors is very close to 0, which is rarely seen in real life.

Figure 6 describes the change trend of stifler over time in the fractional-order rumor propagation model and SIR rumor propagation model. It can be clearly seen from
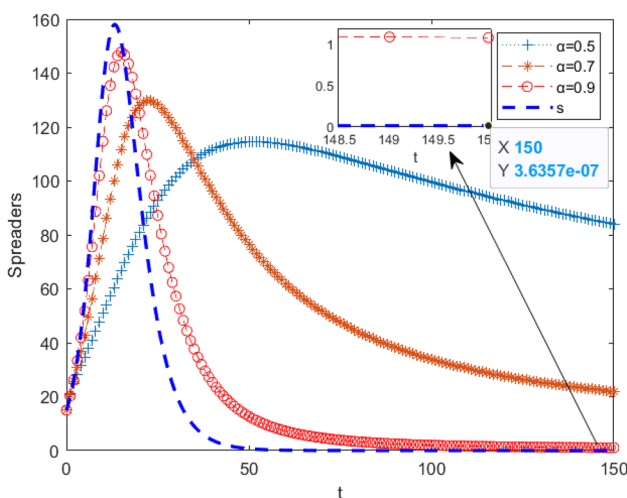

Fig. 6 The changes of stifler over time when $R_0 > 1$ (the black solid line represents the change of R in the SIR model, and the others are the changes of R under different orders) (Color figure online)

Fig. 6 that in the early stage of rumor spread, the number of stifler increased rapidly, and the larger the order, the faster the growth, and the earlier it reached the peak. When $\alpha = 0.5$ and $\alpha = 0.7$, the number of stifler increased steadily. When $\alpha = 0.9$, the number of stifler began to decrease after reaching the peak, which was similar to the change trend of stifler in SIR model.

This may be due to the forgetting mechanism, which makes the stifler believe the rumor again. This situation can be explained by the memory effect. When the order is too large, the memory effect is too weak, resulting in the reduction of the number of stifler. In real life, the memory of netizens will not disappear suddenly in a short time. The change trend of stifler over time should be similar to $\alpha = 0.5$ or $\alpha = 0.7$.

The changes of various populations when $R_0 > 1$ were analyzed, and the situation of $R_0 < 1$ will be discussed below. We suppose that $\beta = 0.5$, $\gamma = 0.6$, $\mu = \nu = 0.001$. So, $R_0 = \frac{\beta\mu}{\nu(\gamma+\nu)} = \frac{0.5}{0.601} < 1$.

An interesting phenomenon is shown in Fig. 7. When $R_0 < 1$, after a period of decline, the number of ignorant in integer-order differential equations will exceed the number of ignorant in partial fractional-order equations. At the same time, it is obvious that in the spread process of rumor, the number of ignorant people is significantly greater than $R_0 > 1$, indicating that the scope of rumor spreading is small and the influence is relatively limited.

As shown in Fig. 8, in the integer-order rumor propagation model, from the definition of R_0, we can see that when $R_0 < 1$, the number of rumors spreader will continue to decrease and eventually become zero. In the fractional-order rumor propagation model, there is a similar

Fig. 5 The change of rumor spreader over time when $R_0 > 1$ (the blue dotted line represents the change of S in the SIR model, and the others are the changes of S under different orders) (Color figure online)
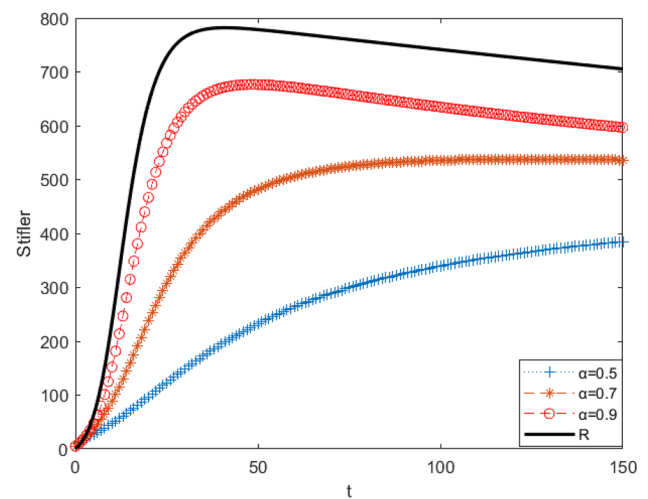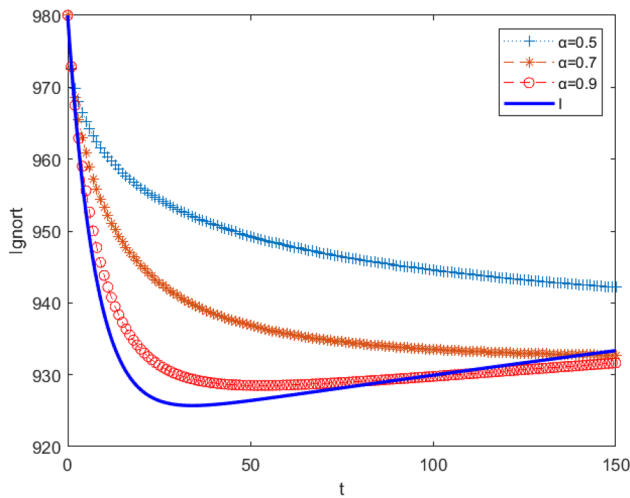
Fig. 7 The change of ignorant over time when $R_0 < 1$ (the blue solid line represents the change of I in the SIR model, and the others are the changes of I under different orders) (Color figure online)
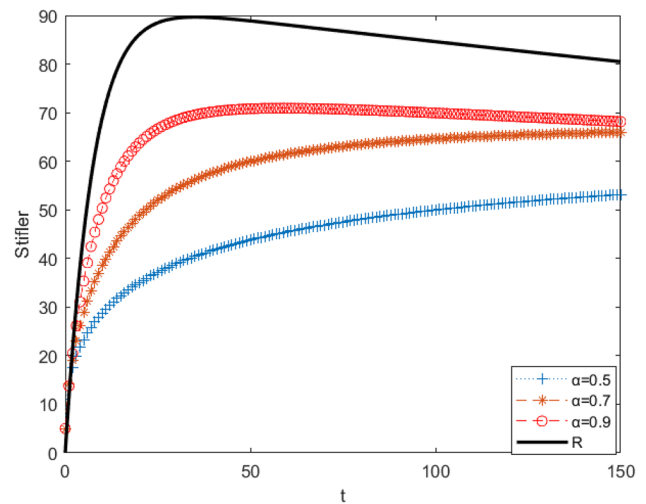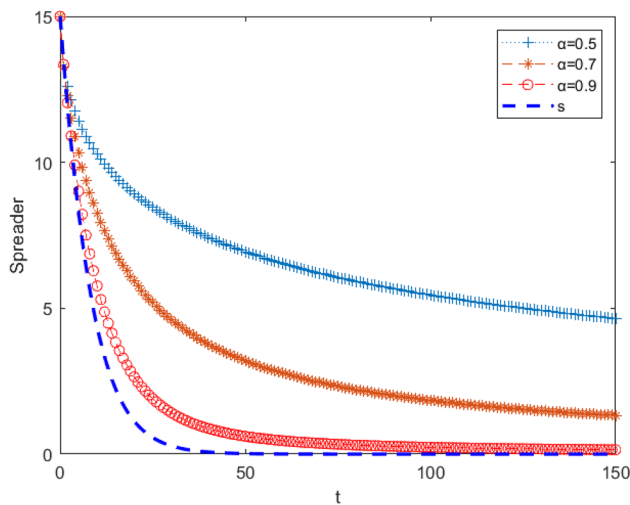

Fig. 9 The change of stifler over time when $R_0 < 1$ (the black solid line represents the change of R in the SIR model, and the others are the changes of R under different orders) (Color figure online)

Fig. 8 The change of rumor spreader over time when $R_0 < 1$ (the blue dotted line represents the change of S in the SIR model, and the others are the changes of S under different orders) (Color figure online)

phenomenon. It is just that under different orders, the speed of the decrease in the number of rumors spreader is different. Specifically, the higher the order, the faster the decrease.

It can be seen from Fig. 9 that the number of stifler under different orders is also increasing. The reason for the decrease in the number of stifler at higher orders is the same as that when $R_0 > 1$. The reason for the small number of people who are immune to rumors is that the number of rumors spreaders has been declining, causing most people to be ignorant of the rumors.

It can be seen from the numerical simulation results that the memory effect plays a vital role in the spread of rumors. Fractional-order differential equations can control the strength of the memory effect through the order. The lower the order, the stronger the memory effect, and the higher the order, the weaker the memory effect, but the first-order differential equation does not have the memory effect. Fractional-order differential equations that can reflect the memory effect are more suitable to study the spread of rumors.

4.2 Compare to real data

We use real data sets to verify the validity of the model. The data come from Bodaghi (2019). They collected 12 latest rumors on Twitter and recorded the rumor tweets sent at each time point. We select "dataset_R2." During the spread of rumors, 497 users sent tweets related to rumors.

In Fig. 10, the abscissa is the time, in hours, and the ordinate is the number. Here, it refers to the person who sends rumor-related tweets. If the user browses the rumors and chooses to believe them, but does not send related tweets, they will not be recorded. It can be seen that the trend of the number of rumor disseminators is similar to that of the model, which shows that the model can well fit the real spread of rumor and verify the effectiveness of the model.

5 Conclusion

Due to the existence of the memory effect, individuals have different judgments on information, and rumors have different effects on different individuals. The memory effect has
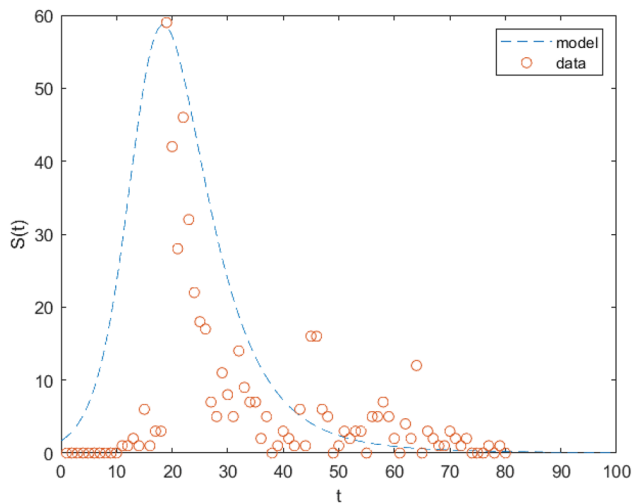
Fig. 10 The fitting results of model and data

an important influence on the process of rumor spreading, so the fractional-order rumor spreading model that considers the memory effect is reasonable.

From the simulation results, the higher the order, the weaker the memory effect, the more people who are exposed to the rumors, and the faster the rumors reach their peak. Although there are many people who are ultimately immune to rumors, they have been affected by the rumors. Therefore, in order to control rumors, it is necessary to strengthen the memory effect, which means that more knowledge should be popularized in daily life and the education level of citizens should be improved. In the face of rumors with common sense errors such as iodized salt can prevent radiation, we can make accurate judgments on information and will not make wrong behaviors. At the lower order, the memory effect is stronger and the spread of rumors is slower. The number of people who know rumors is relatively small, so there are few people who are immune to rumors, but rumors last for a relatively long time. This requires official departments and individuals and organizations with public influence to refute rumors in time to minimize the harm of rumors.

When $R_0 < 1$, it can be seen from the simulation results that the scale and influence of rumors are very small. The value of R_0 is positively correlated with β, and negatively correlated with γ. To decrease R_0, we should decrease β and increase γ. It is necessary to popularize science at ordinary times and refute rumors in time. Rumor is a problem that requires the cooperation of individuals, the government, and society to solve it. Rumors that are not influential are basically harmless to individuals and society.

In this study, a fractional-order rumor propagation model considering memory effect is proposed, which not only provides a new research method of rumor propagation, but also expands the application field of fractional-order calculus. However, dividing the crowd into three categories cannot fully simulate the real situation of rumor spread. We will make up for this shortcoming in the future work. As we all know, there is a similar definition between fractal dimension and fractional order. In our next work, we will explore the relationship between fractal dimension and fractional order of rumor propagation.

Declarations

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