Dyn Games Appl (2013) 3:443–445
DOI 10.1007/s13235-013-0098-3

Preface: DGAA Special Issue on Mean Field Games

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Published online: 25 October 2013

The theory of Mean Field Games (MFG) is a branch of Dynamic Games, which aims at modeling and analyzing complex decision processes involving a large number of indistinguishable rational agents who have individually a very small influence on the overall system and are, on the other hand, influenced by the mass of the other agents. The name comes from particle physics where it is common to consider interactions among particles as an external mean field, which influences the particles. In spite of the optimization made by rational agents, playing the role of particles in such models, appropriate mean field equations can be derived to replace the many particles interactions by a single problem with an appropriately chosen external mean field, which takes into account the global behavior of the individuals. The introduction of a social component in the optimization criteria makes this theory so flexible that it can be applied to various fields and, for this reason, it is attracting increasing interest from economists (micro and macro), engineers, biologists describing animal behavior, and possibly sociologists and urban planners.

The theory originated in the independent work of J.M. Lasry and P.L. Lions [5–7], and of M.Y. Huang, P.E. Caines, and R. Malhamé [2–4]. Lasry and Lions started from the systems of N elliptic partial differential equations (PDEs) that provide a Nash equilibrium in feedback form for N-person stochastic differential games, and found in the limit as $N \to \infty$ a system of two PDEs, where a classical Hamilton–Jacobi–Bellman equation is coupled with a Kolmogorov–Fokker–Planck equation for the density of the players. They also observed that the MFG PDEs have strong links with other important fields of mathematics, such as quantum mechanics, fluid dynamics, optimal transportation, and optimal control of systems driven by PDEs. The more recent developments of the general mathematical theory lead to a PDE in infinite dimensions, called the master equation, whose analysis is a challenging open problem; see P.-L. Lions' courses on the site of the Collège de France http://www.college-de-france.fr/site/en-pierre-louis-lions/.

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Huang, Caines, and Malhamé first derived the MFG equations in the linear-quadratic-Gaussian case for large populations of uniform [2] and nonuniform agents [3] in models motivated by power control problems in cell phone communications and the methods of statistical mechanics. Then, in [4], Huang, Malhamé, and Caines extended their results to nonlinear systems described by McKean–Vlasov SDEs for nonuniform populations of agents. They continued the development of the theory in a series of articles, also with other authors, demonstrating the existence of ϵ-Nash equilibria for masses of agents using MFG PDE derived controls in a range of different settings; these included localized cost functions, social welfare maximization, consensus seeking systems, adaptive control systems, and systems with major and minor agents.

The aim of this special issue of Dynamic Games and Applications and of a companion issue to be published in 2014 is to collect original papers on a wide range of aspects of the MFG theory and applications and, therefore, give an overview of the emerging trends in this fast growing research area. A fairly detailed earlier snapshot of the research on MFG can be found in the special issue of Networks and Hetereogeneous Media [1].

The present issue contains six articles. The paper by Minyi Huang deals with a generalization to the MFG context of models of capital consumption-accumulation in the stochastic theory of economic growth.

Pierre Cardaliaguet considers deterministic MFG and the associated system of first-order PDEs. He shows that the solution over the time-interval $[0, T]$, divided by T, converges as $T \to \infty$ to the solution of an ergodic MFG. The proof relies on delicate arguments about Hamiltonian systems, namely, the recent theory called weak KAM.

The article by Tao Li and Ji-Feng Zhang treats adaptive MFG for large population coupled systems. In this work, an ϵ-Nash equilibrium with respect to long range average costs is obtained by local adaptive control wherein each agent applies a two level adaptive policy: At the upper level, the mean field is recursively estimated and at the lower level the coupling strength is recursively estimated using the mean field estimates and local observations.

Lorenzo Brasco and Guillaume Carlier consider models of congested traffic flows on very dense anisotropic networks leading, in the limit, to a highly degenerate Euler–Lagrange type partial differential equation, which is analyzed from the regularity point of view.

Ermal Feleqi revisits the derivation from the microscopic model of the MFG PDEs for ergodic cost functional under more general conditions than in [5, 7] and for several populations of players.

Finally, the paper by Alain Bensoussan, K.C.J. Sung, and S.C.P. Yam is about a time-inconsistent version of the Linear-Quadratic MFG first studied by Huang, Caines, and Malhamé, with motivations coming from models of financial economics.

In conclusion, we thank Georges Zaccour, Editor-in-Chief of Dynamic Games and Applications, for proposing a special issue on MFG and for his support during its preparation, and all the contributors and the referees. We hope that this and the forthcoming special issue will contribute to the development of new activities in the field.

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