Necessity of Social Distancing in Pandemic Control: A Dynamic Game Theory Approach

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Abstract

We model a society with two types of citizens: healthy and vulnerable individuals. While both types can be exposed to the virus and contribute to its spread, the vulnerable people tend to be more cautious as being exposed to the virus can be fatal for them due to their conditions, e.g., advanced age or prior medical conditions. We assume that both types would like to participate in in-person social activities as freely as possible and they make this decision based on the total number of infected people in the society. In this model, we assume that a local governmental authority imposes and administers social distancing regulations based on the infection status of the society and revises it accordingly in each time period. We model and solve for the steady state in four scenarios: (i) non-cooperative (Nash), (ii) cooperative, (iii) egoistic, and (iv) altruistic. The results show that the Altruistic scenario is the best among the four, i.e., the healthy citizens put the vulnerable citizens' needs first and self-isolate more strictly which results in more flexibility for the vulnerable citizens. We use a numerical example to illustrate that the Altruistic scenario will assist with pandemic control for both healthy and vulnerable citizens in the long run. The objective of this research is not to find a way to resolve the pandemic but to optimally live in a society which has been impacted by pandemic restrictions, similar to what was experienced in 2020 with the spread of COVID-19.

Keywords Dynamic game theory · Pandemic control · Epidemic · Social distancing · Public policy

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1 Introduction

The COVID-19 pandemic, officially declared by the World Health Organization¹ on March 11, 2020, has brought attention to setting the proper yet sustainable regulations in place to control the outbreak. Experts argue that the last pandemic of similar proportion was the so-called Spanish-flu that emerged in 1918. The world was at war and intercontinental traveling was rare. According to the published statistics², as of August 14, 2020, COVID-19 had spread to six continents, with over 750,000 succumbing to the coronavirus that causes the COVID-19 disease. Just over 2 months after, as of October 27, 2020, there were over one million documented deaths globally due to COVID-19³.

Caparrós & Finus [5] claimed that in today's highly connected world, cooperation, and not only coordination, is needed to address such dilemma as an outbreak anywhere in the world can put all other countries at risk. Thus, one region's controlling regulations can impact other regions. Also, in the case of a pandemic, we may not have the luxury of experimenting various strategies to find the right one. Hence, following the right precautions strategies such as social distancing or limiting in person interactions seems to be the logical first step to limit the outbreak. See Reluga [18], Kelso et al. [14], and Caley et al. [4]. That being said, the negative mental and physical impacts of isolation cannot be ignored. A recent empirical study [9] has shown that on average about 10% of the sample (9,565 people from 78 countries) was languishing from low levels of mental health and about 50% had only moderate mental health. Thus, this article aims to find the best strategy that individuals in the society decide to adopt considering the existing lockdown and social distancing regulations set by the authorities, e.g., local government. Individuals in the population affected by epidemics decide independently whether to follow a social regulation or not. In order to estimate the effects of these individual decisions on the overall epidemic spread, we introduce the well-being as a function of this decision.

In this model, we assume two groups of people: healthy and vulnerable. An individual in each group can be infected or susceptible. If an individual from the healthy group is exposed to the virus, the impacts are minor and there is a high chance of full recovery with minimal care. On the contrary, if an individual from the vulnerable group is exposed to the virus, their daily life will be significantly impacted, and the patient is at risk of hospitalization and not fully recovering. The key reasons for an individual to be considered vulnerable are biological factors such as advanced age or pre-existing medical conditions, i.e., chronic diseases.

Over the past century, the use of mathematical biology has been widely adopted by many researchers to assess the spread of disease within societies. Kermack & McKendrick (1929) are recognized as the first to have modeled pandemics using differential equations in their classical contribution known as the susceptible–infected–recovered (SIR) model. Since then, most epidemic models have been developed around a segmentation of the population into smaller groups, in which all individuals share a common characteristic with respect to the disease. Examples include the SEIR model, e.g., Grimm et al. (2021), Berger et al. [2], where E stands for exposed, and the SIS model where there is no certain and permanent treatment for the disease and individuals become susceptible again once cured, Foley et al. [8] and Hethcote [11]. The simplest model that has been considered in the literature is the SI model where the authors do not take into account the recovered individuals, e.g., Hilker et al. [12]

¹ WHO Director-General's Opening Remarks at the Media Briefing on COVID-19, Accessed March 2021

² Number of Novel Coronavirus (COVID-19) Deaths Worldwide by Country, Accessed March 2021

³ COVID-19 Dashboard by the Center for Systems Science and Engineering (CSSE) at Johns Hopkins University (JHU), Accessed March 2021

and Kosmidis & Macheras [16]. For a complete review of the use of mathematics in biology, we refer the reader to the following references: Allen et al. [1], Sietto & Russo [19], and Keeling & Rohani [13]. In fact, the simplicity of the SI model renders it appropriate for studies that do not focus on the extent of disease spread but rather on the regulations that lead to slowing its propagation, or when the study describes a society that is facing some sort of epidemic for the first time and is therefore looking for a way to control the situation in the pre-vaccine period as shown in Demongeot et al. [7].

Furthermore, like many other fields of study where strategic interaction between agents is involved, epidemiology has attracted a large number of game theorists owing to the dynamic structure of models characterized by state variables such as the number of individuals in each group, and by the optimal strategies of each individual, such as whether or not to get vaccinated, Bhattacharyya & Bauch [3]. In this article, the individuals' choice is to isolate oneself or not, while the state variable describes the government regulation that allows more open access (capacity) to public areas including restaurants, cafes, grocery stores, shopping malls, etc. Almost everyone who experienced COVID-19 outbreak faced such a trade-off and decision to make, at least during the first few weeks of its onset. Chang et al. [6] reviewed the literature and classified studies applying game theory in epidemiology. Their main findings suggest that classical compartmental modeling has recently left the scene for network-based modeling with imitation games. For example, Taynitskiy et al. [20] considered in a SWIRS model (W for warned) a combination of two epidemic models where the strategies consist of dissemination of information among healthy nodes and treatment of infected nodes.

At a high level, this research aims to find a behavioral strategy for the citizens to optimally live in a society which has been impacted by pandemic restrictions, similar to what was experienced with the spread of COVID-19 in 2020. To this end, we investigate various behaviors of the people in the society and compare their impacts to overall infection rate and other key indicators. The key research question we are trying to answer is as follows: given an unexpected novel virus outbreak and a benevolent local governmental authority who aims to control the spread and the epidemic, how should the citizens behave to help with epidemic control? While the citizens can be ignorant, self-indulgent, or sympathetic toward others, we try to find the best strategy that results in faster resolution of unexpected and undesirable situations.

This paper is organized as follows. We introduce the model, variables, dynamics, and key assumptions in Sect. 2, followed by various analytical solutions in Sect. 3. We emphasize the results through a simple, yet realistic, numerical example in Sect. 4. Final thoughts and avenues for extensions are provided in Sect. 5 followed by an appendix which includes the detailed proofs of propositions in the study.

2 Model

In this model, we consider a discrete infinite time horizon, and n being the total number of people in the region under study consisting of n_h healthy individuals and n_v vulnerable individuals:

$$n = n_h + n_v; \quad n_l \geq 2 \quad l \in \{h, v\} \quad (1)$$

We define a state variable $x(t)$ set by the local government which dictates the lockdown regulation, as a ratio for the capacity in public areas. $x(t)$ at each point in time has a value between 0 and 1 where $x(t) = 1$ implies no restricting regulation, i.e., all the available public

space can be occupied by the citizens, for example a restaurant with 100 people capacity is allowed to have 100 guests while when $x(t) = 0.5$, the restaurant is allowed to have only 50 guests. Let us denote the decision of an individual in each group, i.e., healthy or vulnerable, by $q_m(t)$ where $m \in \{i, j\}$. For simplicity purposes, we assume $q_i(t)$ is for the v -group (vulnerable population) and $q_j(t)$ for the h -group (healthy population). If an individual decides to be in the public area, they occupy $q_m(t)$ portion of the available capacity at t , defined by the local government, i.e., $x(t)$.

At the beginning of each period, the local government can change the limitation for capacity, $x(t)$. The new capacity is a function of capacity in the previous period and the decisions made by the two groups of citizens. So, we define the state variable dynamics as follows:

$$x_{t+1} = \left(x_t - \sum_{i=1}^{n_v} q_{it} - \sum_{j=1}^{n_h} q_{jt} \right)^\alpha, \quad 0 \leq \alpha \leq 1 \quad (2)$$

where $x(0) = x_0$ is given and α is the rate by which the government changes the capacity ratio from one period to another. This coefficient is time invariant and independent of the state of the game by which the saturation level of the space occupation is normalized to one. Moreover, the smaller it is, the faster the government responds to the spread, i.e., higher restrictions will be in place. When α approaches 1, the government has a slow response and almost no change in socialization restrictions occurs between the periods. It is easy to prove that $x_t = x_{t+1} = 1$, is a stable steady state. For instance, in a recent study, Macdonald et al. [17] argue for a unique model for each state in the United States in their responsiveness to isolation. This pattern is driven by the strong correlation between the magnitude of the epidemic (as a function of the number of the realized tests) and isolation.

Following a similar logic as the common epidemic models, e.g., SI, SIS, SIR, etc., let us assume that regardless of the group they belong to, people monitor the number of infected people at each period, denoted by $I(t)$ and they meet and make contacts enough to result in the spread of the virus at random with a per individual rate $\beta(t)$ at time t . In other words, $\beta(t)$ is the expected number of contacts for an individual (from h or v groups) at each period and we define it as

$$\beta(t) = n_v b_v \sum_{i=1}^{n_v} q_i(t) + n_h b_h \sum_{j=1}^{n_h} q_j(t); \quad t \in [0, \infty). \quad (3)$$

In order to motivate the adoption of the above expression, some assumptions are necessary:

1. $\beta \in [0, n]$ as the maximum number of people each player can meet is the total number of players;
2. $\beta(t) = 0$ when $q_i = q_j = 0$ since there is no meetups when everyone stays at home;
3. $\frac{\partial \beta}{\partial q_m} > 0, \forall m = \{i, j\}$ as the number of people in the street is higher when players decide to leave their homes compared to when they stay more at home;
4. b_v and b_h are positive coefficients having the following relationship $b_v < b_h$.

With the inclusion of b_v and b_h coefficients, a weight is assigned to each group of players describing their social behavior. These coefficients imply that healthy players tend to socialize more freely as they are not concerned about exposure to the virus. On the other hand, vulnerable players are more likely to limit their socialization to essential activities with minimal interactions with others due to their conditions and their potential costly response to exposure, i.e., not fully recovering or dead.

We use $\kappa(t)$ to show the average probability of a person that anyone meets at random being infected which is $\frac{I(t)}{n}$. So, anyone has contact with an average of $\beta(t) \frac{I(t)}{n} = \beta(t)\kappa(t)$ infected people per unit of time.

$$I(t+1) = I(t)\beta(t) \left(1 - \frac{I(t)}{n}\right) \quad (4)$$

which can be written as:

$$\kappa(t+1) = \kappa(t)\beta(t)(1 - \kappa(t)) \quad (5)$$

and gives us:

$$\kappa(t) = \frac{\kappa_0 e^{\beta(t)t}}{1 - \kappa_0 + \kappa_0 e^{\beta(t)t}}. \quad (6)$$

As described, while everyone in the society regardless of their type, i.e., healthy or vulnerable, benefit from in-person socializing, the vulnerable population is more anxious as their exposure to the virus will be very costly (or even tragic) for them. We assume that for a healthy individual, this is not as concerning as they are confident about their full recovery in case of exposure. Hence, we define the dis-utility function for vulnerable individual m which endures from others being outside, as follows:

$$\Theta_m(q_i, q_j) = a \sum_{i \neq m}^{n_v} q_i(t)^{-\theta_v} \sum_{j=1}^{n_h} q_j(t)^{-\theta_h} \quad (7)$$

where $\theta_l \in (0, 1)$; $l = \{h, v\}$ and $a > 0$ are time invariant parameters such that $\theta_v + \theta_h < 1$.

Now, we define the utility of each individual in healthy and vulnerable group with the following functions:

$$\begin{cases} U_i(t) = \ln(q_i(t)\Theta_i(q_i, q_j)); & i \in \{1, \dots, n_v\} \\ U_j(t) = \ln(q_j(t)); & j \in \{1, \dots, n_h\}. \end{cases} \quad (8)$$

With the structure defined above, each individual (or player), maximizes the discounted sum of their utility:

$$J_m(t) = \max_{\{q_m(t) > 0\}} \sum_{t=0}^{\infty} \rho^t U_m(t); \quad m \in \{i, j\} \quad (9)$$

subject to the stock dynamics given by equation (2) where ρ is the discount factor.

3 Solution

Prior to presenting the solutions for different scenarios defined based on each mode of play and to spare on notations, we first provide findings that are applicable for the rest of the paper.

Lemma 1 *The strategies for player m , for $m \in \{i, j\}$, consist of a positive share $\gamma_m \in [0, 1]$ of the state variable x , such that*

$$q_m = \gamma_m x, \quad (10)$$

This lemma reiterates the implication of the decision variable, i.e., if an individual decides to be in the public area, they occupy $q_m(t)$ portion of the available capacity ($x(t)$) at t . The following propositions, respectively, provide the analytical expressions of the trajectory and the steady state of the state variable x .

Proposition 1 *The trajectory of the state variable is given by,*

$$x(t) = x_0 e^{\alpha t} (1 - n_v \gamma_i - n_h \gamma_j) \left[\frac{\alpha(\alpha^t - 1)}{\alpha - 1} \right] \quad (11)$$

Proof The trajectories of the state variables are derived from replacing the strategy q_m in Eq. (10) by its value in Eq. (2), then solving for $x(t)$. $\square$

Proposition 2 *The steady-state value of the state variable is given by,*

$$x_\infty = (1 - n_v \gamma_i - n_h \gamma_j) \left[\frac{\alpha}{1 - \alpha} \right], \quad (12)$$

Proof Equation (12) is obtained by replacing q_m by its value from Eq. (10) then solving for x in Eq. (2). $\square$

In the next section, we solve the model in four different scenarios, so that when deciding to self-isolate:

- Non-cooperative: each player considers their own welfare.
- Cooperative: each player considers the welfare of all, including her/his own.
- Egoistic: While vulnerable players consider the benefits of all, healthy players focus on their own benefits.
- Altruistic: While vulnerable players consider the benefits of all, healthy players consider the welfare of all but themselves.

3.1 Non-cooperative Solution

In a non-cooperative setting, each player individually maximizes the sum of his discounted utility given in Eq. (9). Using a dynamic programming approach, the feedback Nash equilibrium strategies are derived by solving the following HJB equations, for $m \in \{i, j\}$,

$$V_m(x) = \max_{q_m \geq 0} \left(U_m + \rho V_m \left(\left(x - \sum_{i=1}^{n_v} q_i - \sum_{j=1}^{n_h} q_j \right)^\alpha \right) \right), \quad (13)$$

$$\begin{cases} U_i(t) = \ln(q_{iv}(t) \Theta_i(q_{iv}, q_{jh})) \\ U_j(t) = \ln(q_{jh}(t)). \end{cases} \quad (14)$$

Proposition 3 *Assuming an interior solution, the unique feedback Nash equilibrium strategies are given by $\{\gamma_i^N, \gamma_j^N\}$, and the value functions $\{V_i^N(x), V_j^N(x)\}$ are such that,*

$$\begin{cases} \gamma_i^N = \frac{(1-\theta_v)}{(\rho \alpha A_i^N + (1-\theta_v)n_v)} - \frac{\rho \alpha A_i^N n_h (1-\theta_v)}{(\rho \alpha A_j^N + n_h)(\rho \alpha A_i^N + (1-\theta_v)n_v) - n_v n_h (1-\theta_v)} \\ \gamma_j^N = \frac{\rho \alpha A_i^N}{(\rho \alpha A_j^N + n_h)(\rho \alpha A_i^N + (1-\theta_v)n_v) - n_v n_h (1-\theta_v)} \end{cases} \quad (15)$$

$$\begin{cases} V_i^N(x) = A_i^N \ln x + B_i^N \\ V_j^N(x) = A_j^N \ln x + B_j^N \end{cases} \quad (16)$$

where

$$\begin{aligned}
 A_i^N &= \frac{1 - \theta_v - \theta_h}{1 - \rho\alpha} \\
 B_i^N &= \frac{\rho\alpha (1 - \theta_v - \theta_h) \ln \left(1 - n_v \gamma_i^N - n_h \gamma_j^N \right)}{(1 - \rho\alpha)(1 - \rho)} \\
 &\quad + \frac{(1 - \theta_v) \ln \left(\gamma_i^N \right) - \theta_h \ln \left(\gamma_j^N \right) + \ln (n_v - 1) + \ln (n_h) + \ln (a)}{(1 - \rho)} \\
 A_j^N &= \frac{1}{1 - \rho\alpha} \\
 B_j^N &= \frac{\rho\alpha \ln (1 - n_v \gamma_i^N - n_h \gamma_j^N) + (1 - \rho\alpha) \ln (\gamma_j^N)}{(1 - \rho\alpha)(1 - \rho)}.
 \end{aligned}$$

The associated infection rate is

$$\kappa^N(t) = \frac{\kappa_0 e^{\beta(q_i^N, q_j^N)t}}{1 - \kappa_0 + \kappa_0 e^{\beta(q_i^N, q_j^N)t}}.$$

Proof See the appendix. $\square$

3.2 Cooperative, i.e., Social Optimum Solution

In a cooperative setting, players are jointly maximizing the sum of their discounted utility given in equation (9). Using a dynamic programming approach, the cooperative equilibrium strategies are derived by solving the following HJB equation,

$$V(x) = \max_{(q_i, q_j) \geq 0} \left(\sum_{i=1}^{n_v} U_i + \sum_{j=1}^{n_h} U_j + \rho V \left(\left(x - \sum_{i=1}^{n_v} q_i - \sum_{j=1}^{n_h} q_j \right)^\alpha \right) \right). \quad (17)$$

Proposition 4 Assuming an interior solution, the unique cooperative equilibrium strategies are given by $\{\gamma_i^C, \gamma_j^C\}$, and the value function $V^C(x)$ are such that,

$$\begin{cases} \gamma_i^C = \left(\frac{n_v(1-\theta_v)}{\rho\alpha A^C + n_v^2(1-\theta_v)} \right) \left(1 - \frac{(n_h - n_v\theta_h)\rho\alpha A^C}{(\rho\alpha A^C + n_v^2(1-\theta_v))(\rho\alpha A^C + n_h(n_h - n_v\theta_h)) - n_v^2 n_h(1-\theta_v)(n_h - n_v\theta_h)} \right) \\ \gamma_j^C = \frac{(n_h - n_v\theta_h)\rho\alpha A^C}{(\rho\alpha A^C + n_v^2(1-\theta_v))(\rho\alpha A^C + n_h(n_h - n_v\theta_h)) - n_v^2 n_h(1-\theta_v)(n_h - n_v\theta_h)} \end{cases} \quad (18)$$

$$V^C(x) = A^C \ln x + B^C \quad (19)$$

where

$$A^C = \frac{n_v(1 - \theta_v - \theta_h) + n_h}{1 - \rho\alpha}$$

and

$$\begin{aligned}
 B^C &= \frac{\rho\alpha (n_v (1 - \theta_v - \theta_h) + n_h) \ln \left(1 - n_v \gamma_i^C - n_h \gamma_j^C \right)}{(1 - \rho\alpha)(1 - \rho)} \\
 &\quad + \frac{n_v (1 - \theta_v) \ln \left(\gamma_i^C \right) - (n_h - n_v\theta_h) \ln \left(\gamma_j^C \right) + n_v \ln (n_v - 1) + n_v \ln (n_h) + n_v \ln (a)}{(1 - \rho)}.
 \end{aligned}$$

The associated infection rate is

$$\kappa^C(t) = \frac{\kappa_0 e^{\beta(q_i^C, q_j^C)t}}{1 - \kappa_0 + \kappa_0 e^{\beta(q_i^C, q_j^C)t}}.$$

Proof See the appendix. $\square$

3.3 Egoistic Solution

In an egoistic setting, knowing that v -type players will always cooperate, each h -type player individually maximizes the sum of his own discounted utility given in equation (9). Using a dynamic programming approach, the egoistic strategies are derived by solving the following HJB equations,

$$\begin{cases} V_v(x) = \max_{q_i \geq 0} \sum_{i=1}^{n_v} U_i + \rho V_v \left(\left(x - \sum_{i=1}^{n_v} q_i - \sum_{j=1}^{n_h} q_j \right)^\alpha \right) \\ V_j(x) = \max_{q_j \geq 0} \left(U_j + \rho V_j \left(\left(x - \sum_{i=1}^{n_v} q_i - \sum_{j=1}^{n_h} q_j \right)^\alpha \right) \right). \end{cases} \quad (20)$$

Proposition 5 Assuming an interior solution, the unique Egoistic equilibrium strategies are given by $\{\gamma_i^E, \gamma_j^E\}$, and the value functions $\{V_v^E(x), V_j^E(x)\}$ are such that,

$$\begin{cases} \gamma_i^E = \frac{n_v(1-\theta_v-\theta_h)}{\rho\alpha A_v^E + n_v^2(1-\theta_v-\theta_h)} \left[1 - \left(\frac{\rho\alpha A_v^E}{[\rho\alpha A_v^E + n_v^2(1-\theta_v-\theta_h)][\rho\alpha A_j^E + n_h]} - n_v^2 n_h (1-\theta_v-\theta_h) \right) \right] \\ \gamma_j^E = \frac{\rho\alpha A_v^E}{(\rho\alpha A_v^E + n_v^2(1-\theta_v-\theta_h))(\rho\alpha A_j^E + n_h) - n_v^2 n_h (1-\theta_v-\theta_h)} \end{cases} \quad (21)$$

$$\begin{cases} V_v^E(x) = A_v^E \ln x + B_v^E \\ V_j^E(x) = A_j^E \ln x + B_j^E \end{cases} \quad (22)$$

where

$$A_v^E = \frac{n_v(1-\theta_v-\theta_h)}{1-\rho\alpha} \quad (23)$$

$$\begin{aligned} B_v^E = & \frac{\rho\alpha n_v(1-\theta_v-\theta_h) \ln(1 - n_v\gamma_i^E - n_h\gamma_j^E)}{(1-\rho\alpha)(1-\rho)} \\ & + \frac{n_v \left[(1-\theta_v) \ln(\gamma_i^E) - \theta_h \ln(\gamma_j^E) + \ln(n_v-1) + \ln(n_h) + \ln(a) \right]}{(1-\rho)} \end{aligned} \quad (24)$$

$$A_j^E = \frac{1}{1-\rho\alpha} \quad (25)$$

and

$$B_j^E = \frac{\rho\alpha \ln(1 - n_v\gamma_i^E - n_h\gamma_j^E) + (1-\rho\alpha) \ln(\gamma_j^E)}{(1-\rho\alpha)(1-\rho)}. \quad (26)$$

The associated infection rate is

$$\kappa^E(t) = \frac{\kappa_0 e^{\beta(q_i^E, q_j^E)t}}{1 - \kappa_0 + \kappa_0 e^{\beta(q_i^E, q_j^E)t}}. \quad (27)$$

Proof See the appendix. $\square$

3.4 Altruistic Solution

In an altruistic setting, knowing that players will always cooperate, each healthy player will maximize the sum of the discounted utility given in Eq. (9) for all the players, whether vulnerable or healthy, except for himself, i.e., $n_v + n_h - 1$ players. In other words, this scenario implies that the healthy players put the well-being of the vulnerable population ahead of themselves and are willing to stay at home to protect the vulnerable players. To give an example of this scenario, Toxvaerd [21] has shown that before the attainment of herd immunity, susceptible players will engage in socially costly distancing in order to appease the situation by flattening the curve. Using a dynamic programming approach, the altruistic strategies are derived by solving the following HJB equation,

$$V(x) = \max_{(q_i, q_j) \geq 0} \left(\sum_{i=1}^{n_v} U_i + \sum_{j=1}^{n_h-1} U_j + \rho V \left(\left(x - \sum_{i=1}^{n_v} q_i - \sum_{j=1}^{n_h} q_j \right)^\alpha \right) \right). \quad (28)$$

Proposition 6 Assuming an interior solution, the unique Altruistic equilibrium strategies are given by $\{\gamma_i^A, \gamma_j^A\}$, and the value function $\{V^A(x)\}$ are such that,

$$\begin{cases} \gamma_i^A = \left(\frac{n_v(1-\theta_v)}{\rho\alpha A^A + n_v^2(1-\theta_v)} \right) \left(1 - \frac{(n_h-1-n_v\theta_h)\rho\alpha A^A}{(\rho\alpha A^A + n_v^2(1-\theta_v))(\rho\alpha A^A + n_h(n_h-n_v\theta_h)) - n_v^2 n_h(1-\theta_v)(n_h-1-n_v\theta_h)} \right) \\ \gamma_j^A = \left(\frac{n_h-1}{n_h} \right) \left(\frac{(n_h-1-n_v\theta_h)\rho\alpha A^A}{(\rho\alpha A^A + n_v^2(1-\theta_v))(\rho\alpha A^A + n_h(n_h-n_v\theta_h)) - n_v^2 n_h(1-\theta_v)(n_h-1-n_v\theta_h)} \right) \end{cases} \quad (29)$$

$$V^A(x) = A^A \ln x + B^A \quad (30)$$

where

$$A^A = \frac{n_v(1-\theta_v-\theta_h) + n_h - 1}{1 - \rho\alpha} \quad (31)$$

$$\begin{aligned} B^A = & \frac{\rho\alpha(n_v(1-\theta_v-\theta_h) + n_h - 1) \ln(1 - n_v\gamma_i^A - n_h\gamma_j^A)}{(1 - \rho\alpha)(1 - \rho)} \\ & + \frac{n_v(1-\theta_v) \ln(\gamma_i^A) - (n_h - 1 - n_v\theta_h) \ln(\gamma_j^A) + n_v \ln(n_v - 1) + n_v \ln(n_h) + n_v \ln(a)}{(1 - \rho)}. \end{aligned} \quad (32)$$

The associated infection rate is

$$\kappa^A(t) = \frac{\kappa_0 e^{\beta(q_i^A, q_j^A)t}}{1 - \kappa_0 + \kappa_0 e^{\beta(q_i^A, q_j^A)t}}. \quad (33)$$

Proof See the appendix. $\square$

4 Numerical Analysis

Thus far, we have shown our findings in their analytical form. However, the complexity of all the expressions we have derived renders an analytical comparison between the different scenarios far from being obvious. To further illustrate our analytical results, this section presents some tactical insights that can be drawn from the numerical analysis. The twofold results of this model help to derive behavioral patterns in the society by focusing on state and decision variables and also highlight the trend for the key public health indicators, e.g., infection rate. As a baseline scenario, the parameter values are given by

$$n = 4; n_v = 2; n_h = 2; \alpha = 0.9; \rho = 0.95; x_0 = 0.01; \\ a = 6; \theta_v = 0.3; \theta_h = 0.49; \kappa_0 = 0.2; b_v = 0.3; b_h = 2.5.$$

These parameters are selected based on a calibration process that ensures compliance with all the conditions set out in the previous sections of this study. Using this parameter constellation, we run the model for $t \in [0; 50]$ time interval⁴ in order to approximate the following steady-state values:

$$x_{\infty}^A = 0.0671 > x_{\infty}^C = 0.0579 > x_{\infty}^E = 0.0336 > x_{\infty}^N = 0.00036.$$

These numbers show that eventually higher rate of x is set by the local governmental authority in an Altruistic scenario. In other words, in the long run, more public space is available to use for the citizens, whether healthy or vulnerable. This is shown in Fig. 1. In addition, the cooperative approach is considered a second-best option, while the Egoistic scenario is still preferable to the non-cooperative solution. Furthermore, the two groups of players are dissimilar in their preferences for the strategies they adopt. As a result, vulnerable players will prefer the Nash scenario in period 1, but very soon, in period 2, they will be less self-isolating under the Altruistic solution. In the long run, the preferences of vulnerable players are in this sequence, altruistic, cooperative, egoistic, and Nash as can be seen in Fig. 2. As expected, healthy players always choose a higher rate of public space occupation when adopting an egoistic behavior, followed by the cooperative scenario. Despite preferring Nash's mode of play during the first 4 periods, their long-term choice is indeed more altruistic than non-cooperative (Fig. 3).

We now examine the impact of each scenario on the public health risk of infection. We focus on the most relevant indicators, namely, the transmission rate $\beta(t)$, the rate of infection $\kappa(t)$, and the number of active cases $I(t)$. Figure 4 shows the trajectories associated with the different scenarios. We note that the expected number of contacts for an individual is lower when healthy players are altruistic than when they are cooperative or egoistic. Furthermore, we notice that from period 3 onward, the Nash solution is related to the lowest values for $\beta(t)$. A possible explanation is that in this scenario, players' strategies approach zero (i.e., Lockdown) in the long run (Figs. 2 and 3), leading to fewer interactions. Meanwhile, Figs. 5 and 6 show a similar pattern, in accordance with their mathematical relationship given by the following equation $I(t) = n\kappa(t)$. While the egoistic solution shows a lower population infection rate, uncooperative behavior dominates all other scenarios, especially in the long term.

Finally, Table 1 summarizes our results at the steady state by clearly showing that the Altruistic solution is the one to adopt for a successful epidemic control. Indeed, the infection rates will be kept lower while the rate of public space available to all players will be higher.

⁴ The time periods of the simulation are represented by the x-axis in the figures below.

Fig. 1 The level of public space accessibility $x(t)$

Fig. 2 The v -type players' strategies $q_i(t)$

5 Conclusion

In this paper, we modeled a society of rational players as a dynamic game model in Pandemic. The local governmental authority sets the capacity for the public spaces and updates the capacity based on the behavior of the players and infections observed in the previous time period. We assumed that there are two types of individuals in the society, namely, healthy and vulnerable. While the people in both groups can be exposed to the virus and infected, the infection can be fatal for the vulnerable population. As a result, the vulnerable population tracks the status of the society and considers the number of infected people when deciding to whether to be in a public space or not. On the other hand, the healthy population will fully recover with a higher chance, hence, they are less concerned about the number of infected people in the society.

Fig. 3 The h -type players' strategies $q_j(t)$

Fig. 4 The virus transmission rate $\beta(t)$

We solve the model and characterize respective equilibria in various modes of play, i.e., Nash, Cooperative, Egoistic, and Altruistic. Results of our analysis suggest that the last scenario, i.e., Altruistic, is the best for the overall health-being of the society and helps to control the pandemic best. In other words, this scenario implies that the healthy players prioritize the well-being of vulnerable population and are willing to stay at home to protect them. Additionally, this scenario results in higher availability of public spaces and lower rate of infection, compared to the other scenarios studied.

This research is the first attempt to investigate the impact of different behaviors of the people in the society on an ongoing pandemic situation. To do so, some key assumptions were made for mathematical simplification in order to answer the key research questions. Insofar as our results are auspicious, several extensions can be contemplated. To reach a more realistic model, we could first consider uncertain environments, such as a game with an unknown

Fig. 5 The rate of infection within the total population $\kappa(t)$

Fig. 6 The estimated number of infected individuals $I(t)$

Table 1 Summary of results

Variable	Results
x_∞	$A > C > E > N$
$q_{i\infty}$	$A > C > E > N$
$q_{j\infty}$	$E > C > A > N$
β_∞	$E > C > A > N$
κ_∞	$N > E > C > A$
I_∞	$N > E > C > A$

number of players over a random time horizon. Second, a plausible and mathematically feasible setting in which to consider more than one geographic area with multiple variants of the virus, such as countries without border controls or different states/provinces within a country run by decentralized local governments. Third, a complicated but very informative model to explore would be to extend the model to consider other types of players, such as recovered and vaccinated (immune) people in the society as introduced in other well-known epidemiology models in the literature. In short, similar to announcements by various health organizations across the world, this paper emphasizes that a healthy population's response to virus spread in the society is the key in the fight against any pandemic, as witnessed globally during the COVID-19 crisis in the year 2020.

Declarations

Conflict of interest The authors did not receive support from any organization for the submitted work. The authors have no relevant financial or non-financial interests to disclose. The authors have no conflicts of interest to declare that are relevant to the content of this article. All authors certify that they have no affiliations with or involvement in any organization or entity with any financial interest or non-financial interest in the subject matter or materials discussed in this manuscript. The authors have no financial or proprietary interests in any material discussed in this article. This manuscript has no associated data.

6 Appendix

6.1 Proof of Proposition 3

v-type players: The total discounted utility of agent *i* satisfies

$$V_i(x) = \max_{q_i \geq 0} \left(\ln(aq_i \sum_{i=1}^{n_v-1} q_i^{-\theta_v} \sum_{j=1}^{n_h} q_j^{-\theta_h}) + \rho V_i \left(\left(x - \sum_{i=1}^{n_v} q_i - \sum_{j=1}^{n_h} q_j \right)^\alpha \right) \right). \quad (34)$$

We assume the value function to be of the following form,

$$V_i^N(x) = A_i^N \ln x + B_i^N, \quad i = 1, \dots, n_v.$$

The above form yields,

$$\begin{aligned} V_i^N(x) &= (1 - \theta_v) \ln q_i^N - \theta_h \ln q_j^N + \ln a + \ln(n_v - 1) \\ &\quad + \ln(n_h) + \rho \alpha A_i^N \ln \left(x - \sum_{i=1}^{n_v} q_i - \sum_{j=1}^{n_h} q_j \right) + \rho B_i^N. \end{aligned} \quad (35)$$

Assuming symmetric players and replacing q_i^N by their values from Eq. (10), we obtain,

$$\begin{aligned} V_i^N(x) &= \left(1 - \theta_v - \theta_h + \rho \alpha A_i^N \right) \ln(x) + \rho \alpha A_i^N \ln \left(1 - n_v \gamma_i^N - n_h \gamma_j^N \right) \\ &\quad + \rho B_i^N + (1 - \theta_v) \ln \left(\gamma_i^N \right) - \theta_h \ln \left(\gamma_j^N \right) + \ln(n_v - 1) \\ &\quad + \ln(n_h) + \ln(a) \end{aligned}$$

where

$$A_i^N = \frac{1 - \theta_v - \theta_h}{1 - \rho \alpha}$$

and

$$B_i^N = \frac{\rho\alpha(1-\theta_v-\theta_h)\ln(1-n_v\gamma_i^N-n_h\gamma_j^N)}{(1-\rho\alpha)(1-\rho)} + \frac{(1-\theta_v)\ln(\gamma_i^N)-\theta_h\ln(\gamma_j^N)+\ln(n_v-1)+\ln(n_h)+\ln(a)}{(1-\rho)}.$$

Finally, maximizing the right-hand side of Eq. (35) w.r.t q_i gives,

$$\frac{1-\theta_v}{q_i^N} - \frac{\rho\alpha A_i^N}{x-n_vq_i^N-n_hq_j^N} = 0$$

from where we obtain the v -type players best response function,

$$q_i^N(q_j^N) = \frac{(1-\theta_v)(x-n_hq_j^N(q_i^N))}{\rho\alpha A_i^N + (1-\theta_v)n_v}, \quad (36)$$

$$q_i^N(\gamma_j^N) = \frac{(1-\theta_v)(1-n_h\gamma_j^N(\gamma_i^N))}{\rho\alpha A_i^N + (1-\theta_v)n_v}x, \quad (37)$$

$$q_i^N(\gamma_i^N) = \gamma_i^N(\gamma_j^N)x. \quad (38)$$

Similarly, we shall solve for the h -type players.

h-type players: The total discounted utility of agent i satisfies

$$V_j(x) = \max_{q_j \geq 0} \left(\ln q_j + \rho V_j \left(\left(x - \sum_{i=1}^{n_v} q_i - \sum_{j=1}^{n_h} q_j \right)^\alpha \right) \right). \quad (39)$$

We assume the value function to be of the following form,

$$V_j^N(x) = A_j^N \ln x + B_j^N, \quad j = 1, \dots, n_h,$$

The above form yields,

$$V_j^N(x) = \ln q_j^N + \rho\alpha A_j^N \ln \left(x - \sum_{i=1}^{n_v} q_i - \sum_{j=1}^{n_h} q_j \right) + \rho B_j^N \quad (40)$$

Assuming symmetric players and replacing q_i^N by their values from Eq. (10), we obtain,

$$V_j^N(x) = (1 + \rho\alpha A_j^N) \ln(x) + \rho\alpha A_j^N \ln(1 - n_v\gamma_i^N - n_h\gamma_j^N) + \rho B_i^N + \ln(\gamma_j^N)$$

where

$$A_j^N = \frac{1}{1-\rho\alpha}$$

and

$$B_j^N = \frac{\rho\alpha \ln(1 - n_v\gamma_i^N - n_h\gamma_j^N) + (1-\rho\alpha) \ln(\gamma_j^N)}{(1-\rho\alpha)(1-\rho)}.$$

Finally, maximizing the right-hand side of Eq. (40) w.r.t q_j gives,

$$\frac{1}{q_j^N} - \frac{\rho \alpha A_j^N}{x - n_v q_i^N - n_h q_j^N} = 0$$

from where we obtain the h -type players best response function,

$$q_j^N(q_i^N) = \frac{x - n_v q_i^N(q_j^N)}{\rho \alpha A_j^N + n_h}, \quad (41)$$

$$q_j^N(\gamma_i^N) = \frac{1 - n_v \gamma_i^N(\gamma_j^N)}{\rho \alpha A_j^N + n_h} x, \quad (42)$$

$$q_j^N(\gamma_j^N) = \gamma_j^N x. \quad (43)$$

Finally, solving the system of Eqs. (36) and (41),

$$\begin{cases} q_i^N(q_j^N) = \frac{(1-\theta_v)(x - n_h q_j^N(q_i^N))}{\rho \alpha A_i^N + (1-\theta_v)n_v} \\ q_j^N(q_i^N) = \frac{x - n_v q_i^N(q_j^N)}{\rho \alpha A_j^N + n_h} \end{cases}$$

giving,

$$\begin{cases} q_i^N = \left(\frac{(1-\theta_v)}{(\rho \alpha A_i^N + (1-\theta_v)n_v)} - \frac{\rho \alpha A_i^N n_h (1-\theta_v)}{(\rho \alpha A_j^N + n_h)(\rho \alpha A_i^N + (1-\theta_v)n_v) - n_v n_h (1-\theta_v)} \right) x \\ q_j^N = \left(\frac{\rho \alpha A_i^N}{(\rho \alpha A_j^N + n_h)(\rho \alpha A_i^N + (1-\theta_v)n_v) - n_v n_h (1-\theta_v)} \right) x \end{cases}$$

which is equivalent to,

$$\begin{cases} q_i^N = \gamma_i^N x \\ q_j^N = \gamma_j^N x \end{cases}.$$

6.2 Proof of Proposition 4

The total discounted utility of the joint coalition satisfies

$$\begin{aligned} V(x) = \max_{(q_i, q_j) \geq 0} & \left(\sum_{i=1}^{n_v} \ln \left(a q_i \sum_{i=1}^{n_v-1} q_i^{-\theta_v} \sum_{j=1}^{n_h} q_j^{-\theta_h} \right) + \sum_{j=1}^{n_h} \ln q_j \right. \\ & \left. + \rho V \left(\left(x - \sum_{i=1}^{n_v} q_i - \sum_{j=1}^{n_h} q_j \right)^\alpha \right) \right). \end{aligned} \quad (44)$$

We assume the value function to be of the following form,

$$V^C(x) = A^C \ln x + B^C.$$

The above form yields,

$$V^C(x) = n_v (1 - \theta_v) \ln q_i^C - n_v \theta_h \ln q_j^C + n_v \ln a + n_v \ln (n_v - 1)$$

$$+n_v \ln(n_h) + n_h \ln q_j^C + \rho\alpha A^C \ln \left(x - \sum_{i=1}^{n_v} q_i - \sum_{j=1}^{n_h} q_j \right) + \rho B^C. \quad (45)$$

Assuming symmetric players and replacing q_i^C by their values from Eq. (10), we obtain,

$$\begin{aligned} V^C(x) &= \left(n_v (1 - \theta_v - \theta_h) + n_h + \rho\alpha A^C \right) \ln(x) + \rho\alpha A^C \ln \left(1 - n_v \gamma_i^C - n_h \gamma_j^C \right) \\ &+ \rho B^C + n_v (1 - \theta_v) \ln \left(\gamma_i^C \right) - (n_h - n_v \theta_h) \ln \left(\gamma_j^C \right) \\ &+ \ln(n_v - 1) + \ln(n_h) + \ln(a) \end{aligned}$$

where

$$A^C = \frac{n_v (1 - \theta_v - \theta_h) + n_h}{1 - \rho\alpha}$$

and

$$\begin{aligned} B^C &= \frac{\rho\alpha (n_v (1 - \theta_v - \theta_h) + n_h) \ln \left(1 - n_v \gamma_i^C - n_h \gamma_j^C \right)}{(1 - \rho\alpha)(1 - \rho)} \\ &+ \frac{n_v (1 - \theta_v) \ln \left(\gamma_i^C \right) - (n_h - n_v \theta_h) \ln \left(\gamma_j^C \right) + n_v \ln(n_v - 1) + n_v \ln(n_h) + n_v \ln(a)}{(1 - \rho)}. \end{aligned}$$

Finally, maximizing the right-hand side of Eq. (45) w.r.t q_i and q_j gives,

$$\begin{aligned} \frac{n_v (1 - \theta_v)}{q_i^C} - \frac{\rho\alpha A^C}{x - n_v q_i^C - n_h q_j^C} &= 0 \\ \frac{-n_v \theta_h}{q_j^C} + \frac{n_h}{q_j^C} - \frac{\rho\alpha A^C}{x - n_v q_i^C - n_h q_j^C} &= 0 \end{aligned}$$

from where we obtain the v -type players best response function,

$$q_i^C(q_j^C) = \frac{n_v (1 - \theta_v) \left(x - n_h q_j^C(q_j^C) \right)}{\rho\alpha A^C + n_v^2 (1 - \theta_v)}, \quad (46)$$

$$q_j^C(q_i^C) = \frac{(n_h - n_v \theta_h) \left(x - n_v q_i^C(q_i^C) \right)}{\rho\alpha A^C + n_h (n_h - n_v \theta_h)}. \quad (47)$$

Finally, solving the system of Eqs. (46) and (47), yields to the following expressions,

$$\begin{cases} q_i^C = \left(\left(\frac{n_v (1 - \theta_v)}{\rho\alpha A^C + n_v^2 (1 - \theta_v)} \right) \left(1 - \frac{(n_h - n_v \theta_h) \rho\alpha A^C}{(\rho\alpha A^C + n_v^2 (1 - \theta_v))(\rho\alpha A^C + n_h (n_h - n_v \theta_h)) - n_v^2 n_h (1 - \theta_v)(n_h - n_v \theta_h)} \right) \right) x \\ q_j^C = \left(\frac{(n_h - n_v \theta_h) \rho\alpha A^C}{(\rho\alpha A^C + n_v^2 (1 - \theta_v))(\rho\alpha A^C + n_h (n_h - n_v \theta_h)) - n_v^2 n_h (1 - \theta_v)(n_h - n_v \theta_h)} \right) x \end{cases}$$

which is equivalent to,

$$\begin{cases} q_i^C = \gamma_i^C x \\ q_j^C = \gamma_j^C x \end{cases}.$$

6.3 Proof of Proposition 5

v-type players: The total discounted utility of the *v*-group coalition satisfies

$$V_v(x) = \max_{q_i \geq 0} \sum_{i=1}^{n_v} \ln \left(a q_i \sum_{i=1}^{n_v-1} q_i^{-\theta_v} \sum_{j=1}^{n_h} q_j^{-\theta_h} \right) + \rho V_v \left(\left(x - \sum_{i=1}^{n_v} q_i - \sum_{j=1}^{n_h} q_j \right)^\alpha \right). \quad (48)$$

We assume the value function to be of the following form,

$$V_v^E(x) = A_v^E \ln x + B_v^E, \quad i = 1, \dots, n_v,$$

The above form yields,

$$\begin{aligned} V_v^E(x) &= n_v (1 - \theta_v) \ln q_i^E - n_v \theta_h \ln q_j^E + n_v \ln a + n_v \ln (n_v - 1) \\ &\quad + n_v \ln (n_h) + \rho \alpha A_v^E \ln \left(x - \sum_{i=1}^{n_v} q_i - \sum_{j=1}^{n_h} q_j \right) + \rho B_v^E. \end{aligned} \quad (49)$$

Assuming symmetric players and replacing q_i^E by their values from Eq. (10), we obtain,

$$\begin{aligned} V_v^E(x) &= \left(n_v (1 - \theta_v - \theta_h) + \rho \alpha A_v^E \right) \ln(x) + \rho \alpha A_v^E \ln \left(1 - n_v \gamma_i^E - n_h \gamma_j^E \right) + \rho B_v^E \\ &\quad + n_v (1 - \theta_v) \ln \left(\gamma_i^E \right) - n_v \theta_h \ln \left(\gamma_j^E \right) + n_v \ln (n_v - 1) + n_v \ln (n_h) + n_v \ln(a) \end{aligned}$$

where

$$\begin{aligned} A_v^E &= \frac{n_v (1 - \theta_v - \theta_h)}{1 - \rho \alpha} \\ B_v^E &= \frac{\rho \alpha n_v (1 - \theta_v - \theta_h) \ln \left(1 - n_v \gamma_i^E - n_h \gamma_j^E \right)}{(1 - \rho \alpha)(1 - \rho)} \\ &\quad + \frac{n_v \left[(1 - \theta_v) \ln \left(\gamma_i^E \right) - \theta_h \ln \left(\gamma_j^E \right) + \ln (n_v - 1) + \ln (n_h) + \ln(a) \right]}{(1 - \rho)}. \end{aligned}$$

Finally, maximizing the right-hand side of Eq. (49) w.r.t q_i gives,

$$\frac{n_v (1 - \theta_v - \theta_h)}{q_i^E} - \frac{\rho \alpha A_v^E}{x - n_v q_i^E - n_h q_j^E} = 0 \quad (50)$$

from where we obtain the *v*-type players best response function,

$$q_i^E \left(q_j^E \right) = \frac{n_v (1 - \theta_v - \theta_h) \left(x - n_h q_j^E \left(q_i^E \right) \right)}{\rho \alpha A_v^E + n_v^2 (1 - \theta_v - \theta_h)}, \quad (51)$$

$$q_i^E \left(\gamma_j^E \right) = \frac{n_v (1 - \theta_v - \theta_h) \left(1 - n_h \gamma_j^E \left(\gamma_i^E \right) \right)}{\rho \alpha A_v^E + n_v^2 (1 - \theta_v - \theta_h)} x, \quad (52)$$

$$q_i^E \left(\gamma_i^E \right) = \gamma_i^E \left(\gamma_j^E \right) x. \quad (53)$$

Finally, solving the system of Eqs. (51) and (41),

$$\begin{cases} q_i^E(q_j^E) = \frac{n_v(1-\theta_v-\theta_h)(x-n_hq_j^E(q_i^E))}{\rho\alpha A_v^E+n_v^2(1-\theta_v-\theta_h)}, \\ q_j^E(q_i^E) = \frac{x-n_vq_i^E(q_j^E)}{\rho\alpha A_j^E+n_h} \end{cases}$$

gives us,

$$\begin{cases} q_i^E = \left(\frac{n_v(1-\theta_v-\theta_h)}{\rho\alpha A_v^E+n_v^2(1-\theta_v-\theta_h)} \left[1 - \left(\frac{\rho\alpha A_v^E}{[\rho\alpha A_v^E+n_v^2(1-\theta_v-\theta_h)][\rho\alpha A_j^E+n_h]-n_v^2n_h(1-\theta_v-\theta_h)} \right) \right] \right) x \\ q_j^E = \left(\frac{\rho\alpha A_v^E}{[\rho\alpha A_v^E+n_v^2(1-\theta_v-\theta_h)][\rho\alpha A_j^E+n_h]-n_v^2n_h(1-\theta_v-\theta_h)} \right) x \end{cases}$$

which is equivalent to,

$$\begin{cases} q_i^E = \gamma_i^E x \\ q_j^E = \gamma_j^E x \end{cases}.$$

6.4 Proof of Proposition 6

The total discounted utility of the joint coalition satisfies

$$\begin{aligned} V(x) = \max_{(q_i, q_j) \geq 0} & \left(\sum_{i=1}^{n_v} \ln \left(a q_i \sum_{i=1}^{n_v-1} q_i^{-\theta_v} \sum_{j=1}^{n_h} q_j^{-\theta_h} \right) + \sum_{j=1}^{n_h-1} \ln q_j \right. \\ & \left. + \rho V \left(\left(x - \sum_{i=1}^{n_v} q_i - \sum_{j=1}^{n_h} q_j \right)^\alpha \right) \right). \end{aligned} \quad (54)$$

We assume the value function to be of the following form,

$$V^A(x) = A^A \ln x + B^A.$$

The above form yields,

$$\begin{aligned} V^A(x) = n_v(1-\theta_v) \ln q_i^A - n_v\theta_h \ln q_j^A + n_v \ln a + n_v \ln(n_v-1) \\ + n_v \ln(n_h) + (n_h-1) \ln q_j^A + \rho\alpha A^A \ln \left(x - \sum_{i=1}^{n_v} q_i - \sum_{j=1}^{n_h} q_j \right) + \rho B^A. \end{aligned} \quad (55)$$

Assuming symmetric players and replacing q_i^A by their values from Eq. (10), we obtain,

$$\begin{aligned} V^A(x) = & \left(n_v(1-\theta_v-\theta_h) + n_h-1 + \rho\alpha A^A \right) \ln(x) + \rho\alpha A^A \ln \left(1 - n_v\gamma_i^A - n_h\gamma_j^A \right) \\ & + \rho B^A + n_v(1-\theta_v) \ln(\gamma_i^A) - (n_h-1-n_v\theta_h) \ln(\gamma_j^A) + \ln(n_v-1) \\ & + \ln(n_h) + \ln(a) \end{aligned} \quad (56)$$

where

$$A^A = \frac{n_v(1-\theta_v-\theta_h) + n_h-1}{1-\rho\alpha}$$

and

$$B^A = \frac{\rho\alpha(n_v(1-\theta_v-\theta_h)+n_h-1)\ln(1-n_v\gamma_i^A-n_h\gamma_j^A)}{(1-\rho\alpha)(1-\rho)} + \frac{n_v(1-\theta_v)\ln(\gamma_i^A)-(n_h-1-n_v\theta_h)\ln(\gamma_j^A)+n_v\ln(n_v-1)+n_v\ln(n_h)+n_v\ln(a)}{(1-\rho)}.$$
(57)

Finally, maximizing the right-hand side of Eq. (55) w.r.t q_i and q_j gives,

$$\begin{aligned} \frac{n_v(1-\theta_v)}{q_i^A} - \frac{\rho\alpha A^A}{x-n_vq_i^A-n_hq_j^A} &= 0 \\ \frac{-n_v\theta_h}{q_j^A} + \frac{n_h-1}{q_j^A} - \frac{\rho\alpha A^A}{x-n_vq_i^A-n_hq_j^A} &= 0 \end{aligned}$$

from where we obtain the players' best response functions,

$$q_i^A(q_j^A) = \frac{n_v(1-\theta_v)(x-n_hq_j^A(q_i^A))}{\rho\alpha A^A+n_v^2(1-\theta_v)},$$
(58)

$$q_j^A(q_i^A) = \frac{(n_h-1-n_v\theta_h)(x-n_vq_i^A(q_j^A))}{\rho\alpha A^A+n_h(n_h-n_v\theta_h)}.$$
(59)

Finally, solving the system of Eqs. (58) and (59), yields to the following expressions. We adopt a similar approach as in the proof for propositions (3) and (4) in order to derive the following intermediate solution that describe the strategies when only counting for (n_h-1) h -type players,

$$\begin{aligned} \begin{cases} \bar{q}_i^A = \left(\frac{n_v(1-\theta_v)}{\rho\alpha A^A+n_v^2(1-\theta_v)} \right) \left(1 - \frac{(n_h-1-n_v\theta_h)\rho\alpha A^A}{(\rho\alpha A^A+n_v^2(1-\theta_v))(\rho\alpha A^A+n_h(n_h-n_v\theta_h))-n_v^2n_h(1-\theta_v)(n_h-1-n_v\theta_h)} \right) x \\ \bar{q}_j^A = \left(\frac{(n_h-1-n_v\theta_h)\rho\alpha A^A}{(\rho\alpha A^A+n_v^2(1-\theta_v))(\rho\alpha A^A+n_h(n_h-n_v\theta_h))-n_v^2n_h(1-\theta_v)(n_h-1-n_v\theta_h)} \right) x \end{cases} \\ A = \frac{\pi r^2}{2} \\ = \frac{1}{2}\pi r^2 \end{aligned}$$
(60)

which is equivalent to,

$$\begin{cases} \bar{q}_i^A = \bar{\gamma}_i^A x \\ \bar{q}_j^A = \bar{\gamma}_j^A x \end{cases}.$$

Next, we define the final solution as follows:

$$\begin{cases} \gamma_i^A = \bar{\gamma}_i^A \\ \gamma_j^A = \left(\frac{n_h-1}{n_h} \right) \bar{\gamma}_j^A \end{cases}.$$

Finally, the solution is given by,

$$\begin{cases} \gamma_i^A = \left(\frac{n_v(1-\theta_v)}{\rho\alpha A^A+n_v^2(1-\theta_v)} \right) \left(1 - \frac{(n_h-1-n_v\theta_h)\rho\alpha A^A}{(\rho\alpha A^A+n_v^2(1-\theta_v))(\rho\alpha A^A+n_h(n_h-n_v\theta_h))-n_v^2n_h(1-\theta_v)(n_h-1-n_v\theta_h)} \right) \\ \gamma_j^A = \left(\frac{n_h-1}{n_h} \right) \left(\frac{(n_h-1-n_v\theta_h)\rho\alpha A^A}{(\rho\alpha A^A+n_v^2(1-\theta_v))(\rho\alpha A^A+n_h(n_h-n_v\theta_h))-n_v^2n_h(1-\theta_v)(n_h-1-n_v\theta_h)} \right) \end{cases}.$$

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