SERIEs (2013) 4:137–153
DOI 10.1007/s13209-011-0083-2

SERIEs Journal of the Spanish Economic Association

ORIGINAL ARTICLE

Sharing a polluted river through environmental taxes

María Gómez-Rúa

Received: 1 September 2011 / Accepted: 22 November 2011 / Published online: 6 December 2011

Abstract *n* agents located along a river generate residues that then require cleaning to return the river to its natural state, which entails some cost. We propose several rules to distribute the total pollutant-cleaning cost among all the agents. We provide axiomatic characterizations using properties based on water taxes. Moreover, we prove that one of the rules coincides with the weighted Shapley value of a game associated with the problem.

Keywords Cost sharing · Pollutant-cleaning cost · Water taxes

JEL Classification C71 · D61

1 Introduction

The aggravating situation of environmental contamination over the past few years is an important reason for countries to impose taxes on the emission of polluting substances that damage the environment and resources. In Spain, for instance, water and sanitation charges constitute the most representative environmental taxes, these being used by more than two thirds of the governments of the autonomous regions. Sanitation

The author thanks Gustavo Bergantiños for his support and helpful comments during the writing of this paper and acknowledges Miguel Rodríguez for his suggestions. The financial support from Ministerio de Ciencia y Tecnología and FEDER, grant Ref: ECO2008-03484-C02-01/ECON, Ministerio de Ciencia y Educación, grant Ref: ECO2011-23460, and Xunta de Galicia, grants Ref: PGIDIT06PXIB362390PR and Ref: 10 PXIB 362 299 PR, is gratefully appreciated.

M. Gómez-Rúa (✉)
Research Group in Economic Analysis, Facultade de CC. Económicas e Empresariais,
Universidade de Vigo, 36310 Vigo, Pontevedra, Spain
e-mail: mariarua@uvigo.es

charges are mainly a tax instrument used for funding public sewage treatment services (Gago et al. 2006).

An important development in this area is the European Union Water Framework Directive (formally known as Directive 2000/60/EC of the European Parliament and of the Council of 23 October 2000, establishing a framework for Community action in the field of water policy). It is a European Union directive which commits European Union Member States to achieve good qualitative and quantitative status of all water bodies by 2015. This Directive establishes the "polluter-pays" principle, which requires that users pay according to the costs they generate, and determines that Member States will adopt the recovery of costs of water services principle, including environmental and resource costs (see Article 9.1.: recovery of costs for water services).

As such integrated policies among the States of the European Union clarify the link between water use and water pollution, these policies are likely to be more efficient in meeting water management objectives. For example, the costs of cleaning water downstream before it is supplied can be compared with the costs of discouraging pollution upstream. Integrated policies also facilitate cost recovery (OECD 2004). When river-basin authorities have access to the cost of treatment of water supply operators, this provides a wealth of information on the costs of upstream pollution, which can be used to estimate the rates at which the release of pollutants should be charged. River-basin management also facilitates water allocation among competing uses within the basin as well as the control of inter-basin transfers. In Spain, river-basin authorities are purchasing water rights for over-exploited water bodies (OECD 2008).

The Communication "Pricing policies for enhancing the sustainability of water resources", from the European Commission to the Council, the European Parliament and the Economic and Social Committee, states that: "A price directly linked to the water quantities used or pollution produced can ensure that pricing has a clear incentive function for consumers to improve water use efficiency and reduce pollution".

This being the case, the use of economic instruments (taxes, duties, financial assistance, negotiable permits) has gained increasing importance and was fully legitimized in the United Nations' Rio Declaration on the Environment and Development in 1992. The central environmental role played by economic instruments is also recognized at Community level. Furthermore, as we have pointed out, the framework Directive on water advocates a boosting of the part played by pricing to improve the sustainability of water resources.

Given the importance of the task of solving water resource management problems, such as the control of water pollution, the problem of sharing the cleaning costs among all the polluters should be studied formally.

Ni and Wang (2007) developed a theoretical model to study how to allocate pollutant-cleaning costs among the agents that caused this pollution. They consider a river which is divided into n segments from upstream to downstream. In each segment there is an agent who discharges pollutant substances of some kind into the river. The authorities guarantee the cleansing of the water for public use. The authors propose two rules to divide the total river-polluting responsibility among the polluters, the local responsibility sharing (LRS) rule, which charges the agent in a given segment its own local costs, and the upstream equal sharing (UES) rule, which charges an agent the sum of the equal divisions of all downstream costs, including its own local costs.

Littlechild and Owen (1973) studied the airport problem. From a mathematical perspective both models are the same because they both have an order of agents and a cost associated with each agent.

We follow the model of Ni and Wang (2007). We propose several ways to define water taxes mainly following the "polluter-pays" principle, which takes into account the different factors that influence the quality of the water. Assuming that the authority knows for sure the agent responsible for the pollution generated in a particular area, then the agent should be charged with the total cost of cleaning that area. However, in most real-life situations, it is not possible to know exactly who is responsible for the pollution generated in each particular area. This fact will be considered throughout the paper.

One of the most important topics in the cost sharing literature is the axiomatic characterization of rules. The idea is to propose desirable properties and determine which of these characterize every rule. Properties often help agents compare different rules and decide which one is preferred for a particular situation. Following this approach Ni and Wang (2007) characterize both rules with different properties. In particular, they prove that the UES rule is the only one satisfying Efficiency, Additivity, Independence of Upstream Costs (ensuring that no agent is responsible for the pollution caused in the upstream segments) and Upstream Symmetry (which states that for any given downstream cost, all upstream polluters share it equally).

Efficiency and Independence of Upstream Costs are very appealing properties. The European Union Water Framework Directive notes that the objectives of water pricing policies include the full recovery of financial costs, this being established under the property of Efficiency. As stated before, one of the main objectives of charging water use is to reduce pollution. Following Ni and Wang (2007) we implicitly assume that agents pollute the river during some period of time and this pollution must be cleaned. However, we are not able to know who is responsible for the particular pollution being generated at each segment. If we were, we would simply charge agents the cleaning cost they are responsible for. In our model, we only know the cost associated with cleaning each area. Given the direction of the water body, the water transfers the pollution from upstream to downstream. Hence, agents are partially responsible for the pollution costs downstream but not those upstream. The property of Independence of Upstream Costs reflects this fact. A property of Independence of Downstream Costs could make sense in some other situations like when you want to assure that the only pollution found at one area was generated only in that area.

Additivity is a standard property in the literature. Mathematically, it is an appealing property because if a rule is additive, then we only need to be concerned with minor problems, usually easier to solve. Moreover, the Additivity axiom has a clear meaning in this context. Assuming that to clean each area different technologies, with associated costs, may be needed, then, the cost of cleaning each area will be the sum of the costs of using each technology in that area. So, it seems reasonable that the amount that an agent should pay to clean any area of the river is equal to the sum of the shares that the agent should pay to use each technology.

Focusing on a real-life situation, the Communication from the Commission to the Council, European Parliament and Economic and Social Committee: Pricing and sustainable management of water resources, COM (2000) 477, reports that: "The overall price P paid by a given user can be computed as F + a.Q + b.Y, with F: an element related to fixed costs, general taxes, etc.; a: a charge per unit of water used; b: a charge per unit of pollution produced; Q: the total quantity of water used; Y: the total pollution produced. A reduction in the quantity of water used (Q) and/or the pollution produced (Y) will then lead to a reduction in the overall water price P paid by the user. Thus, it provides an incentive for users to increase water use efficiency and reduce pollution."

Therefore, it is clear that the assumption that a tax should be additive is quite reasonable in our context.

With respect to the property of Upstream Symmetry, as we will explain further on in the paper, many situations exist where this axiom cannot be applied. The main objective of this paper is to change Upstream Symmetry and use other more suitable properties. Thus, we will obtain rules applicable to these situations.

Our first result is a characterization of the set of rules satisfying Efficiency, Additivity and Independence of Upstream Costs. Later on, three more characterizations are provided by adding in each case a different property to those in the first result.

To define the first property we take into account that in many cases all the agents dump the same kind of residues into the water. Moreover, all the residues are biodegradable, hence, they decompose into natural chemical elements through the action of biological processes involving light, air, water, bacteria, plants or animals. In this case, pollution disappears with time. Thus, it seems reasonable to assume that the further away the area is from agent i, the smaller the part of the cost that agent i should pay for cleaning this area. This is the statement of Upstream Monotonicity.

The second property follows from the fact that on several occasions when the residues are biodegradable, it is possible to know their biodegradation rate, that is, the rate of deterioration of materials in question under normal, natural conditions, say δ. Hence, the cost that an agent pays for a polluted area should depend on this biodegradation rate. In many countries, for instance, Belgium, Finland and South Africa, among others, water taxes depend on the quality of pollution (OECD 2006). A way to measure the quality of pollution could be through the biodegradation rate: the smaller the rate is, the less hazardous the pollution, as it disappears quicker, and hence, produces less responsibility for agents downstream. This is the idea of $\delta-$Biodegradation Rate.

The motivation of the third property is that in many countries there are several alternatives in the design of water tax rates. There is a variable component which depends on different factors, such as, volume of water consumed, pollution load, population of the municipality, type of residue, etc. This is the case of Austria, Canada, Finland, France, Germany, Greece, Hungary, Italy, Korea, Spain, Sweden, USA, among many others (Gago et al. 2005, 2006; OECD 2006). Thus, the amount paid by each agent depends proportionally on such a factor, for instance, the pollution load. This idea is collected within the property of Proportional Tax.

Ni and Wang (2007) prove that the UES rule coincides with the Shapley value (Shapley 1953b) of a cooperative game c' where the value of a coalition S represents the pollutant-cleaning costs of all segments for which agents in S are partially responsible, namely, the segments that are downstream of some agent in S. In this paper we introduce another cooperative game c where the value of a coalition S represents the pollutant-cleaning costs of segments for which only agents in S are responsible, i.e., the segments polluted only by agents belonging to this coalition. We prove that the rule characterized by using Proportional Tax coincides with the weighted Shapley value (Shapley 1953a; Kalai and Samet 1987) of the game c.

The paper is organized as follows. Section 2 introduces the model and the main axioms. Section 3 presents the characterization results for different rules. We also prove that one of the rules coincides with the weighted Shapley value of a particular cooperative game. The concluding remarks and a comparison of the axiomatized rules are presented in Sect. 4 while the proofs appear in the Appendix.

2 The mathematical model

The model presented by Ni and Wang (2007) was followed.

Consider a river which is divided into n segments indexed in a given order $i = 1, 2, \ldots, n$ from upstream to downstream. There are n agents (for instance, firms) located along the river, each located in one of these segments in the above order. We assume that each firm generates a certain amount of pollutants.

In every segment $i\,(i = 1, \ldots, n)$ an environmental authority sets a standard of the degree of pollutants in segment i so that the quality of the water body satisfies the environmental standard.

In this problem we try to find rules that allocate the total costs of cleaning the pollution among all the firms generating this pollution.

Formally, let $\mathcal{N} = \{1, 2, \ldots\}$ be the set of all possible agents. Let $N \subset \mathcal{N}$ be a finite set of agents. Usually we take $N = \{1, \ldots, n\}$. Let $C = (c_1, \ldots, c_n) \in \mathbb{R}^n_+$ be the pollutant-cleaning cost vector, where c_i represents the cost incurred by agent i. (c_i also captures the costs of using a particular technology for cleaning the segment i).

A *pollution cost sharing problem* is a pair (N, C). When N is fixed the problem is denoted as C.

A *solution* to a problem (N, C) is a vector $x = (x_1, \ldots, x_n) \in \mathbb{R}^n_+$ such that $\sum_{i \in N} x_i = \sum_{i \in N} c_i$, where x_i represents the cost share assigned to agent i.

A *rule* is a mapping x that assigns to each problem (N, C) a solution $x(N, C)$.

The main axioms used in the characterization results are:

Efficiency (*Eff*) $\sum_{i \in N} x_i = \sum_{i \in N} c_i$.
A rule is efficient if it distributes the total cost of cleaning the pollution.
Additivity (*Add*) For any $C^1 = (c^1_1, \ldots, c^1_n) \in \mathbb{R}^n_+$ and $C^2 = (c^2_1, \ldots, c^2_n) \in \mathbb{R}^n_+$ we have

$$x_i(C^1 + C^2) = x_i(C^1) + x_i(C^2)$$

for each $i \in N$, where $C^1 + C^2 = (c^1_i + c^2_i)_{i \in N}$.
Additivity indicates that dividing the total cost among agents is the same as dividing one part of the cost first and then dividing the remaining cost.
Independence of upstream costs (*IUC*) Let $l \in N$ and $C, C' \in \mathbb{R}^n_+$ such that $c_i = c'_i$ for all $i > l$. Then,

$$x_i(C) = x_i(C')$$

for each $i > l$.

The cost paid by an agent only depends on its own pollution cost and all downstream costs, but not on upstream costs for which it has no control or responsibility for.

3 Main results

Ni and Wang (2007) characterized the UES rule using four axioms: Additivity, Efficiency, Independence of Upstream Costs and Upstream Symmetry. The last one ensures that all the upstream agents have equal responsibility for a given downstream pollution cost. However, situations exist where this axiom does not apply. For instance, in some regions of Spain like Valencia and Catalonia, for water taxes applicable to households, the population size of each municipality where the house is located is taken into account. In this case, it is not possible to assume that all upstream agents are symmetric agents due to the pollution caused.

Now, we present the family of rules satisfying Add, Eff and IUC. These rules divide the cost of cleaning each segment among the agents responsible for the pollution at that segment, proportionally to a weight vector. For a particular area, the agents responsible for this pollution are those located upstream.

Theorem 1 *A rule x satisfies Eff, Add and IUC if and only if for each $j = 1, \ldots, n$ there exists a weight system $\left(p_i^j\right)_{i \in N} \in \mathbb{R}_+^n$ such that $p_i^j = 0$ when $i > j$, $\sum_{i=1}^n p_i^j = 1$ and*

$$x_i(C) = \sum_{j=1}^{n} p_i^j c_j$$

for all $C \in \mathbb{R}_+^n$ and all $i \in N$.

Proof See Appendix. □

An example of the family of rules characterized in Theorem 1 is presented:

Example E1 Let $N = \{1, 2, 3\}$, and $c_i = 1$ for all $i \in N$. Consider that the proportions that agent 1 should pay for each of the costs are the following: $p_1^1 = 1$, $p_1^2 = 0.3$, $p_1^3 = 0.5$; the proportions of agent 2: $p_2^1 = 0$, $p_2^2 = 0.7$, $p_2^3 = 0.3$ and those of agent 3 are given by: $p_3^1 = 0$, $p_3^2 = 0$, $p_3^3 = 0.2$. In that case, the rule introduced in Theorem 1 gives the following assignments:

$$\begin{aligned} x_1(N, C) &= p_1^1 c_1 + p_1^2 c_2 + p_1^3 c_3 = 1.8 \\ x_2(N, C) &= p_2^1 c_1 + p_2^2 c_2 + p_2^3 c_3 = 1 \\ x_3(N, C) &= p_3^1 c_1 + p_3^2 c_2 + p_3^3 c_3 = 0.2 \end{aligned}$$

Next we study what happens if we add new properties to those in Theorem 1. These properties are based on possible and real-life taxes over pollution.

Assume that the agents dump the same kind of residues into the water. Moreover, these residues are biodegradable, so the pollution disappears with time. Several biodegradable residues exist, for instance, organic food waste, garden waste, forest residues, some industrial waste, etc. Therefore, it seems reasonable to assume that the further away the area is from agent i, the smaller the part of the cost that agent i should pay for cleaning this area.

A new property in this context is introduced:

Upstream monotonicity (*UM*) Given $j \in N$, for any $i, k \in N$ such that $i < k \leq j$

$$x_i(0, \ldots, 0, c_j, 0, \ldots 0) \leq x_k(0, \ldots, 0, c_j, 0, \ldots 0).$$

This property states that the further away the agent is from an area, the less responsibility the agent has for the pollution of that area.

In the next theorem we study what happens if we add UM to the properties in Theorem 1. The family of rules that we characterize shares the cost of cleaning each polluted area among all the agents responsible for this pollution, proportionally to a vector of weights. These weights satisfy that the further away the responsible agent is from an area, the lower the proportion of the cost that the agent has to pay.

Theorem 2 *A rule x satisfies Add, Eff, IUC and UM if and only if for each* $j = 1, \ldots, n$ *there exists a weight system* $\left(p_i^j\right)_{i \in N} \in \mathbb{R}_+^n$ *such that* $p_i^j = 0$ *when* $i > j$, $p_i^j \leq p_k^j$ *for any* $i < k \leq j$, $\sum_{i=1}^n p_i^j = 1$ *and* $x_i(C) = \sum_{j=1}^n p_i^j c_j$ *for all* $C \in \mathbb{R}_+^n$ *and all* $i \in N$.

Proof See Appendix. □

Now an example of the rules introduced in Theorem 2 is presented:

Example E2 Let $N = \{1, 2, 3\}$, and $c_i = 1$ for all $i \in N$. Consider that the proportions that agent 1 should pay for each of the costs are the following: $p_1^1 = 1$, $p_1^2 = 0.3$, $p_1^3 = 0.2$; the proportions of agent 2: $p_2^1 = 0$, $p_2^2 = 0.7$, $p_2^3 = 0.3$ and those of agent 3: $p_3^1 = 0$, $p_3^2 = 0$, $p_3^3 = 0.5$. In this case, the rule introduced in Theorem 2 assigns the following:

$$\begin{aligned} x_1(N, C) &= p_1^1 c_1 + p_1^2 c_2 + p_1^3 c_3 = 1.5 \\ x_2(N, C) &= p_2^1 c_1 + p_2^2 c_2 + p_2^3 c_3 = 1 \\ x_3(N, C) &= p_3^1 c_1 + p_3^2 c_2 + p_3^3 c_3 = 0.5 \end{aligned}$$

Note that the weight system given in Example E1 is not valid in this context because it does not satisfy the condition: $p_i^j \leq p_k^j$ for any $i < k \leq j$.

Sometimes, when the agents dump biodegradation residues, it is possible to know their biodegradation rate, say δ. Articles on the computation of biodegradation rate of a particular residue are numerous. We now introduce a new property, $\delta-$ Biodegradation Rate, which ensures that the taxes paid by agents depend on this rate.

δ-Biodegradation rate (δ -BR) Let $\delta \in [0, 1]$. Given $j \in N$, for any $i \in N$ such that $i < j$,

$$x_i(0, \ldots, 0, c_j, 0, \ldots 0) = \delta^{j-i} x_j(0, \ldots, 0, c_j, 0, \ldots 0).$$

Note that $\delta = 0$ means that the residue of agent i only affects its own area. In this case BR indicates that every agent pays the cost corresponding to its area, namely $x_i(C) = c_i$ for all C and $i \in N$. Additionally, $\delta = 1$ means that the residue is not biodegradable at all. In this case δ-BR coincides with Upstream Symmetry.

δ-BR is stronger than UM (if a rule x satisfies δ-BR, it also satisfies UM).

In the next theorem we study the effects of adding δ-BR to the properties in Theorem 1. We prove that there is a unique rule satisfying these properties. This rule shares the cost of cleaning each segment among the agents responsible for this pollution with respect to a particular vector of weights, such that the proportion that each agent pays for cleaning a particular area takes into account the biodegradation rate of the residue thrown into the water.

Theorem 3 *A rule x satisfies Add, Eff, IUC and δ-BR if and only if for each $j = 1, \ldots, n$ there exists a weight system $\left(p_i^j\right)_{i \in N} \in \mathbb{R}_+^n$ such that $p_i^j = 0$ when $i > j$, $p_i^j = \delta^{k-i} p_k^j$ for any $i < k \leq j$, $\sum_{i=1}^n p_i^j = 1$ and $x_i(C) = \sum_{j=1}^n p_i^j c_j$ for all $C \in \mathbb{R}_+^n$ and all $i \in N$.*

Proof See Appendix. □

Note that the rule characterized in Theorem 3 belongs to the family of rules characterized in Theorem 1.

An example of this rule is presented.

Example E3 Let $N = \{1, 2, 3\}$, and $c_i = 1$ for all $i \in N$. Assume that the biodegradation rate of the residue is $\delta = 0.1$. Now, we deduce the proportions of costs that agents should pay for each area.

By Theorem 3, we have that $p_1^2 = \delta p_2^2$ and $p_1^1 + p_2^1 = 1$, therefore, $p_2^2 = \frac{1}{1.1} = 0.909$ and $p_1^2 = 0.0909$. Moreover, $p_1^3 = \delta^2 p_3^3$, $p_1^3 = \delta p_2^3$, $p_2^3 = \delta p_3^3$ and $p_1^3 + p_2^3 + p_3^3 = 1$. Then, we can deduce that $p_1^3 = 0.00909$, $p_2^3 = 0.09$ and $p_3^3 = 0.9090$. Now we can compute the allocation given by the rule introduced in Theorem 3:

$$x_1(N, C) = p_1^1 c_1 + p_1^2 c_2 + p_1^3 c_3 = 1.1089$$
$$x_2(N, C) = p_2^1 c_1 + p_2^2 c_2 + p_2^3 c_3 = 0.999$$
$$x_3(N, C) = p_3^1 c_1 + p_3^2 c_2 + p_3^3 c_3 = 0.9090$$

In many countries, the water taxes are modulated considering different factors such as pollution load, population of the cities, monthly water consumption, etc. (see Gago et al. 2006; OECD 2006). These ideas are captured by the following axiom:

Proportional tax with respect to w (PT-w) Let $w = (w_i)_{i \in N} \in \mathbb{R}_{++}^N$. Given $i, j, k \in N$ such that $i < k \leq j$,

$$\frac{x_i(0, \ldots, 0, c_j, 0, \ldots, 0)}{x_k(0, \ldots, 0, c_j, 0, \ldots, 0)} = \frac{w_i}{w_k}.$$

This property states that the amount that each agent pays for a polluted area is given by some factor w. For instance, w_i could represent the population of the municipality.

$PT{-}w$ generalizes Upstream Symmetry because when $w_i = w_j$ for all $i, j \in N$ both properties coincide.

In the next theorem we study the effects of adding this property to the properties in Theorem 1. For each $w \in \mathbb{R}^N_{++}$ we prove that there is a unique rule satisfying these properties. This rule, as well as the ones introduced in the previous Theorems, also distributes the cost of cleaning each segment proportionally among all the agents that are responsible for this pollution. However, unlike the others, in this rule, the proportion that each agent pays for cleaning each area considers an exogenous factor that is fixed for each agent. This weight is applied to distribute the cost of cleaning each area the agent is responsible for polluting, whether directly or from the agent's pollution traveling downstream to other areas. For instance, if the agents represent firms, the weight could be as a function of their size, such that the bigger the firm, the higher the weight assigned.

Theorem 4 *A rule x satisfies Add, Eff, IUC and PT-w if and only if for each $j = 1, \ldots, n$ there exists a weight system $\left(p_i^j\right)_{i \in N} \in \mathbb{R}^n_+$ such that $p_i^j = 0$ when $i > j$, $p_i^j = \frac{w_i}{\sum_{l=1}^{j} w_l}$ for all $i \leq j$, $\sum_{i=1}^{n} p_i^j = 1$ and $x_i(C) = \sum_{j=1}^{n} p_i^j c_j$ for all $C \in \mathbb{R}^n_+$ and all $i \in N$.*

Proof See Appendix. □

Note that the rule characterized in Theorem 4 belongs to the family of rules characterized in Theorem 1.

Example E4 Let $N = \{1, 2, 3\}$, and $c_i = 1$ for all $i \in N$. Assume that the weight system is given by: $\omega_1 = 0.25$, $\omega_2 = 0.5$, and $\omega_3 = 0.25$.

To compute the proportions of costs that agents should pay for each area, the condition given in Theorem 4 is taken into account, and therefore, we deduce that those proportions for agent 1 are: $p_1^1 = \frac{\omega_1}{\omega_1} = 1$; $p_1^2 = \frac{\omega_1}{\omega_1+\omega_2} \backsimeq 0.34$, $p_1^3 = \frac{\omega_1}{\omega_1+\omega_2+\omega_3} = 0.25$; for agent 2: $p_2^1 = 0$, $p_2^2 = \frac{\omega_2}{\omega_1+\omega_2} \backsimeq 0.67$, $p_2^3 = \frac{\omega_2}{\omega_1+\omega_2+\omega_3} = 0.5$; and for agent 3: $p_3^1 = p_3^2 = 0$, $p_3^3 = \frac{\omega_3}{\omega_1+\omega_2+\omega_3} = 0.25$.

Then, the final assignment is:

$$x_1(N, C) = p_1^1 c_1 + p_1^2 c_2 + p_1^3 c_3 \backsimeq 1.6$$
$$x_2(N, C) = p_2^1 c_1 + p_2^2 c_2 + p_2^3 c_3 \backsimeq 1.15$$
$$x_3(N, C) = p_3^1 c_1 + p_3^2 c_2 + p_3^3 c_3 = 0.25$$

Remark As observed, Theorem 4 proposed taxes that are linear-wise with respect to the weight system. Several real examples show this. For instance, in Spain, in the autonomous region of Galicia, both the taxes for industrial and domestic uses have a variable component that is linear with respect to aspects such as water consumption, volume of residues, etc. (see Law 14/2006). This is also observed in Navarra (Law Foral 17/2006), La Rioja (Law 11/2006), Catalonia (Law 16/2003), Aragon (Law 18/2006), Asturias (Law 11/2006), among others. For more information, see OECD (2010).

Below, the rule characterized in Theorem 4 is related to a well-known solution in transferable utility games. Before presenting the result, some notation:

A *transferable utility game*, *TU game*, is a pair (N, c) where $N \subset \mathcal{N}$ is finite and $c : 2^N \to \mathbb{R}$ satisfies $c(\emptyset) = 0$.

Given a finite subset $N \subset \mathcal{N}$, let $\Pi(N)$ denote the set of all orders in N. Given $\pi \in \Pi(N)$, let $Pre(i, \pi)$ denote the set of elements of N which come before i in the order given by π, *i.e.*,

$$Pre(i, \pi) = \{j \in N : \pi(j) < \pi(i)\}.$$

One of the most important values in TU games is the *weighted Shapley value* (Shapley 1953a). Given a vector of weights $w \in \mathbb{R}^N_{++}$, we denote the *weighted Shapley value* with weights given by w as $\phi^w_i(N, c)$. It can be expressed (Kalai and Samet 1987) as:

$$\phi^w_i(N, c) := \sum_{\pi \in \Pi(N)} p_\omega(\pi)[c(Pre(i, \pi) \cup i) - c(Pre(i, \pi))] \quad \text{for all } i \in N$$

where $p_\omega(\pi) = \prod_{j=1}^{|N|} \frac{\omega_{\pi(j)}}{\sum_{k=1}^{j} \omega_{\pi(k)}}$.

Ni and Wang (2007) prove that the solution they propose is related to some natural TU games they introduce. In particular, the *Upstream Equal Sharing* rule coincides with the Shapley value (Shapley 1953b) of the TU game $\left(N, c'\right)$, where $c'(S) = \sum_{i=min\, S}^{n} c_i$ for all $S \subset N$. Namely $c'(S)$ represents the pollutant-cleaning costs in the downstream segments of some agent of S.

We now relate the solutions given by Theorem 4 with the weighted Shapley values of another TU game. First we introduce the following TU game:

Given a pollution cost sharing problem (N, C), we define the TU game (N, c) where

$$c(S) = \sum_{\{1,\ldots,i\} \subset S} c_i$$

for all $S \subset N$. Namely $c(S)$ represents the pollutant-cleaning costs in the segments polluted only by agents in S.

We now compare the game c' of Ni and Wang (2007) with our game c. $c'(S)$ takes the pessimistic approach for coalition S and computes the cost of all segments for which agents in S are partially responsible for, namely, the segments that are downstream of some agent in S. In this case, if segment i is downstream of agent $j \in S$, agent j is responsible for part of the pollution in segment i (note that we are assuming that the water could move the pollution from one segment to another). Nevertheless, $c(S)$ takes an optimistic approach for coalition S and computes the cost of the segments for which only agents in S are responsible for. Since we are assuming that pollution could move from one segment to any downstream segment, the pollution of segment i is caused by all its upstream agents (namely, agents in $\{1, \ldots, i\}$). Thus, agents in S are responsible only for the pollution in segment i if and only if $\{1, \ldots, i\} \subset S$. It is trivial to see that $c(S) \leq c'(S)$ for each $S \subset N$.

To clarify the idea of the game c, consider the following example:

Example E5 Let $N = \{1, 2, 3\}$ and $c_i = 1$ for all $i \in N$. We will compute $c(S)$ for all $S \subset N$. Let $S = \{1\}$, since segment 1 is polluted only by agent 1 and $S = \{1\}$, we have that $c(\{1\}) = c_1 = 1$. Consider now $S = \{2\}$. The pollution of segment 2 is responsibility not only for agent 2, but also for agent 1, and since $1 \notin S$, $c(\{2\}) = 0$. Similarly, we conclude that $c(\{3\}) = 0$.

Consider $S = \{1, 2\}$. Segments 1 and 2 are polluted only by agents 1 and 2, since both belong to S, $c(\{1, 2\}) = c_1 + c_2 = 2$. Let $S = \{1, 3\}$, now, segment 1 is polluted only by agent 1, who belongs to S, but segment 3 is polluted by agents 1, 2 and 3. Since $2 \notin S$, we do not take into account c_2 to compute $c(\{1, 3\})$, then, $c(\{1, 3\}) = c_1 = 1$. For $S = \{2, 3\}$, we have that agent 1 is also responsible for the pollution in segments 2 and 3, and since 1 does not belong to S, then $c(\{2, 3\}) = 0$. Using similar arguments, we obtain that $c(N) = c_1 + c_2 + c_3 = 3$.

In the following table we summarize the results and we compare them with the ones given by c':

S	$c'(S)$	$c(S)$
$\{1\}$	$c_1 + c_2 + c_3 = 3$	$c_1 = 1$
$\{2\}$	$c_2 + c_3 = 2$	0
$\{3\}$	$c_3 = 1$	0
$\{1, 2\}$	$c_1 + c_2 + c_3 = 3$	$c_1 + c_2 = 2$
$\{1, 3\}$	$c_1 + c_2 + c_3 = 3$	$c_1 = 1$
$\{2, 3\}$	$c_2 + c_3 = 2$	0
N	$c_1 + c_2 + c_3 = 3$	$c_1 + c_2 + c_3 = 3$

Theorem 5 *Let x^w be the solution given by Theorem 4. Then, x^w coincides with the weighted Shapley value of (N, c) with weights given by $w \in \mathbb{R}^N_{++}$, $\phi^w(N, c)$.*

Proof See Appendix. □

The *Upstream Equal Sharing* rule coincides with the rule given by Theorem 4 when $w_i = w_k$ for all $i, k \in N$. By Theorem 5 the *Upstream Equal Sharing* rule also coincides with the Shapley value of the TU game c.

4 Concluding remarks

The important task of solving water resource management problems such as the control of water pollution is addressed in this work. It then seems reasonable to study the problem of sharing the cleaning costs among all the polluters in a formal way, this being one of the aims of our model. We simplify the situation by considering a river, divided into many segments, as many polluters are along the river. These polluters may represent a town, a firm, a household, a region or even a country, depending on the particular situation studied. We propose several ways to define water taxes, mainly following the “polluter-pays” principle, which takes into account different factors that influence the quality of the water. Assuming that an environmental authority knows exactly who the agent responsible for the pollution generated in a particular area is, then the agent should be charged with the total cost of cleaning that area, however, in most real-life situations, this is not possible. Thus, we propose several rules that reduce the incentive for agents to cause pollution, which is the main objective of water taxes in most countries.

In this paper we characterize several rules that have a common structure: all proportionally distribute the cost of cleaning each area among all the agents that are responsible for this pollution. However, the weights assigned to the agents are different in each case.

In Theorem 1, we characterize a family of rules that distribute the costs with respect to a general vector of weights, such that the cost of a particular area is totally shared among all the agents that are responsible for the pollution present in that segment, *i.e.*, the agents located upstream.

In Theorem 2, the family of rules characterized distributes the costs with respect to a vector of weights that satisfies the following condition: the further away an agent is from a polluted area (for which it is responsible), the lower the proportion of the cost of cleaning that area.

The rule characterized in Theorem 3 could be applied when the agents throw the same kind of residue and it is biodegradable. The biodegradation rate of the residue represents the amount of pollution that is transferred from one area to the following ones. There, the weights assigned to each agent depend on the biodegradation rate of the residue. Therefore, for a specific biodegradation rate, there will be a specific vector of weights.

The rule characterized in Theorem 4 takes into account a weight system in which the weight assigned to each agent is fixed a priori. Then, the cost of cleaning each area is distributed among all the agents responsible for the pollution there, proportionally to these weights.

Finally, in Theorem 5 we relate the last rule characterized with one of the most important values in TU games. We prove that this rule coincides with the weighted Shapley value of a particular game.

5 Appendix

In this section we include the formal proofs of the results presented previously.

Theorem 1 *A rule x satisfies Eff, Add and IUC if and only if for each $j = 1, \ldots, n$ there exists a weight system $\left(p_i^j\right)_{i \in N} \in \mathbb{R}_+^n$ such that $p_i^j = 0$ when $i > j$, $\sum_{i=1}^n p_i^j = 1$ and*

$$x_i(C) = \sum_{j=1}^n p_i^j c_j.$$

Proof of Theorem 1 Let x be a rule defined as above. We first prove that x satisfies Eff, Add and IUC.

x satisfies Eff:

$$\sum_{i=1}^n x_i(C) = \sum_{i=1}^n \sum_{j=1}^n p_i^j c_j = \sum_{j=1}^n c_j \left(\sum_{i=1}^n p_i^j \right) = \sum_{j=1}^n c_j.$$

x satisfies Add: Let C and $C' \in \mathbb{R}_+^n$ and $i \in N$. Thus,

$$\begin{aligned} x_i(C + C') &= \sum_{j=1}^{n} x_i^j (C + C') = \sum_{j=1}^{n} p_i^j (c + c')_j \\ &= \sum_{j=1}^{n} p_i^j \left(c_j + c'_j\right) = \sum_{j=1}^{n} p_i^j c_j + \sum_{j=1}^{n} p_i^j c'_j \\ &= x_i(C) + x_i(C'). \end{aligned}$$

x satisfies IUC: Let $l \in N$ and $C, C' \in \mathbb{R}_+^n$ such that $c_i = c'_i$ for all $i > l$. Let $i \in N, i > l$. Since $p_i^j = 0$ when $i > j$,

$$\begin{aligned} x_i(C) &= \sum_{j=1}^{n} p_i^j c_j = \sum_{j=i}^{n} p_i^j c_j = \sum_{j=i}^{n} p_i^j c'_j \\ &= \sum_{j=1}^{n} p_i^j c'_j = x_i(C'). \end{aligned}$$

We now prove the reciprocal. Assume that x is a solution satisfying Eff, Add and IUC. For each $j \in N$, let $1_j = (y_1, \ldots, y_n) \in \mathbb{R}_+^n$ such that $y_j = 1$ and $y_i = 0$ when $i \neq j$. We define $p^j = x(1_j)$. It is trivial to see that $\left\{p^j\right\}_{j \in N}$ is a weight system.

Let x^p be the rule induced by the weight system $\left\{p^j\right\}_{j \in N}$. We will prove that $x = x^p$. We first prove two claims following Bergantiños and Vidal-Puga (2004).

Claim 1 *Let $c_j \in \mathbb{Q}_+$ (a non-negative rational number), then*

$$x_i(0, \ldots, c_j, \ldots, 0) = c_j x_i(0, \ldots, 1, \ldots, 0).$$

Proof of Claim 1 Let $c_j = 1/q$, where $q \in \mathbb{N}$. By Add, $x_i(0, \ldots, 1, \ldots, 0) = \sum_{k=1}^{q} x_i(0, \ldots, \frac{1}{q}, \ldots, 0) = q x_i(0, \ldots, \frac{1}{q}, \ldots, 0)$. Thus,

$$x_i\left(0, \ldots, \frac{1}{q}, \ldots, 0\right) = \frac{x_i(0, \ldots, 1, \ldots, 0)}{q} = c_j x_i(0, \ldots, 1, \ldots, 0). \quad (1)$$

Let $c_j \in \mathbb{Q}_+$, say $c_j = \frac{p}{q}$. By Add, $x_i\left(0, \ldots, \frac{p}{q}, \ldots, 0\right) = p x_i\left(0, \ldots, \frac{1}{q}, \ldots, 0\right)$. Then by (1), $x_i\left(0, \ldots, \frac{p}{q}, \ldots, 0\right) = \frac{p}{q} x_i(0, \ldots, 1, \ldots, 0)$. □

Claim 2 *Let $c_j \in \mathbb{R}_+ \backslash \mathbb{Q}_+$ (a non-negative irrational number), then*

$$x_i(0, \ldots, c_j, \ldots, 0) = c_j x_i(0, \ldots, 1, \ldots, 0).$$

Proof of Claim 2 Let $c_j \in \mathbb{R}_+ \backslash \mathbb{Q}_+$. Then, there exists $\{b_l\}_{l=1}^{\infty}$ such that $b_l \in \mathbb{Q}_+$, $b_l < c_j$ and $\lim_{l \to \infty} b_l = c_j$.

Let $l \in \mathbb{N}$. Since $x(0, \ldots, c_j - b_l, \ldots, 0) \in \mathbb{R}^n_+$ and $\sum_{i \in N} x_i(0, \ldots, c_j - b_l, \ldots, 0) = c_j - b_l$,

$$0 \le x_i(0, \ldots, c_j - b_l, \ldots, 0) \le c_j - b_l.$$

By Add, $x_i(0, \ldots, c_j, \ldots, 0) = x_i(0, \ldots, c_j - b_l, \ldots, 0) + x_i(0, \ldots, b_l, \ldots, 0)$. So,

$$0 \le x_i(0, \ldots, c_j, \ldots, 0) - x_i(0, \ldots, b_l, \ldots, 0) \le c_j - b_l.$$

Since $b_l \in \mathbb{Q}_+$, $x_i(0, \ldots, b_l, \ldots, 0) = b_l x_i(0, \ldots, 1, \ldots, 0)$. Then,

$$0 \le x_i(0, \ldots, c_j, \ldots, 0) - b_l x_i(0, \ldots, 1, \ldots, 0) \le c_j - b_l.$$

Thus,

$$0 \le \lim_{l \to \infty} \left[x_i(0, \ldots, c_j, \ldots, 0) - b_l x_i(0, \ldots, 1, \ldots, 0) \right] \le \lim_{l \to \infty} \left[c_j - b_l \right].$$

So,

$$0 \le x_i(0, \ldots, c_j, \ldots, 0) - c_j x_i(0, \ldots, 1, \ldots, 0) \le 0.$$

Therefore,

$$x_i(0, \ldots, c_j, \ldots, 0) = c_j x_i(0, \ldots, 1, \ldots, 0).$$

□

Let $C = (c_1, \ldots, c_n) \in \mathbb{R}^n_+$. We now prove that $x_i(C) = x_i^p(C)$ for each $i \in N$. Let $i \in N$

$$\begin{aligned}
x_i(C) &= \sum_{j=1}^{n} x_i(0, \ldots, c_j, \ldots, 0) && x \textit{ satisfies Add} \\
&= \sum_{j=1}^{n} c_j x_i(0, \ldots, 1, \ldots, 0) && \textit{Claims } 1 \textit{ and } 2 \\
&= \sum_{j=1}^{n} c_j p_i^j && \textit{Definition of } p^j \\
&= x_i^p(C). && \textit{Definition of } x^p.
\end{aligned}$$

This finishes the proof of the Theorem. □

Theorem 2 *A rule x satisfies Add, Eff, IUC and UM if and only if for each* $j = 1, \ldots, n$ *there exists a weight system* $\left(p_i^j\right)_{i \in N} \in \mathbb{R}^n_+$ *such that* $p_i^j = 0$ *when* $i > j$, $p_i^j \le p_k^j$ *for any* $i < k \le j$, $\sum_{i=1}^n p_i^j = 1$ *and* $x_i(C) = \sum_{j=1}^n p_i^j c_j$ *for all* $C \in \mathbb{R}^n_+$ *and all* $i \in N$.

Proof of Theorem 2 By Theorem 1 the rule x defined as above satisfies Add, Eff, and IUC. Now we prove that x satisfies UM. Let $i, j, k \in N$ such that $i < k \leq j$. Let $(0, \ldots, c_j, \ldots, 0) \in \mathbb{R}^n_+$. Then,

$$\begin{aligned} x_i(0\ldots, c_j, \ldots, 0) &= \sum_{l=1}^{n} p_i^l c_l = p_i^j c_j \\ &\leq p_k^j c_j = \sum_{l=1}^{n} p_k^l c_l \\ &= x_k(0\ldots, c_j, \ldots, 0). \end{aligned}$$

We now prove the reciprocal. Let x be a rule satisfying Add, Eff, IUC and UM. By Theorem 1 for each $j = 1, \ldots, n$ there exists a weight system $\left(p_i^j\right)_{i\in N} \in \mathbb{R}^n_+$ such that $p_i^j = 0$ when $i > j$, $\sum_{i=1}^n p_i^j = 1$ and $x_i(C) = \sum_{j=1}^n p_i^j c_j$ for all $C \in \mathbb{R}^n_+$ and all $i \in N$. We now prove that $p_i^j \leq p_k^j$ for any $i < k \leq j$.

Let $i, j, k \in N$ such that $i < k \leq j$. By the proof of Theorem 1, $p^j = x\left(1_j\right)$. Since x satisfies UM, $x_i\left(1_j\right) \leq x_k\left(1_j\right)$. Thus, $p_i^j \leq p_k^j$. □

Theorem 3 *A rule x satisfies Add, Eff, IUC and δ -BR if and only if for each $j = 1, \ldots, n$ there exists a weight system $\left(p_i^j\right)_{i\in N} \in \mathbb{R}^n_+$ such that $p_i^j = 0$ when $i > j$, $p_i^j = \delta^{k-i} p_k^j$ for any $i < k \leq j$, $\sum_{i=1}^n p_i^j = 1$ and $x_i(C) = \sum_{j=1}^n p_i^j c_j$ for all $C \in \mathbb{R}^n_+$ and all $i \in N$.*

Proof of Theorem 3 By Theorem 1 the rule x defined as above satisfies Add, Eff, and IUC.

Now we prove that x satisfies δ-BR. Let $i, j, k \in N$ such that $i < k \leq j$. Let $(0, \ldots, c_j, \ldots, 0) \in \mathbb{R}^n_+$. Then,

$$\begin{aligned} x_i(0\ldots, c_j, \ldots, 0) &= \sum_{l=1}^{n} p_i^l c_l = p_i^j c_j = \delta^{j-i} p_j^j c_j \\ &= \delta^{k-i}\delta^{j-k} p_j^j c_j = \delta^{k-i} p_k^j c_j \\ &= \delta^{k-i} x_k(0\ldots, c_j, \ldots, 0). \end{aligned}$$

We now prove the reciprocal. Let x be a rule satisfying Add, Eff, IUC and δ-BR. By Theorem 1 for each $j = 1, \ldots, n$ there exists a weight system $\left(p_i^j\right)_{i\in N} \in \mathbb{R}^n_+$ such that $p_i^j = 0$ when $i > j$, $\sum_{i=1}^n p_i^j = 1$ and $x_i(C) = \sum_{j=1}^n p_i^j c_j$ for all $C \in \mathbb{R}^n_+$ and all $i \in N$. We now prove that $p_i^j = \delta^{k-i} p_k^j$ for any $i < k \leq j$.

Let $i, j, k \in N$ such that $i < k \leq j$. By the proof of Theorem 1, $p^j = x\left(1_j\right)$. Since x satisfies δ-BR,

$$p_i^j = x_i\left(1_j\right) = \delta^{j-i} x_j\left(1_j\right) = \delta^{k-i}\delta^{j-k} x_j\left(1_j\right) = \delta^{k-i} x_k\left(1_j\right) = \delta^{k-i} p_k^j.$$

□

Theorem 4 *A rule x satisfies Add, Eff, IUC and PT-w if and only if for each $j = 1, \ldots, n$ there exists a weight system $\left(p_i^j\right)_{i \in N} \in \mathbb{R}_+^n$ such that $p_i^j = 0$ when $i > j$, $p_i^j = \frac{w_i}{\sum_{l=1}^j w_l}$ for all $i \leq j$, $\sum_{i=1}^n p_i^j = 1$ and $x_i(C) = \sum_{j=1}^n p_i^j c_j$ for all $C \in \mathbb{R}_+^n$ and all $i \in N$.*

Proof of Theorem 4 By Theorem 1 the rule x defined as above satisfies Add, Eff, and IUC. It is not difficult to prove that x satisfies PT-w.

We now prove the reciprocal. By Theorem 1 it is enough to prove that $x_i\left(1_j\right) = \frac{w_i}{\sum_{l=1}^j w_l}$ for any $i < k \leq j$. Since x satisfies Eff and PT-w,

$$\frac{1}{p_j^j} = \frac{\sum_{l=1}^j p_l^j}{p_j^j} = \sum_{l=1}^j \frac{x_l\left(1_j\right)}{x_j\left(1_j\right)} = \sum_{l=1}^j \frac{w_l}{w_j} = \frac{\sum_{l=1}^j w_l}{w_j}.$$

By PT-w,

$$p_i^j = x_i\left(1_j\right) = \frac{w_i}{w_j} x_j\left(1_j\right) = \frac{w_i}{w_j} p_j^j = \frac{w_i}{\sum_{l=1}^j w_l}.$$

Theorem 5 *Let x^w be the solution given by Theorem 4. Then, x^w coincides with the weighted Shapley value of (N, c) with weights given by $w \in \mathbb{R}_{++}^N$, $\phi^w(N, c)$.*

Proof of Theorem 5 Let $w = (w_i)_{i \in N} \in \mathbb{R}_{++}^N$.

Let $\{u_S\}_{S \subset N}$ be a family of TU games such that $u_S(T) = 1$ if $S \cap T \neq \varnothing$ and $u_s(T) = 0$ otherwise. It is well known, for instance, in Kalai and Samet (1987), that $\{u_S\}_{S \subset N}$ is a basis for the set of all TU games.

Kalai and Samet (1987) define the value ϕ^{w*} as the unique linear value satisfying that for each $S \subset N$, $\phi_i^{w*}(u_S) = \frac{w_i}{\sum_{k \in S} w_k}$ if $i \in S$ and $\phi_i^{w*}(u_S) = 0$ otherwise. Besides, they prove that for each $w \in \mathbb{R}_{++}^N$ and each TU game v, $\phi^{w*}(v) = \phi^w(v^*)$ where $v^*(S) = v(N) - v(N \backslash S)$ for all $S \subset N$.

For each $j = 1, \ldots, n$, let $\left(N, v^j\right)$ be the TU game where for all $S \subset N$, $v^j(S) = c_j$ if $S \cap \{1, \ldots, j\} \neq \varnothing$ and $v^j(S) = 0$ otherwise. Note that $v^j = c_j u_{\{1,\ldots,j\}}$ for all $j \in N$.

Given $i \in N$,

$$\begin{aligned} x_i^w(C) &= \sum_{j=1}^n p_i^j c_j = \sum_{j=i}^n \frac{w_i}{\sum\limits_{k=1}^j w_k} c_j = \sum_{j=i}^n \phi_i^{w*}\left(v^j\right) \\ &= \sum_{j=1}^n \phi_i^{w*}\left(v^j\right) = \sum_{j=1}^n \phi_i^w\left(v^{j*}\right) = \phi_i^w\left(\sum_{j=1}^n v^{j*}\right). \end{aligned}$$

Let $S \subset N$. Then, $v^{j*}(S) = v^j(N) - v^j(N \backslash S) = c_j - v^j(N \backslash S)$. Since $v^j(N \backslash S) = c_j$ when $N \backslash S \cap \{1, \ldots, j\} \neq \varnothing$ and $v^j(N \backslash S) = 0$ when $N \backslash S \cap \{1, \ldots, j\} = \varnothing$,

$$v^{j*}(S) = \begin{cases} c_j & \text{if } \{1, \ldots, j\} \subset S \\ 0 & \text{otherwise.} \end{cases}$$

Now it is trivial to prove that for all $S \subset N, c(S) = \sum_{j=1}^{n} v^{j*}(S)$. Hence, $x_i^w(N, C) = \phi_i^w(N, c)$. □

References

Bergantiños G, Vidal-Puga JJ (2004) Additive rules in bankruptcy problems and other related problems. Math Soc Sci 47:87–101

Bergantiños G, Vidal-Puga JJ (2009) Additivity in minimum cost spanning tree problems. J Math Econ 45:38–42

Gago A, Labandeira X, Picos F, Rodríguez M (2005) La Imposición Ambiental Autonómica (Regional Environmental Taxation). In: Bosch N, Durán JM (eds) La Financiaci ón de las Comunidades Autónomas: Políticas Tributarias y Solidaridad Interterritorial. Edicions i Publicacions de la Universitat de Barcelona, Barcelona (Spain)

Gago A, Labandeira X, Picos F, Rodríguez M (2006) Environmental Taxes in Spain: A Missed Opportunity". In: Martínez-V ázquez J, Sanz JF (eds) Fiscal reform in Spain: accomplishments and challenges. Edward Elgar, Northampton

Kalai E, Samet D (1987) On weighted Shapley values. Int J Game Theory 16:205–222

Littlechild SC, Owen G (1973) A simple expression for the Shapley value in a special case. Manag Sci 20:370–372

Ni D, Wang Y (2007) Sharing a polluted river. Games Econ Behav 60:176–186

OECD (2004) Sustainable development in OECD countries: getting the policies right. OECD, Paris

OECD (2006) Database on economic instruments. OECD, Paris. http://www2.oecd.org/ecoinst/queries/index.htm

OECD (2008) Freshwater. OECD Environmental Outlook to 2030. OECD, Paris

OECD (2010) Database on instruments used for environmental policy and natural resources management. OECD, Paris

Shapley LS (1953a) Additive and non-additive set functions. PhD thesis. Princeton University

Shapley LS (1953b) A value for n-person games. Kuhn HW, Tucker AW (eds) Contributions to the theory of Games II. Princeton University Press, Princeton, pp 307–317