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ORIGINAL ARTICLE

Sequential licensing with several competing technologies

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Abstract

We assume a multistage oligopoly wherein a given number of innovators compete by selling their substitutive technologies. Each innovator sequentially and independently chooses how many licenses to sell, and subsequently, all licensees compete à la Cournot in the product market. We show that, in equilibrium, the total number of licensees grows exponentially with the number of innovators. In addition, this sequential outcome is also obtained as a subgame perfect Nash equilibrium in pure strategies of a game with endogenous timing. Interestingly, by extending the duopoly model of Badia et al. (Math Soc Sci 108:8–13, 2020) to the case of more than two innovators and exploring pure strategy equilibria instead of mixed strategy equilibria, we derive drastically different policy implications, in terms of patent regulations. Our results suggest that more competition in the upstream market (e.g., by relaxing patent protection against the appearance of similar technologies) tends to increase downstream competition and welfare instead of discouraging or delaying technology adoption. In addition, our analysis is extended to explore the strategic role of public investment in basic R&D.

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1 Introduction

Patents award innovators the right to exploit their inventions in exclusivity, so that other firms cannot use them freely. This static inefficiency is justified to give innovators ex-ante incentives to innovate. However, a patent holder may find it profitable to undertake some degree of innovation diffusion by licensing its newly developed technology to third parties.

The question of the degree of diffusion of innovations obtained through licensing has been considered in the seminal papers on licensing. For example, Kamien and Tauman (1986) find that patented innovation is not always licensed to all firms if it is traded through a license fee. Katz and Shapiro (1986) obtain a similar result when the patent holder sells the technology through auctions (possibly with a minimum bid).

Sen and Tauman (2007) extend this issue to more general types of licensing contracts that are a combination of upfront fees and royalties, showing, under the assumption of homogenous goods and a finite number of potential licensees, that non-drastic process innovations are licensed (practically) to all potential licensees.[1] Exploring the same question, Fauli-Oller et al. (2013) assume product differentiation in the market for the final good and find that both drastic and non-drastic innovations are licensed to all firms.

These studies have mainly focused on cost-reducing innovations and, most importantly, assumed a monopolist patent holder. We consider product innovations and assume competition among multiple patent holders, as in Arora and Fosfuri's (2003) seminal paper, where they note (p. 278 footnote 4):

"The presence of competing technologies is not a mere theoretical possibility. The chemical industry is a rich source of examples. As reported in Arora (1997), Union Carbide, Himont and Mobil compete with each other in selling polypropylene licenses; BP and Du Pont compete in polyethylene process technology; UOP, Mobil-BP and Phillips Petroleum in methyl tert-butyl ethers (MTBE)."

Fosfuri (2006) also explains the case of the ethylene glycol market, in which three firms license different ways to produce ethylene glycol: Union Carbide, Shell, and Scientific Design (external patentee).

Arora and Fosfuri (2003, p. 278) also note that "[t]he noteworthy feature is that often these firms license their technology to other firms that could potentially compete with them." Arora (1997) studies the extent of technology licensing during 1980-90, finding that "of these reported licensing agreements, a little over 80% involve sales of

[1] We say "practically" because Sen and Tauman (2007) determine the upfront through an auction, where sometimes it is profitable to exclude one of the potential licensees. In the same vein, Erutku and Richelle (2007) prove that the innovation is sold to all potential licensees if the patentee uses two-part tariff licensing contracts.

technology between firms not linked through ownership ties" (p. 398), which again opens the possibility that firms were effectively licensing to competitors.

Coherent with these observations, Arora and Fosfuri (2003) develop a model of licensing competition, revealing that the holders of similar technologies sign a multiplicity of licensing contracts. Intuitively, under an oligopoly, the incentives to sell technological licensing to competitors appear when the *rent dissipation effect* owing to the proliferation of multiple competitors is outweighed by a strategic *market share effect,* which allows the patent holder to obtain a larger portion of the total industry profits. This trade-off is similar to that obtained in the strategic divisionalization literature (for the case of quantity competition and homogeneous product, see Corchón 1991; Baye et al. 1996; and Corchón and González-Maestre 2000; for the case of competition in prices with differentiation à la Salop, see González-Maestre 2001).

Technically, our model works as a model of strategic divisionalization; however, we formulate it as a model of patent licensing because our research question is to address several issues pertaining to the effects of patent competition on technology diffusion. In particular, we show that our approach helps clarify some aspects of the debate on the optimal regulation of patents, particularly regarding how easily patent authorities should allow new patents of similar existing technologies. We explain this issue in this section and in more detail in Sect. 2. Moreover, we extend this analysis to consider also the role of R&D subsidies under endogenous technological entry (Sect. 3), and the interplay between the technological policies regulating innovators' entry (patent and R&D policies) and the public investment in basic R&D (Sect. 4). We explain our results, in these extensions, later on.

Our approach extends the analysis of Arora and Fosfuri (2003), who focused on simultaneous licensing, to the case of sequential licensing.[2] Assuming Cournot competition at product market with linear demand and cost functions, we show that the equilibrium number of licensees in the market increases exponentially with the number of innovators. This result is connected to the subgame perfect equilibrium strategy of innovators in the sequential game: A firm choosing its number of licenses at the t^{th} position wants to sell one more license than those sold by the firms that preceded it. This is obvious for the firm choosing in last place, but curiously enough, also holds for the rest of the firms. The resulting snowball effect explains why the total number of licenses increases exponentially with the number of innovators. A similar result holds under price competition and spatial product differentiation à la Salop (1979), assuming linear transport costs.

Interestingly, our analysis shows that, in a game with endogenous timing, sequentiality is the resulting equilibrium outcome for any number of innovators equal to or greater than two. From the patent regulation perspective, our analysis of the non-exclusive licensing of similar technologies yields new and meaningful insights. The endogenous appearance of a sequential licensing choice shows the pro-competitive and welfare-enhancing effects of allowing competition among similar technologies. It is then interesting to compare our results with those derived by Badia et al. (2020), who focus on the particular case of dynamic competition between two innovators selling

2 Zhou (2013) was the first to introduce sequentiality in a model of divisionalization for the case of two firms. Our model can be viewed as an extension of Zhou's model to a general number of firms for the particular case of constant marginal costs.

technologies that are perfect substitutes. Besides the endogenous timing of licensing decisions and time discounting, they assume that each licensee is able to produce from the period at which the licensing contract is signed. Focusing on mixed strategies, they show the existence of a symmetric equilibrium such that the probability that the two innovators inefficiently delay the time at which they sign their licensing contracts increases with the discount factor. Badia, Tauman, and Tumendemberel conclude that more competition in the innovation market might yield a lower diffusion of technology and lower welfare. Considering the general case of more than two innovators and exploring the existence of asymmetric equilibria in pure strategies, we show that competition and welfare both strictly increase with the number of innovators. Our model differs from Badia, Tauman, and Tumendemberel's in that we assume that production can only take place after all the innovators have signed their licensing contracts. Nevertheless, as explained in Sect. 2, the qualitative aspects of our results remain in the scenario in which each licensee is able to already produce from the period in which the contract is signed.

We further extend our model to analyze the interaction between the diffusion mechanism and innovation incentives. Specifically, we explicitly consider the case of an endogenous number of innovations. In this extended setup, we consider the welfare effects associated with the appearance of new technologies in the market under sequential licensing. We show that the relationship between the free-entry number of innovators and the socially optimal level of this variable is qualitatively different both from the case of simultaneous licensing and from the case we call the exclusive licensing setup, in which the number of independent sellers associated with a particular patent is restricted to one. In particular, for large markets, the degree of excessive entry, measured by the ratio of (equilibrium number of innovators)/(socially optimal number of innovators), is smaller under sequential licensing than in the exclusive licensing setup. Nevertheless, this ratio is greater than that under simultaneous licensing because, in contrast to the simultaneous setup, the equilibrium number of licensees does not tend to infinity with more than one active innovator. The basic intuition behind these properties is that, in contrast to private entry incentives, social preferences for entry increase in post-entry competition intensity, which, in turn, increases with the sensitivity of licensing proliferation to entry. This sensitivity increases when we shift from the exclusive licensing setup to sequential licensing, and then from sequential to simultaneous licensing. Notably, those results have important policy implications, in terms of patent regulations, suggesting, in particular, that a more relaxing patent protection is more likely to be optimal the more competitive the licensing stage is.

In addition, our analysis is extended to explore the strategic role of public investment in basic R&D under product differentiation. In this extended scenario, we show that the optimal level of public investment in basic R&D must be higher (respectively, smaller) under sequential licensing than under exclusive licensing if the degree of product differentiation is small (respectively, large).

The remainder of this paper is organized as follows. In the next section, we develop a model with an exogenously specified number of innovators. In Sect. 3, we determine the number of innovators endogenously through a free-entry condition. We then compare the equilibrium number of innovators with that which maximizes social welfare.

Section 4 extends the model to the case of endogenous public investment in basic R&D. The final section concludes.

2 The model with no entry

Assume $k \geq 2$ innovators owning technologies that are perfect substitutes, following the spirit of Badia et al. (2020). Each of those technologies can be used to produce a homogenous good the inverse demand of which will be specified later. The innovators license their technology to independent firms in a perfectly competitive pool through a take-it-or-leave-it fixed fee licensing contract.[3] Therefore, each innovator's payoff will be the sum of all its licensees' profits. The licensing contract involves no fixed cost, and the good is produced at a constant marginal cost c. Firms choose the number of licensing contracts they want to create sequentially. In particular, we consider a $T+1$ stage game, where $T = k$. Each innovator, $t = 1, .., T$ chooses its number of licensees, $m_t \geq 1$, at time t. At time $T+1$, all the independent licensees, determined in the previous periods, compete simultaneously à la Cournot. We look for the subgame perfect equilibrium of this game using backward induction. In "Appendix 1", we show that this order of moves can be obtained as the equilibrium outcome of a game, wherein innovators choose freely at which date they commit to the number of independent licensing contracts they want to sign.

For simplicity, we assume that innovators do not have productive facilities to sell goods to consumers. However, things would not change if some of them could (thereafter, productive firms). They would choose to sell the same number of licenses as innovators minus one, so that the total number of sellers of the good remains unchanged. This is because the same profit is obtained by selling directly to consumers rather than by licensing the technology to a firm because we have assumed that competition between firms to obtain a license allows the patent holder to extract all the rents from the licensee. This is not the case in Arora and Fosfuri (2003), wherein the possibility of extracting rents from licensees is limited because contracting involves transaction costs. Thus, more profit is obtained by selling directly than by licensing the technology. The *exclusive* licensing case, which is used as a benchmark, refers to the case wherein innovators can only sell one license and productive firms are not allowed to sell the technology.

2.1 The case of linear demand

In this case, demand of the final good is given by $p = a - x$, where p denotes the price and x the quantity sold. In the rest of the analysis, we assume the same constant marginal production cost, c for each producer. Standard calculations show that in stage $T+1$, if M licenses have been signed, the profit per licensee is given by $\pi_i(M) = \dfrac{(a-c)^2}{(M+1)^2}$, where $i = 1, .., M$.

[3] In particular, we do not allow the use of per-unit royalties.

Let us define $M_t \equiv \sum_{h=1}^{t} m_h$ for $t \geq 1$, and because in period 1, there are no previously created licensees, $M_0 = 0$. The following proposition describes the firms' equilibrium strategies.

Proposition 1 *With linear demand, and sequential licensing, the subgame perfect Nash equilibrium (SPNE) strategy, of any innovator* $t = 1, ..., T$ *is given by* $m_t = M_{t-1}+1$.

Proof To show that this is the optimal strategy for each innovator, consider, first, the profit-maximization decision of the last innovator, T:

$$\begin{aligned}
\pi_T(m_T, M_{T-1}) &= \frac{(a-c)^2 m_T}{(M_{T-1}+m_T+1)^2} \\
&\Rightarrow \frac{\partial \pi_T}{\partial m_T} = \frac{(a-c)^2(m_T + M_{T-1} + 1 - 2m_T)}{(M_{T-1}+m_T+1)^3} = 0 \\
&\Rightarrow m_T = M_{T-1}+1.
\end{aligned}$$

Therefore, the property formulated in the proposition follows for the last innovator (T). Now, assume that, given $t < T$, the property is satisfied for any t' such that $T \geq t' > t$. Under this assumption and using the fact that $M_t \equiv m_t + M_{t-1}$, innovator t expects that

$$\begin{aligned}
M_T + 1 &= M_t + 1 + M_t + 1 + 2(M_t + 1) + 4(M_t + 1) + 8(M_t + 1) + ... \\
&\quad +2^{T-t-1}(M_t + 1) \\
&= 2^{T-t}(M_t + 1),
\end{aligned}$$

and we have the following profit-maximization decision of innovator t:

$$\begin{aligned}
\pi_t(m_t, M_{t-1}) &= \frac{(a-c)^2 m_t}{2^{2(T-t)}(M_t+1)^2} = \frac{(a-c)^2 m_t}{2^{2(T-t)}(m_t + M_{t-1}+1)^2} \Rightarrow \\
\frac{\partial \pi_t(m_t, M_{t-1})}{\partial m_t} &= \frac{(a-c)^2(m_t + M_{t-1} + 1 - 2m_t)}{2^{2(T-t)}(m_t + M_{t-1}+1)^3} = 0 \Rightarrow m_t = M_{t-1}+1.
\end{aligned}$$

Therefore, given t, if $m_{t'} = M_{t'-1}+1$ for any $t' > t$ then $m_t = M_{t-1}+1$. Because we have proved that this property is satisfied for $t' = T$, mathematical induction implies that $m_t = M_{t-1}+1$ for any $t = 1, ..., T$, which completes the proof. □

The previous proposition states that the innovator that chooses its number of licenses in the t^{th} position wants to create one more contract than those created by innovators who preceded it. This result is straightforward for the innovator choosing in the last place, because it comes from direct profit maximization, taking the number of licenses of the remaining competitors as given. Interestingly, the same logic applies for the remaining innovators.

The next proposition specifies the number of licenses signed by each innovator and the total number of licenses, in the sequential game.

Proposition 2 *In the sequential licensing game, with linear demand, the following properties hold: (a) The SPNE number of licenses of firm* $t = 1, .., T$ *is given by* $m_t = 2^{t-1}$*; (b) the total SPNE number of licenses of the* T *innovators is given by* $M_T = 2^T - 1$.

Proof Part (a) follows from the recursive substitution in the general expression $m_t = M_{t-1} + 1$, bearing in mind $M_0 = 0$. Part (b) is obtained by adding each innovator's number of licenses and observing that m_t follows a geometric progression. □

To compare our sequential setup with the simultaneous licensing game, the following remark explains the SPNE outcome in the case of simultaneous moves.

Remark 1 With linear demand and simultaneous licensing, the SPNE implies perfect competition.

The result summarized in Remark 1 is similar to that obtained by Corchón (1991) in the context of divisionalization and is obtained by observing that, according to the proof of Proposition 1, the best reply of each innovator is to sign a number of licenses equal to that signed by the remaining firms plus one. In contrast with Remark 1, Proposition 2 shows that, under sequential moves, the competitive use of licensing is substantially reduced with respect to the simultaneous game for a small number of innovators. However, as the number of innovators increases, the SPNE of the sequential game exponentially converges to that of the simultaneous setup.

One of the main insights of Badia et al. (2020) is that, with only two innovators, the symmetric equilibrium in mixed strategies, in a setup with time discounting and endogenous timing, implies that, for a sufficiently large discount factor, the social welfare associated with competition among innovators is smaller than in the case of a monopolistic innovator. However, in part (i) of "Appendix 1", we show that Proposition 2 still holds under the assumption of endogenous timing. The intuition behind this result follows directly from the previous paragraph: If two or more firms sign their contracts simultaneously, then the total number of contracts tends to infinity, which involves zero profits. Therefore, an SPNE in pure strategies is consistent only with sequential moves. In addition, part (ii) of this appendix explains that extending our model to a setup in which production can take place in different periods does not qualitatively change our results if the discount factor is sufficiently large. This case is particularly emphasized by Badia et al. (2020) because it corresponds to an SPNE in mixed strategies in which competition between two innovators yields lower welfare than a technological monopoly.

2.2 The case of unitary elasticity demand

Similar to the divisionalization setup (see Corchón and González-Maestre 2000), for a general inverse demand $p(x)$, licensing incentives grow with the degree of demand concavity, as defined by $\dfrac{p''(x)x}{p'(x)}$. In the context of our sequential licensing game, we illustrate this issue by comparing the results we have obtained with linear demand (degree of concavity equal to 0) with the ones with unit-elastic demand (degree of concavity equal to -2).

In this case, demand is given by $p = A/x$, where p denotes the price and x the quantity sold. Standard calculations show that in stage $T+1$, if M licenses have been created, the profit per licensee is given by $\pi_i(M) = \dfrac{A}{M^2}$, where $i = 1, .., M$. The following proposition describes the equilibrium strategies of innovators:

Proposition 3 *With unitary elasticity demand, and sequential licensing, the subgame perfect Nash equilibrium (SPNE) strategy, of any innovator $t = 1, ..., T$ is given by $m_t = M_{t-1}$, with $M_0 = 1$.*

Proof *See Appendix 2.* □

The next proposition specifies the number of licenses signed by each innovator and the total number of licenses.

Proposition 4 *In the sequential licensing game, with unitary elasticity demand, the following properties hold: (a) The SPNE number of licenses of firm $t = 1, .., T$ is given by $m_t = 2^{t-2}$, for $t \geq 2$, and $m_1 = 1$. (b) The total SPNE number of licenses of the T firms is given by $M_T = 2^{T-1}$.*

Proof Part (a) follows from the recursive substitution in the general expression $m_t = M_{t-1}$. Part (b) is obtained by adding each firm's number of licenses, and observing that m_t follows a geometric progression. □

Therefore, as previously anticipated, because the degree of demand concavity is higher under linear demand than under a unitary elasticity demand, more licenses are sold in the linear case $(2^T - 1)$ than in the unit-elasticity case (2^{T-1}), whenever T is higher than one.

The following remark explains the SPNE outcome, under simultaneous moves, in the case of unitary elasticity demand.

Remark 2 Assuming simultaneous licensing, under unitary elasticity demand, the SPNE implies perfect competition with more than two innovators, and any symmetric number of licenses per innovator with two innovators.

This result is similar to that obtained by Corchón (1991) in the context of divisionalization with unitary elasticity demand and is obtained by observing that, according to the proof of Proposition 3, the best reply of each innovator is to sign a number of licenses equal to that signed by the rest of the firms. Comparing Remark 2 with Proposition 4, it follows that, under sequential moves, the competitive use of licensing is substantially reduced, with respect to the simultaneous game for a small number of innovators (except for the case of two innovators, which is consistent with an equilibrium with only two competing licenses). However, as the number of innovators increases, the SPNE of the sequential game exponentially converges to that obtained in the simultaneous setup.

The equilibrium profit function is the same in the case of unitary elasticity demand and in the Salopś model (1979) with linear transportation costs (t) and constant marginal cost of production replacing A by t. So, the results in Propositions 3, 4 and Remark 2 hold for the case of spatial product differentiation a la Salop.

3 Welfare analysis under endogenous entry

In this section, we compare the free-entry equilibrium number of innovators with the number of innovators that maximizes social welfare, considering that entry involves a fixed cost F, which is the investment cost of creating a new technology. Entry decisions are made simultaneously. As, after entry, licensing decisions are taken sequentially, the profits from entering depend on the position firms are expected to have at the licensing stage. To maintain symmetry, we assume that innovators that enter are assigned to each position with probability $\frac{1}{k}$, where k is the number of innovators that enter. The expected profits from entering are the profit per license multiplied by the average number of licenses per innovator. We assume that innovators are risk neutral, and consider three different cases: the linear demand model, the unitary elasticity demand model and the Salop model.

3.1 The case of linear demand

According to Proposition 2, if k innovators enter the market in the sequential game, the equilibrium level of licenses is

$$M(k) = 2^k - 1.$$

The free-entry equilibrium level for k is given by:

$$\pi_i(k) = \frac{(2^k - 1)(a - c)^2}{(2^k)^2 k} - F = 0,$$

where $\frac{(a-c)^2}{(2^k)^2}$ is the profit per license and $\frac{(2^k - 1)}{k}$ is the average number of licenses per firm. By manipulating the expression and defining $Z \equiv \frac{(a-c)^2}{F}$, we obtain:

$$\begin{aligned} Z &= k\frac{(2^k)^2}{2^k - 1} \\ 2^{k^*} &= Z^{1/2}\left(\frac{2^{k^*} - 1}{k^*}\right)^{1/2}, \end{aligned} \tag{1}$$

where k^* is the number of innovators under free-entry, without considering the integer constraint.

We express social welfare as a function of the number of innovators. It is written as the difference between welfare with perfect competition and deadweight loss owing to imperfect competition, which decreases with the number of licensees, minus the fixed costs.

$$
\begin{aligned}
W(k) &= \frac{(a-c)^2}{2} - \frac{1}{2}\frac{(a-c)^2}{(2^k)^2} - kF \\
W'(k) &= \frac{(a-c)^2}{(2^k)^2}(\ln 2) - F = 0 \Rightarrow Z = \frac{\left(2^{k^o}\right)^2}{\ln 2} \Rightarrow \\
2^{k^o} &= Z^{1/2}(\ln 2)^{1/2},
\end{aligned}
\tag{2}
$$

where k^o is the number of innovators that maximizes social welfare without considering the integer constraint.

Combining the expressions (1) and (2), we obtain:

$$
\begin{aligned}
2^{k^*-k^o} &= \frac{1}{(\ln 2)^{1/2}}\left(\frac{2^{k^*}-1}{k^*}\right)^{1/2} \Rightarrow \\
k^* - k^o &= \frac{1}{2(\ln 2)}\left(\ln(2^{k^*}-1) - \ln k^* - \ln(\ln 2)\right).
\end{aligned}
$$

The previous expression is positive and increases in k^* for the relevant values of k^*. Therefore, by treating the number of innovators as a continuous variable, we find that we have excess entry, that is, private incentives to enter exceed social incentives. By further manipulating the expression, we obtain:

$$
\begin{aligned}
k^o &= k^* - \frac{1}{2(\ln 2)}\left(\ln(2^{k^*}-1) - \ln k^* - \ln(\ln 2)\right) \Rightarrow \\
\frac{k^o}{k^*} &= 1 - \frac{1}{2(\ln 2)}\frac{\left(\ln(2^{k^*}-1) - \ln k^* - \ln(\ln 2)\right)}{k^*}.
\end{aligned}
$$

Because $\ln(2^{k^*}-1)$ can be approximated by $\ln(2^{k^*}) = k^* \ln 2$ when k^* is large, we find that the ratio $\frac{k^o}{k^*}$ tends to $1/2$ as k^* tends to infinity, that is, when Z tends to infinity. Equivalently, the degree of excessive entry, measured by $\frac{k^*}{k^o}$, approaches 2 as Z tends to infinity. In the exclusive licensing setup, this ratio tends to infinity when the size of the market tends to infinity.[4] According to this result, although in our model there is still a discrepancy between private and social incentives to enter, which results in excessive entry, it is not as exaggerated as in the exclusive licensing model. Intuitively, the exponential competitive effect of non-exclusive licensing reduces, to a large extent, the private entry incentives while it increases social preferences for new entry.

4 Standard calculations in the exclusive licensing setup yield $k^*/k^o = \left(Z^{1/2}-1\right)/\left(Z^{1/3}-1\right)$, which tends to infinity as Z tends to infinity.

3.2 The case of unitary elasticity demand

According to Proposition 4, in the sequential model, the equilibrium level of licensing, given the number of innovators, k is given by:

$$M(k) = 2^{k-1}.$$

Free-entry equilibrium for k^* is given by:

$$\pi_i(k) = \frac{2^{k^*-1}A}{k^*\left(2^{k^*-1}\right)^2} - F = 0 \Longrightarrow \frac{A}{F} = g(k^*) \equiv k^* 2^{k^*-1},$$

where $g(k)$ indicates the relative market size $(\frac{A}{F})$, such that k is the equilibrium number of innovators.

Social welfare given k innovators is:

$$W(k) = \int_{\frac{A}{2c}}^{X(M(k))} \frac{A}{X} dX - cX(M(k)) - kF,$$

where $X(M) = \dfrac{A(M-1)}{Mc}$ is the equilibrium output as a function of the number of licenses M. The optimal number of innovators k^o, satisfies:

$$W'(k^o) = \frac{A \ln 2}{2^{k^o-2}(2^{k^o}-2)} - F = 0$$
$$\frac{A}{F} = f(k^o) \equiv \frac{2^{k^o-2}(2^{k^o}-2)}{\ln 2},$$

where function $f(k)$ tells us the relative size of the market $(\frac{A}{F})$ such that k is the optimal number of innovators. In order to compare $k^*$with k^o, we use functions $f(k)$ and $g(k)$. Both $f(k)$ and $g(k)$ are strictly increasing in k. It is possible to check that:

$$g(k) > f(k) \text{ iff } k < 2.4213$$

This implies that $k^* < k^o$ (insufficient entry) only for $\dfrac{A}{F} < f(2.4213) = 6.4852$. The ratio between the optimal number of innovators and the equilibrium number of innovators, $\dfrac{k^o}{k^*}$ tends to $\frac{1}{2}$ when k^* tends to infinity, that is, $\dfrac{A}{F}$ tends to infinity. This property can be shown combining conditions $\frac{A}{F} = g(k^*)$ and $\frac{A}{F} = f(k^o)$:

$$\frac{2^{k^o-2}(2^{k^o}-2)}{\ln 2} = k^* 2^{k^*-1} \Rightarrow$$

$$\frac{2^{k^o-k^*}(2^{k^o-1}-1)}{\ln 2} = k^* \Rightarrow$$
$$\left(k^o-k^*\right)\ln 2+\ln(2^{k^o-1}-1)-\ln(\ln 2)=\ln k^* \Rightarrow$$
$$\left(\frac{k^o}{k^*}-1\right)\ln 2+\frac{\ln(2^{k^o-1}-1)}{k^*}-\frac{\ln(\ln 2)}{k^*}=\frac{\ln k^*}{k^*}.$$

Taking the limit of the two sides of this expression as k^* tends to infinity, and bearing in mind that $\ln(2^{k^o-1}-1)$ can be approximated by $\ln(2^{k^o-1}) = (k^o-1)\ln 2$ when k^o is large, we obtain:

$$\lim_{k^*\to\infty}\left(\frac{k^o}{k^*}\ln 2-\ln 2+\frac{k^o}{k^*}\ln 2\right)=0 \Rightarrow \lim_{k^*\to\infty}\frac{k^o}{k^*}=\frac{1}{2}\Rightarrow \lim_{k^*\to\infty}\frac{k^*}{k^o}=2.$$

Again, for large markets, the degree of excessive entry, $\frac{k^*}{k^o}$, is much smaller under sequential licensing than in the case of exclusive licensing because, in the second case, $\frac{k^*}{k^o}$ approaches infinity as the market size grows.[5]

3.3 The Salop model

The transportation cost is linear in distance (D) and is given by $C(D) = tD$. Then, if the total number of licensees is M, the profit per licensee is $\pi_i(M) = \frac{t}{M^2}$. As it is the same profit as in the previous case (only replacing A by t), we obtain the same equilibrium number of licenses as a function of k

$$M(k) = 2^{k-1},$$

and the same free-entry condition and the same number of active innovators k^*satisfying

$$\frac{t}{F} = k^* 2^{k^*-1}. \tag{3}$$

In Salop (1979), all consumers buy one unit of the good (we assume that the market is covered); hence, the socially optimal number of innovators k^o is obtained by minimizing the sum of total transportation costs plus fixed costs. For a given number of licenses M, the total transportation cost is

$$T(M) = 2M\int_0^{1/(2M)} tD\,dD = \frac{t}{4M}.$$

[5] Routine calculations in the exclusive licensing setup yield $k^*/k^o = \left(k^o-1\right)^{1/2}$, which tends to infinity as A/F tends to infinity because k^o tends to infinity as A/F tends to infinity.

Then, the total cost as a function of the number of innovators k is given by

$$S(k) = kF + \frac{t}{4M(k)}. \tag{4}$$

The optimal number of innovators k^o satisfies:

$$\begin{aligned} S'(k^o) &= F - \frac{t(\ln 2)}{2^{k^o+1}} = 0 \Longrightarrow \\ \frac{t}{F} &= \frac{2^{k^o+1}}{\ln 2}. \end{aligned} \tag{5}$$

Dividing the expression (3) by expression (5), we obtain:

$$\begin{aligned} \frac{k^* 2^{k^*-1} \ln 2}{2^{k^o+1}} &= \frac{k^* 2^{k^*-1} \ln 2}{2^{k^o-1} 4} = 1 \\ \frac{2^{k^*-1}}{2^{k^o-1}} &= \frac{4}{k^* \ln 2} \\ k^* - k^o &= \frac{\ln 4 - \ln(\ln 2) - \ln(k^*)}{\ln 2}. \end{aligned} \tag{6}$$

Then, we have $k^* - k^o < 0$ (insufficient entry) iff $k^* > \frac{4}{\ln 2} \approx 5.77$, that is, iff $\frac{t}{F} > \frac{4}{\ln 2} 2^{\frac{4}{\ln 2}-1} \approx 157.53$. Therefore, in this case, ignoring the integer constraint, we may have excessive entry when $\frac{t}{F}$ is low and insufficient entry when $\frac{t}{F}$ is high. Manipulating (6), we obtain the ratio between the optimal number of innovators and equilibrium number of licenses,

$$\frac{k^o}{k^*} = 1 - \frac{\ln 4 - \ln(\ln 2) - \ln(k^*)}{k^* \ln 2},$$

which tends to 1 when k^* tends to infinity, that is, when $\frac{t}{F}$ tends to infinity.

In general, models with product differentiation can produce excessive or insufficient entry, depending on the degree of preference for variety, as shown by Mankiw and Whinston (1986). Interestingly, we find that the discrepancy between k^o and k^* changes sign when the game structure changes while maintaining the same type of product differentiation. For example, in Salop's (1979) exclusive licensing game with linear transport costs, the equilibrium number of firms is twice the number of optimal firms, that is, we have excessive entry.[6] However, in the sequential licensing game, there is insufficient entry when t is sufficiently high. In both cases, when t increases, the number of entrants also increases, but t only plays a role in determining whether there is excessive or insufficient entry in the sequential case, wherein the number

[6] Specifically, in the exclusive licensing setup, $k^* = t/F$, and $k^o = (t/F)/2$.

Table 1 Values of the ratio between the size of the market and the fixed cost for which we have excess entry ($\frac{k^*}{k^o} > 1$) or insufficient entry ($\frac{k^*}{k^o} < 1$) for different demand types

Demand	Excessive vs. insufficient entry
Linear	$\frac{k^*}{k^o} > 1$ always
Unitary elasticity	$\frac{k^*}{k^o} \gtreqless 1 \ iff \ \frac{A}{F} \gtreqless 6.48$
Salop	$\frac{k^*}{k^o} \gtreqless 1 \ iff \ \frac{t}{F} \lesseqgtr 157.53$

Table 2 Excessive entry comparisons, measured by the limit of $\frac{k^*}{k^o}$ as the ratio between market size and fixed cost tends to infinity, for different market games and demand types

	Game structure		
Demand	Exclusive licensing	Sequential licensing	Simultaneous licensing
Linear	$\frac{k^*}{k^o} \to \infty$	$\frac{k^*}{k^o} \to 2$	$\frac{k^*}{k^o} = 1/2$
Unitary elasticity	$\frac{k^*}{k^o} \to \infty$	$\frac{k^*}{k^o} \to 2$	$\frac{k^*}{k^o} = 2/3$
Salop	$\frac{k^*}{k^o} = 2$	$\frac{k^*}{k^o} \to 1$	$\frac{k^*}{k^o} = 2/3$

of licensees increases exponentially with the number of entrants. The effect of entry differs significantly when t is small or high. If t is small, we have few entrants, and we are in the flat part of the exponential function; therefore, new entry has a small impact on the number of competitors, and the situation is close to that of Salop (1979), where we have excessive entry. However, if t is high, we have many entrants and we are in the steep part of the exponential function so that new entry increases by much the number of licensees, which benefits society but hurts innovators, leading to an insufficient entry result.

Therefore, our sequential licensing version of Salop (1979) shows that the occurrence of excessive versus insufficient entry depends on the degree of product differentiation, which contrasts with the exclusive licensing setup.

The main results of Sect. 3 are summarized in Table 1. We identify whether we have excessive entry $\left(\frac{k^*}{k^o} > 1\right)$ or insufficient entry $\left(\frac{k^*}{k^o} < 1\right)$ depending on the ratio between market size and fixed cost for the different types of demand we have considered.

Next, we focus on the asymptotic results when the ratio of the market size to fixed costs tends toward infinity. We compare the results obtained in our setup (Sequential licensing) with those obtained in two alternative market games: exclusive licensing (Exclusive licensing) and the one in which non-exclusive licenses are chosen simultaneously (Simultaneous licensing). Table 2 presents these results

In Table 2, an arrow indicates asymptotic results, and equality means that the result always holds independent of market size. The only requisite is that the market size is sufficiently high to ensure $\max\{k^*, k^o\} \geq 2$. The results in the fourth column are based on Remarks 1, 2, and 2', after considering the following observations: i) in the linear case, $k^* = 1$ because potential innovators anticipate negative profits (net of entry innovation costs) with more than one active innovator, while $k^o = 2$ ensures the maximum level of competition with the minimum innovation costs; ii) under the unitary elasticity demand and in the spatial differentiation model in Salop (1979), we assume, first, that if $k = 2$, innovators are able to coordinate at the SPNE with the minimum level of competition (i.e., one signed license per innovator), which implies, by a similar reasoning to that used in the linear demand case, that $k^* = 2$, and $k^o = 3$.

The basic intuition of Table 2 is that the less competitive the post-entry market game the higher the excessive entry coefficient. On the one hand, less competition implies higher profits and higher private incentives to enter. This pushes k^* up. On the other hand, less competition implies lower social gains obtained by entry because its effects on price and/or variety are small. This pushes k^o down and explains that the market game with sequential licensing yields intermediate values for the limit of $\frac{k^*}{k^o}$ because its degree of competitiveness, in the post-entry market game, is midway between the simultaneous licensing game and the exclusive licensing setup.

The comparison between the sequential and simultaneous setups is particularly interesting. In the simultaneous case, there is always insufficient entry (see the fourth column of Table 2); in the sequential game (third column of Table 2), the ratio $\frac{k^*}{k^o}$ approaches 2 under homogeneous goods, and tends to 1 under product differentiation à la Salop, as market size increases. This substantial decrease in that ratio as we shift from the sequential to the simultaneous game is mainly owing to the drastic increase in licensing incentives in the simultaneous licensing setup, reflected in the fact that the equilibrium price equals the marginal cost with only two or three firms, depending on the particular demand structure.

These results imply that, from the social welfare perspective, the incentives to induce entry of new innovators are higher the more competitive the licensing stage is. Entry can be regulated by both adjusting the level of R&D subsidies[7] given to firms and by choosing the degree of tightness by which patent protection laws are implemented. In the former case, one induces entry by making more generous the policy of R&D subsidies given to firms while one reduces entry by making it less generous. In the latter case, inducing entry calls for a "relaxed" patent policy that is more permissive in allowing patents of analog technologies while a more "stringent" patent policy is needed when one wants to reduce the number of technological competitors.

In the next section we elaborate more on the specific role of patent policies, by extending our setup to the case in which patent regulation is combined with public investment in basic R&D.

[7] They usually take the form of cost subsidies. However, they may also subsidize the patent, that is, to give a prize to a firm that successfully innovates (Pérez-Castrillo and Sandonís 1996). In our case, as the outcome of the R&D investment is deterministic, both types of subsidies are equivalent.

Table 3 Destination of public R&D&i funds in 2012. Source: Eurostat. Cited from Xifré and Kasperskaya (2016)

	Percentages		
	Higher education	Government	Business enterprises
Finland	65,4	25,8	8,8
Italy	59,0	30,4	10,6
UK	58,7	21,9	19,4
France	52,4	31,7	15,9
Germany	49,1	40,8	10,1
Spain	46,3	38,1	15,5

Funds financed by the Government and executed by: higher education institutions, government and business enterprises and private non-profit firms

4 Extension: the role of public investment in basic R&D

In the previous section, we have discussed the potential use of public R&D subsidies given to private innovators to achieve the optimal number of entrants. However, most funds, financed by the government to promote R&D, are allocated to public institutions (either belonging to the government itself or higher education institutes). Table 3 illustrates this issue.

Then, the percentage of public R&D funds that goes to business enterprises and private non-profit firms ranges from 8,8 percentage points in Finland to 19,4 in the UK. So, the largest portion of this expenditure goes to universities and to research institutes belonging to the government.

It seems natural to study how the R&D developed by public institutions can be combined with the public R&D subsidies given to private firms to improve the innovation policy of the Government. The peculiarity of the R&D developed by public institutions is that it is mainly devoted to basic research, which is intended to improve the understanding of fundamental principles, usually with no immediate commercial benefits but serving as the basis of subsequent applied research leading to the appearance of new products in the market.

Different papers have found a positive impact of publicly funded science on private firms' performance. For example, Cockburn and Henderson (2001) find that public research had a very positive impact on the productivity of the US pharmaceutical industry in the last two decades of the twentieth century. In a more recent work, García-Vega and Vicente-Chirivella (2020) find that the technology transfer from universities stimulates private firms' innovation, using a large sample of Spanish firms. Anecdotal evidence can also be provided about the profitable link between basic research and the industry, such as NASA's innovations in materials that are subsequently used in industrial production processes and the Department of Energy (DOE) Office of Science that initiated the Human Genome Project and developed the noninvasive detection of cancers and other diseases, providing the base for further developments in the pharmaceutical industry (for more examples of this kind, see footnote 3 in González-Maestre and Granero 2013).

To include these ideas in the model, we assume that the applied technological entry cost of potential innovators, F, can be reduced by the public investment in basic R&D, given by $G(F) = \beta F^{-1}$. In words, if the public sector spends $G(F)$ in basic R&D, the technological entry cost of potential innovators will be F. Given that $G'(F) < 0$, the higher the level of basic R&D, the lower the entry cost. This simple formulation incorporates the idea that basic research works as a public good in the sense that it can simultaneously reduce the R&D cost of all innovators. The R&D expenditure of each innovator instead is applied, because it can only be used to develop the project of each particular firm.

In order to investigate the role of product differentiation, we focus on the spatial model, in the spirit of Salop (1979), analyzed in the previous section. In our analysis, we will make a distinction between the exclusive licensing game and the sequential setup, to investigate the impact of the intensity of competition in the licensing stage on the socially optimal level of basic R&D.

4.1 Exclusive licensing and basic R&D

We assume that the social planner chooses, simultaneously, the level of active innovators (which can be achieved by adjusting both the level of R&D subsidies given to firms and the degree to which the patent policy is implemented as explained in the previous section) and the level of public investment in basic R&D, given by $G(F) = \beta F^{-1}$.

From the calculations made in the previous sections (see Eq. (4) in Sect. 3.3), the social cost associated to a vector of choices (k, F) is given by

$$S_E(k, F) = kF + \frac{t}{4k} + \beta F^{-1} \tag{7}$$

The first-order condition of cost minimization with respect to F yields

$$\frac{\partial S_E}{\partial F} = k - \beta \frac{1}{F^2} \Rightarrow F = \left(\frac{\beta}{k}\right)^{1/2}. \tag{8}$$

As the second-order condition with respect to F is satisfied, we can substitute this expression in (7) to obtain the reduced social cost function as depending only on k

$$S_E(k, F(k)) = 2\beta^{1/2}k^{1/2} + \frac{t}{4k}.$$

From the derivative of this expression with respect to k, we obtain

$$\frac{dS_E}{dk} = \frac{1}{k^{1/2}}\left(\beta^{1/2} - \frac{t}{4k^{3/2}}\right) \lesseqqgtr 0 \Rightarrow \frac{t^2}{\beta} \gtreqqless H(k) = 16k^3.$$

Therefore, the second-order conditions are satisfied[8] and the equation function $\frac{t^2}{\beta} = H(k)$ indicates the relationship between parameters t and β and the socially optimal level of k.

4.2 Sequential licensing and basic R&D

By applying a similar reasoning to the previous subsection, the social cost in the sequential licensing setup is given by

$$S_L(k, F) = kF + \frac{t}{4 \times (2^{k-1})} + \beta F^{-1}.$$

Again, the first-order condition with respect to F implies that the relationship in expression (8) still holds, which implies that the following reduced social cost function holds:

$$S_L(k, F(k)) = 2\beta^{1/2}k^{1/2} + \frac{t}{4 \times (2^{k-1})}$$

The first derivative of this function with respect to k yields

$$\frac{dS_L}{dk} = \frac{1}{4k^{1/2}(2^{k-1})}\left(4\beta^{1/2}2^{k-1} - tk^{1/2}\right) \lesseqgtr 0 \Leftrightarrow \frac{t^2}{\beta} \gtreqless R(k) = \frac{16(2^{k-1})^2}{(\ln 2)^2\, k}.$$

Given that $R(k)$ is strictly increasing in k,[9] the restriction $k \geq 2$ is satisfied for $\frac{t^2}{\beta} \geq R(2) = 66.6$.

4.3 Comparisons between the exclusive licensing game and the sequential licensing game

Routine calculations show that $R(k) \gtreqless H(k) \Leftrightarrow \frac{t^2}{\beta} \gtreqless 2332.5$. This comparison is reflected in Fig. 1, where $y \equiv \frac{t^2}{\beta}$, $H(k)$ is the thin line, and $R(k)$ is the thick line. Define k^o (respectively, k^{oo}) as the optimal number of innovators in the exclusive licensing game (respectively, sequential licensing game). From our previous calculations, and the observation of Fig. 1, it turns out that $k^{oo} \gtreqless k^o \Leftrightarrow \frac{t^2}{\beta} \lesseqgtr 2332.5$. Therefore, optimal entry is larger (respectively, smaller) in the sequential licensing game for small (respectively, large) degrees of product differentiation. Notably, according to expression (8) higher levels of optimal k are associated with smaller levels of F and, consequently, with larger levels of basic innovation investment. Intuitively, for small degrees of product differentiation, entry tends to be small, which implies a

[8] Given that $H'(k) > 0$, the first derivative is negative (respectively, positive) for low (respectively, high) values of k. Then $S_E(F, F(k))$ is quasi-convex in k.

[9] This implies for the same argument, as in the previous footnote that, $S_L(F, F(k))$ is quasi-convex.

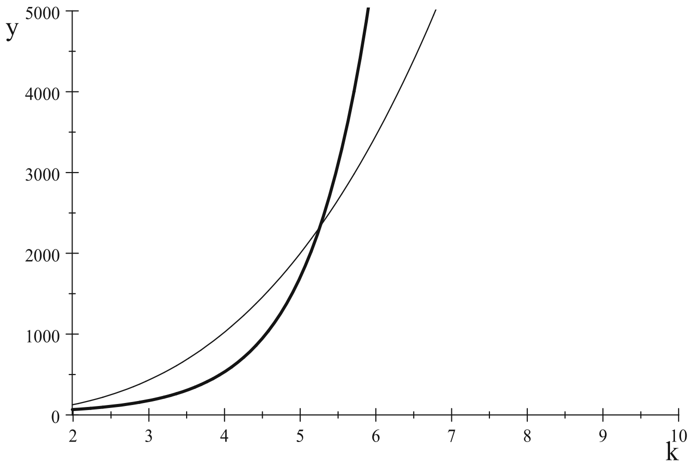

Fig. 1 Comparison between the optimal level of k in the non-licensing game (thin line) and in the sequential licensing game (thick line) as functions of the relative degree of product differentiation y

relatively restricted pro-competitive effect of the exponential licensing proliferation associated with the sequential licensing game. In consequence, the social planner has a substantial comparative incentive to increase entry (through a combination of higher basic innovation and large R&D subsidies), to favor such exponential effect. By contrast, under large levels of product differentiation, entry is large and this exponential effect does not need much stimulus from the planner, in comparison with the game with exclusive licensing.

Therefore, the main insight in this section can be summarized as follows. In product markets with small levels of product differentiation, social incentives to increase entry are higher in the more competitive licensing game (sequential versus exclusive licensing). However, this result is reversed in the case of high levels of product differentiation, where we should expect smaller levels of basic innovation under more competitive licensing markets.

5 Conclusions and final remarks

The literature on licensing has mainly analyzed the case where the number of licenses is chosen simultaneously by patent holders. We have analyzed instead, as Badia et al. (2020), the case wherein non-exclusive licenses are sequentially chosen by innovators. As obtained in the model of endogenous timing of non-exclusive licensing contracts developed by Badia et al. (2020), in the particular case of two innovators, the competitive effect of licensing is smaller when the number of licenses are chosen sequentially rather than simultaneously. However, by extending Badia, Tauman, and Tumendemberel's two innovators model to a more general number of innovators, we find that this difference in the level of competition narrows dramatically with the number of innovators because the incentives to create new licenses exponentially grow with that number. This sequential outcome is also obtained as an SPNE in pure strategies of a game with endogenous timing. More interestingly, by extending Badia, Tauman, and Tumendemberel's endogenous timing to the case of more than two innovators and exploring pure strategy equilibria instead of mixed strategy equilibria, our analysis yields drastically different policy implications. Our results suggest that more competition in the upstream market (e.g., by relaxing patent protection against the appearance of similar technologies) tends to increase downstream competition and welfare instead of discouraging or delaying technology adoption.

Our analysis has been extended to explore the interaction between the mentioned public policies intended to regulate optimal innovators' entry, with the strategic role of public investment in basic R&D under product differentiation. In this extended scenario, we show that the optimal level of public investment in basic R&D must be higher (respectively, smaller) under sequential licensing than under exclusive licensing if the degree of product differentiation is small (respectively, large).

For tractability, our model was simplified into some dimensions. We ignored the possibility that each licensee must pay an exogenous entry cost, apart from the fee paid to the licensor. This simplifying assumption outweighs the additional complexity of combining sequential licensing with an arbitrarily large number of innovators. Nevertheless, first-sight intuition suggests that the equilibrium number of competing licenses would decrease with entry costs because of decreasing licensing incentives. In general, the equilibrium number of licenses is somewhere between ours and an exclusive licensing game.

Our model also assumes that innovators are risk neutral, implying that some innovators might obtain negative profits in the post-entry game. Under the alternative assumption of risk aversion, it is easy to argue that the higher the level of risk aversion, the smaller is the number of active innovators. This conclusion follows directly by observing that, at the free-entry equilibrium of our model, the expected profit is equal to zero, and risk-averse potential innovators prefer the zero profit level with certainty, associated with remaining non-active, to an expected zero pay-off under entry. Therefore, the higher the risk aversion level, the higher must be the expected nonzero profit, which directly implies a lower number of equilibrium entrants. A similar argument applies when innovators are subject to loss aversion, as described in behavioral economics literature. This literature shows that loss aversion is the most important factor in determining risk aversion, and that people are most risk-averse in risky situations involving both gains and losses from a reference point. An extreme case of this deviation from the standard rational approach would imply, in the context of our model, that the free-entry condition should warrant nonnegative profits for any entrant. In such situation, because the innovator that acts as the first mover in the post-entry licensing game only obtains the profits of one single producer, the free-entry condition would easily imply the same number of independent final product sellers as in the *exclusive* licensing game. In general, for moderate levels of loss aversion, the equilibrium number of active sellers in the production stage lies somewhere between the scenario with exclusive licensing and that with sequential licensing.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

6 Appendix 1 (endogenous timing)

(i) Proof that sequential licensing is a SPNE under endogenous timing

Assume that, at the licensing stage, each innovator decides the period t (arbitrarily large) at which it commits to a particular number of licenses. We show that the following strategy profile is an SPNE at the licensing stage: Each firm $i = 2, .., k$ announces that it will choose its number of licenses in the immediate period after innovator $i - 1$ makes its choice, and innovator $i = 1$ announces that it will make its choice at period 1. Notably, if this strategy profile is implemented, yields a sequential timing which is equivalent to the one assumed in our model with exogenous timing. To show that no innovator is interested in unilaterally deviating from its strategy, observe that, if innovator i delays its choice till period $t \geq i + 1$, then it will trigger a credible delay of $(t - i) \geq 1$ periods for any innovator $j \geq i + 1$, in accordance with its announced strategy. In consequence, the deviating innovator does not reach any advantage from its deviation strategy as the order of moves remains the same.

(ii) Extension to the case of sequential production

Consider the assumption that production occurs immediately after signing a contract and that the discount factor δ is sufficiently large. By using an argument similar to that in part (i) in this appendix and using the same type of strategies as those used to sustain the SPNE in the game considered in that part, it is easy to infer that these strategies are still an SPNE under the new assumptions. This result is obtained by observing, first, that the presence of time discounting reduces innovators' incentives to delay their licensing decisions. Second, because δ is assumed to be sufficiently large, no innovator is interested in anticipating its licensing contracts, with respect to the timing implied by the strategies described in part (i) of this appendix. Therefore, the result for part (i) holds under sequential production.

7 Appendix 2

Proof of Proposition 3 To show that this is the optimal strategy, for each innovator, consider, first, the profit-maximization decision of the last firm, T:

$$\pi_T(m_T, M_{T-1}) = \frac{Am_T}{(M_{T-1} + m_T)^2} \Rightarrow \frac{\partial \pi_T}{\partial m_T} = \frac{A(m_T + M_{T-1} - 2m_T)}{(M_{T-1} + m_T)^3} = 0$$
$$\Rightarrow m_T = M_{T-1}.$$

This property follows for the last innovator (T). Now, assume that, given $t < T$, the property is satisfied for any t', such that $T \geq t' > t$. Under this assumption, and considering $M_t \equiv m_t + M_{t-1}$, we have the following profit-maximization decision for innovator t:

$$\begin{aligned}
\pi_t(m_t, M_{t-1}) &= \frac{Am_t}{\left(M_t + M_t + 2M_t + 4M_t + 8M_t + ... + 2^{T-t-1}M_t\right)^2} \\
&= \frac{Am_t}{2^{2(T-t)}(m_t + M_{t-1})^2} \Rightarrow \\
\frac{\partial \pi_t(m_t, M_{t-1})}{\partial m_t} &= \frac{A(m_t + M_{t-1} - 2m_t)}{2^{2(T-t)}(m_t + M_{t-1})^3} = 0 \Rightarrow m_t = M_{t-1}.
\end{aligned}$$

Therefore, we have shown that, given t, if $m_{t'} = M_{t'-1}$, for any $t' > t$ then $m_t = M_{t-1}$. Because we have proved that this property is satisfied for $t' = T$, mathematical induction implies that $m_t = M_{t-1}$ for any $t = 1, ..., T$, which completes the proof. □

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