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ORIGINAL ARTICLE

Horizontal licensing in vertically related markets

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Abstract
We study the incentives of a downstream firm that sources its core input from a vertically integrated supplier to license its patented technology to an external firm. Licensing transforms the licensee into both a direct downstream competitor and a customer of the supplier. The vertically integrated supplier trades with his competitors/customers through a two-part tariff contract. We find that the incumbent opts for licensing when the supplier provides the input to both the licensor and the licensee. Although licensing intensifies downstream competition, the licensor benefits from the lower input cost and the supplier from the expansion of the market. Licensing incentives in a vertical market are weaker compared to a one-tier market. Moreover, licensing has a positive impact on both consumer and total welfare. Finally, licensing incentives and welfare enhancement continue to occur under a wider set of conditions such as different types of licensing contracts, input trading contracts and market structures.

Keywords Licensing · Vertical relations · Vertical integration · Entry · Two-part tariffs

JEL Classification D45 · L13 · L22 · L24 · L42

1 Introduction

Licensing patented technology is a common practice in many industries and can potentially transform the licensee into a competitor of the licensor. Often both the licensor and the licensee operate in the downstream sectors of vertically related markets and source their inputs from vertically integrated suppliers. For instance, in the car market, Tesla, the American automotive corporation that specializes in electric vehicles, provides lithium-ion batteries and other powertrain components to Toyota, the Japanese

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automotive manufacturer, for its electric vehicle RAV4. In 2010, Toyota announced that it licensed its hybrid technology to Daimler, the German automotive manufacturer. In 2012, Tesla started supplying lithium-ion batteries and electric powertrain components to Daimler for the Mercedes-Benz B-class electric car.[1]

Similarly, in the smartphone market, the Japanese conglomerate Sony supplies Apple, the American technology company, with camera sensors for its smartphones (i-phones). In 2012, Apple and HTC, the Taiwanese consumers electronics company, reached a 10-year licensing agreement covering current and future patents and in 2016, HTC announced it will use Sony camera sensors for its new smartphone.[2]

In this paper, we examine the incentives for horizontal licensing in vertically related markets. We focus on the case in which the licensor and his potential licensee source an input from a vertically integrated firm and address a number of questions, such as: When does a downstream firm license its technology to a potential competitor? Does a vertically integrated firm have incentives to provide the input to both the licensor and the licensee? How does the presence of a vertically integrated firm affect the incentives for licensing? Is licensing in a vertically related market welfare enhancing?

We consider a framework in which a downstream firm sources an input from a vertically integrated competitor. The downstream firm considers licensing its patented technology to an external firm for a fixed licensing fee, transforming the latter into both a downstream competitor and a customer of the vertically integrated supplier. The vertically integrated firm decides whether it will supply its competitor(s)-customer(s). In the case where the supplier serves the downstream firm(s), input trading takes place through a two-part tariff contract whose terms are determined through bargaining. Then, firms compete in quantities in the final goods market.

Licensing triggers the entry of the licensee in the final goods market, and thus, by increasing the number of the downstream firms, it cannibalizes the demand of the licensor's product. We refer to this as the *cannibalization effect* of licensing. Moreover, licensing results in the *business-stealing effect* and the *market expansion effect*. The *business-stealing effect* refers to the fact that the licensee 'steals' part of the sales and the market share of the vertically integrated firm. The *market expansion effect* refers, instead, to the fact that the licensee's entry increases product variety, and thus, expands demand in the final goods market. We find that the vertically integrated supplier prefers the provision of the input to both the licensor and the licensee when the final product differentiation is high and its bargaining power is not too low. This is due to the fact that then the *market expansion effect* is strong and dampens the negative impact of the *business-stealing effect*, while the supplier can extract a large share of its customers' profits through the fixed fee of the two-part tariff contract.

Importantly, licensing in a vertical market brings about the *input pricing effect*. This refers to the fact that now the licensor obtains the input at a lower wholesale price. One would expect that the vertically integrated supplier has incentives to raise its rivals' cost in order to enlarge its own downstream market share. However, there is another force

[1] For more on this, see, e.g., Toyota May Supply Daimler with Hybrid Parts: Report, *Reuters* (September 16, 2010), and What Do Toyota and Mercedes See In Tesla? A Bit Of Themselves, *Forbes* (June 1, 2013).

[2] For more on this, see, e.g., Apple, HTC Settle Patent Suits; Sign 10-Year Licensing Deal, *Forbes* (November 11, 2012) and HTC One M10 Has Sony IMX377 Camera Sensor, Same as Nexus 6P/5X, *Softpedia News* (February 11, 2016).

in action, namely, the inability of the supplier to publicly commit to specific contract terms to all downstream customers. This inability makes him behave opportunistically and set a lower wholesale price. This is called the commitment problem and in contrast to many studies (e.g., Rey and Tirole 2007; Reisinger and Tarantino 2015), it cannot be perfectly solved by the supplier's vertical integration. Intuitively, the licensor benefits from his lower cost and, by extracting the licensee's profits through the fixed licensing fee, takes full advantage of both the *business-stealing effect* and the *market expansion effect*. This offsets the negative impact of the *cannibalization effect*, rendering licensing desirable when the vertically integrated supplier provides the input to both the licensor and the licensee. Indeed, the *input pricing effect*, which is absent in a market that firms are only horizontally related, allows the supplier—that is, the firm that is not included in the licensing agreement—to affect his rivals' cost, by improving his position even more in a vertical market rather than in a market where there is no vertical contract. This makes licensing incentives more likely to occur in a one-tier market rather than in a vertically related market.

Licensing is beneficial not only for the licensor, but also for the consumers and the economy as a whole. Consumers benefit due to the new entry into the downstream market, which intensifies competition, increases product variety and decreases the firms' cost, thus, resulting in lower final prices. Also, licensing has a positive impact on both the licensor and the supplier's profits, which, in turn, enhances total welfare.

After analyzing various extensions of the main model such as different types of licensing contracts, input trading contracts, market structures and downstream competition, we find that licensing in a vertically related market can still be beneficial for both the licensor and total welfare. Licensing through a per-unit royalty brings about the *input price discrimination effect*, which refers to the charging of different wholesale prices among the licensor and the licensee, because of the downstream cost asymmetry due to the per-unit royalty. Although the licensor pays a higher wholesale price than the licensee, licensing incentives still exist and the impact on total welfare continues to be positive, but both are weaker compared to licensing through a fixed fee. Licensing continues to emerge and enhance total welfare when the supplier is not a final goods producer, when input trading takes place through a wholesale price contract, when the supplier bargains sequentially with his customers, when downstream competition is in prices, as well as, when the final products of the licensor and the licensee differ less between them than they differ from the supplier's product.

The paper proceeds as follows. In Sect. 2, we cover the related literature, and in Sect. 3, we describe our main model. In Sect. 4, we present the equilibrium analysis. In Sect. 5, we determine the licensing incentives. In Sect. 6, we evaluate the welfare implications of a horizontal licensing in a vertical market. In Sects. 7 and 8, we extend our main model, and in Sect. 9, we conclude. All the proofs are included in the Appendix.

2 Literature review

Most of the literature on technology licensing analyzes various aspects of licensing in one-tier markets, such as the choice among licensing fee and royalties (Kamien and

Tauman 1986; Muto 1993; Wang 1998; Colombo et al. 2023), the use of licensing for entry deterrence (Gallini 1984), the impact of licensing on innovation (Gallini and Winter 1985), the impact of competition among patent holders on licensing incentives (Arora and Fosfuri 2003) and the choice among licensing and merger (Fauli-Oller and Sandonis 2003). Our paper is closely related to the limited literature that examines both horizontal and vertical licensing in vertically related markets. This literature shows that downstream horizontal licensing is profitable if it enhances competition in the upstream and the downstream market (Mukherjee 2003; Mukherjee and Ray 2007) or if it creates a weak rival (Arya and Mittendorf 2006). Moreover, vertical licensing can create a seller–buyer relationship among the licensor and the licensee that can be profitable if it intensifies downstream competition (Rey and Salant 2012; Bakaouka and Milliou 2018), but it may also be welfare reducing (Lin et al. 2022). In contrast to these papers, we consider horizontal licensing in the downstream market that creates a new rival who competes with both the licensor and the input supplier in the final goods market. We highlight the role of licensing and its impact on vertical input trading under different specifications of contracting. We show that licensing causes vertical interactions leading the licensor to achieve more efficient input pricing and the input supplier to not passively accept the other firms' decisions, which is crucial for the survival of the licensee in the market.

Our paper also has common elements with the literature of horizontal divisionalization (Corchón 1991; Baye et al. 1996; Ziss 1998; Creane and Davidson 2004). In particular, the increase in the number of competing firms through divisionalization resembles the licensee's entry in the downstream market and the multidivisional firm's gains from divisionalization resemble the licensor's licensing revenues. However, this literature does not focus on vertical relationships. Exceptions include the papers by Bru et al. (2001) and Mizuno (2009), which examine downstream divisionalization in vertically related markets. However, in contrast to our paper, they do not allow for vertically integrated firms and do not examine the role of input trading.[3]

Our paper is also related to the literature on vertical contracting that considers settings in which an upstream firm trades with multiple competing downstream firms (Rey and Vergé 2004; Milliou and Petrakis 2007; Rey and Tirole 2007). However, these branches of the literature do not allow for vertically integrated input suppliers.[4] Exceptions are the papers by Arya et al. (2007) and Reisinger and Tarantino (2015) which analyze vertical integration of the upstream monopolist with an existing downstream firm and the incentives for supplying a rival retailer. However, they do not consider a supplier who trades with multiple competing downstream firms and, at the same time, has his own direct channel in the downstream market, and thus, ignore that the commitment problem can still be present with a vertically integrated supplier. The analysis of a vertically integrated firm's incentives to supply the input to a downstream entrant is also considered by Ordover and Shaffer (2007), Brito and Pereira (2010) and

[3] Bru et al. (2001) examined an efficient upstream firm's incentives to vertically integrate with a downstream firm and serve the rival firm's divisions. They show that if vertical integration occurred, the rivals would have stronger incentives for divisionalization and thus vertical integration would be undesirable.

[4] The literature on outsourcing also deals with vertical integration and vertical contracting (Chen 2001; Chen et al. 2004; Sappington 2005; Arya et al. 2008). However, these papers focus on downstream firms' 'make or buy' decision.

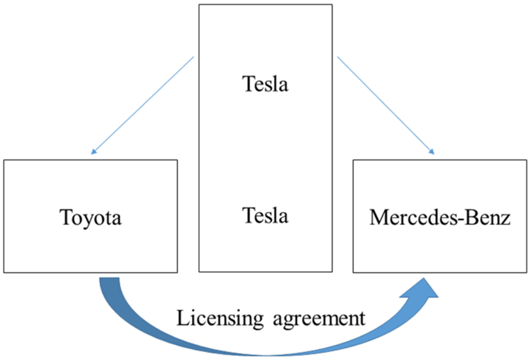


Fig. 1 Example of licensing agreement and input provision among Tesla, Toyota and Mercedes-Benz

Bourreau et al. (2011). However, these papers do not focus on licensing incentives and consider different market structures with multiple vertically integrated firms trading with only one downstream firm.

3 The model

We consider a market consisting initially of two firms, a downstream firm D (e.g., Toyota in Fig. 1) and a vertically integrated firm I (e.g., Tesla in Fig. 1). Each firm has a patented technology and produces a differentiated final good using, in a one-to-one proportion, a core input that firm I produces in-house at marginal cost $c > 0$. Firm D sources the input from firm I through a two-part tariff contract that includes a fixed fee, T_D, and a wholesale price per unit, w_D.

Firm D considers licensing its patented technology to an external firm E (e.g., Mercedes-Benz in Fig. 1)—this technology being an essential input—for a fixed licensing fee, $F \geq 0$.[5] Licensing will allow firm E to produce a differentiated version of the final good, transforming it into a downstream competitor of both firm D and firm I. Firm E will also source the core input from firm I, after paying a fixed fee, T_E, and a wholesale price per unit, w_E.

The (inverse) demand function for firm i's final good is:

$$p_i(q_i, Q_{-i}) = \alpha - q_i - \gamma Q_{-i},\ 0 < \gamma < 1,\ a > c > 0,$$

where p_i and q_i are the price and the quantity of firm i's final good, respectively, and Q_{-i} is the quantity of its rival(s)' final good(s). Specifically, $Q_{-i} = q_j$, with $i,j = D, I$ and $i \neq j$ in the no licensing case, while $Q_{-i} = q_j + q_k$, with $i, j, k = D, I, E$ and $i \neq j \neq k$, in the licensing case. The parameter γ measures the degree of product differentiation; namely, the higher γ is, the closer substitutes the final goods are.

[5] In Sects. 7 and 8, we provide a series of extensions. In Sect. 7, we examine what happens when licensing takes place through a per-unit royalty and discuss the cases of licensing through a two-part tariff and ad valorem profit contracts. Furthermore, in Sect. 8 we examine what happens when firms bargain over F and when firm I, instead of firm D, considers licensing its own-patented technology.

The timing of moves is as follows. First, firm D decides whether to license its patented technology to firm E. In the licensing case, it sets the licensing fee F and, in turn, firm E signs or not the licensing agreement. In the following stage, in the no licensing case, firm I decides whether to adjust its input production capabilities so as to supply firm D. Similarly, in the licensing case, it decides whether to adjust its input production capabilities so as to supply both firm D and firm E, one of them, or none of them.[6] In the third stage, firm I bargains with its customer(s) over the contract terms.[7] To model the bargaining game, we invoke the Nash equilibrium of simultaneous generalized Nash bargaining games, in which the bargaining power of firm I is given by β and that of the downstream firm(s) by $1-\beta$, with $0<\beta<1$. We assume that firm I bargains with each downstream firm simultaneously and separately and that during the negotiations of a bargaining pair, each of its agents takes as given the outcome of the simultaneously run negotiations of the other bargaining pair.[8] To avoid the multiple equilibria that may arise due to the multiplicity of the beliefs that the downstream firms can form when they receive out-of-equilibrium offers, we impose pairwise proofness on the equilibrium contracts. That is, we assume that a contract between a bargaining pair is immune to a bilateral deviation of a rival pair's contract, holding the former contract constant. Pairwise proofness is closely related to the passive beliefs assumption (Hart and Tirole 1990; McAfee and Schwartz 1994, 1995). Downstream firms have passive beliefs, i.e., when firm i receives an out-of-equilibrium offer, it keeps on believing that firm j has received the equilibrium offer (Hart and Tirole 1990; Rey and Vergé 2004). Another assumption of our bargaining game is that the contract terms of a bargaining pair are not contingent on the disagreement of a rival pair. In particular, a bargaining pair is unable to write and implement a contract specifying different contract terms depending on the successful termination or breakdown of a rival pair's negotiations. This assumption captures the idea that bargaining parties are unable to commit to a permanent and irrevocable breakdown in their negotiations and is in a similar vein with the assumption of separate and simultaneous negotiations. This assumption is also adopted by Horn and Wolinsky (1988), O'Brien and Shaffer (1992), McAfee and Schwartz (1994 and 1995), Caprice (2006), and Milliou and Petrakis (2007). In the last stage, the firms which are active in the downstream market choose their quantities simultaneously and separately, after observing each other's

[6] We implicitly assume that in order to supply a downstream customer, firm I has to undertake investments in order to increase the size of its input production facilities or to open new input production lines. Without these investments, firm I will not be in the position to serve its downstream customer(s). Another justification of firm I's decision not to commit to serve its customer(s) is that it is equivalent to a refusal to supply the downstream competitor(s) (Fumagalli and Motta 2020). The effect of the inclusion of stage 2—in which firm I has the ability to exclude firms from the input market—is discussed in Sect. 5.

[7] The justification for this order of moves is that the contract terms are easier to change than the input production capabilities of firm I and their determination is a shorter run decision and is adjusted to actual market conditions.

[8] Such an assumption is common in the literature on multilateral contracting (Cremer and Riordan 1987; Horn and Wolinsky 1988; Hart and Tirole 1990; O'Brien and Shaffer 1992; McAfee and Schwartz 1994, 1995; Rey and Vergé 2004; Milliou and Petrakis 2007; Alipranti et al. 2014).

contract terms. Therefore, we assume that the contract terms are *interim observable*.[9] We solve for the subgame perfect Nash equilibrium of this game.

Our notational convention will be as follows: we will use the superscript N and L to denote whether we are in the no licensing or in the licensing case, respectively.

4 Downstream competition and bargaining

In this section, we perform the equilibrium analysis with and without licensing.

4.1 No licensing

When firm D does not license its patented technology to firm E, there are two possible third-stage subgames. In the first subgame, firm I commits to not serving firm D, and thus, makes monopoly profits, $\pi_{\mathrm{I}}^{\mathrm{M}} = \frac{(\alpha-c)^2}{4}$. In the second subgame, firm I can serve firm D, thus, both firms can be present in the downstream market. In such a case, in the last stage, firm D and firm I choose their quantities, q_D and q_I, in order to maximize their (gross from T_D) profits:

$$\pi_D(q_D, q_I, w_D) = (\alpha - q_D - \gamma q_I - w_D)q_D; \tag{1}$$

$$\pi_I(q_D, q_I, w_D) = (\alpha - q_I - \gamma q_D - c)q_I + (w_D - c)q_D. \tag{2}$$

Solving the system of the first order conditions, we find:

$$q_D(w_D) = \frac{\alpha(2-\gamma) + c\gamma - 2w_D}{4-\gamma^2}; \tag{3}$$

$$q_I(w_D) = \frac{\alpha(2-\gamma) - 2c + \gamma w_D}{4-\gamma^2}. \tag{4}$$

In the previous stage, firm I and firm D negotiate over (w_D, T_D). In particular, they solve the following generalized Nash bargaining problem:

$$\max_{w_D, T_D} [\pi_I(w_D) - d_I + T_D]^{\beta}[\pi_D(w_D) - T_D]^{1-\beta} \tag{5}$$

where $\pi_D(w_D)$ and $\pi_I(w_D)$ are found after substituting (3) and (4) into (1) and (2), respectively. The disagreement payoff of firm D is null since firm D does not have an outside option. On the other hand, firm I has an outside option in its bargaining with firm D: in case of disagreement, firm I can gain monopoly profits from its own sales in the final goods market, given by the disagreement payoff $d_I \equiv \pi_{\mathrm{I}}^{\mathrm{M}}$. Maximization

[9] This is in line with Horn and Wolinsky (1988), McAfee and Schwartz (1995), and Milliou and Petrakis (2007), where contract terms are secret in the contracting stage, but are observed by the downstream firms before the downstream competition stage.

of (5) with respect to T_D, yields:

$$T_D = \beta\pi_D(w_D) - (1-\beta)[\pi_I(w_D) - d_I]. \tag{6}$$

Using the above expression, we find:

$$\pi_I(w_D) - d_I + T_D = \beta[\pi_I(w_D) + \pi_D(w_D) - d_I]; \tag{7}$$

$$\pi_D(w_D) - T_D = (1-\beta)[\pi_I(w_D) + \pi_D(w_D) - d_I]. \tag{8}$$

Substituting (7) and (8) into (5), we note that the latter reduces to an expression proportional to the joint profits of firms *I* and *D* minus firm *I*'s disagreement payoff. Thus, in the setting of the wholesale price, firm *D* and firm *I* behave as a multiproduct monopolist. The wholesale price that maximizes this expression is:

$$w_D^N = \frac{\alpha(2-\gamma)^2\gamma + c(8-\gamma(4+\gamma(2+\gamma)))}{8-6\gamma^2}. \tag{9}$$

One can easily check that $w_D^N > c$. Setting a positive mark-up, firm *I* decreases the aggressiveness of its rival in the final goods market. Moreover, we can observe that w_D^N is decreasing in the degree of product differentiation, $\frac{\partial w_D^N}{\partial \gamma} > 0$. This means that the closer substitutes the two final products are, and thus, the fiercer the downstream market competition is, the higher the wholesale price is. In other words, when downstream market competition becomes fiercer, firm *I*'s incentives to decrease firm *D*'s aggressiveness get stronger.

Using (9), (6), (4) and (3), we obtain the equilibrium outputs and the fixed fee, when firm *I* serves firm *D* in the no licensing case:

$$q_D^N = \frac{2(\alpha-c)(1-\gamma)}{4-3\gamma^2}, \quad q_{\mathrm{I}}^N = \frac{(\alpha-c)(4-\gamma(2+\gamma))}{8-6\gamma^2}; \tag{10}$$

$$\mathrm{T}_D^{\mathrm{N}} = \frac{(\alpha-c)^2(1-\gamma)^2(4\beta+3(1-\beta)\gamma^2)}{(4-3\gamma^2)^2}. \tag{11}$$

The resulting firms' equilibrium profits are included in Table 1 of the Appendix.

In the second stage, firm *I* decides whether to provide the input to its customer or to foreclose him from the market. Importantly, firm *I* is always better off when it faces downstream competition than when it is a monopolist in the final products market, namely, $\pi_{\mathrm{I}}^{\mathrm{N}} > \pi_{\mathrm{I}}^{\mathrm{M}}$. Even though the presence of firm *D* in the final goods market increases the number of downstream firms, the fact that firm *I* and firm *D* set the wholesale price in a collusive way, leads to a decrease of the negative impact of intensified competition. In this way, firm *I*'s benefit from the input sales increment, due to the greater input demand, is larger than the loss from the fiercer downstream competition. In addition, both firms *D* and *I* behave independently in the competition stage, and due to $w_D^N > c$, firm I produces more than firm D in the final goods market ($q_I^N > q_D^N$). Thus, firm I manages to generate profits that exceed the profits of a single-product monopolist, but they are lower than those of a multiproduct monopolist. As Reisinger and Tarantino (2015) also note in their paper, if the products were homogeneous, w_D^N would be such that firm D would be foreclosed from the market. In such a case, firm I would make monopoly profits. However, when products are even slightly differentiated, foreclosure is not profitable. In case that final products are independent ($\gamma = 0$), and thus, there is no downstream competition, it holds that $w_D^N = c$, since there are no incentives in distorting efficiency.

4.2 Licensing

We examine now the case in which firm D licenses its patented technology to firm E. When firm E signs the licensing agreement, there are four possible third-stage subgames. In the first subgame, firm I commits to not serving firms D and E, and thus, gains the monopoly profits π_I^M. In the second and third subgames, firm I serves only firm D or only firm E, and the equilibrium outcomes coincide with the respective ones in the no licensing case, i.e., w_i^N, π_i^N, π_I^N. In the last subgame, which we analyze next, firm I serves both firm D and firm E.

In the last stage, firms D, I and E choose their outputs in order to maximize their profits given by:

$$\pi_D(q_D, q_I, q_E, w_D, w_E) = (\alpha - q_D - \gamma q_I - \gamma q_E - w_D)q_D; \tag{12}$$

$$\pi_I(q_D, q_I, q_E, w_D, w_E) = (\alpha - q_I - \gamma q_D - \gamma q_E - c)q_I + (w_D - c)q_D + (w_E - c)q_E; \tag{13}$$

$$\pi_E(q_D, q_I, q_E, w_D, w_E) = (\alpha - q_E - \gamma q_D - \gamma q_I - w_E)q_E. \tag{14}$$

Solving the system of the first order conditions, we obtain:

$$q_D(w_D, w_E) = \frac{\alpha(2-\gamma) - 2w_D + \gamma(c - w_D + w_E)}{2(2-\gamma)(1+\gamma)}; \tag{15}$$

$$q_I(w_D, w_E) = \frac{\alpha(2-\gamma) - c(2+\gamma) + \gamma(w_D + w_E)}{2(2-\gamma)(1+\gamma)}; \tag{16}$$

$$q_E(w_D, w_E) = \frac{\alpha(2-\gamma) + \gamma(c + w_D) - (2+\gamma)w_E}{2(2-\gamma)(1+\gamma)}. \tag{17}$$

In the previous stage of the game, firm I bargains with each downstream firm i, with $i = D{,}E$, over (w_i, T_i) taking as given the outcome of the simultaneously run two-part tariff negotiations with the downstream rival j, where $j = D, E$, with $i \neq j$. Letting (w_j^L, T_j^L) denote the equilibrium bargaining outcome offered to rival firm j, w_i and T_i are chosen to solve the following maximization problem:

$$\max_{w_i, T_i} [\pi_I\left(w_i, w_j^L\right) - d_I^O + T_i + T_j^L]^{\beta} [\pi_i\left(w_i, w_j^L\right) - d_i^O - T_i]^{1-\beta} \tag{18}$$

where $\pi_I\left(w_i, w_j^L\right)$ and $\pi_i\left(w_i, w_j^L\right)$ are found after substituting (15), (16) and (17) into (12), (13) and (14), respectively. If an agreement between firms I and i is not reached, then firm I's disagreement payoff is $d_I^O\left(w_j^L, T_j^L\right) \equiv \left(a - q_I^O\left(w_j^L\right) - \gamma q_j^O\left(w_j^L\right) - c\right) q_I^O\left(w_j^L\right) + \left(w_j^L - c\right) q_j^O\left(w_j^L\right) + T_j^L$ where $q_j^O\left(w_j^L\right)$ and $q_I^O\left(w_j^L\right)$ are given by (3) and (4) when $w = w_j^L$, respectively. However, each downstream firm has a different disagreement payoff, d_i^O, when it does not reach the agreement with firm I. In particular, firm D's disagreement payoff is $d_D^O \equiv F$, i.e., the licensing revenues that firm D can extract through the fixed licensing fee since firm E would be present in the market. Instead, firm E's disagreement payoff is null, $d_E^O \equiv 0$, since firm E does not have an outside option. Maximizing (18) with respect to T_i, we get:

$$T_i = \beta\left[\pi_i\left(w_i, w_j^L\right) - d_i^O\right] - (1-\beta)\left[\pi_I\left(w_i, w_j^L\right) - d_I^O\right]. \tag{19}$$

Using the above expression, we find:

$$\pi_I\left(w_i, w_j^L\right) - d_I^O + T_i = \beta\left[\pi_I\left(w_i, w_j^L\right) + \pi_i\left(w_i, w_j^L\right) - d_I^O - d_i^O\right]; \tag{20}$$

$$\pi_i\left(w_i, w_j^L\right) - d_i^O - T_i = (1-\beta)\left[\pi_I\left(w_i, w_j^L\right) + \pi_i\left(w_i, w_j^L\right) - d_I^O - d_i^O\right]. \tag{21}$$

Substituting (20) and (21) into (18), we note that the latter reduces to an expression proportional to the joint profits of firms I and i minus their disagreement payoffs. Due to the problem's symmetry, the wholesale prices that maximize this expression are:

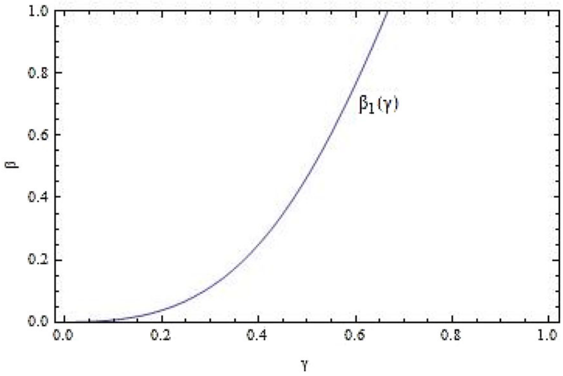
$$w^L = w_i^L = w_j^L = \frac{\alpha(2-\gamma)\gamma + c(2+\gamma)^2}{4+6\gamma}. \tag{22}$$

We find that $w^L > c$ for the same reason as in the no licensing case. However, we now find that an increase in product differentiation decreases the wholesale price when the product differentiation is not sufficiently low ($\gamma < \frac{2}{3}$). In other words, when the final market competition is not too fierce, the higher are firm I's incentives to enlarge its own market share by increasing its customers' input cost. When, instead, market competition becomes too fierce ($\gamma > \frac{2}{3}$), the wholesale price decreases, as a further increase in w^L can lead both downstream customers to a foreclosure.[10]

We can obtain the equilibrium fixed fees T_i^L after substituting (22) into (19). Substituting T_i^L and w^L into (20) and (21), we can obtain firm D and firm E's equilibrium profits gross from the fixed licensing fee, $\pi_D^L\left(w^L\right) - T_D^L$ and $\pi_E^L\left(w^L\right) - T_E^L$, and firm I's equilibrium profits, π_I^L.

[10] As we will see in Proposition 1, the foreclosure of both downstream firms is not desirable for firm I. As we saw in Sect. 4.1, firm I is always better off when there is at least one downstream competitor than being a monopolist in the final goods market.

Fig. 2 Input provision incentives



In the previous stage, firm *I* decides whether it will supply its competitors-customers. We find that firm *I* decides to always serve at least one of the downstream firms instead of being a monopolist. The reason is the same as in the no licensing case.

Proposition 1 *The supplier provides the input to both the licensor and the licensee if and only if* $\beta > \beta_1(\gamma) \equiv \frac{\gamma^2(4-3\gamma^2)(2+8\gamma+3\gamma^2)}{16-\gamma^2(32-23\gamma^2-3\gamma(8-6\gamma^2-3\gamma^3))}$. *Otherwise, he opts to supply one downstream firm only.*

This result can be gleaned from Fig. 2, which depicts line $\beta_1(\gamma)$, i.e., all the cases in which the supplier is indifferent between providing the input to one of the downstream firms or to both of them. That is, it depicts the case where the profits from supplying the input to one or two firms are equal. As is evident from Fig. 2, firm *I* will supply both downstream firms for all the combinations of β and γ that lay on the left of curve$\beta_1(\gamma)$, that is, for all cases where its bargaining power is not sufficiently low ($\beta > \beta_1(\gamma)$). Additionally, it should be noted that for $\gamma > \frac{2}{3}$, i.e., for sufficiently low degrees of product differentiation, firm *I* will always decide to serve one downstream firm instead of two.[11] Given that firms *D* and *E* have the same cost function (i.e., they are symmetric), firm *I* is indifferent between selling the input to either firm *D* or firm *E*.

The intuition is as follows. The entry of the licensee into the final goods market gives rise to the *market expansion effect*. This effect refers to the fact that the licensee's entry increases the number of differentiated final products, and thus, it increases the final product variety in the market.[12] Moreover, when firm *I* serves both downstream

[11] For more details, see the proof of Proposition *1* in the Appendix.

[12] The global smartphone market has been expanding over the recent years and, since 2014, annual shipments exceed one billion units. Sony, one of the largest camera sensor manufacturers, has since increased its production in order to keep up with demand for sensors used in mobile phone cameras. For more on this, see, e.g., Global Smartphone Production expected to reach 1.36 billion units in 2021, *Bloomberg*, (January 5, 2021) and Sony can't make image sensors fast enough to keep up with demand, *Bloomberg*, (December 23, 2019). The *market expansion effect* is also present in the electric car market. According to the International Energy Agency (IEA), sales of electric cars hit 6.6 million in 2021, more than tripling their market share from two years earlier. In 2012, about 130.000 electric cars were sold worldwide. In 2021, the same amount of electric cars was sold in the space of a single week. For more on this, see, e.g., electric cars fend off supply challenges to more than double global sales, *IEA*, (January 30, 2022).

firms, licensing also brings about the *business-stealing effect*. This refers to the fact that the entry of the licensee decreases firm I's output, namely, firm E 'steals' part of firm I's market share in the final goods market. When product differentiation is low, the *business-stealing effect* is strong and the *market expansion effect* is weak, making the input provision to both downstream firms undesirable for firm I. However, firm I prefers serving both downstream firms when product differentiation is higher, because then the *market expansion effect* gets stronger and a firm I's bargaining power that is sufficiently high, allows firm I to extract a large share of its customers' profits through the fixed fee of the contract, alleviating the negative impact of the *business-stealing effect*.

In the continuation of the game, when firm I serves both downstream firms, firm D knows that firm E will reject the licensing agreement if and only if its profits without the agreement exceed its profits with the agreement. Since the former are equal to zero, firm D optimally sets $F^L = \pi_{\mathrm{E}}^L(w^L) - T_E^L$. Hence, firm D's net equilibrium profits (included in Table 1 of the Appendix) are $\pi_D^L = \pi_D^L(w^L) - T_D^L + F^L = 2(\pi_D^L(w^L) - T_D^L)$ since $\pi_D^L(w^L) - T_D^L = \pi_{\mathrm{E}}^L(w^L) - T_E^L$. Therefore, through licensing, firm D captures all of its joint profits with firm E.

5 Licensing incentives

Having analyzed the vertically integrated firm's incentives to provide the input to its competitors, we are now able to examine the licensing incentives.

Proposition 2 *The licensee's entry into the final goods market decreases the wholesale price.*

Interestingly, licensing results in a lower wholesale price for the licensor, $w^L < w^N$. We refer to this as the *input pricing effect* of licensing. One might expect that as the final goods market becomes more intense, firm I would have incentives to raise its rivals' cost in order to enlarge its own downstream market share. However, there is another force in action. Although firm I is in a monopoly position, its inability to publicly commit to specific contract terms to all downstream customers when negotiations are secret gives incentives for opportunistic behavior and prevents it from inducing the maximum overall industry profits. This is called commitment problem (O'Brien and Shaffer 1992; McAfee and Schwartz 1994, 1995; Rey and Vergé 2004). In that way, firm I promotes its rivals' position in the downstream market via lower wholesale prices while it uses the fixed fee of the contract in order to extract part of the greater profits of its customers. The literature in vertical contracting has explained the commitment problem of a monopolist upstream supplier when it provides an input to his customers, inducing the supplier to charge a wholesale price that is even lower than his own marginal cost and uses the fixed fee of the contract to transfer upstream the higher gross profits of the downstream firms (Milliou and Petrakis 2007; Alipranti et al. 2014). As the literature has addressed (Rey and Tirole 2007; Reisinger and Tarantino 2015), the commitment problem has been solved by the supplier's vertical integration with one of his retailers, while at the same time either the supplier forecloses the rest of the retailers or keeps the more efficient firm in the downstream market. However, in our setting, the supplier's commitment problem cannot be perfectly solved, because the supplier does not integrate with one of the downstream firms, but he has his own direct channel in the final goods market and the secret contracting still exists. Thus, the wholesale price is above the supplier's marginal cost but it is lower than the one in the no licensing case in which the commitment problem is absent.[13]

Proposition 3 *Firm D always opts for licensing when firm I serves both the licensor and the licensee. Otherwise, firm D is indifferent between licensing and no licensing.*

When $\beta > \beta_1(\gamma)$, and thus, firm *I* serves both downstream firms, firm *D* always prefers to license its technology to firm *E*. The intuition is as follows. The licensee's entry brings about the *cannibalization effect* that corresponds to the new final product that partially cannibalizes firm *D*'s output and market share. However, the *input pricing effect* of licensing benefits firm *D*. At the same time, both the *business-stealing effect* and the *market expansion effect* have a positive impact on firm *D*'s profits. Intuitively, the *business-stealing effect* works in favor of firm *D*, since the latter extracts firm *E*'s profits through the fixed licensing fee, that are part of firm *I*'s 'stolen' market share. Therefore, the negative impact of the *cannibalization effect* on firm *D*'s profits is dampened by the positive impact of the *input pricing effect*, the *market expansion effect* and the *business-stealing effect*. These effects increase the joint profits of firms *D* and *E*, rendering licensing desirable. This result is in contrast to Arya and Mittendorf (2006), who find that a fixed fee licensing contract is not desirable because it prevents the licensor from using the new entry as a vehicle for obtaining a lower input price, and thus, the *input pricing effect* is absent; at the same time, the positive impact of the *market expansion effect* cannot dominate the negative impact of the stronger downstream competition. Bakaouka and Milliou (2018) show that fixed fee licensing is beneficial only if it creates a seller–buyer relationship between the licensor and the licensee. In their setup, both the licensor and the licensee use the vertical contract as a means of setting a higher input price in order to decrease the negative impact of the increased downstream competition. It follows that licensing in vertically related markets can significantly affect the input trading outcomes, and thus, the conditions under which licensing incentives might exist under such market structures. When $\beta < \beta_1(\gamma)$, and thus, firm *I* serves only one of its downstream firms, firm *D*'s profits are equal to π_D^N, and thus, it is indifferent between licensing and no licensing.

One might wonder how the presence of a vertical market structure can affect the licensing incentives. To examine this, we consider the case in which all firms are vertically integrated and produce their input in-house, and thus, they are only horizontally related to each other. Intuitively, in a one-tier market, licensing results in the *cannibalization effect*, the *business-stealing effect* and the *market expansion effect*. In this case, licensing is always profitable (Corchón 1991). However, licensing in a one-tier market does not result in the *input pricing effect*. Thus, it does not allow firm *I* to reap the benefits from the input sales and alleviate the *business-stealing effect*. Licensing with a new entry is similar to the case where one of the firms splits profitably into two firms—a reverse merger. Salant et al. (1983) showed that the non-merged firm

[13] In Sect. 8, we also examine what happens when the supplier is a non-integrated firm and he does not participate in the final goods market. In this case, the commitment problem is even stronger.

will increase its output as a reaction to the output reduction by the merging firms. In contrast to their paper, the firms in our setting are affected by the *input pricing effect*. Consequently, the *input pricing effect*—the vertical contract—is an instrument that allows firm I to intervene in both of its rivals' input provision cost. In fact, it works in such a way that firm I can be better off with licensing (reverse merger) than without licensing (merger), thus making the licensing incentives weaker compared to the case in which no vertical market structure occurs.

Another interesting question is whether the existence of stage 2—where firm I has the ability to exclude firms from the input market—may affect our main results. Here, we assume that the second stage of the game is suppressed and the timing of the game moves from the licensing decision to the bargaining stage of the input contract terms. In this case, firm I remains passive to changes in the market conditions and will always supply the input to both the licensor and the licensee. This allows both downstream competitors to gain profits against the supplier. In fact, licensing will hurt firm I when the degree of product differentiation is low, because supplying both downstream firms will lower the profits of firm I compared to the no licensing case. On the other hand, licensing is always profitable for firm D. Under the assumption that firm I cannot commit to excluding firms from the input market, licensing incentives are even more pronounced as there will be cases where licensing hurts the vertically integrated competitor. Moreover, when the second stage is suppressed, licensing incentives are still greater in a one-tier market compared to a vertically related market. As is already known in the literature (see Corchón 1991), licensing in a one-tier market is always profitable even when the final products are homogeneous ($\gamma = 1$). As such, higher degrees of product differentiation weaken the *cannibalization effect* and enhance the *market expansion effect*, leading to greater licensing incentives.

If input trading takes place through a per-unit wholesale price contract (there is no fixed fee part in the contract), in contrast to conventional wisdom that increased competition will harm the licensor, the *input pricing effect* of licensing still exists. One might expect that a vertically integrated supplier will increase the per-unit wholesale price in order to increase his competitive position in the downstream market. However, the supplier sets a lower per-unit wholesale price under licensing in order to counterbalance the cost advantage of his own direct channel in the downstream market, as he produces the input at marginal cost c. The fact that the input price is lower reduces the downstream profits of the supplier, but it increases the wholesale profits more by keeping input demand up, and in turn, the licensor benefits from the supplier's presence in the final goods market. If this cost advantage was not alleviated by a lower input price, the licensor's input demand would decrease. Thus, under a per-unit wholesale price contract, our main results are robust, and firm D has licensing incentives, which still would be weaker in a vertically related market rather than in a horizontally related market.[14]

[14] Irrespective of whether there is a discriminatory per-unit wholesale price contract or a uniform per-unit wholesale price contract, the results will be the same—the reason is that the equilibrium per-unit wholesale price is the same for both cases. This is so because there is no cost asymmetry between the licensor and the licensee, and thus, no different outcomes arise.

6 Welfare implications

Here, we examine the impact of licensing in a vertically related market on welfare and discuss the policy implications of our findings.

Proposition 4 *Horizontal licensing in a vertically related market always has a positive impact on consumer surplus and on total welfare.*

Licensing is always beneficial for the consumers as it results in lower final prices, because it triggers a new entry into the final goods market that leads to a decrease of the downstream firms' cost and to an increase of the final product variety. Licensing also has a positive impact on firms' profits. The licensor benefits due to the lower wholesale price while the vertically integrated supplier from the expansion of the market. Thus, licensing is welfare enhancing for the economy as a whole. As we demonstrate in Sects. 7 and 8, the positive impact of licensing can still exist when licensing takes place through a per-unit royalty, when input trading takes place sequentially, when firms compete in prices, when intrabrand differentiation is smaller than interbrand differentiation and when the supplier is not active in the final goods market.

Many papers show that licensing is beneficial for the consumers due to the positive impact of the market expansion effect and the lower final prices triggered by a new entry. In vertical markets, licensing should also lead to the creation of a weaker rival (Arya and Mittendorf 2006) or to changes on both upstream and downstream market structure (Mukherjee 2003; Bakaouka and Milliou 2018) in order to continue to have a positive impact on welfare. In market structures similar to our setting, licensing can still be welfare enhancing either when the licensor is equally efficient with the new rival or even when the licensor is less efficient than the licensee is, e.g., when the bargaining over contract terms takes place sequentially.[15]

The welfare conclusions could be of use in the treatment of mergers and licensing agreements by the competition policy authorities. In our setting, licensing corresponds to a reverse merger which increases the number of downstream firms, intensifies the final goods competition and enhances firms' profits, rendering licensing desirable both for the consumers and the industry. Given this, the competition authorities can consider licensing agreements among competitors pro-competitive even in cases in which firms have large market shares and when the licensing's positive impact is also spread out to firms outside of the agreement which are still both horizontally and vertically related with them.[16]

[15] The positive impact of horizontal licensing on welfare can be ambiguous when its effect depends on factors related to the size of the innovation (Faulí-Oller and Sandonís 2002; Ding and Ko 2021), on both the kind of goods produced and the degree of product differentiation (San Martin and Saracho 2015), on cases using complementary inputs for the final goods production (Lin et al. 2022) and when licensing occurs in multiple market tiers (Tsao et al. 2023).

[16] EU and US antitrust authorities consider licensing agreements pro-competitive by expediting technology transfer; however, their regulations are applying exemptions for licensing agreements among competitors. According to the EU Regulation N. 316/2014 and the US Antitrust Guidelines for the Licensing of Intellectual Property, in the case of licensing agreements between competitors, the block exemption applies when the combined market share of the parties does not exceed 20% on the relevant market.

7 Licensing through a per-unit royalty

One might wonder what happens if licensing takes place through a per-unit royalty, $r \geq 0$, imposed on firm E's output, instead of licensing through a fixed fee. In what follows, we briefly analyze the possible subgames and examine the licensing incentives.[17]

7.1 Serving both downstream firms or only firm E

First, we discuss the subgame in which firm I serves both firm D and firm E. Due to the existence of firm E, and thus of the per-unit royalty, both firms' wholesale prices depend on the royalty. We find that a higher per-unit royalty leads to higher wholesale prices for both firm D and firm E. Inderst and Shaffer (2009) demonstrated that firm i's wholesale price is decreasing in its rival's marginal cost. However, in our setting, unlike in Inderst and Shaffer (2009), firm I's presence in the final goods market reinforces the downstream competition, and thus, firm I has incentives to dampen the aggressiveness of its downstream rivals.

In the first stage of the game, assuming that in the continuation of the game firm I serves both downstream firms, the maximization of firm D's profits leads to $\widehat{r}^L$ with $\frac{\partial \widehat{r}^L}{\partial \gamma} < 0$, namely, a decrease in product differentiation has a negative impact on the per-unit royalty. Thus, as competition becomes fiercer, a lower per-unit royalty is imposed in order to alleviate the positive impact on the wholesale prices of both the licensor and the licensee.[18]

Proposition 5 *When* firm *I provides the input to both downstream firms and the final products are sufficiently differentiated ($\gamma < \frac{2}{3}$), the licensor pays a higher wholesale price than the licensee, $\widehat{w}_D^L > \widehat{w}_E^L$. For lower degrees of product differentiation ($\gamma > \frac{2}{3}$), the licensor pays a lower wholesale price than the licensee, $\widehat{w}_D^L < \widehat{w}_E^L$.*

As Proposition 5 states, the downstream firms get different wholesale prices. This is due to the cost asymmetry generated by the per-unit royalty. We refer to this as the *input price discrimination effect* of the licensing. When product differentiation is low ($\gamma > \frac{2}{3}$), the more efficient firm receives a lower wholesale price than the less efficient firm does. The supplier has incentives to induce the aggressiveness of his more efficient customer in the downstream market and extract the greater surplus through the fixed fee of the contract. If this was the only force, we would obtain the same result with Inderst and Shaffer (2009).[19] However, there is another force in action. Importantly,

[17] A detailed equilibrium analysis is included in the Appendix.

[18] For the rest of the analysis in this subsection, we make Assumption 1, namely, $\beta > \overline{\beta}(\gamma) \equiv \frac{8+16\gamma-12\gamma^2-30\gamma^3+4\gamma^4+13\gamma^5}{\gamma^2(-2-2\gamma+2\gamma^2+\gamma^3)}$. This is a sufficient condition that guarantees that the per-unit royalty and the wholesale prices are nonnegative and that an equilibrium exists in this subgame. When $\beta < \overline{\beta}(\gamma)$, there is no equilibrium in which firm I serves both downstream firms. More details are available in the Appendix.

[19] In contrast to Inderst and Shaffer (2009) and our findings, DeGraba (1990) and Yoshida (2000) demonstrate that if the input trading takes place through a per-unit wholesale price contract, the supplier will always charge the more efficient downstream firm with a higher wholesale price. Also, Arya and Mittendorf (2006) note that the supplier charges a lower wholesale price to the less efficient licensee than to the more efficient licensor, in order to keep up input demand.

the per-unit royalty is part of the licensor's revenues and it is used as an instrument in the vertical trading contract that allows the licensor to self-sabotage by increasing his own wholesale price so as to decrease the positive impact of the per-unit royalty on the licensee's wholesale price. The licensor does so, because he wants to induce an increase in firm *E*'s output, and thus, in firm *D*'s licensing revenues. For higher degrees of product differentiation ($\gamma < \frac{2}{3}$), this second force dominates and leads the licensor to receive a higher wholesale price than the licensee does. When, however, product differentiation is lower and competition becomes too fierce, the first force dominates, as an even higher wholesale price could lead to firm *D*'s foreclosure.[20]

In the subgame in which firm *I* decides to serve only firm *E*, firm *D*'s profits come from its licensing revenues, $\pi_D(r) = rq_E$. In line with our previous analysis, a per-unit royalty always has a positive impact on firm *E*'s wholesale price. In the first stage of the game, assuming that in the continuation of the game firm *I* serves only firm *E*, the maximization of firm *D*'s profits leads to $\widehat{r}^{LE}$ with $\frac{\partial \widehat{r}^{LE}}{\partial \gamma} < 0$.

7.2 Licensing incentives

Before we analyze the licensing incentives, we should examine if firm *I* prefers serving both, one or none of its customers in the downstream market.

Lemma 1 *Under a per-unit royalty licensing, firm I serves at least one downstream firm. When it serves one downstream firm, it prefers serving firm D to firm E if and only if* $r < \overline{r}(\gamma) \equiv 2(\alpha - c)(1-\gamma)$, *with* $\frac{\partial \overline{r}}{\partial \gamma} < 0$.

Proof Calculating the difference,$\pi_{\rm I}^{\rm N} - \pi_{\rm I}^{\rm M} = \frac{\beta(\alpha-c)^2(1-\gamma)^2}{4-3\gamma^2}$, we find that it is always positive. Thus, $\pi_{\rm I}^{\rm N} > \pi_{\rm I}^{\rm M}$. Calculating the difference, $\widehat{\pi}_{\rm I}^{LE}(r) - \pi_{\rm I}^{\rm M} = \frac{\beta((\alpha-c)(1-\gamma)-r)^2}{4-3\gamma^2}$, we find that it is positive, unless $r = (\alpha - c)(1-\gamma)$, where $\widehat{\pi}_{\rm I}^{LE}(r) = \pi_{\rm I}^{\rm M}$. Calculating the difference, $\pi_{\rm I}^{\rm N} - \widehat{\pi}_{\rm I}^{LE}(r) = \frac{\beta r(2(\alpha-c)(1-\gamma)-r)}{4-3\gamma^2}$, we find that it is positive if and only if $r < \overline{r}(\gamma) \equiv 2(\alpha - c)(1-\gamma)$. Hence,$\pi_{\rm I}^{\rm N} > \widehat{\pi}_{\rm I}^{LE}(r) > \pi_{\rm I}^{\rm M}$ if and only *if* $r < \overline{r}(\gamma)$. Otherwise, $\widehat{\pi}_{\rm I}^{LE}(r) > \pi_{\rm I}^{\rm N} > \pi_{\rm I}^{\rm M}$ if and only if $r > \overline{r}(\gamma)$.

In the second stage, firm *I* always decides to supply at least one of the downstream firms instead of being a monopolist. The reason is the same as in the no licensing case. Also, we find that in both subgames in which firm *D* sets the per-unit royalty, it holds that $\widehat{r}^{L} < \overline{r}(\gamma)$ and $\widehat{r}^{LE} < \overline{r}(\gamma)$. From Lemma 1 emerges that firm *I* decides between trading with both downstream firms and trading only with firm *D*, while firm *I*'s incentives to serve only firm *E* are distorted.[21] In fact, firm *I* prefers supplying the more efficient downstream firm, as the input revenues that firm *I* will extract from the licensor are greater than those that could extract from the less efficient firm. Resorting

[20] If the products were homogeneous, the wholesale price would be so high that firm *D* would be foreclosed from the market. When products are slightly differentiated, the supplier prefers to induce firm *D*'s aggressiveness and obtain the input revenues.

[21] Reisinger and Tarantino (2015) show a similar result, in which a vertically integrated firm prefers to serve a more efficient downstream firm and foreclose a less efficient downstream firm's access to the input.

to numerical simulations, we find that for $\gamma > 0.218164$, firm I prefers serving only firm D instead of both downstream firms. For lower values of γ, and thus, when competition is weak, firm I prefers serving both its downstream rivals.

Proposition 6 *Under a per-unit royalty licensing, firm I provides the input to both the licensor and the licensee when final product differentiation is sufficiently high (i.e.,* $\gamma < 0.218164$*). In this case, firm D always opts for licensing. Otherwise, firm D is indifferent between licensing and no licensing.*

The intuition is as follows. The licensee's entry gives rise to the *cannibalization effect*, the *market expansion effect*, the *business-stealing effect* and the *input price discrimination effect*. When product differentiation is high, and thus, firm I serves both downstream firms, the negative impact of the *cannibalization effect* and the *input price discrimination effect* on firm D's profits is weak while firm D by capturing the licensing revenues, takes advantage of the *market expansion effect* and the *business-stealing effect*, rendering licensing desirable. This comes in contrast to the result of Bakaouka and Milliou (2018), who find that licensing through a per-unit royalty is not desirable, because in their setting the input supplier cannot interact with more than one downstream firms, and thus, to eliminate the negative impact of the *cannibalization effect*. However, when firm I decides not to serve firm E, and thus there is no new entry in the downstream market, firm D is indifferent between licensing and no licensing.

It emerges from the above analysis that the type of the licensing contract affects both firm I's input provision incentives and firm D's licensing incentives. In contrast to the case of licensing through fixed fee, a per-unit royalty licensing brings about the *input price discrimination effect*, and makes the input sourcing terms being higher then, namely, $\widehat{w}_D^L > w^L$ and $\widehat{w}_E^L > w^L$. Thus, a per-unit royalty licensing is less likely to occur because of the *input price discrimination effect* and the fact that firm D cannot take full advantage of the *market expansion effect* and the *business-stealing effect*. Licensing through a per-unit royalty is welfare enhancing, but, because of the *input price discrimination effect* and the higher input prices, it is less desirable than licensing through a fixed fee for both consumers and total welfare.[22]

In fact, we show that our results are in contrast to the patent licensing literature that considers a licensor who is active in the final goods market. This literature shows that licensing by means of a per-unit royalty is superior to licensing by means of a fixed fee from the viewpoint of the licensor.[23] This happens because the licensor can reap the reward of licensing while he still enjoys the benefit of his cost advantage under a per-unit royalty contract (Wang 1998). The same logic also holds in the paper of Arya and Mittendorf (2006) which considers a vertical market in which both the licensor and the licensee source the required input from a common supplier. In their setting, a per-unit royalty licensing creates a new weaker rival who is more sensitive to the supplier's input pricing. In fact, in order to keep input demand up, the supplier charges

[22] The comparison takes place within the common set of parameters under which licensing incentives occur under both licensing contracts. In other words, licensing through a fixed fee is preferred to licensing through a per-unit royalty contract, using the same set of parameters and parameter values for the comparison.

[23] One exception is the paper of Colombo et al. (2023) which examines the case of a capacity constrained licensor that charges a fixed fee. They show that the fixed fee is preferred even if the licensor competes with the licensee, provided that he is able to produce a relatively small quantity.

a lower wholesale price to the less efficient licensee compared to the more efficient licensor who continues to receive the same wholesale price as in the no licensing case. As a result, the licensor benefits from the licensee's increased profits which he can partially extract. However, in line with Bakaouka and Milliou (2018), in our setting a per-unit royalty licensing leads to worse input sourcing terms, and thus, into lower downstream profits for the licensor. The fact that the licensor cannot take full advantage of both the *market expansion effect* and the *business-stealing effect* renders a per-unit royalty licensing less desirable compared to a fixed fee licensing. It follows that both the market structure and the vertical contracting can substantially affect the superiority of the licensing contract between a fixed fee and a per-unit royalty licensing.

In what follows, we examine how a two-part tariff licensing contract can affect the licensing incentives. In this scenario, the licensor has two instruments (fixed fee and per-unit royalty) to capture the licensee's profits and to take full advantage of the *market expansion effect* and the *business-stealing effect*. Licensing incentives still occur, rendering the two-part tariff licensing contract the more desirable compared to both the per-unit royalty and the fixed fee licensing contracts. From a welfare point of view, licensing through a fixed fee is the most desirable one because of the *input pricing effect* and the lower final prices. Under a two-part tariff licensing contract, the *input price discrimination effect* still occurs, but is weaker compared to the per-unit royalty licensing as it leads to lower input prices and, as such, to lower final prices. In turn, the positive impact of licensing on both consumer and total welfare is greater under the two-part tariff than under the per-unit royalty licensing.

Regarding the optimality of a two-part tariff licensing contract, the literature has shown that it can indeed be optimal under alternative scenarios: indicatively, when there is cost asymmetry among the licensor and the licensee (Poddar and Sinha 2010), when the magnitude of the innovation size is large (Tsao et al. 2023), and when the firms operate in a homogeneous or a differentiated Cournot oligopoly (Fauli-Oller and Sandonis 2003; Sen and Tauman 2007). We add to the existing literature by showing that a two-part tariff licensing contract can still be optimal in vertically related markets similar to our setting.

A number of recent contributions to the patent licensing theory (see, among others, San Martin and Saracho 2010; Colombo and Filippini 2016; Hsu et al. 2019) investigate the implications of an alternative contract, namely, the ad valorem profit-licensing contract. This type of contract allows the licensor to extract a quota of the licensee's profits. Under ad valorem licensing, the licensor tends to have financial interests in the licensee's profits and thus, behaves less aggressively. That is, the licensor keeps final production low in order to soften competition and extract a share of the licensee's increased profits.

In our setting, an increase in the ad valorem royalty leads to higher input prices for both downstream firms, while the licensor's input price is lower than the one of the licensee. These results hold for the following reasons: firstly, firm I has incentives to raise its rivals' cost in order to enlarge its own downstream market share and, secondly, firm I charges a higher wholesale price to the licensee in order to mitigate the strategic behavior of the licensor (that is, the licensor's effort to raise the profits of the licensee). Numerical simulations show that under ad valorem licensing, firm I prefers to supply both downstream firms when product differentiation is high. In this case, licensing continues to be desirable from the licensor's viewpoint due to the *market expansion effect* and the *business-stealing effect*, and enhances both consumer and total welfare.

8 Extensions

In this section, we briefly discuss a number of further extensions of our main model to gain additional insights.

8.1 Licensing by the vertically integrated firm

In this subsection, we examine the case where the vertically integrated firm I opts for licensing, giving rise to the entry of firm E into the final goods market. An interesting illustration of this market structure can be found in the virtual reality (VR) headset market. In particular, Samsung—one of the largest OLED display manufacturers—-supplies Apple with OLED panels for different i-phone models and VR headsets. In 2004, Samsung and Sony entered in a long-term cross-license agreement for patents, which, at the time, were considered basic technologies necessary for product development. Patents that were excluded from the agreement were the ones related to Sony's Digital Reality Creation (DRC) and PlayStation architecture, and Samsung's Digital Natural Image Engine (DNIe) and TFT-LCD and organic light emitting diode (OLED) display patents. Almost twenty years later, in 2023, Sony launched its next generation VR headset, PlayStation VR2, which features OLED panels manufactured by Samsung, thus directly competing with Samsung's Gear VR headset.[24] As such, we observe that firm I (in this case, Samsung) licenses firm E (in this case, Sony) which ends up entering the market and competing with both firms I and D (that is, Samsung and Apple).

In this case, it is firm D that suffers from the *business-stealing effect*, while now the *cannibalization effect* hurts firm I. Similarly to the main model, when $\beta > \beta_1(\gamma)$, firm I provides the input to both downstream firms and always opts for licensing, since it takes full advantage of the *input pricing effect*, the *business-stealing effect* and the *market expansion effect*. When $\beta < \beta_1(\gamma)$, firm I still has incentives for licensing and serves only one downstream firm, the licensee. In the latter case, firm I, by capturing the licensee's profits, benefits from the *market expansion effect* and eliminates the negative impact of the *cannibalization effect*.

In contrast to the main model, firm I always has incentives to license its technology independently of either it will serve both downstream firms or only the licensee. Still, when $\beta > \beta_1(\gamma)$, due to the *market expansion effect*, firm I would have incentives to license its technology to firm E even for free. Licensing by the vertically integrated firm leaves welfare unaffected.

[24] For more information, see, e.g., Samsung, Sony enter cross-license agreement, *Forbes* (December 27, 2004) and Sony plans PSVR successor for next holiday season, *The Verge* (June 16, 2021).

8.2 Non-vertically integrated firm

The fact that the supplier is a vertically integrated firm gives rise to the question of what would happen if, alternatively, firm I was a non-integrated upstream firm. An interesting illustration of this market structure comes from the electronics industry. In 2021, Huawei began giving access to its portfolio of 5G patents to Apple and Samsung, creating a lucrative revenue source. Huawei, Apple, Samsung and other smartphone brands were vying to secure components for their devices, such as ceramic capacitors from Murata, the world's biggest maker of multi-layer ceramic capacitors (MLCC). According to Murata, the demand for fifth-generation wireless devices was expected to surpass 500 million units in the following financial year.[25]

Following the above framework, in the no licensing case, there would be two downstream firms, firm D and firm K, and thus, three possible third-stage subgames.[26] In the first and the second subgame, firm I can serve only firm D or only firm K and the equilibrium outcomes coincide with the case in which the downstream firm is a monopolist in the market. As it is well established in the literature, firm I and the downstream firm would set $w^{SM} = c$ and would achieve the profits of a monopolist splitting them up according to their bargaining power. In the third subgame in which firm I serves both downstream firms, the equilibrium wholesale price would be $w^{SN} = c - \gamma^2 \frac{(a-c)}{2(2-\gamma^2)} < c$, namely, firm I would subsidize both downstream firms, as it suffers from the commitment problem.[27]

In the licensing case, the downstream market consists of three firms, D, K and E, and the equilibrium wholesale price is $w^{SL} = \frac{a\gamma^2 - c(2-\gamma)(1+\gamma)}{2\gamma^2-\gamma-2} < w^{SN} < c$. With the increase in the number of the downstream firms, the commitment problem of the supplier becomes more severe (Rey and Tirole 2007), rendering the *input pricing effect* even stronger when the supplier is not a vertically integrated firm. Importantly, firm D would opt for licensing when firm I supplies all downstream firms. This occurs when the final products are more differentiated and firm I's bargaining power is not sufficiently low.[28] Similarly to the main model, we observe that when product differentiation is high, the negative impact of the *cannibalization effect* is weak and the

[25] For more information, see, e.g., Huawei to start demanding 5G royalties from Apple, Samsung, *Bloomberg* (March 16, 2021), and Apple supplier Murata expects half billion 5G smartphones in new year, *Bloomberg* (January 5, 2021).

[26] In stage 2, if firm I decided not to serve none of the downstream firms, then its profits would be equal to zero, and thus, it would never decide to foreclose both downstream firms. For the rest of the subgames, a detailed equilibrium analysis is included in the Appendix.

[27] When firm I is non-integrated and trades with firm D, it cannot commit to not offering better trading terms to firm K. Firm D knows that firm I has incentives to behave opportunistically and make firm K an aggressive competitor in the final goods market, via a lower wholesale price, because it can use the fixed fee in order to transfer upstream the higher gross profits of firm K. Firm D anticipates this and it will never agree on a wholesale price that is not below the marginal cost of firm I. See, e.g., McAfee and Schwartz (1995), Rey and Vergé (2004), de Fontenay and Gans (2005), Milliou and Petrakis (2007), Alipranti et al. (2014).

[28] Firm D would opt for licensing when $\beta > \beta_5(\gamma) \equiv \frac{\gamma^3(40-12\gamma-38\gamma^2+15\gamma^3+8\gamma^4-4\gamma^5)}{2(2-\gamma)(1-\gamma)(2-\gamma^2)(1-\gamma-\gamma^2)(2+\gamma-2\gamma^2)}$. More details are available in the Appendix.

positive impact of the *market expansion effect* is strong and along with a larger subsidy, and thus, a stronger *input pricing effect*, reinforce firm D's licensing incentives and increase consumer and total welfare. The latter effect actually makes licensing more desirable from both a welfare and a licensor's point of view, when the supplier is a non-integrated rather than a vertically integrated firm. In other words, vertical integration of the supplier removes firm D's incentives to license to a new firm.

8.3 Intrabrand product differentiation

One might think that since under licensing, firm D and firm E use the same patented technology, their final products can differ less between them than they differ from firm I's final product. This possibility can be captured by assuming that while the degree of interbrand differentiation among firm I's product and the products of firm D and firm E is γ, the degree of intrabrand differentiation among the products of firm D and firm E is given by δ, with $0 < \gamma < \delta < 1$. In such a case, as expected, the *cannibalization effect* is stronger and the *market expansion effect* is weaker. The *business-stealing effect* is also weaker, because the fact that the licensee competes more fiercely with its licensor, induces the entrant to produce even less than firm I. Importantly, the *input pricing effect* is stronger, as the wholesale price decreases with a decrease in intrabrand differentiation, and thus, the more intense is competition among the licensor and the licensee. Only if intrabrand differentiation is not sufficiently low ($\delta < \frac{2}{3}$), the *market expansion effect* and the *business-stealing effect,* which, although they are weaker, are still present, and along with the *input pricing effect,* make providing the input to both downstream firms desirable for firm I. In this case, licensing incentives still exist, although they are weaker when intrabrand differentiation is lower than interbrand differentiation. Moreover, licensing continues to have positive welfare implications when there is intrabrand differentiation, and its impact on consumer surplus is even greater due to the stronger *input pricing effect,* but the impact on total welfare is smaller due to the smaller increase on firms' profits.

8.4 Bargaining over the licensing fee

In the main model, in the first stage of the game, firm D makes a take-it-or-leave-it offer to firm E for the licensing fee. Here, we relax this assumption and examine what happens if the licensor bargains with the licensee over the licensing fee. We assume that the bargaining power of the licensor is m and that of the licensee is $1 - m$. When the bargaining power of firm E is positive, firm D is not able to fully extract firm E's profits through F. Still, even then, firm D always opts for licensing. This holds because during the negotiations over F, the licensee has to compensate the licensor for the profits that it would make without licensing. Because of this, in the extreme case in which firm D has no bargaining power ($m = 0$), it is indifferent among licensing and no licensing. In all other cases, it prefers licensing to no licensing, and the higher its bargaining power is, the more it prefers licensing. The conclusions regarding the welfare implications of licensing remain unchanged, since the allocation of the licensing fee does not affect consumer and total welfare.

8.5 Sequential bargaining

In settings in which the licensor is the incumbent firm and the licensee is the entrant, one might wonder how the sequential bargaining in vertical contracting could affect our findings. In order to examine this, we consider that the supplier bargains over the contract terms first with firm D and then with firm E. When firm D opts for licensing, it faces a higher wholesale price than the licensee does, namely, $w_D^B > w_E^B > w^N$. Thus, *the input pricing effect* of licensing disappears under sequential bargaining, and importantly, the competition between firms is transformed from Cournot to Stackelberg. The reason is that now the supplier does not have the ability to commit to an opportunistic behavior and prefers to use the sequential bargaining in order to generate a cost asymmetry between his rivals. Intuitively, the supplier deteriorates the position of the licensor by charging a higher wholesale price because he has an outside option to mitigate his profit loss. On the other hand, the supplier has incentives to induce the aggressiveness of the licensee by charging a wholesale price that is lower than the licensor's input price (McAfee and Schwartz 1994; Bedre-Defolie 2012), and thus, transforming him into a Stackelberg leader. In that way, the licensee faces a weaker competition that leads to increase his product, and thus, the supplier benefits by enlarging his own profits from the input sales, but also from the greater surplus that extracts through the fixed fee of the contract.

Licensing continues to be desirable from the licensor's viewpoint due to the *business-stealing effect* and the *market expansion effect* and welfare enhancing, unless the final products are close substitutes. However, licensing is more likely to occur under simultaneous bargaining than under sequential bargaining because then the *input pricing effect* occurs, which can induce a greater positive impact on both consumer and total welfare.

8.6 Divisionalization

Licensing with a new entry resembles the horizontal divisionalization case that has been examined in the literature. One might wonder whether firm D's decision to be divided into two firms which act as a multiproduct firm can affect our findings. In order to examine this, we consider a case where in the first stage of the game firm D decides—instead of licensing—to proceed with a divisionalization. We note here that the divisionalization of firm D does not result in two independent firms. Rather as mentioned above, they behave as a multiproduct firm. When firm D is divided, the *input pricing effect*, the *cannibalization effect*, the *market expansion effect* and the *business-stealing effect*, although they still exist, they are weaker than the licensing case. The main difference with our main analysis is that now firm D and firm E choose their quantities in a collusive way, and when the supplier and the downstream firms set the wholesale prices, reinforce the collusive behavior of the downstream firms through the setting of a higher wholesale price compared to the licensing case. This leads firm I to be better off with a multiproduct customer rather than with a single downstream firm, only when the downstream competition intensity is weak (γ < 0.349946) and its bargaining power is not too low. In turn, firm D has incentives for divisionalization; however, it would be more likely to prefer licensing rather than divisionalization because of the stronger effects. Based on the above, divisionalization is welfare enhancing both for consumers and the economy as a whole, but its positive impact is smaller than the licensing case.

8.7 Price competition

We now discuss what happens if the firms compete in prices in the final goods market. Similar to the main analysis, licensing arises in equilibrium when firm I serves both the licensor and the licensee. This holds when the final products are not close substitutes and firm I's bargaining power is not low.[29] Interestingly, in contrast to a one-tier market, a vertically related market is more competitive when downstream firms compete in quantities than when they compete in prices (Alipranti et al. 2014). This is so because the wholesale prices are lower under quantity than under price downstream competition. In particular, since prices are strategic complements while quantities are strategic substitutes, the supplier's incentives to behave opportunistically are more pronounced in the latter case, and thus, the *input pricing effect* is stronger then. Hence, the stronger positive impact of both the *input pricing effect* and the *market expansion effect* dominate the negative impact of the *cannibalization effect,* and thus, the licensing incentives are stronger under quantity competition than under price competition. Just as in our main analysis, licensing enhances consumer and total welfare, but it does more under quantity than under price competition.

9 Concluding remarks

This paper analyzed the incentives of a downstream firm that sources its input from a vertically integrated firm, to license its patented technology to an external firm, thus transforming the licensee both into a downstream competitor, as well as into a customer of the vertically integrated supplier. We used a framework in which, after the licensing agreement is signed, the vertically integrated supplier decides which of the downstream firm(s) he will serve, and then firms compete in the final goods market.

We showed that although the licensee's entry intensifies downstream competition and cannibalizes the demand of the licensor's product, licensing can be desirable; this is due to the fact that the licensor benefits from the lower input cost, the expansion of the final goods market and the business stealing from the rival vertically integrated firm. This holds when the supplier has incentives to serve both the licensor and the licensee unless the final products are not close substitutes and the supplier's bargaining power is not low. Licensing in a vertically related market is welfare enhancing both for the consumers and the economy as a whole, as it triggers new entry into the downstream market, leads to an increase in competition intensity and in product variety and decreases both input costs and final prices. However, the vertical market structure can diminish licensing incentives and welfare enhancement compared to a market in which no vertical relations among firms exist.

[29] More details are available in the Appendix.

Various extensions of the main model demonstrated that there are still licensing incentives under alternative licensing contracts, when downstream competition is in prices, when intrabrand product differentiation is less than interbrand differentiation, when input trading takes place sequentially, as well as when the supplier is not active in the downstream market or is the one who does the licensing. The positive impact of licensing on consumers and total welfare continues to exist, and it is smaller when licensing is through a per-unit royalty, when vertical contracting is sequential or there is downstream price competition, while it is larger when the supplier is non-vertically integrated compared to the main analysis.

Summing up, we analyzed the common business practice of licensing in vertically related markets where licensing transforms the licensee into a direct downstream competitor of the integrated supplier and the licensor. The analysis was based on strategic considerations without exogenously assumed asymmetries, e.g., cost efficiencies of the licensor or the licensee and/or costly investments in input production, which might result in less surprising results than in our setting. The examination of licensing incentives and their welfare implications under different market structures and specifications of vertical contracting is left for future work.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Appendix

See Tables 1 and 2

Table 1 Equilibrium values under no licensing and under licensing through fixed fee when firm *I* serves both firm *D* and firm *E*

$$\pi_D^N = \frac{(\alpha-c)^2(1-\beta)(1-\gamma)^2}{4-3\gamma^2};\ \ \pi_I^N = \frac{(\alpha-c)^2\left(4+4\beta(1-\gamma)^2-3\gamma^2\right)}{4\left(4-3\gamma^2\right)}$$

$$q_D^L = q_E^L = \frac{(\alpha-c)}{2+3\gamma};\ \ q_I^L = \frac{(\alpha-c)(2+\gamma)}{4+6\gamma}$$

$$\pi_D^L(q_D^L) - T_D^L = \frac{2(1-\beta)(\alpha-c)^2(2-\gamma(-2+\gamma(2+\gamma)))}{\left(4+8\gamma+3\gamma^2\right)^2}$$

$$\pi_I^L = \frac{(\alpha-c)^2\left(16+16\beta\left(2+2\gamma-2\gamma^2-\gamma^3\right)+\gamma\left(64+80\gamma+16\gamma^2-3\gamma^3\right)\right)}{4(2+\gamma)^2(2+3\gamma)^2}$$

$$F^L = \frac{2(1-\beta)(\alpha-c)^2(2-\gamma(-2+\gamma(2+\gamma)))}{\left(4+8\gamma+3\gamma^2\right)^2};\ \ \pi_D^L = \frac{4(1-\beta)(\alpha-c)^2(2-\gamma(-2+\gamma(2+\gamma)))}{\left(4+8\gamma+3\gamma^2\right)^2}$$

Table 2 Equilibrium values under licensing through per-unit royalty when firm *I* serves both firm *D* and firm *E*

$$\hat{w}_D^L = \frac{A}{(2\left(2+2\gamma-2\gamma^2-\gamma^3\right)\left(32+32\gamma-72\gamma^2-40\gamma^3+47\gamma^4+\beta\gamma^4\right)}$$

$$\hat{w}_E^L = \frac{B}{2(2+\gamma)\left(2+2\gamma-2\gamma^2-\gamma^3\right)\left(32+32\gamma-72\gamma^2-40\gamma^3+47\gamma^4+\beta\gamma^4\right)}$$

$$\hat{q}_D^L = \frac{2(\alpha-c)(1-\gamma)\left(2+\gamma-2\gamma^2\right)\left(8+8\gamma-9\gamma^2-5\gamma^3\right)}{\left(2+2\gamma-2\gamma^2-\gamma^3\right)\left(32+32\gamma-72\gamma^2-40\gamma^3+47\gamma^4+\beta\gamma^4\right)}$$

$$\hat{q}_{\mathrm{I}}^L = \frac{(\alpha-c)\left(64+80\gamma-160\gamma^2-176\gamma^3+4\beta\gamma^3+126\gamma^4+2\beta\gamma^4+122\gamma^5-6\beta\gamma^5-24\gamma^6-33\gamma^7+\beta\gamma^7\right)}{2\left(2+2\gamma-2\gamma^2-\gamma^3\right)\left(32+32\gamma-72\gamma^2-40\gamma^3+47\gamma^4+\beta\gamma^4\right)}$$

$$\hat{q}_E^L = \frac{2(\alpha-c)(1-\gamma)\left(8+8\gamma-14\gamma^2-2\beta\gamma^2-5\gamma^2-2\beta\gamma^3+9\gamma^4+2\beta\gamma^4-3\gamma^5+\beta\gamma^5\right)}{\left(2+2\gamma-2\gamma^2-\gamma^3\right)\left(32+32\gamma-72\gamma^2-40\gamma^3+47\gamma^4+\beta\gamma^4\right)}$$

$$\hat{\pi}_D^L = \frac{8(\alpha-c)^2(1-\gamma)^2\left(2+\gamma-2\gamma^2\right)\left(6-4\beta+11\gamma-6\beta\gamma+2\beta\gamma^2-7\gamma^3+4\beta\gamma^3-3\gamma^4+\beta\gamma^4\right)}{(2+\gamma)^2\left(2+2\gamma-2\gamma^2-\gamma^3\right)\left(32+32\gamma-72\gamma^2-40\gamma^3+47\gamma^4+\beta\gamma^4\right)}$$

$$\hat{\pi}_{\mathrm{I}}^L = \frac{(\alpha-c)^2\Gamma}{4(2-\gamma)^2(2+\gamma)^4\left(2+2\gamma-2\gamma^2-\gamma^3\right)^2\left(32+32\gamma-72\gamma^2-40\gamma^3+47\gamma^4+\beta\gamma^4\right)^2}$$

$$\hat{\pi}_{\mathrm{E}}^L = \frac{4(1-\beta)(\alpha-c)^2(1-\gamma)^2\left(2+\gamma-2\gamma^2\right)\Delta}{(2-\gamma)^2(2+\gamma)^4\left(2+2\gamma-2\gamma^2-\gamma^3\right)^2\left(32+32\gamma-72\gamma^2-40\gamma^3+47\gamma^4+\beta\gamma^4\right)^2}$$

where

$$\begin{aligned}\mathrm{A} = {} & 128\mathrm{c} + 96\alpha\gamma + 160\mathrm{c}\gamma + 32\alpha\gamma^2 - 320\mathrm{c}\gamma^2 - 320\alpha\gamma^3 + 8\alpha\beta\gamma^3 \\ & - 320\mathrm{c}\gamma^3 - 8\beta\mathrm{c}\gamma^3 + 8\alpha\gamma^4 + 244\mathrm{c}\gamma^4 + 4\beta\mathrm{c}\gamma^4 + 334\alpha\gamma^5 - 14\alpha\beta\gamma^5 \\ & + 158\mathrm{c}\gamma^5 + 18\beta\mathrm{c}\gamma^5 - 102\alpha\gamma^6 + 6\alpha\beta\gamma^6 - 6\mathrm{c}\gamma^6 - 10\beta\mathrm{c}\gamma^6 - 82\alpha\gamma^7 \\ & + 2\alpha\beta\gamma^7 - 12\mathrm{c}\gamma^7 - 4\beta\mathrm{c}\gamma^7 + 33\alpha\gamma^8 - \alpha\beta\gamma^8 - 33\mathrm{c}\gamma^8 + \beta\mathrm{c}\gamma^8 > 0,\end{aligned}$$

$$\begin{aligned}
\mathrm{B} = {} & 256\mathrm{c} + 128\alpha\gamma + 512\mathrm{c}\gamma + 192\alpha\gamma^2 - 512\mathrm{c}\gamma^2 - 288\alpha\gamma^3 - 1280\mathrm{c}\gamma^3 \\
& - 496\alpha\gamma^4 + 360\mathrm{c}\gamma^4 + 32\alpha\beta\gamma^4 - 24\beta\mathrm{c}\gamma^4 + 188\alpha\gamma^5 + 4\alpha\beta\gamma^5 + 1048\mathrm{c}\gamma^5 \\
& + 8\beta\mathrm{c}\gamma^5 + 466\alpha\gamma^6 - 58\alpha\beta\gamma^6 - 190\mathrm{c}\gamma^6 + 54\beta\mathrm{c}\gamma^6 - 54\alpha\gamma^7 + 14\alpha\beta\gamma^7 \\
& - 242\mathrm{c}\gamma^7 - 22\beta\mathrm{c}\gamma^7 - 172\alpha\gamma^8 + 12\alpha\beta\gamma^8 + 78\mathrm{c}\gamma^8 - 14\beta\mathrm{c}\gamma^8 + 33\alpha\gamma^9 \\
& - \alpha\beta\gamma^9 - 33\mathrm{c}\gamma^9 + \beta\mathrm{c}\gamma^9 > 0, \\
\Gamma = {} & 262144 + 327680\beta + 1310720\gamma + 589824\beta\gamma + 819200\gamma^2 - 2162688\beta\gamma^2 \\
& - 32768\beta^2\gamma^2 - 5750784\gamma^3 - 3620864\beta\gamma^3 - 32768\beta^2\gamma^3 - 8544144\gamma^4 \\
& + 6602752\beta\gamma^4 + 204800\beta^2\gamma^4 + 4096\beta^3\gamma^4 + 9707520\gamma^5 + 9478144\beta\gamma^5 \\
& + 135168\beta^2\gamma^5 + 4096\beta^3\gamma^5 + 21912576\gamma^6 - 11889664\beta\gamma^6 - 559104\beta^2\gamma^6 \\
& - 22528\beta^3\gamma^6 - 7085056\gamma^7 - 3531136\beta\gamma^7 - 177152\beta^2\gamma^7 - 16384\beta^3\gamma^7 \\
& - 28749824\gamma^8 + 13357056\beta\gamma^8 + 817408\beta^2\gamma^8 + 51712\beta^3\gamma^8 + 342528\gamma^9 \\
& + 11342336\beta\gamma^9 + 40192\beta^2\gamma^9 + 23040\beta^3\gamma^9 + 21849024\gamma^{10} - 9276544\beta\gamma^{10} \\
& - 648832\beta^2\gamma^{10} - 60096\beta^3\gamma^{10} + 3240064\gamma^{11} - 5676864\beta\gamma^{11} + 51520\beta^2\gamma^{11} \\
& - 12928\beta^3\gamma^{11} - 9941488\gamma^{12} + 3809152\beta\gamma^{12} + 286912\beta^2\gamma^{12} + 34048\beta^3\gamma^{12} \\
& - 2437808\gamma^{13} + 1611488\beta\gamma^{13} + 4272\beta^2\gamma^{13} + 3392\beta^3\gamma^{13} + 2634676\gamma^{14} \\
& - 807368\beta\gamma^{14} - 105420\beta^2\gamma^{14} - 7616\beta^3\gamma^{14} + 770200\gamma^{15} \\
& - 182608\beta\gamma^{15} - 15368\beta^2\gamma^{15} - 1920\beta^3\gamma^{15} - 321364\gamma^{16} + 8376\beta\gamma^{16} \\
& + 35164\beta^2\gamma^{16} + 592\beta^3\gamma^{16} - 119052\gamma^{17} + 504\beta\gamma^{17} + 52\beta^2\gamma^{17} \\
& + 512\beta^3\gamma^{17} - 1712\gamma^{18} + 24512\beta\gamma^{18} - 4192\beta^2\gamma^{18} + 16\beta^3\gamma^{18} + 12496\gamma^{19} \\
& - 4080\beta\gamma^{19} + 192\beta^2\gamma^{19} - 32\beta^3\gamma^{19} + 165\gamma^{20} - 170\beta\gamma^{20} + 5\beta\gamma^{20} > 0 \text{ and} \\
\Delta = {} & 2048 + 5120\gamma - 4068\gamma^2 - 1024\beta\gamma^2 - 15616\gamma^3 - 2560\beta\gamma^3 - 6656\gamma^4 \\
& + 1536\beta\gamma^4 + 128\beta^2\gamma^4 + 19712\gamma^5 + 6528\beta\gamma^5 + 320\beta^2\gamma^5 - 10048\gamma^6 \\
& - 704\beta\gamma^6 - 96\beta^2\gamma^6 - 12256\gamma^7 - 6432\beta\gamma^7 - 656\beta^2\gamma^7 + 10888\gamma^8 \\
& + 112\beta\gamma^8 - 136\beta^2\gamma^8 + 3564\gamma^9 + 3152\beta\gamma^9 + 420\beta^2\gamma^9 \\
& - 7556\gamma^{10} + 440\beta\gamma^{10} + 160\beta^2\gamma^{10} - 592\gamma^{11} - 1088\beta\gamma^{11} - 60\beta^2\gamma^{11} \\
& + 3562\gamma^{12} - 444\beta\gamma^{12} - 52\beta^2\gamma^{12} + 72\gamma^{13} + 272\beta\gamma^{13} - 12\beta^2\gamma^{13} \\
& - 792\gamma^{14} + 108\beta\gamma^{14} - 2\beta^2\gamma^{14} + 9\gamma^{15} - 6\beta\gamma^{15} + \beta^2\gamma^{15} > 0.
\end{aligned}$$

Licensing through a per-unit royalty

The outcomes of the no licensing case are the same with the main model. When licensing takes place through a per-unit royalty, $r \geq 0$, imposed on firm E's output, there are four possible third-stage subgames. In the first subgame, firm I does not serve none of the downstream firms and thus gains the monopoly profits, π_I^{M}. In the second subgame, firm I decides to serve only firm D and the equilibrium outcomes coincide with the respective ones of the no licensing case, i.e., w_D^N, π_D^N, π_I^N. In the rest of the subgames, which we analyze next, firm *I* either serves both firm *D* and firm *E* or serves only firm *E*.

(a) Serving both firm *D* and firm *E*

In the last stage, firms *D*, *I* and *E* choose their outputs in order to maximize their profits given by:

$$\pi_D(q_D, q_I, q_E, w_D, w_E, r) = (\alpha - q_D - \gamma q_{\mathrm{I}} - \gamma q_{\mathrm{E}} - w_D)q_D + rq_E; \tag{23}$$

$$\pi_{\mathrm{I}}(q_D, q_I, q_E, w_D, w_E, r) = (\alpha - q_I - \gamma q_D - \gamma q_{\mathrm{E}} - c)q_I + (w_D - c)q_D + (w_E - c)q_E; \tag{24}$$

$$\pi_E(q_D, q_I, q_E, w_D, w_E, r) = (\alpha - q_E - \gamma q_D - \gamma q_I - w_E - r)q_E. \tag{25}$$

Solving the system of the first order conditions, we obtain:

$$q_D(w_D, w_E, r) = \frac{\alpha(2-\gamma) - 2w_D + \gamma(c + r - w_D + w_E)}{2(2-\gamma)(1+\gamma)}; \tag{26}$$

$$q_{\mathrm{I}}(w_D, w_E, r) = \frac{\alpha(2-\gamma) - c(2+\gamma) + \gamma(r + w_D + w_E)}{2(2-\gamma)(1+\gamma)}; \tag{27}$$

$$q_E(w_D, w_E, r) = \frac{\alpha(2-\gamma) - 2(r + w_E) + \gamma(c - r + w_D - w_E)}{2(2-\gamma)(1+\gamma)}. \tag{28}$$

In the previous stage of the game, firm *I* bargains with each downstream firm *i*, with $i = D, E$, over (w_i, T_i) taking as given the outcome of the simultaneously run two-part tariff negotiations with the downstream rival *j*, where $j = D, E$, with $i \neq j$. Letting $(\widehat{w}_j^L, \widehat{T}_j^L)$ denote the equilibrium bargaining outcome offered to rival *j*, w_i and T_i are chosen to solve the following maximization problem:

$$\max_{w_i, T_i} [\pi_I\left(w_i, \widehat{w}_j^L, r\right) - d_{I_i} + T_i + T_j]^{\beta} [\pi_i\left(w_i, \widehat{w}_j^L, r\right) - d_i - T_i]^{1-\beta} \tag{29}$$

where $\pi_I\left(w_i, \widehat{w}_j^L, r\right)$ and $\pi_i\left(w_i, \widehat{w}_j^L, r\right)$ are found after substituting (26), (27) and (28) into (23), (24) and (25), respectively. If an agreement between firms *I* and *i* is not reached, then their disagreement payoffs differ depending on the bargaining pair (*I, i*). In particular, when an agreement between firm *I* and firm *D* is not reached, firm *I*'s disagreement payoff is given by $d_{I_D}\left(\hat{w}_E^L, r, \hat{T}_E^L\right) \equiv \left(a - q_I^B\left(\hat{w}_E^L, r\right) - \gamma q_E^B\left(\hat{w}_E^L, r\right) - c\right) q_I^B\left(\hat{w}_E^L, r\right) + \left(\hat{w}_E^L - c\right) q_E^B\left(\hat{w}_E^L, r\right) + \hat{T}_E^L$, where $q_I^B\left(\hat{w}_E^L, r\right) = \frac{\alpha(2-\gamma) - 2c + \gamma\left(r + \hat{w}_E^L\right)}{4-\gamma^2}$ and $q_E^B\left(\hat{w}_E^L, r\right) = \frac{\alpha(2-\gamma) + c\gamma - 2\left(r + \hat{w}_E^L\right)}{4-\gamma^2}$. Firm *D*'s disagreement payoff is given by $d_D(r) \equiv rq_E^B$, namely, its licensing revenues. Instead, when an agreement between firm *I* and firm *E* is not reached, firm *I*'s disagreement payoff is given by $d_{I_{\mathrm{E}}}\left(\hat{w}_D^L, r, \hat{T}_D^L\right) \equiv \left(a - q_I^C\left(\hat{w}_D^L, r\right) - \gamma q_D^C\left(\hat{w}_D^L, r\right) - c\right)$

$q_I^C(\hat{w}_D^L, r) + (\hat{w}_D^L - c) q_D^C(\hat{w}_D^L, r) + \hat{T}_D^L$, where $q_D^C(\hat{w}_D^L, r)$ and $q_I^C(\hat{w}_D^L, r)$ are given by (3) and (4) when $w_D = \hat{w}_D^L$, respectively. Firm E's disagreement payoff is null, $d_E \equiv 0$, since firm E does not have an outside option. Maximizing (29) with respect to T_i, we get:

$$T_i = \beta\Big[\pi_i\Big(w_i, \widehat{w}_j^L, r\Big) - d_i\Big] - (1-\beta)\Big[\pi_I\Big(w_i, \widehat{w}_j^L, r\Big) - d_{I_i}\Big]. \tag{30}$$

Using the above expression, we find:

$$\pi_I\Big(w_i, \widehat{w}_j^L, r\Big) - d_{I_i} + T_i = \beta\Big[\pi_I\Big(w_i, \widehat{w}_j^L, r\Big) + \pi_i\Big(w_i, \widehat{w}_j^L, r\Big) - d_{I_i} - d_i\Big]; \tag{31}$$

$$\pi_i\Big(w_i, \widehat{w}_j^L, r\Big) - d_i - T_i = (1-\beta)\Big[\pi_I\Big(w_i, \widehat{w}_j^L, r\Big) + \pi_i\Big(w_i, \widehat{w}_j^L, r\Big) - d_{I_i} - d_i\Big]. \tag{32}$$

Substituting (31) and (32) into (29), we note that the latter reduces to an expression proportional to the joint profits of firm I and firm i minus their disagreement payoffs. The wholesale prices that maximize this expression are:

$$w_D(r) = \frac{(1-\gamma)\big(\alpha(2-\gamma)\gamma + c(2+\gamma)^2\big)\big(2+\gamma-2\gamma^2\big) + \big(4\gamma-\gamma^3\big)\big(1+\gamma-\gamma^2\big)r}{2(1-\gamma)(2+3\gamma)(2+\gamma(1-2\gamma))}; \tag{33}$$

$$w_{\mathrm{E}}(r) = \frac{(1-\gamma)\big(\alpha(2-\gamma)\gamma + c(2+\gamma)^2\big)\big(2+\gamma-2\gamma^2\big) + \gamma^2\big(8+4\gamma-10\gamma^2+\gamma^3\big)r}{2(1-\gamma)(2+3\gamma)(2+\gamma(1-2\gamma))}. \tag{34}$$

We find that a higher per-unit royalty has a positive impact on firm E's wholesale price and on firm D's wholesale price, $\frac{\partial w_i}{\partial r} > 0$.

After substituting (33) and (34) into (26), (27) and (28), we obtain $q_i(r)$ and substituting them into (23), (24), (25) and (30), we obtain $\widehat{\pi}_I^L(r)$, $\widehat{\pi}_i^L(r)$ and $\widehat{T}_i^L(r)$.

In the first stage of the game, assuming that in the continuation of the game firm I serves both downstream firms, firm D chooses the level of the per-unit royalty that maximizes its profits $\widehat{\pi}_D(r) - \widehat{\mathrm{T}}_D(r)$. The equilibrium per-unit royalty is:

$$\widehat{r}^L = \frac{4(\alpha-c)(1-\gamma)\big(2+\gamma-2\gamma^2\big)\mathrm{Z}}{(2+\gamma)(2+2\gamma-2\gamma^2-\gamma^3)(32+32\gamma-72\gamma^2-40\gamma^3+47\gamma^4-\beta\gamma^4} \tag{35}$$

where $\mathrm{Z} = (8 + 16\gamma - 12\gamma^2 + 2\beta\gamma^2 - 30\gamma^3 + 2\beta\gamma^3 + 4\gamma^4 - 2\beta\gamma^4 + 13\gamma^5 - \beta\gamma^5)$.

We can easily check that a decrease in product differentiation has a negative impact on the per-unit royalty. For the rest of the analysis of this subgame, we make the following assumption that is a sufficient condition that guarantees that the per-unit royalty and the wholesale prices are nonnegative and that an equilibrium exists in this subgame:

Assumption 1 $\beta > \overline{\beta}(\gamma) \equiv \frac{8+16\gamma-12\gamma^2-30\gamma^3+4\gamma^4+13\gamma^5}{\gamma^2(-2-2\gamma+2\gamma^2+\gamma^3)}$ where $\overline{\beta}(\gamma)$ is increasing in γ with $\overline{\beta}(0.948409) = 0$ and $\overline{\beta}(1) = 1$. When $\beta < \overline{\beta}(\gamma)$, there is no equilibrium in which firm I serves both downstream firms.

After making all the suitable substitutions, we obtain the equilibrium wholesale prices,$\widehat{w}_D^L$ and $\widehat{w}_E^L$, the equilibrium quantities, $\widehat{q}_D^L$, $\widehat{q}_I^L$and$\widehat{q}_E^L$, and the equilibrium profits, $\widehat{\pi}_D^L$, $\widehat{\pi}_I^L$ and$\widehat{\pi}_E^L$, which are included in Table 2 of the Appendix.

(b) Serving only firm E

In this subgame, firm I decides to serve only firm E, which means that firm D's profits come from its licensing revenues, $\pi_D(r) = rq_E$. Therefore, in the last stage, only firm I and firm E choose their outputs in order to maximize their profits:

$$\pi_{\mathrm{I}}(q_I, q_E, w_E, r) = (\alpha - q_I - \gamma q_{\mathrm{E}} - c)q_I + (w_E - c)q_E; \tag{36}$$

$$\pi_E(q_I, q_E, w_E, r) = (\alpha - q_E - \gamma q_I - w_E - r)q_E. \tag{37}$$

Solving the system of the first order conditions, we obtain:

$$q_{\mathrm{I}}\left(w_{E,}, r\right) = \frac{\alpha(2-\gamma) + c\gamma - 2(r + w_E)}{4 - \gamma^2}; \tag{38}$$

$$q_E(w_E, r) = \frac{\alpha(2-\gamma) - 2c + \gamma(r + w_E)}{4 - \gamma^2}. \tag{39}$$

In the previous stage of the game, firm I and firm E solve the following maximization problem:

$$\max_{w_{\mathrm{E}}, T_{\mathrm{E}}} [\pi_I(w_{\mathrm{E}}, r) - d_{\mathrm{I}} + T_{\mathrm{E}}]^{\beta}[\pi_{\mathrm{E}}(w_{\mathrm{E}}, r) - T_{\mathrm{E}}]^{1-\beta} \tag{40}$$

where $\pi_I(w_{\mathrm{E}}, r)$ and $\pi_{\mathrm{E}}(w_{\mathrm{E}}, r)$ are found after substituting (38) and (39) into (36) and (37), respectively. The disagreement payoff of firm E is null, while firm I's disagreement payoff is given by $d_{\mathrm{I}} = \pi_I^{\mathrm{M}}$. Maximizing (40) with respect to T_{E}, yields:

$$T_{\mathrm{E}} = \beta\pi_{\mathrm{E}}(w_{\mathrm{E}}, r) - (1-\beta)[\pi_I(w_{\mathrm{E}}, r) - d_{\mathrm{I}}]. \tag{41}$$

Using the above expression, we find:

$$\pi_I(w_{\mathrm{E}}, r) - d_{\mathrm{I}} + T_{\mathrm{E}} = \beta[\pi_I(w_{\mathrm{E}}, r) + \pi_{\mathrm{E}}(w_{\mathrm{E}}, r) - d_{\mathrm{I}}]; \tag{42}$$

$$\pi_{\mathrm{E}}(w_{\mathrm{E}}, r) - T_{\mathrm{E}} = (1-\beta)[\pi_I(w_{\mathrm{E}}, r) + \pi_{\mathrm{E}}(w_{\mathrm{E}}, r) - d_{\mathrm{I}}]; \tag{43}$$

Substituting (42) and (43) into (40), we note that the latter reduces to an expression proportional to the joint profits of firms I and E minus firm I's disagreement payoff. The wholesale price that maximizes this expression is:

$$w_{\mathrm{E}}(r) = \frac{c(8 - \gamma(4 + \gamma(2+\gamma))) + \gamma(\alpha(2-\gamma)^2 + 4\gamma r}{8 - 6\gamma^2}. \tag{44}$$

Table 3 Equilibrium values under licensing through per-unit royalty when firm I serves only firm E

$\hat{w}_{\mathrm{E}}^{L\mathrm{E}}$	$= \frac{\alpha\gamma(4-\gamma(2+\gamma))+c(8-\gamma(4+(4-\gamma)\gamma))}{8-6\gamma^2}$
$\hat{q}_{I}^{L\mathrm{E}}$	$= \frac{(\alpha-c)\left(4-\gamma-2\gamma^2\right)}{8-6\gamma^2}$
$\hat{q}_{\mathrm{E}}^{L\mathrm{E}}$	$= \frac{(\alpha-c)(1-\gamma)}{4-3\gamma^2}$
$\hat{\pi}_{D}^{L\mathrm{E}}$	$= \frac{(\alpha-c)(1-\gamma)^2}{8-6\gamma^2}$
$\hat{\pi}_{I}^{L\mathrm{E}}$	$= \frac{(\alpha-c)^2\left(4+\beta(1-\gamma)^2-3\gamma^2\right)}{4\left(4-3\gamma^2\right)}$
$\hat{\pi}_{E}^{L\mathrm{E}}$	$= \frac{(1-\beta)(a-c)^2(1-\gamma)^2}{4\left(4-3\gamma^2\right)}$

In line with our previous analysis, a per-unit royalty always has a positive impact on firm E's wholesale price. After substituting (44) into (38) and (39), we obtain $q_i(r)$ and substituting them into (36), (37) and (41), we obtain $\widehat{\pi}_{I}^{L\mathrm{E}}(r)$, $\widehat{\pi}_{E}^{L\mathrm{E}}(r)$ and $\widehat{T}_{E}^{L\mathrm{E}}(r)$, respectively.

In the first stage of the game, assuming that in the continuation of the game firm I serves only firm E, firm D chooses the level of the per-unit royalty that maximizes its profits $\pi_D(r)$. The resulting equilibrium per-unit royalty is:

$$\widehat{r}^{LE} = \frac{(\alpha - c)(1-\gamma)}{2}. \tag{45}$$

After making all the suitable substitutions, we obtain the equilibrium wholesale price, $\widehat{w}_{E}^{L\mathrm{E}}$, the equilibrium quantities, $\widehat{q}_{I}^{L\mathrm{E}}$ and $\widehat{q}_{E}^{L\mathrm{E}}$, and the equilibrium profits, $\widehat{\pi}_{I}^{L\mathrm{E}}$, $\widehat{\pi}_{D}^{L\mathrm{E}}$, $\widehat{\pi}_{E}^{L\mathrm{E}}$ which are included in Table 3 of the Appendix.

Non-integrated supplier

In the no licensing case, in the subgame in which firm I serves only firm D or only firm K, thus, in equilibrium, a monopoly exists in the downstream market: $w^{SM} = c$, $\pi_i^{SM} = \frac{(1-\beta)(\alpha-c)^2}{4}$ with $i = D, K$ and $\pi_I^{SM} = \frac{\beta(\alpha-c)^2}{4}$.

In the no licensing case, in the subgame in which firm I can serve both firm D and firm K, in the last stage, each firm i chooses its quantity to maximize: $\pi_i\left(q_i, q_j, w_i\right) = (\alpha - q_i - \gamma q_j)q_i - w_i q_i$, with $i, j = D, K$ and $i \neq j$. This results in:$q_i\left(w_i, w_j\right) = \frac{a(2-\gamma)-2w_i+\gamma w_j}{4-\gamma^2}$.

Letting w_j^{SN} denote the equilibrium outcome of the negotiations of firm I and firm j, w_i is chosen to maximize:

$$\max_{w_i, T_i} [\pi_I\left(w_i, w_j^{SN}\right) + T_i + T_j - d_{I_1}\left(w_j^{SN}, T_j^{SN}\right)]^{\beta} [\pi_i\left(w_i, w_j^{SN}\right) - T_i]^{1-\beta}, \tag{46}$$

where $\pi_I\left(w_i, w_j^{SN}\right) = (w_i - c)q_i\left(w_i, w_j^{SN}\right) + \left(w_j^{SN} - c\right)q_j\left(w_i, w_j^{SN}\right)$ and $\pi_i\left(w_i, w_j^{SN}\right) = \pi_i\left(q_i(w_i, w_j^{SN}\right), q_j\left(w_i, w_j^{SN}\right), w_i, w_j^{SN})$ and the disagreement payoff of firm I is given by $d_{I_1}\left(w_j^{SN}, T_j^{SN}\right) = \left(w_j^{SN} - c\right)q_j^{\text{mon}}\left(w_j^{SN}\right) + T_j^{SN}$, where $q_j^{\text{mon}}\left(w_j^{SN}\right) = (a - w_j^{SN})/2$. From the first order conditions of (45), we obtain: $w^{SN} = c - \frac{\gamma^2(\alpha-c)}{2(2-\gamma^2)}$, where $w^{SN} < c$. The respective equilibrium quantities are: $q_i^{SN} = \frac{(\alpha-c)(2-\gamma)}{2(2-\gamma^2)}$. In turn, the equilibrium net profits are:

$$\pi_i^{SN} = \pi_i\left(q_i^{SN}\right) - \mathrm{T}_i^{SN} = \frac{(1-\beta)(\alpha-c)^2(2-\gamma)^2}{8(2-\gamma)^2}; \tag{47}$$

$$\pi_{\mathrm{I}}^{SN} = \frac{(\alpha-c)^2(2-\gamma)\left(-\gamma^3 + \beta(2-\gamma)\left(2-\gamma^2\right)\right)}{4\left(2-\gamma^2\right)^2}. \tag{48}$$

Note that $\pi_{\mathrm{I}}^{SN} > 0$ if and only if $\beta > \beta_2(\gamma) \equiv \frac{\gamma^3}{4-2\gamma-2\gamma^2+\gamma^3}$, where $\frac{\partial\beta_2(\gamma)}{\partial\gamma} > 0$, $\beta_2(0) = 0$and$\beta_2(1) = 1$. When $\beta < \beta_2(\gamma)$, firm I cannot benefit from its bargaining power since it is not so strong to extract a large fixed fee. The same also holds when γ is low enough and β is small. This is so, because then firm I's disagreement payoff becomes negative due to the subsidy and cannot be outweighed by a positive fixed fee. In the second stage of the game, firm I would prefer to serve two downstream firms instead of one, namely, when it holds that $\pi_I^{SN} > \pi_I^{SM}$, if and only if $0 < \gamma < 0.653986$ and $\beta > \beta_3(\gamma) \equiv \frac{2\gamma^3-\gamma^4}{4-8\gamma+2\gamma^2+4\gamma^3-2\gamma^4}$ where $\frac{\partial\beta_3(\gamma)}{\partial\gamma} > 0$, $\beta_3(0) = 0$and$\beta_3(0.653986) = 1$ and $\beta_3(\gamma) > \beta_2(\gamma)$.

In the licensing case, in the subgame in which firm I serves two downstream firms, the equilibrium outcomes coincide with those of the no licensing case, which analyzed above.

In the licensing case, in the subgame in which firm I serves all three downstream firms, in the last stage, each firm i chooses its quantity to maximize: $\pi_i(q_i, q_j, q_k, w_i) = (\alpha - q_i - \gamma q_j - \gamma q_k)q_i - w_i q_i$, with $i, j, k = D, E, K$ and $i \neq j \neq k$. This results in: $q_i\left(w_i, w_j, w_k\right) = \frac{a(2-\gamma)-(2+\gamma)w_i+\gamma(w_j+w_k)}{2(2-\gamma)(1+\gamma)}$.

Letting w_j^{SL} and w_k^{SL} denote the equilibrium outcomes of the negotiations of firm I and firms j and k, w_i is chosen to maximize:

$$\max_{w_i, T_i}[\pi_I\left(w_i, w_j^{SL}, w_k^{SL}\right) + T_i + T_j + T_k - d_{I_1}\left(w_j^{SL}, w_k^{SL}, T_j^{SL}, T_k^{SL}\right)]^{\beta}$$
$$[\pi_i\left(w_i, w_j^{SL}, w_k^{SL}\right) - T_i]^{1-\beta}, \tag{49}$$

where $\pi_I\left(w_i, w_j^{SL}, w_k^{SL}\right) = (w_i - c)q_i\left(w_i, w_j^{SL}, w_k^{SL}\right) + \left(w_j^{SL} - c\right)$ $q_j\left(w_i, w_j^{SL}, w_k^{SL}\right) + (w_k^{SL} - c)q_k\left(w_i, w_j^{SL}, w_k^{SL}\right)$, $\pi_i\left(w_i, w_j^{SL}, w_k^{SL}\right) =$

$\pi_i\left(q_i\left(w_i, w_j^{SL}, w_k^{SL}\right), q_j\left(w_i, w_j^{SL}, w_k^{SL}\right), w_i, w_j^{SL}, w_k^{SL}\right)$ and the disagreement payoff of firm I is given by $d_{I_1}\left(w_j^{SL}, w_k^{SL}, T_j^{SL}, T_k^{SL}\right) = \left(w_j^{SL} - c\right)q_j\left(w_j^{SL}, w_k^{SL}\right) + \left(w_k^{SL} - c\right)q_k\left(w_j^{SL}, w_k^{SL}\right) + T_j^{SL} + T_k^{SL}$, where $q_j\left(w_j^{SL}, w_k^{SL}\right) = \frac{a(2-\gamma)-2w_j^{SL}+\gamma w_k^{SL}}{4-\gamma^2}$. From the first order conditions of (49), we obtain: $w^{SL} = \frac{\alpha\gamma^2-c(2-\gamma)(1+\gamma)}{2\gamma^2-\gamma-2}$, where $w^{SL} < w^{\mathrm{SN}} < c$. The respective equilibrium quantities are: $q_i^{SL} = \frac{(a-c)(2-\gamma)}{4+2\gamma-4\gamma^2}$. In turn, the equilibrium net profits are:

$$\pi_D\left(q_D^{SL}\right) - T_D^{SL} = \pi_E\left(q_E^{SL}\right) - T_E^{SL} = \pi_K\left(q_K^{SL}\right) - T_K^{SL} = \frac{(1-\beta)(\alpha-c)^2(2-\gamma)^2}{4(2+\gamma)(2+\gamma(1-2\gamma))}; \tag{50}$$

$$\pi_I^{SL} = \frac{3(\alpha-c)^2(2-\gamma)\left(4\beta - 5\beta\gamma^2 - 2(2-\beta)\gamma^3\right)}{4(2+\gamma)\left(2+\gamma-2\gamma^2\right)^2}. \tag{51}$$

Note that $\pi_I^{SL} > 0$ if and only if $0 < \gamma < 0.780776$ and $\beta > \beta_4(\gamma) \equiv \frac{4\gamma^3}{(2-\gamma)(2+\gamma-2\gamma^3)}$, where $\frac{\partial\beta_4(\gamma)}{\partial\gamma} > 0$, $\beta_4(0) = 0$and$\beta_4(0.780776) = 1$ and it holds that $\beta_4(\gamma) > \beta_2(\gamma)$. In the second stage of the game, firm I decides whether it will provide the input to all three downstream firms or only to two downstream firms: $\pi_I^{SL}-\pi_I^{\mathrm{SN}}=\frac{1}{4}(\alpha-c)^2(2-\gamma)(\frac{3(4\beta-5\beta\gamma^2-2(2-\beta)\gamma^3)}{(2+\gamma)(2+\gamma-2\gamma^2)^2} - \frac{\beta(2-\gamma)(2-\gamma^2)-\gamma^3}{(2-\gamma^2)^2})>0$ if and only if $\beta > \beta_5(\gamma) \equiv \frac{\gamma^3(40-12\gamma-38\gamma^2+15\gamma^3+8\gamma^4-4\gamma^5)}{2(2-\gamma)(1-\gamma)(2-\gamma^2)(1-\gamma-\gamma^2)(2+\gamma-2\gamma^2)}$, where $\frac{\partial\beta_5(\gamma)}{\partial\gamma} > 0$, $\beta_5(0) = 0$and$\beta_5(0.437465) = 1$, and it holds that $\beta_5(\gamma) > \beta_3(\gamma) > \beta_4(\gamma) > \beta_2(\gamma)$. Thus, when $\beta > \beta_5(\gamma)$, firm I prefers to serve three instead of two downstream firms. If $\beta_3(\gamma) < \beta < \beta_5(\gamma)$ occurs, firm I would provide the input to two downstream firms and if $\beta<\beta_3(\gamma)$ occurs, firm I would provide the input to one downstream firm.

In the first stage of the game, in case of licensing, firm D optimally sets $F^{SL} = \pi_E\left(q_E^{SL}\right) - T_E^{SL}$ and its profits are: $\pi_D^{SL} = \frac{(1-\beta)(\alpha-c)^2(2-\gamma)^2}{(2+\gamma)(2+\gamma(1-2\gamma))}$. Therefore, firm D decides whether it will license its technology to firm E: $\pi_D^{SL} - \pi_D^{SN} = \frac{(1-\beta)(\alpha-c)^2(2-\gamma)^2(4-\gamma(4+\gamma(1-2\gamma)))}{8(2+\gamma)(2-\gamma^2)(2+\gamma(1-2\gamma))}$. The difference is positive and hence, firm D prefers licensing compared to no licensing if and only if $\beta > \beta_5(\gamma)$. Otherwise, firm D is indifferent between licensing and no licensing.

Checking when firm D is more likely to license its technology, with a vertically integrated firm I or with a non-vertically integrated firm I, we find that the difference: $\left(\pi_D^L - \pi_D^N\right) - \left(\pi_D^{SL} - \pi_D^{SN}\right) = \frac{(1-\beta)(\alpha-c)^2\gamma H}{8(2+\gamma)^2(2+3\gamma)^2(2-\gamma^2)(2+\gamma-2\gamma^2)(-4+3\gamma^3)} < 0$, where $H = 512 + 256\gamma - 1920\gamma^2 + 672\gamma^3 + 2608\gamma^4 + 784\gamma^5 - 1448\gamma^6 - 446\gamma^7 + 279\gamma^8 + 90\gamma^9 > 0$, namely, licensing is more likely to occur when the supplier is non-integrated than a vertically integrated firm.

Sequential bargaining

In the licensing case, if firm D has not reached an agreement with the supplier, then firm I serves only firm E, and the equilibrium outcomes coincide with the no licensing case. In the licensing case, solving by backward induction, if firm D reaches an agreement with the supplier, then firm I and firm E solve the following: $\max_{w_E, T_E} [\pi_I(w_D, w_E) - d_I^O + T_D + T_E]^{\beta} [\pi_E(w_D, w_E) - T_E]^{1-\beta}$. Taking this into account, solving the previous stage of the game in which firm I negotiates with firm D over(w_D, T_D), they solve

$$\max_{w_D, T_D} [\pi_I(w_D, w_E(w_D)) - d_I^O + T_D + T_E(w_D)]^{\beta} [\pi_D(w_D, w_E(w_D)) - T_D]^{1-\beta}.$$

This leads to $w_D^B = \frac{\Theta}{2\Lambda}$ and $w_E^B = \frac{\Theta - \mathrm{O}}{2\Lambda}$, where $\Theta = 128c + 128\alpha\gamma + 128c\gamma + 64\alpha\gamma^2 - 32\alpha\beta\gamma^2 - 288c\gamma^2 + 32\beta c\gamma^2 - 288\alpha\gamma^3 - 256c\gamma^3 - 56\alpha\gamma^4 + 64\alpha\beta\gamma^4 + 184c\gamma^4 - 72\beta c\gamma^4 + 200\alpha\gamma^5 - 20\alpha\beta\gamma^5 + 184c\gamma^5 + 12\beta c\gamma^5 + 8\alpha\gamma^6 - 20\alpha\beta\gamma^6 + 6c\gamma^6 + 28\beta c\gamma^6 - 49\alpha\gamma^7 + 4\alpha\beta\gamma^7 - 47c\gamma^7 + 2\alpha\beta\gamma^8 - 24\gamma^8 - 2\beta c\gamma^8 + 4\alpha\gamma^9 - 4c\gamma^9$, $\Lambda = (64 + 128\gamma - 112\gamma^2 - 272\gamma^3 + 64\gamma^4 - 4\beta\gamma^4 + 192\gamma^5 - 4\beta\gamma^5 + 7\gamma^6 + 4\beta\gamma^6 - 48\gamma^7 + 2\beta\gamma^7 - 12\gamma^8)$ and $\mathrm{O} = 4(\alpha - \mathrm{c})(2 - \gamma)(1 - \gamma)\gamma(8 + 16\gamma - 4\beta\gamma + 2\gamma^2 - 4\beta\gamma^2 - 11\gamma^3 + 4\beta\gamma^3 - 4\gamma^4 + 2\beta\gamma^4)$ and it holds that $w_D^B > w_E^B > w^N$.

Firm D prefers licensing rather than no licensing when $\pi_D^B - \pi_D^N = \frac{\Xi}{(4-3\gamma^2)^2\Sigma^2} > 0$, where $\Xi = ((1 - \beta)\ (\alpha - c)^2\ (1 - \gamma)^2\ (16384 + 32768\gamma - 81920\gamma^2 - 159744\gamma^3 + 4096\beta\gamma^3 + 192512\gamma^4 + 12288\beta\gamma^4 - 1024\beta^2\gamma^4 + 315392\gamma^5 - 4096\beta\gamma^5 - 2048\beta^2\gamma^5 - 293632\gamma^6 - 41984\beta\gamma^6 + 2816\beta^2\gamma^6 - 332288\gamma^7 - 15616\beta\gamma^7 + 6400\beta^2\gamma^7 + 317312\gamma^8 + 57344\beta\gamma^8 - 3328\beta^2\gamma^8 + 220032\gamma^9 + 36480\beta\gamma^9 - 8000\beta^2\gamma^9 - 241184\gamma^{10} - 37632\beta\gamma^{10} + 2096\beta^2\gamma^{10} - 113472\gamma^{11} - 31488\beta\gamma^{11} + 5024\beta^2\gamma^{11} + 120064\gamma^{12} + 9976\beta\gamma^{12} - 560\beta^2\gamma^{12} + 52836\gamma^{13} + 12664\beta\gamma^{13} - 1616\beta^2\gamma^{13} - 33165\gamma^{14} + 616\beta\gamma^{14} - 64\beta^2\gamma^{14} - 18096\gamma^{15} - 1916\beta\gamma^{15} + 224\beta^2\gamma^{15} + 2808\gamma^{16} - 576\beta\gamma^{16} + 44\beta^2\gamma^{16} + 2880\gamma^{17} - 48\beta\gamma^{17} + 432\gamma^{18}))$ > 0 if and only if the final products are not close substitutes, namely when $\gamma < 0.840326$ and $\beta > \beta_B(\gamma) = \frac{\Phi}{\mathrm{X}} - 2\sqrt{\frac{\Psi}{\Omega}}$, where $\frac{\partial\beta_B(\gamma)}{\partial\gamma} > 0$, $\beta_B(0.840326) = 0$and$\beta_B(0.968449) = 1$, where $\Phi = 0.5(512 + 1024\gamma - 1024\gamma^2 - 2944\gamma^3 + 480\gamma^4 + 3232\gamma^5 + 336\gamma^6 - 1568\gamma^7 - 416\gamma^8 + 263\gamma^9 + 120\gamma^{10} + 12\gamma^{11})$, $X = \gamma(128 + 128\gamma - 352\gamma^2 - 256\gamma^3 + 384\gamma^4 + 184\gamma^5 - 190\gamma^6 - 62\gamma^7 + 34\gamma^8 + 11\gamma^9)$, $\Psi = (65{,}536 + 131072\gamma - 425984\gamma^2 - 786432\gamma^3 + 1.441792{*}10^6\gamma^4 + 2.097152{*}10^6\gamma^5 - 3.325952{*}10^6\gamma^6 - 3.299328{*}10^6\gamma^7 + 5.501696{*}10^6\gamma^8 + 3.49184{*}10^6\gamma^9 - 6.506944{*}10^6\gamma^{10} - 2.768{*}10^6\gamma^{11} + 5.453552{*}10^6\gamma^{12} + 1.812272{*}10^6\gamma^{13} - 3.196476{*}10^6\gamma^{14} - 1.01028{*}10^6\gamma^{15} + 1.272625{*}10^6\gamma^{16} + 446117\gamma^{17} - 321699\gamma^{18} - 137879\gamma^{19} + 43192\gamma^{20} + 25608\gamma^{21} - 1068\gamma^{22} - 2124\gamma^{23} - 288\gamma^{24})$ and $\Omega = \gamma^4\ (2 + 2\gamma - 2\gamma^2 - \gamma^3)^2\ (64 - 112\gamma^2 + 16\ \gamma^3 + 64\gamma^4 - 12\gamma^5 - 11\gamma^6)^2$.

Price competition

In the licensing case, firms maximize these profits: $\pi_i^{LP} = (p_i - w_i)\left(\frac{a(1-\gamma)-(1+\gamma)p_i+\gamma(p_j+p_I)}{1+\gamma-2\gamma^2}\right)$ and $\pi_{\mathrm{I}}^{LP} = (p_{\mathrm{I}} - c)\left(\frac{a(1-\gamma)-(1+\gamma)p_I+\gamma(p_i+p_j)}{1+\gamma-2\gamma^2}\right) + (w_i - c)\left(\frac{a(1-\gamma)-(1+\gamma)p_i+\gamma(p_j+p_I)}{1+\gamma-2\gamma^2}\right) + (w_j - c)\left(\frac{a(1-\gamma)-(1+\gamma)p_j+\gamma(p_i+p_I)}{1+\gamma-2\gamma^2}\right)$ with $i,\ j = D,\ E$ and $i \neq j$. Firm D prefers licensing rather than no licensing when firm I serves both the licensor and the licensee, namely, when $\pi_{\mathrm{I}}^{LP} - \pi_{\mathrm{I}}^{NP} = \frac{(\alpha-c)^2(\beta\Sigma)-\Upsilon}{(2-\gamma)^2(1+\gamma)(2+\gamma)^2(1+2\gamma)(4+5\gamma^2)\left(2+5\gamma+8\gamma^2+4\gamma^3\right)^2} > 0$, where $\Sigma = 64 + 256\gamma + 592\gamma^2 + 656\gamma^3 + 412\gamma^4 - 176\gamma^5 - 657\gamma^6 - 961\gamma^7 - 769\gamma^8 + 61\gamma^9 + 378\gamma^{10} + 96\gamma^{11} - 16\gamma^{12} + 32\gamma^{13} + 32\gamma^{14}$ and $\Upsilon = 32\gamma^2 + 64\gamma^3 + 224\gamma^4 + 424\gamma^5 + 618\gamma^6 + 578\gamma^7 + 297\gamma^8 - 11\gamma^9 - 243\gamma^{10} - 181\gamma^{11} + 22\gamma^{12} + 80\gamma^{13} + 40\gamma^{14}$.

This holds when $\gamma < 0.814489$ and $\beta > \beta_6(\gamma) = \frac{\Upsilon}{\Sigma}$, where $\beta_6(\gamma)$ is increasing in γ, with $\beta_6(0) = 0$, and $\beta_6(0.814489) = 1$.

Proof of Proposition 1 Under licensing through fixed fee, we find that the difference: $\pi_{\mathrm{I}}^{L} - \pi_{\mathrm{I}}^{N} = \frac{(\alpha-c)^2(\gamma^2(-4+3\gamma^2)(2+\gamma(8+3\gamma))+\beta(16-32\gamma^2+24\gamma^3+23\gamma^4-18\gamma^5-9\gamma^6))}{(2+\gamma)^2(2+3\gamma)^2(4-3\gamma^2)}$ is positive if and only if $\gamma \in (0,\frac{2}{3})$ and $\beta > \beta_1(\gamma) \equiv \frac{\gamma^2(4-3\gamma^2)(2+8\gamma+3\gamma^2)}{16-\gamma^2(32-23\gamma^2-3\gamma(8-6\gamma^2-3\gamma^3))}$, where $\frac{\partial\beta_1(\gamma)}{\partial\gamma} > 0,\ \beta_1(0) = 0and\beta_1\left(\frac{2}{3}\right) = 1$. $\beta_1(\gamma)$ depicts all the cases under which the difference$\pi_{\mathrm{I}}^{L} - \pi_{\mathrm{I}}^{N} = 0$, which implies that firm I is indifferent between serving one or both downstream rivals.

Proof of Proposition 2 Calculating the difference in the wholesale prices with and without licensing: $w^L - w^{\mathrm{N}} = -\frac{2(\alpha-c)(2-\gamma)\gamma^2}{(2+3\gamma)(4-3\gamma^2)}$, we find that it is always negative. Thus, the wholesale price with licensing is lower than without licensing.

Proof of Proposition 3 Calculating the difference in firm D's profits between licensing through fixed fee and no licensing, we find: $\pi_D^L - \pi_D^N = \frac{(1-\beta)(\alpha-c)^2(16-32\gamma^2+24\gamma^3+23\gamma^4-18\gamma^5-9\gamma^6))}{(2+\gamma)^2(2+3\gamma)^2(4-3\gamma^2)}$, which is always positive. Hence, when $\beta > \beta_1(\gamma)$, firm D always has incentives to license its technology to firm E.

Proof of Proposition 4 In the no licensing case, consumer surplus is:

$$CS^N = aq_D^N + aq_I^N - \frac{1}{2}\left[\left(q_D^N\right)^2 + \left(q_I^N\right)^2 + 2\gamma q_D^N q_I^N\right] - p_D q_D^N - p_I q_I^N$$
$$= \frac{(\alpha - c)^2\left(8 - 4\gamma - 3\gamma^2\right)}{8\left(4 - 3\gamma^2\right)}.$$

In the case of licensing, consumer surplus is:

$$CS^L = aq_D^L + aq_I^L + aq_E^L - \frac{1}{2}\left[\left(q_D^L\right)^2 + \left(q_I^L\right)^2 + \left(q_E^L\right)^2 + 2\gamma q_D^L q_I^L + 2\gamma q_D^L q_E^L + 2\gamma q_I^L q_E^L\right]$$
$$- p_D q_D^L - p_I q_I^L - p_E q_E^L = \frac{(\alpha - c)^2\left(12 + 28\gamma + 9\gamma^2\right)}{8(2 + 3\gamma)^2}.$$

Calculating $CS^L - CS^N = \frac{(\alpha-c)^2(2+\gamma)(2+3\gamma-3\gamma^2)}{2(2+3\gamma^2(4-3\gamma^2)}$, we find that it is always positive.

Total welfare is defined as the sum of consumer and producer surplus: $W^k = CS^k + \pi_D^k + \pi_I^k + \pi_E^k$ where $k = N, L$. Calculating $W^L - W^N = \frac{(\alpha-c)^2(12-17\gamma^2+9\gamma^3)}{2(2+3\gamma)^2(4-3\gamma^2)}$, we find that it is always positive.

Proof of Proposition 5 Under a per-unit royalty licensing, calculating the difference: $\widehat{w}_D^L - \widehat{w}_E^L = \frac{2(a-c)(1-\gamma)\gamma(2-3\gamma)Z}{(2+\gamma)(2+2\gamma-2\gamma^2-\gamma^3)(32+32\gamma-72\gamma^2-40\gamma^3+47\gamma^4+\beta\gamma^4)}$, we find that it is positive if and only if $\gamma < \frac{2}{3}$. Thus, firm D has greater wholesale price than firm E if and only if $\gamma < \frac{2}{3}$. Otherwise, firm D pays a lower wholesale price than firm E.

Proof of Proposition 6 When licensing takes place through a per-unit royalty, the difference $\widehat{\pi}_I^L - \pi_I^N$ (see Tables 1 and 2 for the expressions $\widehat{\pi}_I^L$ and π_I^N) is positive for $\gamma < 0.218164$. Calculating the difference in firm D's profits when firm I serves both downstream firms and in the no licensing case, we find:$\widehat{\pi}_D^L - \pi_D^N = \frac{(\alpha-c)^2(1-\gamma)^2 M}{(2+\gamma)^2(4-3\gamma^2)(2+2\gamma-2\gamma^2-\gamma^3)(32+32\gamma-72\gamma^2-40\gamma^3+47\gamma^4+\beta\gamma^4)}>0$, where $M = 128 + 128\gamma + 256\beta\gamma - 320\gamma^2 + 384\beta\gamma^2 - 96\gamma^3 - 640\beta\gamma^3 + 528\gamma^4 - 1008\beta\gamma^4 + 8\beta^2\gamma^4 + 48\gamma^5 + 400\beta\gamma^5 + 16\beta^2\gamma^5 - 390\gamma^6 + 780\beta\gamma^6 + 2\beta^2\gamma^6 - 106\gamma^7 + 20\beta\gamma^7 - 10\beta^2\gamma^7 + 98\gamma^8 - 188\beta\gamma^8 - 6\beta^2\gamma^8 + 47\gamma^9 - 46\beta\gamma^9 - \beta^2\gamma^9 > 0$. Therefore, firm D has incentives to license its technology as long as firm I serves both downstream firms.

References

Aliprant M, Milliou C, Petrakis E (2014) Price vs. quantity competition in a vertically related market. Econ Lett 124:122–126. https://doi.org/10.1016/j.econlet.2014.05.002

Arora A, Fosfuri A (2003) Licensing the market for technology. J Econ Behav Organ 52:277–295. https://doi.org/10.1016/S0167-2681(03)00002-7

Arya A, Mittendorf B (2006) Enhancing vertical efficiency through horizontal licensing. J Regul Econ 29:333–342. https://doi.org/10.1007/s11149-006-7403-7

Arya A, Mittendorf B, Sappington D (2007) The bright side of supplier encroachment. Market Sci 26:651–659. https://doi.org/10.1287/mksc.1070.0280

Arya A, Mittendorf B, Sappington D (2008) Outsourcing, vertical integration, and price vs. quantity competition. Int J Ind Organ 26:1–16. https://doi.org/10.1016/j.ijindorg.2006.10.006

Bakaouka E, Milliou C (2018) Vertical licensing, input pricing, and entry. Int J Ind Organ 59:66–96. https://doi.org/10.1016/j.ijindorg.2018.03.007

Baye M, Crocker K, Ju J (1996) Divisionalization, franchising, and divestiture incentives in oligopoly. Am Econ Rev 86:223–236

Bedre-Defolie Ö (2012) Vertical coordination through renegotiation. Int J Ind Organ 30:553–563. https://doi.org/10.1016/j.ijindorg.2012.05.002

Bourreau M, Hombert J, Pouyet J, Schutz N (2011) Upstream competition between vertically integrated firms. J Ind Econ 59:677–713. https://doi.org/10.1111/j.1467-6451.2011.00469.x

Brito D, Pereira P (2010) Access to bottleneck inputs under oligopoly: a prisoners' dilemma? Southern Econ. J. 76:660–677. https://doi.org/10.4284/sej.2010.76.3.660

Bru L, Fauli-Oller R, de Haro JMO (2001) Divisionalization in vertical structures. Available at SSRN 289563. https://ssrn.com/abstract=289563

Caprice S (2006) Multilateral vertical contracting with an alternative supply: the welfare effects of a ban on price discrimination. Rev Ind Org 28:63–80. https://doi.org/10.1007/s11151-006-0008-6

Chen Y, Ishikawa J, Zhihao Y (2004) Trade liberalizatiion and strategic outsourcing. J Int Econ 63:419–436. https://doi.org/10.1016/S0022-1996(03)00056-4

Chen Y (2001) On vertical mergers and their competitive effects. Rand J Econ 32:667–685

Colombo S, Filippini L (2016) Revenue royalties. J Econ 118:47–76. https://doi.org/10.1007/s00712-015-0459-z
Colombo S, Filippini L, Sen D (2023) Patent licensing and capacity in a cournot model. Rev Ind Organ 62:45–62. https://doi.org/10.1007/s11151-022-09886-7
Corchón L (1991) Oligopolistic competition among groups. Econ Lett 36:1–3. https://doi.org/10.1016/0165-1765(91)90045-M
Creane A, Davidson C (2004) Multidivisional firms, internal competition, and the merger paradox. Can J Econ 37:951–977. https://doi.org/10.1111/j.0008-4085.2004.00255.x
Cremer J, Riordan MH (1987) On governing multilateral transactions with bilateral contracts. Rand J Econ 18:436–451
deFontenay CC, Gans JS (2005) Vertical integration in the presence of upstream competition. Rand J Econ 36:544–572
DeGraba P (1990) Input market price discrimination and the choice of technology. Am Econ Rev 80:1246–1253
Ding R, Ko CY (2021) Does licensing improve welfare with rent dissipation? Econ Model 105:105655. https://doi.org/10.1016/j.econmod.2021.105655
Fauli-Oller R, Sandonis J (2003) To merge or to license: implications of competition policy. Int J Ind Organ 21:655–672. https://doi.org/10.1016/S0167-7187(02)00122-4
Fumagalli C, Motta M (2020) Dynamic vertical foreclosure. J Law Econ 63:763–812. https://doi.org/10.1086/709584
Gallini N, Winter R (1985) Licensing in the theory of innovation. Rand J Econ 16:237–252
Gallini N (1984) Deterrence by market sharing: a strategic incentive for licensing. Am Econ Rev 74:931–941
Hart O, Tirole J (1990) Vertical integration and market foreclosure. Brook Pap Eco Ac Microecon 1990:205–286. https://doi.org/10.2307/2534783
Horn H, Wolinsky A (1988) Bilateral monopolies and incentives for merger. Rand J Econ 19:408–419
Hsu L, Liu L, Wang XH, Zeng C (2019) Ad valorem versus per-unit royalty licensing in a Cournot duopoly model. Manch Sch 87:890–901. https://doi.org/10.1111/manc.12280
Inderst R, Shaffer G (2009) Market power, price discrimination, and allocative efficiency in intermediate-goods markets. Rand J Econ 40:658–672. https://doi.org/10.1111/j.1756-2171.2009.00083.x
Kamien M, Tauman Y (1986) Fee versus royalties and the private value of a patent. Q J Econ 101:471–492. https://doi.org/10.2307/1885693
Lin YJ, Lin YS, Shih PC (2022) Welfare reducing vertical licensing in the presence of complementary inputs. J Econ 137:121–143. https://doi.org/10.1007/s00712-022-00782-y
McAfee P, Schwartz M (1994) Opportunism in multilateral vertical contracting: nondiscrimination, exclusivity, and uniformity. Am Econ Rev 84:210–230
McAfee P, Schwartz M (1995) The non-existence of pairwise-proof equilibrium. Econ Lett 49:251–259. https://doi.org/10.1016/0165-1765(95)00681-5
Milliou C, Petrakis E (2007) Upstream horizontal mergers, vertical contracts and bargaining. Int J Ind Organ 25:963–987. https://doi.org/10.1016/j.ijindorg.2006.11.007
Mizuno C (2009) Divisionalization and horizontal mergers in a vertical relationship. Manch Sch 77:317–336. https://doi.org/10.1111/j.1467-9957.2009.02099.x
Mukherjee A, Ray A (2007) Strategic outsourcing and R&D in a vertical structure. Manch Sch 75:297–310. https://doi.org/10.1111/j.1467-9957.2007.01018.x
Mukherjee A (2003) Licensing in a vertically separated industry. University of Nottingham Discussion Paper No. 03/01. https://doi.org/10.2139/ssrn.401502
Muto S (1993) On licensing policies in bertrand competition. Games Econ Behav 5:257–267. https://doi.org/10.1006/game.1993.1015
O'Brien D, Shaffer G (1992) Vertical control with bilateral contracts. Rand J Econ 23:299–308
Ordover J, Shaffer G (2007) Wholesale access in multi-firm markets: When is it profitable to supply a competitor? Int J Ind Organ 25:1026–1045. https://doi.org/10.1016/j.ijindorg.2007.01.001
Poddar S, Sinha UB (2010) Patent licensing from a high-cost firm to a low-cost firm. Econ Rec 86:384–395. https://doi.org/10.1111/j.1475-4932.2010.00633.x
Reisinger M, Tarantino E (2015) Vertical integration, foreclosure, and productive efficiency. Rand J Econ 46:461–479. https://doi.org/10.1111/1756-2171.12093
Rey P, Tirole J (2007) A primer on foreclosure. In: Armstrong M, Porter RH, eds. Handbook of Industrial Organization, vol 3, pp 2145–2220. https://doi.org/10.1016/S1573-448X(06)03033-0
Rey P, Vergé T (2004) Bilateral control with vertical contracts. Rand J Econ 35:728–746

Rey P, Salant D (2012) Abuse of dominance and licensing of intellectual property. Int J Ind Organ 30:518–527. https://doi.org/10.1016/j.ijindorg.2012.05.003

Salant S, Switzer S, Reynolds R (1983) Losses from horizontal merger: the effects of an exogenous change in industry structure on cournot-nash equilibrium. Q J Econ 98:185–199. https://doi.org/10.2307/1885620

San Martin M, Saracho AI (2010) Royalty licensing. Econ Lett 107:284–287. https://doi.org/10.1016/j.econlet.2010.02.010

San Martin M, Saracho AI (2015) Optimal two-part tariff licensing mechanisms. Manch Sch 83:288–306. https://doi.org/10.1111/manc.12059

Sappington D (2005) On the irrelevance of input prices for make-or-buy decisions. Am Econ Rev 95:1631–1638. https://doi.org/10.1257/000282805775014344

Sen D, Tauman Y (2007) General licensing schemes for a cost-reducing innovation. Games Econ Behav 59:163–186. https://doi.org/10.1016/j.geb.2006.07.005

Tsao KC, Hu JL, Hwang H, Lin YS (2023) More licensed technologies may make it worse: a welfare analysis of licensing vertically two-tier foreign technologies. J Econ. https://doi.org/10.1007/s00712-023-00818-x

Wang X (1998) Fee versus royalty licensing in a cournot duopoly model. Econ Lett 60:55–62. https://doi.org/10.1016/S0165-1765(98)00092-5

Yoshida Y (2000) Third-degree price discrimination in input markets: output and welfare. Am Econ Rev 90:240–246. https://doi.org/10.1257/aer.90.1.240

Ziss S (1998) Divisionalization and product differentiation. Econ Lett 59:133–138. https://doi.org/10.1016/S0165-1765(98)00017-2