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Portfolio choice and the effects of liquidity

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Abstract This paper discusses how to introduce liquidity into the well known mean-variance framework of portfolio selection using a representative sample of Spanish equity portfolios. Either by estimating mean-variance liquidity constrained frontiers or directly estimating optimal portfolios for alternative levels of risk aversion and preference for liquidity, we obtain strong effects of liquidity on optimal portfolio selection. In particular, portfolio performance, measured by the Sharpe ratio relative to the tangency portfolio, varies significantly with liquidity. When the investor shows no preference for liquidity, the performance of optimal portfolios is relatively more favorable. However, it is also the case that, under no preference for liquidity, these portfolios display lower levels of liquidity. Finally, we also study how the aggregate level of illiquidity affects optimal portfolio selection.

Keywords Liquidity · Mean-variance frontiers · Performance · Portfolio selection

JEL Classification G10 · G11

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1 Introduction

It is clear that liquidity is a very complex concept. We may think about liquidity as the ease of trading any amount of a security without affecting its price. This already suggests that liquidity has two key dimensions; its price and quantity characteristics.[1] It is very common to proxy these two dimensions by the relative bid-ask spread and depth, respectively.[2]

Liquidity has been mostly discussed on a direct microstructure context, where one of the main concerns is to understand the effects of market design on liquidity. However, there has also been an interest on the relationship between liquidity and the behavior of asset prices. In particular, a very important research connects the cross-sectional relationship between expected return and risk to microstructure issues by explicitly recognizing the level of liquidity on the asset pricing model. Most papers employ the relative bid-ask spread as a measure of the level liquidity, and study the existence of an illiquidity premium on stock returns. A classic example of this literature is Amihud and Mendelson (1986), who show that expected stock returns are an increasing function of illiquidity costs, and that the relationship is concave due to the clientele effect.[3] Another classic paper is Brennan and Subrahmanyam (1996), who use Kyle (1985) lambda estimated from intraday trade and quote data, as the proxy for the level of liquidity. Their evidence is also consistent with a positive illiquidity effect. Finally, a closely related literature analyzes information risk, rather than the level of liquidity, as the determinant of the cross-sectional variation of stock returns. The paper by Easley et al. (2002), show that adverse selection costs do affect asset prices, and O'Hara (2003) argues that symmetric information-based asset pricing models do not work because they assume that the underlying problems of liquidity and price discovery have been solved. She develops an asymmetric information asset pricing model that incorporates these effects, and shows how important informed-based trading becomes to explain the cross-section of stock prices.

Interestingly, once we recognize that there is commonality in liquidity, that is, individual liquidity shocks co-vary significantly with innovations in market-wide liquidity, as documented by Chordia et al. (2000) and Hasbrouck and Seppi (2001), researchers have become interested in analyzing liquidity as an aggregate risk factor. This literature basically studies whether aggregate illiquidity shocks convey a risk premium.[4] Along these lines, Amihud (2002), Pastor and Stambaugh (2003), Acharya and Pedersen

[1] A very intuitive but also rigorous discussion on the two dimensions of liquidity may be found in Lee et al. (1993). It should also be pointed out that, following Kyle (1985), some authors consider a third dimension of liquidity called resiliency, which refers to the speed with which prices return to their efficient level after an uninformative shock. We thank both referees for pointing out this additional dimension of liquidity. Moreover, there are at least two nice surveys on liquidity. The paper by Amihud et al. (2005) which covers a discussion not only on stocks, but also on bonds and options, and the paper by Pascual (2003) which also discusses key econometric issues on estimating liquidity. Moreover, a general and relevant survey on microstructure is provided by Biais et al. (2005).

[2] Depth is the sum of total shares available for trading at the demand and supply sides of the limit order book. An empirical application of both dimensions to Spanish data may be found in Martínez et al. (2005).

[3] The longer the holding period, the lower compensation investors require for the costs of illiquidity.

[4] Note that we refer to aggregate liquidity shocks as either aggregate liquidity or market-wide liquidity.

(2005), Korajczyk and Sadka (2008), Watanabe and Watanabe (2008), and Márquez et al. (2009), using US data, show significant pricing effects of liquidity as a risk factor. On the other hand, Martínez et al. (2005), using Spanish data, compare alternative measures of aggregate liquidity risk. They employ the measures of Pastor and Stambaugh (2003), Amihud (2002), and the return differential between portfolios of stocks with high and low sensitivity to changes in their relative bid-ask spread. They show that when aggregate liquidity is measured as suggested by Amihud (2002), higher (absolute) liquidity-related betas lead to higher expected returns.

By jointly analyzing the previous empirical evidence, it seems reasonable to conclude that there is positive illiquidity premium on stock returns. This suggests that optimal portfolio choices by investors should be affected by liquidity. Surprisingly, however, very little academic attention has been paid to directly consider the impact of liquidity on the optimal portfolio formation process. This paper covers this gap by extending the well known mean-variance approach to solve for the optimal portfolio problem based on the simultaneous trade-off between mean-variance and liquidity.[5]

The paper employs two approaches to better understand the effects of liquidity on the optimal portfolio choices of investors. First, we solve for the mean-variance liquidity frontier by introducing an additional constraint on the traditional optimization problem. In particular, we obtain the mean-variance frontier subject not only to the typical constraint that the portfolio has a minimum required average return, but also subject to the constraint that our optimal portfolio has a minimum level of liquidity. Secondly, we directly solve for the optimal portfolio by changing the traditional objective function, where the expected portfolio return is penalized by the variance of the portfolio given a level of risk aversion. In this case, we also place some weight on the preference for liquidity we assume on investors. This implies that we are able to find the optimal portfolios for (simultaneously) different levels of risk aversion and preference for liquidity. Hence, we can easily analyze the impact of the two preference parameters on the optimal decision of investors. We also justify this approach by a simple theoretical model in which we obtain the optimal portfolio weights by maximizing expected utility under a *CARA* utility function and normally distributed returns.

We employ 116 stocks trading in the Spanish Stock Market at some point from January 1991 to December 2004. Our liquidity measure is based on Amihud (2002) measure of individual stocks illiquidity, which is calculated as the ratio of the absolute value of daily return over the euro volume, a measure that is closely related to the notion of price impact. The main advantage of Amihud (2002) illiquidity ratio is that can be computed using daily data and, consequently, allows us to study a long time period which is clearly relevant for sensible conclusions on portfolio optimal decisions.[6]

[5] In independent work, Lo (2008), in the context of hedge funds, recently discusses how to construct portfolio frontiers by taking simultaneously into account average returns, volatility and liquidity. In a previous work, Lo et al. (2003) also solve for the mean-variance liquidity-constrained frontier with a sample of 50 stocks from the U.S. market. They show that similar portfolios, in the sense of the mean-variance classic frontier, can differ significantly in their liquidity characteristics.

[6] Unfortunately, given the lack of available data, this long sample period does not allow us to use the relative bid-ask spread as an alternative measure of illiquidity.

We find strong support for the impact of liquidity on portfolio choice. In fact, we show that, for levels of relative risk aversion lower than 10, mean-variance optimal portfolios have higher Sharpe ratios when the preference for liquidity is not taken into account. It is also the case that, independently of the level of risk aversion, optimal portfolios are characterized by higher illiquidity. In other words, if we do not impose any preference for liquidity in the maximization problem, optimal portfolios are always less liquid than the corresponding optimal portfolios when there is an explicit preference for liquidity. We also report that the specific relationship between liquidity and average returns and between liquidity and the Sharpe ratio seem to depend on the market-wide level of liquidity.

This paper is organized as follows. Section 2 discusses the data employed in the paper, and some preliminary results. Section 3 presents the optimization problem imposing a restriction on the required liquidity level and reports the corresponding empirical results, while Sect. 4 discusses alternative characteristics of optimal portfolios for different levels of risk aversion and preference for liquidity. Section 5 analyzes systematic liquidity, and Sect. 6 concludes.

2 Data and preliminary results

We employ daily rates of returns on 116 stocks trading in the Spanish Stock Market at some point from January 1991 to December 2004. We also collect the daily euro volume of trading for the available 116 individual stocks.[7] From these data, we calculate Amihud (2002) illiquidity ratio. In particular, for each asset i and day d, we calculate,

$$Amihud_{i,d} = \frac{|R_{i,d}|}{Vol_{i,d}} \quad (1)$$

where $R_{i,d}$ is the daily rate of return of stock i, and $Vol_{i,d}$ is the euro volume traded on day d.

This measure is aggregated over all days for each month in the sample period to obtain an individual illiquidity measure for each stock at month t,

$$Amihud_{i,t} = \frac{1}{D_{i,t}} \sum_{d=1}^{D_{i,t}} Amihud_{i,d} \quad (2)$$

where $D_{i,t}$ is the number of days in which we have data on stock i during month t.[8]

Among others, this ratio has been used by Amihud (2002), Acharya and Pedersen (2005), Korajczyk and Sadka (2008), Kamara et al. (2008), Watanabe and Watanabe (2008), and Márquez et al. (2009). As mentioned above, the main advantage of Amihud (2002) illiquidity ratio is that can be easily computed using daily data during

[7] In particular, returns are calculated from daily closing prices. All data are provided by the Spanish Stock Exchange. All stocks in the sample belong to the Spanish Stock Exchange Official Index.

[8] At least ten observations of the ratio within the considered month are required for asset i to be included in the sample.

long periods of time. Moreover, Hasbrouck (2009) shows that, at least for US data, Amihud (2002) ratio better approximates Kyle's lambda relative to competing measures of illiquidity.[9]

Finally, using all N available stocks, we obtain the market-wide illiquidity measure as the cross-sectional average of expression (1) for each day in the sample period as,[10]

$$Amihud_{m,d} = \frac{1}{N}\sum_{i=1}^{N} Amihud_{i,d} \tag{3}$$

The monthly market-wide measure is easily obtained aggregating daily observations throughout Eq. (2). From daily returns, and the corresponding compounding given the number of trading days for each month in our sample, we calculate monthly returns for each stock. Finally, the monthly 1-year treasury bill from the secondary market is employed as the risk-free rate in the optimization problems, in which we always use monthly data.

From the 116 stocks in the sample, we construct 30 liquidity-sorted portfolios using all available stocks for the corresponding month. We consistently have enough data to form portfolios with at least two stocks, and all 30 portfolios tend to have the same number of stocks. When this restriction cannot be satisfied, extreme portfolios have one additional stock relative to the rest of portfolios. We then calculate equally-weighted monthly portfolio returns. In any case, to check for robustness in the empirical results, we also employ 15 liquidity-sorted portfolios. The monthly returns of these two portfolio sets are the final assets employed in our optimization exercises.

Table 1 contains monthly illiquidity given by expression (2), monthly average returns, monthly volatility, skewness and kurtosis. Moreover, we report the return-based illiquidity beta, the infrequent-trading-adjusted market beta following the well known estimation procedure proposed by Dimson (1979), and the average trading volume in euros. Panel A displays average data on 30 portfolios, while Panel B contains the descriptive characteristics of 15 portfolios.

The return-based illiquidity beta is estimated by running, for each portfolio, the following OLS time-series regression with monthly data from January 1991 to December 2004,

$$R_{p,t} = \alpha_p + \beta_p \Delta Amihud_{m,t} + \varepsilon_{p,t} \tag{4}$$

[9] Although it is clear that the use of Amihud (2002) ratio responds to the enlargement of the database relative to the available period of bid and asks prices, it is also the case that appropriately reflects the notion of price impact as discussed by Kyle (1985). In order to approximate the idea of price impact, it must be noted that we need both price changes and trading volume. This also makes Amihud (2002) ratio relatively close to other and more complex measures of liquidity in which both spreads and depths are taking simultaneously into account. Finally, Amihud (2002) ratio is also a natural proxy for information asymmetry, in the sense of Wang (1994) who shows that the correlation between absolute return and dollar volume increases in information asymmetry.

[10] In Eq. (3), the "m" sub-index refers to market-wide. This aggregation procedure to obtain a representative measure of market-wide illiquidity is a very common approach in literature. See Amihud (2002), and Watanabe and Watanabe (2008) among many others.

Table 1 Sample descriptive statistics

Portfolios	Illiquidity	β_p (Illiq)	Return	Volatility	Skewness	Kurtosis	Volume	β_{pm}^{Dimson}
Panel A: 30 portfolios								
1	0.0005	−0.09	0.0131	0.0653	0.0301	6.9535	2600665848.5	0.76
2	0.0010	−0.10	0.0156	0.0631	−0.1983	7.8861	1072238556.8	0.92
3	0.0018	−0.08	0.0145	0.0591	−0.3657	9.5180	428386085.7	0.97
4	0.0033	−0.09	0.0143	0.0619	−0.1215	6.6017	214114432.8	0.74
5	0.0049	−0.07	0.0181	0.0651	0.2524	11.3553	141288173.1	0.82
6	0.0067	−0.09	0.0157	0.0633	−0.1904	5.4905	105883760.1	0.76
7	0.0090	−0.08	0.0127	0.0658	−0.1609	6.9542	85858281.1	0.92
8	0.0115	−0.08	0.0159	0.0657	0.0712	7.6119	68456943.9	0.99
9	0.0144	−0.08	0.0149	0.0679	−0.1083	8.2642	57527080.7	1.04
10	0.0180	−0.10	0.0159	0.0679	−0.3514	11.0573	46097868.4	1.26
11	0.0226	−0.09	0.0161	0.0650	0.5358	18.4780	34710504.8	1.12
12	0.0285	−0.06	0.0104	0.0619	0.1171	7.8039	24768345.6	1.08
13	0.0349	−0.11	0.0196	0.0630	−0.1008	7.6487	18460709.5	1.15
14	0.0419	−0.07	0.0136	0.0598	−0.1792	6.6389	17005712.1	1.10
15	0.0516	−0.09	0.0132	0.0654	0.1683	11.1456	14447210.7	1.13
16	0.0638	−0.11	0.0125	0.0642	0.5293	9.7504	11636357.4	1.12
17	0.0778	−0.06	0.0185	0.0618	0.4945	12.0921	10578576.1	1.32
18	0.0940	−0.08	0.0177	0.0604	0.3747	13.9941	8385329.6	1.04
19	0.1141	−0.06	0.0102	0.0650	0.0910	11.7579	7339078.5	0.89
20	0.1405	−0.07	0.0108	0.0601	0.0577	8.4092	6082856.4	1.06
21	0.1733	−0.08	0.0095	0.0602	0.4035	10.4388	4943454.5	0.90
22	0.2167	−0.04	0.0113	0.0562	0.6305	10.5921	5287035.3	0.81
23	0.2666	−0.06	0.0106	0.0648	0.3711	13.5561	3716006.5	1.13
24	0.3390	−0.07	0.0151	0.0566	0.3913	8.3996	2919455.0	1.21
25	0.4515	−0.08	0.0121	0.0555	0.5979	8.2439	2339546.0	0.98
26	0.6386	−0.05	0.0115	0.0574	0.1825	10.3773	1884289.9	0.88
27	0.9399	−0.06	0.0030	0.0593	0.0600	7.3883	1445103.6	1.04
28	1.3963	−0.07	0.0142	0.0599	0.8716	9.5369	1352915.9	0.92
29	2.3742	−0.06	0.0075	0.0688	0.5765	9.5150	692155.6	1.04
30	7.8535	−0.05	0.0105	0.0860	0.0719	5.7133	720595.2	1.12
Correlation coefficients		β_p (Illiq)	Return	Volatility	Skewness	Kurtosis	Volume	β_{pm}^{Dimson}
Illiquidity		0.39	−0.31	0.72	0.11	−0.24	−0.47	0.14
Panel B: 15 portfolios								
1	0.0007	−0.09	0.0136	0.0608	−0.1229	7.3326	1953437974.4	0.86
2	0.0020	−0.09	0.0159	0.0531	−0.2719	6.2937	393826513.7	0.80
3	0.0048	−0.08	0.0135	0.0576	−0.2707	7.6739	141977263.0	0.88
4	0.0087	−0.08	0.0176	0.0567	−0.4170	8.9138	85446406.3	0.85
5	0.0144	−0.09	0.0162	0.0571	−0.4324	11.5401	58723491.2	1.11
6	0.0235	−0.09	0.0111	0.0523	−0.2762	8.0430	32155999.6	1.26

Table 1 continued

Portfolios	Illiquidity	β_p (Illiq)	Return	Volatility	Skewness	Kurtosis	Volume	β_{pm}^{Dimson}
7	0.0371	−0.08	0.0172	0.0473	−0.3623	7.0397	17965578.3	1.10
8	0.0577	−0.10	0.0135	0.0504	0.0720	7.9357	13409789.6	1.19
9	0.0894	−0.07	0.0162	0.0483	−0.0968	17.3188	8883860.2	1.08
10	0.1386	−0.07	0.0097	0.0486	−0.1107	8.4791	6436817.4	1.05
11	0.2247	−0.06	0.0122	0.0450	0.2429	8.8161	4527085.8	0.81
12	0.3656	−0.07	0.0129	0.0467	0.2549	10.1277	3118782.1	1.20
13	0.6704	−0.06	0.0096	0.0452	0.1833	7.8661	1768115.4	0.95
14	1.3277	−0.07	0.0110	0.0486	0.3575	7.7270	1355517.3	0.98
15	5.4855	−0.06	0.0072	0.0598	0.1220	5.8697	670877.3	1.06

Correlation coefficients	$\beta_p(Illiq)$	Return	Volatility	Skewness	Kurtosis	Volume	β_{pm}^{Dimson}
Illiquidity	0.47	−0.63	0.29	0.38	−0.30	−0.55	0.08

Sample monthly average returns, volatility, skewness, kurtosis, illiquidity, trading volume in euros, adjusted market betas and return-based illiquidity betas of 30 (15) portfolios sorted by the level of illiquidity from January 1991 to December 2004. The illiquidity betas are estimated by an OLS regression of monthly returns on the monthly variation of market-wide illiquidity. The market beta employs Dimson's adjustment for infrequent trading. These statistics are calculated using monthly returns and monthly variations of illiquidity from a sample of 116 stocks traded in the Spanish continuous stock market at some point during the sample period. Illiquidity is obtained using the Amihud (2002) measure given by the ratio of the absolute return of a given stock to the euro volume of the stock. The correlation coefficient between illiquidity and trading volume is calculated by taking first the logarithm of trading volume

where $R_{p,t}$ is the monthly return of portfolio p in month t, and $\Delta Amihud_{m,t}$ is the monthly variation of market-wide illiquidity during month t. As expected, given the economic implications of the market-wide illiquidity factor, we obtain negative and significant coefficients for all portfolios.[11] All stock returns are negatively affected by adverse illiquidity shocks. Interestingly, however, there is a positive correlation coefficient between return-based illiquidity betas and the level of illiquidity for both 30 and 15 portfolios. This suggests that the sensitivity of adverse market-wide illiquidity shocks affect more to highly liquid firms than to illiquid firms. On the other hand, it seems that highly illiquid portfolios tend to have higher market betas.

Although we find the expected negative relationship between (the logarithm of) trading volume and illiquidity, we report a surprising negative (positive) relation between average return and illiquidity (liquidity). In other words, at least for our sample period, highly liquid firms tend to have higher average returns. Even if we measure liquidity as systematic liquidity—the beta coefficient of an OLS regression of monthly changes of individual illiquidity on the monthly variation of market-wide illiquidity—we tend to find a negative relationship between returns and illiquidity.[12] Moreover, generally

11 Market-wide illiquidity as measured by Amihud (2002) ratio tends to be high in recessions and low in expansions.

12 For the shorter time-period between January 1996 and December 2000, and using only 29 stocks for which data are available, we find the same negative relationship when we measure illiquidity either by the

speaking, we report negative skewness for highly liquid firms, while positive skewness characterizes highly illiquid stocks.[13] Finally, as expected, there is a positive correlation between illiquidity and volatility.

Table 2 displays some general relationships presented in our data. The discussion based on the results reported in this table facilitates the interpretation of some of the key results we discuss later in the paper. Panel A contains the sample characteristics by liquidity-sorted portfolios. Using the complete time period, and from the 30 (15) portfolios ranked by the average Amihud (2002) illiquidity ratio, we form three new portfolios where the first one (*High Liquidity*) includes 10 (5) portfolios with the lowest average Amihud's ratio, the second portfolio (*Medium Liquidity*) employs 10 (5) portfolios with intermediate Amihud's ratio, while the third one (*Low Liquidity*) contains the 10 (5) most illiquid portfolios. Once again, we observe that portfolios with higher liquidity also have the highest average return.[14] However, the medium liquidity firms have the highest Sharpe ratio. This is because, on average, medium liquidity portfolios have much lower volatility than high liquidity assets. This compensates the higher average returns of the highly liquid firms. Interestingly, the lowest Sharpe ratio is reported for highly illiquid firms. The explanation may be related to the positive skewness reported above for these types of firms. On average, given the positive skewness of these stocks, investors do not seem to require a particularly high premium per unit of volatility risk.[15]

Panel B of Table 2 displays the average Amihud (2002) illiquidity ratio for 9 portfolios defined by intersections of average return and volatility levels. Three portfolios are then formed according to either average return (*Low Average Return*, *Medium Average Return* and *High Average Return*) or volatility (*Low Volatility*, *Medium Volatility* and *High Volatility*). Then, nine portfolios based on intersections are obtained. We report the average illiquidity of the nine intersection portfolios. It is again the case that the relationship between average return and liquidity is positive. Moreover, independently of the level of volatility, portfolios with low average returns tend to have high illiquidity, while portfolios with high average returns present low levels of illiquidity. Portfolios with the highest volatility and lowest average return are the most illiquid assets in our sample.

Finally, Panel C of Table 2 reports the average returns for nine portfolios defined by intersections of average illiquidity and volatility levels. The construction is similar to the previous two panels, and the results tend to confirm our initial empirical evidence.

Footnote 12 continued
bid-ask spread or by Amihud (2002) illiquidity ratio. This finding is surprising given the generally positive illiquidity risk premium reported in the literature relating asset pricing and microstructure. See Amihud et al. (2005) for a survey.

13 This is, by itself, an interesting result which deserves further future research. It should be pointed out that these statistics are calculated with daily returns of either 30 or 15 portfolios. When we use monthly returns, all correlation signs are maintained with the exception of kurtosis. When using monthly returns we obtain a positive correlation between kurtosis and illiquidity.

14 It is important to note that during the nineties, the size effect in Spain changed surprisingly its sign, and highly liquid stocks also tend to be the stocks with the largest capitalization.

15 Of course, this is just a casual observation. Before reaching pricing conclusions, formal tests should be performed. This is outside the scope of this paper.

Table 2 Liquidity, average returns and volatility

	Average return	Volatility of returns	Sharpe ratio	Illiquidity Amihud ratio
Panel A				
30 Portfolios				
High liquidity	0.0151	0.0487	0.2050	0.0071
Medium liquidity	0.0143	0.0382	0.2398	0.0670
Low liquidity	0.0105	0.0315	0.1721	1.4650
15 Portfolios				
High liquidity	0.0154	0.0491	0.2089	0.0061
Medium liquidity	0.0135	0.0381	0.2219	0.0693
Low liquidity	0.0106	0.0315	0.1737	1.6148
Illiquidity Amihud ratio	Low volatility	Medium volatility	High volatility	
Panel B				
30 Portfolios				
Low average return	0.4218	0.1475	3.4473	
Medium average return	0.4729	0.0335	0.0189	
High average return	0.3390	0.0395	0.0115	
15 Portfolios				
Low average return	0.6704	0.4966	5.4855	
Medium average return	0.2952	0.0577	0.0027	
High average return	0.0632	0.0020	0.0115	
Portfolio average return	Low volatility	Medium volatility	High volatility	
Panel C				
30 Portfolios				
High liquidity	0.0145	0.0152	0.0151	
Medium liquidity	0.0122	0.0158	0.0117	
Low liquidity	0.0110	0.0106	0.0090	
15 Portfolios				
High liquidity	NA[a]	0.0159	0.0152	
Medium liquidity	0.0167	0.0114	NA[a]	
Low liquidity	0.0116	0.0110	0.0072	

Panel A: sample characteristics by liquidity-sorted portfolios. Using the complete time period from January 1991 to December 2004, all 116 stocks traded at some point during the sample period are ranked by the Amihud (2002) average illiquidity ratio. Three portfolios are then formed where the first one (*High Liquidity*) includes ten portfolios (5 portfolios) with the lowest illiquidity ratio, the second portfolio (*Medium Liquidity*) employs ten (5) portfolios with intermediate ratio, while the third one (*Low Liquidity*) contains the ten (5) most illiquid portfolios. *Panel B*: It contains the Amihud (2002) average illiquidity ratio for 9 portfolios based on intersections between 30 (15) portfolios sorted (separately) by average return and volatility, using the complete time period from January 1991 to December 2004. Three portfolios are then formed according to either average return (*Low Average Return*, *Medium Average Return* and *High Average Return*) or volatility (*Low Volatility*, *Medium Volatility* and *High Volatility*). We finally calculate Amihud (2002) average illiquidity ratio of the nine intersection portfolios. *Panel C*: It reports average returns for 9 portfolios based on intersections between 30 (15) portfolios sorted (separately) by the Amihud (2002) average illiquidity ratio and volatility, using the complete time period from January 1991 to December 2004. Three portfolios are then formed according to either average illiquidity ratio (*High Liquidity*, *Medium Liquidity* and *Low Liquidity*) or volatility (*Low Volatility*, *Medium Volatility* and *High Volatility*). Then, we calculate the average returns of the nine intersection portfolios

[a] Portfolio is empty

Independently of the level of volatility, portfolios with low liquidity tend to obtain low average returns.

3 The mean-variance liquidity-constrained frontier

In this section, we obtain the minimum variance frontier by imposing not only the traditional constraint on average return, but also an additional constraint on a minimum required level of liquidity.

The mean-variance liquidity constrained frontier is obtained by solving the following optimization problem:

$$\min_{\omega} \frac{1}{2}\omega' V\omega$$

$$subject\ to \begin{cases} \omega'\mu = \mu_p \\ \omega' Amihud = Amihud_p \\ \omega' 1_N = 1 \\ \omega \geq 0 \end{cases} \quad (5)$$

where V is the $N \times N$ variance-covariance matrix of monthly portfolio returns, μ is the N-vector of monthly mean portfolio returns, *Amihud* is the N-vector of monthly portfolio illiquidity, μ_p and $Amihud_p$ are the required levels of average return and illiquidity on the minimum variance liquidity constrained portfolio, and ω are the non-negative weights of each portfolio on the minimum variance liquidity constrained portfolio. We solve this problem for $N = 30$ and $N = 15$ illiquidity-sorted portfolios.[16]

Panel A of Fig. 1 displays the three-dimensional mean-variance liquidity constrained frontier, while Panel B contains the mean-variance frontier for alternative levels of liquidity. Both figures contain the results for 30 and 15 portfolios, and Panel A shows the three-dimensional frontiers from two different perspectives. For each portfolio illiquidity level between 5×10^{-4} and 7.85, for $N = 30$, and between 7×10^{-4} and 5.49 for $N = 15$, we obtain two different frontiers depending upon the behaviour of returns, volatility and liquidity. For high levels of liquidity, given by Amihud (2002) illiquidity ratio between 5×10^{-4} and 0.95, for $N = 30$, and between 7×10^{-4} and 1, for $N = 15$, the frontier in terms of average return depends not only on illiquidity but also on volatility. For low levels of volatility, the frontier moves as expected. This means that higher illiquidity is associated with higher average returns. On the other hand, for high levels of volatility, higher illiquidity is accompanied with lower average returns. This also occurs for high levels of illiquidity in which Amihud (2002) ratio is between 0.95 and 7.85 for $N = 30$, and between 1 and 5.49 for $N = 15$. Once again, in this case, the frontier moves contrary to our expectation. These two

16 We understand that the liquidity restriction is assumed to be linear, to affect all stocks, and to be binding. There may be alternative specifications to the problem given by Eq. (5) above. For example, as pointed out by one of the referees, it may be the case that investors were only interested in guarantying a minimum liquidity level for some of the stocks. In fact, Sect. 4 below recognizes explicitly the preference for liquidity in the objective function.

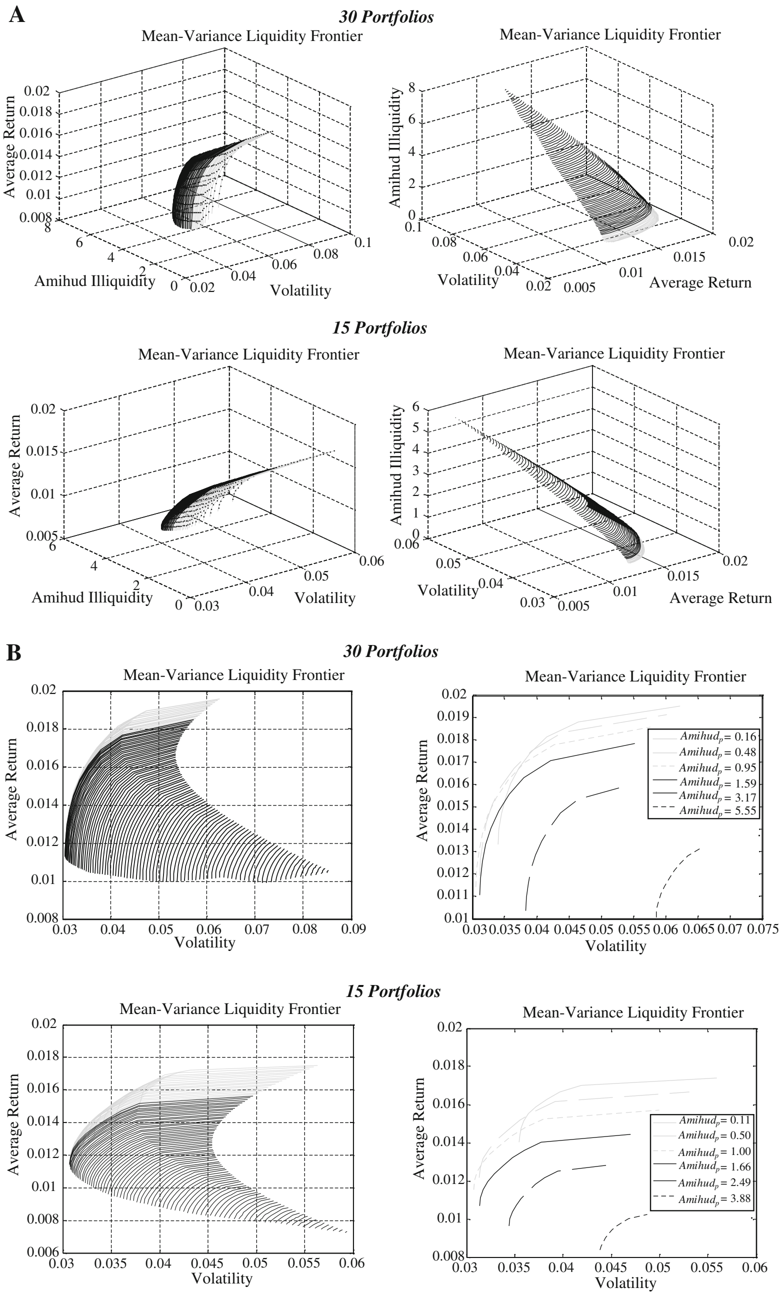

Fig. 1 The mean-variance liquidity constrained frontier January 1991–December 2004. All data are monthly returns of either 30 or 15 portfolios constructed every month using 116 stocks traded at some point during the sample period from January 1991 to December 2004. **a** Three dimensional mean-variance-liquidity constrained frontier from alternative perspectives. **b** Mean-variance frontier for alternative levels of liquidity as measured by Amihud (2002) ratio

different types of behaviour of the three-dimensional frontier are represented by a dark area in the first case, and a light zone in the second case.

These results suggest that it is important to simultaneously consider the interplay between average returns, volatility and illiquidity. This is an interesting result which already implies that liquidity as a characteristic plays a role on determining optimal portfolios.

To be more precise, we calculate the tangency portfolio for each efficient frontier given a level of liquidity. In particular, we maximize the Sharpe ratio as follows,

$$\max_{\omega} \frac{\mu_p - r_f}{\sigma_p}$$
$$subject\ to \begin{cases} \omega'\mu + \left(1 - \omega' 1_N\right) r_f = \mu_p \\ \sqrt{\omega' V \omega} = \sigma_p \\ \omega' Amihud = Amihud_p \\ \omega' 1_N = 1 \\ \omega \geq 0 \end{cases} \tag{6}$$

where r_f is the monthly risk-free rate and σ_p is the volatility of the portfolio.

The results contained in Table 3 and Fig. 2 are consistent with our previous discussion. Only for the very high levels of liquidity [low Amihud (2002) illiquidity ratio], the Sharpe ratio increases as a function of illiquidity. Thus, the performance of the tangency portfolio is better on average the higher the illiquidity level imposed. In other words, illiquidity incorporates a premium on performance. However, the opposite results are obtained when illiquidity is medium or high. In fact, in most cases, the Sharpe ratio decreases with the level of illiquidity.[17]

4 Optimal portfolios, risk aversion and the preference for liquidity

We now introduce explicitly risk aversion and preference for liquidity in the objective function of the investor. The optimization problem becomes,[18]

$$\max_{\omega} \mu_p - \frac{\gamma}{2}\sigma_p^2 + \eta \frac{1}{Amihud_p} \Leftrightarrow \max_{\omega} \omega'\mu - \frac{\gamma}{2}\omega' V \omega + \eta \frac{1}{Amihud_p}$$
$$subject\ to \begin{cases} \omega' 1_N = 1 \\ \omega \geq 0 \end{cases} \tag{7}$$

[17] In order to perform a test for the statistical differences among Sharpe ratios reported in Table 3, we run an OLS regression of the Sharpe ratios on a constant and the illiquidity level of each tangency portfolio. The slope coefficient turns out to be negative and highly significant. This suggests that illiquidity is significantly (negatively) related to the Sharpe ratio. Moreover, we also perform a Wilcoxon–Mann–Whitney test in which we divide the Sharpe ratios reported in four groups and test for the equality of pairs of Sharpe ratios. We reject the null across all pairs. Finally, we perform the Kruskal–Wallis test which is the extension of the previous test for k samples. We then test for equality across all samples, and once again, we clearly reject the null hypothesis. We conclude that the differences across Sharpe ratios are statistically significant.

[18] The Appendix justifies this approach assuming a negative exponential *CARA* utility function with normally distributed returns. The authors thank one of the referees for pointing out this possibility.

Table 3 Sharpe ratios for alternative levels of liquidity

Amihud (2002) illiquidity ratio for tangency portfolios	Volatility for tangency portfolios	Average return for tangency portfolios	Sharpe ratio for tangency portfolios
30 Portfolios			
0.0799	0.0414	0.0180	0.3115
0.3178	0.0376	0.0171	**0.3199**
1.0317	0.0366	0.0165	0.3118
1.9043	0.0382	0.0159	0.2827
2.7768	0.0423	0.0153	0.2411
3.6494	0.0484	0.0147	0.1983
4.5219	0.0560	0.0141	0.1602
5.3945	0.0637	0.0133	0.1284
6.1084	0.0698	0.0125	0.1060
6.8223	0.0761	0.0117	0.0863
7.7742	0.0852	0.0105	0.0639
15 Portfolios			
0.0561	0.0400	0.0168	0.2916
0.1115	0.0386	0.0164	**0.2920**
0.7209	0.0366	0.0153	0.2783
1.3304	0.0366	0.0144	0.2542
1.9398	0.0377	0.0135	0.2217
2.5492	0.0402	0.0125	0.1850
3.1586	0.0432	0.0115	0.1479
3.7681	0.0468	0.0104	0.1132
4.2667	0.0501	0.0095	0.0872
4.7653	0.0540	0.0086	0.0640
5.4301	0.0593	0.0073	0.0370

Data correspond to monthly returns of 30 (15) portfolios constructed every month according to Amihud (2002) illiquidity ratio. We employ 116 stocks traded at some point during the sample period from January 1991 to December 2004. To obtain the tangency portfolio we maximize the Sharpe ratio for a given level of average returns, volatility and liquidity. The table contains the volatility, average return and Sharpe ratio for alternative levels of illiquidity measured by Amihud (2002) ratio. The maximum Sharpe ratio obtained by the optimization is reported in bold.

where $\gamma > 0$ is the risk aversion parameter and $\eta \geq 0$ represents the preference for liquidity. We solve the problem for several values of γ and η. In particular, we allow risk aversion to take values within the following set $\gamma \in \{1, 2, 2.188, 4, 5, 10, 20\}$ where the value of 2.188 is the risk aversion estimated by León et al. (2007) for the Spanish stock market from January 1988 to December 2004. On the hand, the values for the liquidity preference parameter are $\eta \in \{0,\ 5\times10^{-6},\ 5\times10^{-5},\ 5\times10^{-4}, 5\times10^{-3}\}$. Of course, when $\eta = 0$, problem (7) reduces to the traditional mean-variance optimization problem.

Since the results are very similar for all levels of η different from zero, we just report the evidence obtained for cases in which the investor is either indifferent to liquidity,

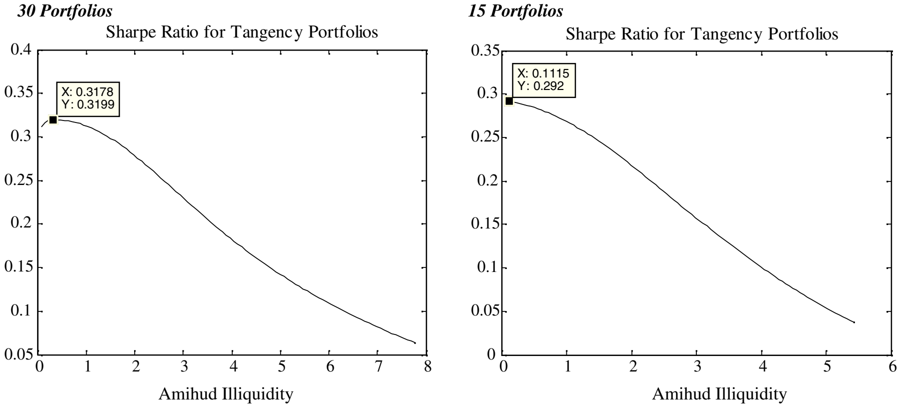

Fig. 2 The Sharpe ratio for alternative levels of illiquidity January 1991–December 2004. All data are monthly returns of either 30 or 15 portfolios constructed every month using 116 stocks traded at some point during the sample period from January 1991 to December 2004. To obtain the tangency portfolio we maximize the Sharpe ratio for a given level of return, volatility, and liquidity. The figure shows the behavior of the Sharpe ratio of the tangency portfolio for alternative levels of liquidity as measured by Amihud (2002) ratio. The *black square point* shows the maximum Sharpe ratio obtained in the optimization process

$\eta = 0$, or has preference for liquidity, $\eta > 0$. The results are displayed in Fig. 3, where we analyze four cases, Panel A to Panel D, in which we study the relationship between average return, volatility, illiquidity and Sharpe ratio of optimal portfolios respectively as a function of the risk aversion parameter and for either $\eta = 0$ or $\eta > 0$. In particular, for $N = 30$, $\eta = 5 \times 10^{-5}$ while for $N = 15$, we report the results for $\eta = 5 \times 10^{-6}$.

As expected and independently of the preference for liquidity, Panels A and B show a declining average return and volatility as risk aversion increases. This suggests that, as risk aversion becomes more important, the optimal portfolio becomes less risky and, consequently, average returns are also negatively affected. When we do not place any weight on liquidity, $\eta = 0$, we find that, except for low levels of risk aversion coefficients (<1.5 when $N = 30$, and less than 4.5 when $N = 15$), optimal portfolios have lower average returns than cases in which $\eta > 0$. At the same time, independently of risk aversion, optimal portfolios always have higher volatility when we impose a positive preference for liquidity. This higher volatility is compensated with higher average returns only when risk aversion is sufficiently high.

The results of Panel C show that, when we do not impose any preference for liquidity, investors are willing to accept higher illiquidity in their optimal portfolios. This is true independently of the level of risk aversion.

Finally, Panel D contains the results regarding the Sharpe ratio of optimal portfolios. The effects of the preference for liquidity on performance are very clear. For reasonable levels of risk aversion, $\gamma < 10$, the Sharpe ratio of the optimal portfolios is higher when we do not show any preference for liquidity. This is especially important for $N = 15$, where we obtain Sharpe ratios 20% lower when $\eta > 0$. Of course, this favorable result in terms of average returns and volatility when $\eta = 0$ is also accompanied by lower levels of liquidity in the optimal portfolios. Thus, for risk aversion lower than 5, when $\eta = 0$ and for $N = 30$ ($N = 15$), the liquidity of optimal portfolios is

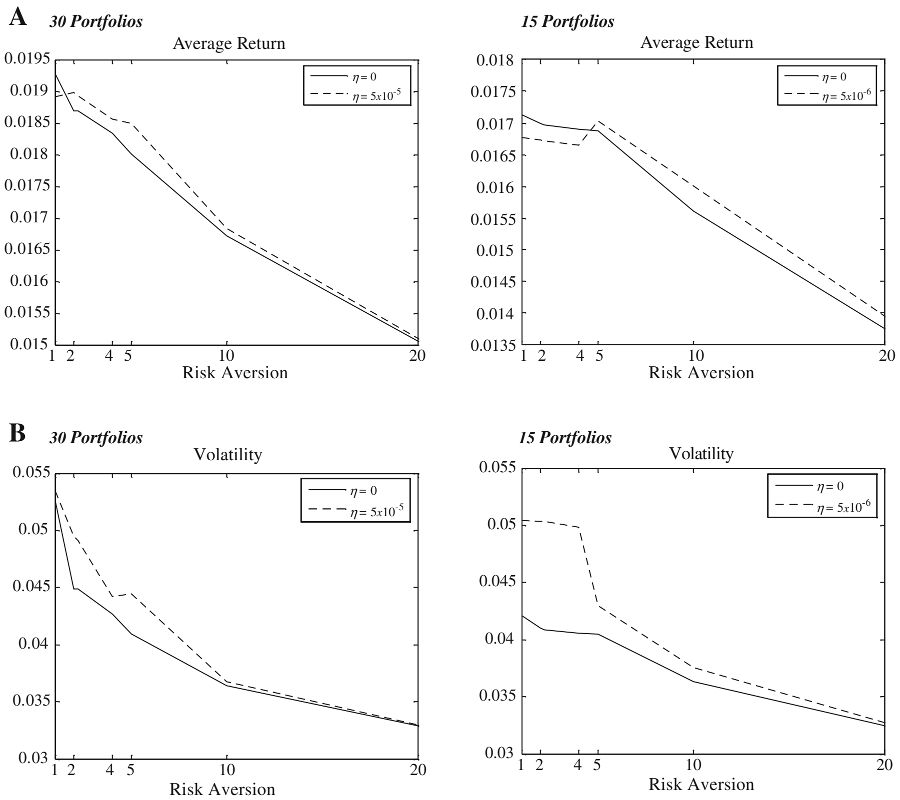

Fig. 3 Characteristics of optimal portfolios for alternative levels of risk aversion and preference for liquidity January 1991–December 2004. All data are monthly returns of either 30 or 15 portfolios constructed every month using 116 stocks traded at some point during the sample period from January 1991 to December 2004. **a** Average return of the optimal portfolio as a function of risk aversion and preference for liquidity. **b** Volatility of the optimal portfolio as a function of risk aversion and preference for liquidity. **c** Illiquidity of the optimal portfolio as a function of risk aversion and preference for liquidity. **d** Sharpe ratio of the optimal portfolio as a function of risk aversion and preference for liquidity

50% (88%) lower than in the cases in which $\eta > 0$.[19] Therefore, it seems that investors are willing to accept lower average returns per unit of volatility risk in order to obtain higher liquidity in their optimal portfolios.

5 Market-wide illiquidity

5.1 Systematic illiquidity over time

As pointed out in the introduction, the cross-sectional variation in liquidity commonality has received a great deal of attention in literature. In this section we first analyze

19 When León et al. (2007) allow for asymmetric negative and positive shocks on the conditional variance, the risk aversion coefficient for the Spanish stock market becomes 3.40.

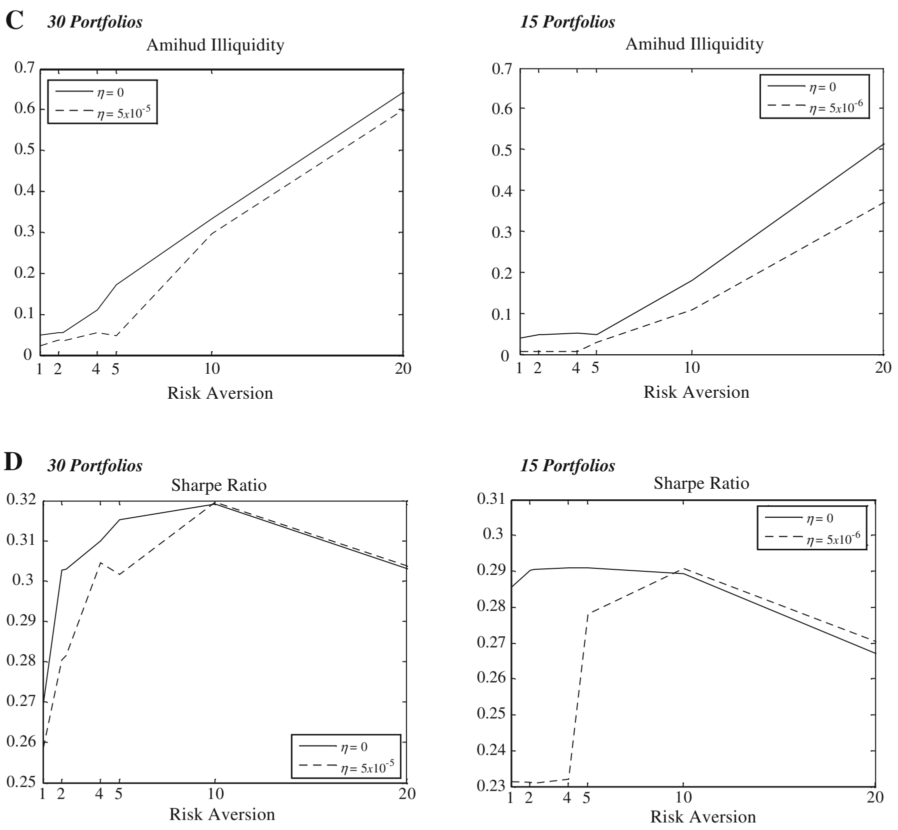

Fig. 3 continued

the behavior over time of the market-wide component of illiquidity or systematic illiquidity from January 1991 to December 2004. Systematic illiquidity is defined as the sensitivity of the stock's illiquidity to market-wide illiquidity. Recently Kamara et al. (2008) demonstrate that systematic illiquidity has decreased for small-cap firms, but increased for large-cap stocks. Since this implies that the ability to diversify aggregate liquidity shocks by holding large stocks has declined, the US market seems to be more fragile to unanticipated aggregate credit and/or liquidity shocks. Given the importance of this issue, and before discussing the effects of market-wide illiquidity on portfolio decisions, we study the evolution over time of systematic illiquidity for our sample of stocks.

Due to the non-stationary nature of the time series of Amihud (2002) illiquidity ratio, we employ the change in Amihud (2002) measure (in logs) as our daily illiquidity measure. Hence, for each stock i and day d, we calculate the following variation:

$$\Delta Amihud_{i,d} = \log\left(Amihud_{i,d}/Amihud_{i,d-1}\right) \quad (8)$$

As in Chordia et al. (2000), and Amihud (2002), we discard firm-days outliers whose $\Delta Amihud_{i,d}$ measure is in the lowest and highest 1% percentiles. The variation of the

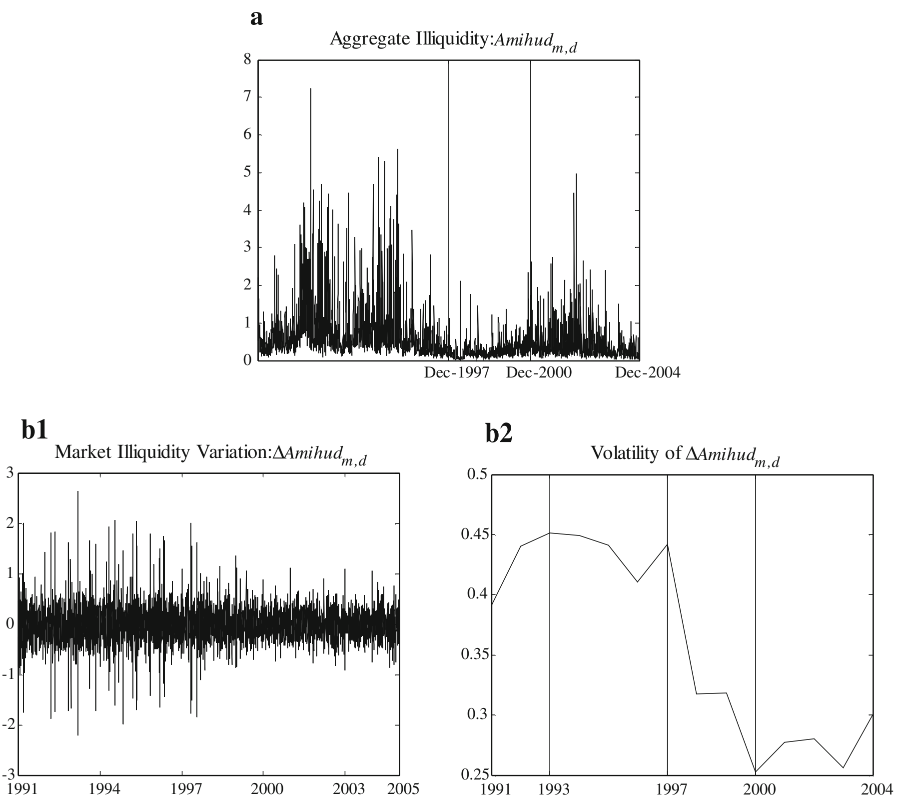

Fig. 4 The behavior of aggregate illiquidity January 1991–December 2004. $Amihud_{m,d}$ is the aggregate daily illiquidity calculated as the average of individual illiquidity ratios. $\Delta Amihud_{i,d}$ is the daily variation (in logs) of Amihud (2002) illiquidity ratio of stock i between day $d-1$ and day d. $\Delta Amihud_{m,d}$ is the equally weighted cross-sectional average variation of the illiquidity ratios of all stocks in the sample. We use 116 stocks traded at some point during the sample period from January 1991 to December 2004. **a** The daily aggregate illiquidity ratio. **b1** The daily variation of the aggregate illiquidity ratio. **b2** The volatility of the variation of the aggregate illiquidity ratio

market-wide measure of illiquidity, $\Delta Amihud_{m,d}$, is the equally weighted average of $\Delta Amihud_{i,d}$ across all stocks. Panel A of Fig. 4 contains the behavior over time of market-wide illiquidity. As expected, we observe a strong decline in the levels of illiquidity (higher market-wide liquidity) between the mid-nineties until 2000, and then again from 2002 to the end of sample. Panel B.1 shows no time trend in the mean of market's daily variation of illiquidity. However, Panel B.2 shows a strong decline in the volatility of the variation of market-wide illiquidity during the second part of the sample period.

For each year in the sample, we run the following regression with daily data for each stock i:

$$\Delta Amihud_{i,d} = \alpha_i + \beta_i \Delta Amihud_{m,d} + \varepsilon_{i,d} \tag{9}$$

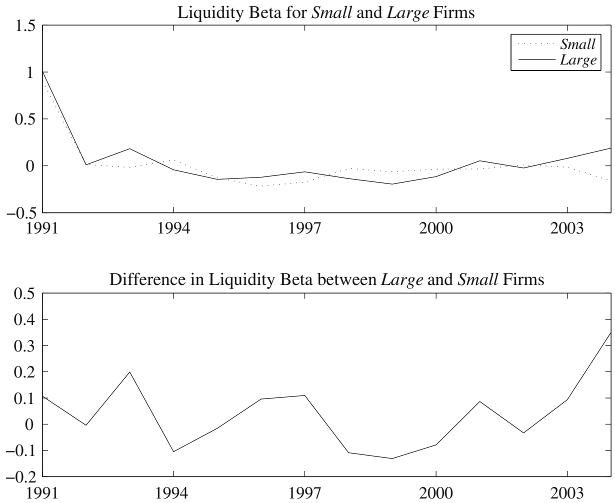

Fig. 5 Time series of liquidity beta by subperiods January 1991–December 2004. For each stock i and year t, we run the following time-series regressions: $\Delta Amihud_{i,d} = \alpha_i + \beta_i \Delta Amihud_{m,d} + \varepsilon_{i,d}$, where d denotes the days in year t, $\Delta Amihud_{i,d}$ is the daily variation (in logs) of Amihud (2002) illiquidity ratio of stock i between day $d-1$ and day d, and $\Delta Amihud_{m,d}$ is the equally-weighted cross-sectional average variation of the illiquidity ratios of all stocks in the sample. From the sample of 116 stocks traded at some point during the sample period from January 1991 to December 2004, each year, only firms with at least one hundred observations are retained. Stocks are sorted into five groups each year based on trading volume at the end of the prior year. Small and large portfolios are firms in the smallest and largest volume quintile

where β_i is the sensitivity of changes in stock i's illiquidity to changes in market-wide illiquidity or systematic illiquidity.[20]

Using all stocks in the sample we construct five volume-sorted portfolios for each year from 1991 to 2004. Portfolio 1 contains the less traded stocks, while Portfolio 5 the highly traded firms. For these two extremes portfolios, which we call small and large, and for each year, we calculate the equally weighted average systematic illiquidity as the cross-sectional mean of the individual betas estimated from regression (9). Panel A of Fig. 5 displays the evolution over time of both series of systematic illiquidity betas, while Panel B plots its difference over time. It seems that after 1999, highly traded firms (large) have become relatively more sensitive to market-wide illiquidity shocks than low traded stocks (small). It is surprising that blue chips stocks have become more fragile with respect to illiquidity shocks precisely when the economy has been characterized by a period of high liquidity both at the micro and macro levels.

Finally, given the strong decline in the volatility of the variation of market-wide illiquidity shown in Panel B.2 of Fig. 4, we calculate systematic illiquidity for all available stocks in the sample, and for the small and large portfolios in two sub-periods from 1991 to 1997 and from 1998 to 2004. The results are reported in Table 4. The volatility of the variation of market-wide illiquidity has indeed decreased from

[20] We employ stocks with at least 100 observations of the daily variation of illiquidity for each of the years in the sample period.

Table 4 Systematic Liquidity

Sub-period	Average liquidity beta				Average R^2 (All, %)	Volatility of the variation of the market-wide illiquidity
	All stocks	Small	Big	Big minus smalls		
1991–1997	0.950	0.705	1.120	0.415	7.75	0.432
1998–2004	−0.037	−0.018	−0.090	−0.072	0.18	0.287

For each sub-period and every stock in the sample, we estimate the following time-series regressions: $\Delta Amihud_{i,d} = \alpha_i + \beta_i \Delta Amihud_{m,d} + \varepsilon_{i,d}$, where $\Delta Amihud_{i,d}$ is the daily variation (in logs) of Amihud (2002) illiquidity ratio of stock i and day d, and $\Delta Amihud_{m,d}$ is the equally weighted cross-sectional average variation of the illiquidity ratios of all stocks in the sample. For each sub-period, 1991–1997 and 1998–2004, we select all stocks with at least 100 days of the variation of the illiquidity ratio. We construct five size-sorted portfolios, and we report the average liquidity beta for the smallest and largest quintile, and the difference between both betas. We also report the cross-sectional average of the R^2 from all stocks in each sub-period, and the volatility of the variation of the market-wide illiquidity ratio

0.43 to 0.29 from one sub-period to the other. It is also interesting to observe that commonality in liquidity has declined considerably during the second part of the sample period. It seems that the volatility of aggregate illiquidity shocks is so low in the second sub-period that the variation of illiquidity across all stocks, and the variation of liquidity of the two extreme portfolios, are not sensitive to these relatively small changes in market-wide illiquidity. This suggests that the increasing pattern of the systematic illiquidity divergence between large and small portfolios observed at the end of the sample period may not be economically relevant.

5.2 Market-wide illiquidity and portfolio choice

Given the empirical relevance of liquidity as a risk factor on recent asset pricing literature and the evolution of systematic illiquidity, we next analyze the portfolio performance in terms of the Sharpe ratio for alternative levels of aggregate liquidity. We separate our measure of market-wide illiquidity into three time periods depending upon the level of aggregate illiquidity and we again solve the optimization problem given by expression (6) for each of the three sub-periods separately. The results are displayed in Fig. 6. As in Table 3, the Sharpe ratio decreases when market-wide illiquidity increases. However, for $N = 30$, we observe that the Sharpe ratio goes up monotonically with illiquidity as long as market-wide illiquidity (liquidity) is sufficiently low (high). For these levels of liquidity, it seems that an increase in market-wide illiquidity is compensated with a higher Sharpe ratio. This suggests that the unexpected relations we reported above between liquidity and average returns and between liquidity and the Sharpe ratio may depend on the aggregate level of liquidity. At least, between 1991 and 2004, the illiquidity premium, as given by the Sharpe ratio, it seems to be present only as long as market-wide liquidity is high enough. Unfortunately, this theoretically appealing result is not obtained for all levels of market-wide illiquidity.

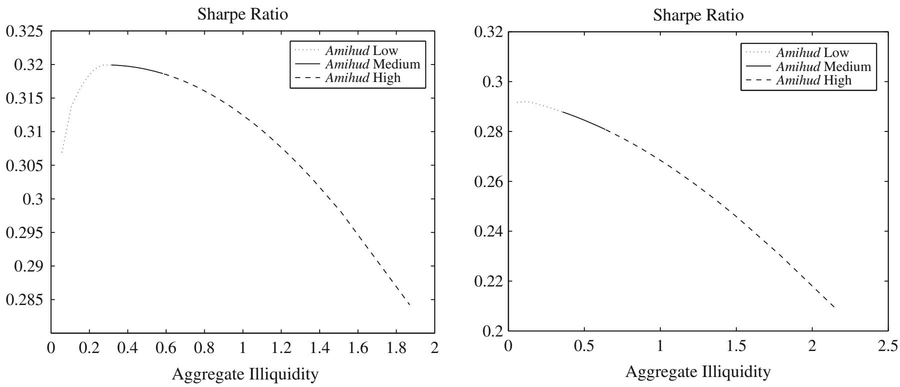

Fig. 6 The Sharpe ratio for alternative levels of aggregate illiquidity January 1991–December 2004. We represent the relationship between the Sharpe ratio and illiquidity once the sample period has been divided in three sub-periods classified according to the aggregate level of illiquidity. All data are monthly returns of either 30 or 15 portfolios constructed every month using 116 stocks traded at some point during the sample period from January 1991 to December 2004. To obtain the tangency portfolio we maximize the Sharpe ratio for a given level of return, volatility, and aggregate illiquidity. The figure shows the behavior of the Sharpe ratio of the tangency portfolio for alternative levels of aggregate illiquidity as measured by Amihud (2002) ratio

6 Conclusions

This paper shows strong effects of liquidity on optimal portfolio selection. Complex simultaneous relations are found between average returns, volatility and liquidity that should be taken into account when selecting optimal portfolios. Portfolio performance, as measured by the Sharpe ratio relative to the tangency portfolio, varies significantly with liquidity. Moreover, this relationship depends upon the market-wide level of liquidity. As long as aggregate liquidity is high enough, the Sharpe ratio increases with illiquidity suggesting that, on average, there is required illiquidity premium when taking optimal portfolio decisions. Finally, when the investor shows no preference for liquidity the performance of optimal portfolios is clearly better, at least for risk aversion coefficients lower than 10. However, these portfolios display a much lower level of liquidity than the optimal portfolios obtained when recognizing explicitly preference on liquidity.

Appendix: Optimal portfolio selection with preference for liquidity

If we interpret the effect of illiquidity as a transaction cost, expected utility may be written as,

$$E[u(R_p - \eta Amihud_p)] \Leftrightarrow E\left[u\left(R_p + \eta \frac{1}{Amihud_p}\right)\right]$$

where R_p is the return of portfolio p, η is the preference parameter for liquidity, and $Amihud_p$ is the illiquidity cost measured by the Amihud's ratio (2002).

We assume a negative exponential utility function, $u(\cdot) \sim CARA$, where γ is the absolute risk aversion coefficient, and R_p is normally distributed. We maximize expected utility for a given level of illiquidity. Then,

$$\max_{\omega} E\left[u\left(R_p + \eta\frac{1}{Amihud_p}\right)\right]$$

where,

$$\begin{aligned} &E\left[u\left(R_p + \eta\frac{1}{Amihud_p}\right)\right] \\ &= E\left[u\left(R_p\right)\right] + \eta\frac{1}{Amihud_p} \underset{u(\cdot)\sim CARA}{=} \eta\frac{1}{Amihud_p} - E\left[\exp\left(-\gamma R_p\right)\right] \\ &\underset{\substack{R_p\sim normal \\ \exp(R_p)\sim log-normal}}{=} \eta\frac{1}{Amihud_p} - \exp\left(E\left[-\gamma R_p\right] + \frac{Var\left[-\gamma R_p\right]}{2}\right) \\ &= \eta\frac{1}{Amihud_p} - \exp\left(-\gamma\mu_p + \frac{\gamma^2}{2}\sigma_p^2\right) \end{aligned}$$

Therefore,

$$\max_{\omega} E\left[u\left(R_p + \eta\frac{1}{Amihud_p}\right)\right] \Leftrightarrow \max_{\omega} \mu_p - \frac{\gamma}{2}\sigma_p^2 + \eta\frac{1}{Amihud_p}$$

This holds because,

$$\begin{aligned} \max_{\omega} E\left[u\left(R_p + \eta\frac{1}{Amihud_p}\right)\right] &\Leftrightarrow \eta\frac{1}{Amihud_p} - \max_{\omega}\exp\left(-\gamma\mu_p + \frac{\gamma^2}{2}\sigma_p^2\right) \\ &\Leftrightarrow \eta\frac{1}{Amihud_p} + \max_{\omega}\left(\mu_p - \frac{\gamma}{2}\sigma_p^2\right) \\ &\Leftrightarrow \max_{\omega} \mu_p - \frac{\gamma}{2}\sigma_p^2 + \eta\frac{1}{Amihud_p} \end{aligned}$$

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