



A new derivative-based method for determination of bubble point pressure of hydrocarbon systems

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Abstract

The bubble point pressure is essential for planning and managing oil field development and production strategies. The conventional procedure of the determination of bubble point pressure and volume is a trial-and-error method. Consequently, this leads to the lack of uniqueness, accuracy, and repeatability of the solution. This paper describes a new technique that utilizes the pressure–volume (PV) data obtained from the constant-composition expansion (CCE) test to determine the bubble point pressure of hydrocarbon systems. This method is a derivative-based procedure where consecutive derivative ratios form peaks. The highest peak always exists at the inflection of PV data to traverse into a two-phase region. A new mathematical model based on the exponential-power function is introduced to accurately describe the PV data above and below the bubble point. The new model leads to the direct determination of both bubble point pressure and volume simultaneously. Uniqueness, accuracy, and repeatability in the new method are guaranteed regardless of who performs the calculation.

Keyword Bubble point pressure · Constant-composition expansion test · Smoothing pressure–volume data · Finite difference derivative · Exponential-power function

Abbreviations

a, b	Fitting constants	\hat{v}	Predicted value of the dependent variable v
E_a	Average absolute relative error above p_b , Eq. 23	v_b	Bubble point volume, stb (cm ³)
E_b	Average absolute relative error below p_b , Eq. 26	v_i	Volume at point i
ε_{oi}	Relative error above p_b , Eq. 22	v_o	Oil volume
ε_{ii}	Relative error below p_b , Eq. 25	v_r	Relative volume
f	Function	v_t	Two-phase fluid volume
f'	Function first derivative	x_{\max}	Maximum value of function, Eq. 18
fratio	First derivative ratio	y	Y-function, Eq. 1
g	Function	y_i	Y-function at point i , Eq. 2
i	Index		
n	Total number of PV data points		
n_o	Number of PV data points above p_b		
n_t	Number of PV data points below p_b		
p	Pressure, psi (kPa)		
p_b	Bubble point pressure		
p_i	Pressure at point i		
p_o	Oil pressure		
p_t	Two-phase fluid pressure		
v	Volume, stb (cm ³)		

Introduction

The bubble point pressure of hydrocarbon systems is the pressure at which the first bubble of gas evolves from the liquid phase at a specific temperature. The property is obtained experimentally from the CCE test. The bubble point pressure is essential for planning and managing oil field development and production strategies.

The conventional procedure of the determination of bubble point pressure is a trial-and-error. It consists of two steps: the estimation of bubble point pressure by visual graphical analysis. Then smoothing PV data below bubble point pressure utilizing y -function, described by Standing (1952) and Williams (2011), to refine the first estimate.

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The y-function smoothing method has several disadvantages. Firstly, the visual graphical estimation of bubble point pressure or pressure–volume traverse into the two-phase region is subjective. Secondly, only the experimental data at pressures below the bubble point are utilized to obtain the bubble point pressure, while the PV data above the bubble point is ignored. Thirdly, the smoothing procedure is a time-consuming trial-and-error process and might not yield the optimal result. Y-function with an error in the bubble point volume may yield a straight line but with the wrong bubble point pressure. Also, the Y-function straight line can be obtained with several combinations of bubble point volume and bubble point pressure. Fourthly, the bubble point pressure is adjusted while bubble point volume is not, leading to dislocation of the intersection point. Therefore, the smoothing method with y-function leads to the non-uniqueness of bubble point pressure and volume values. The lack of repeatability and accuracy of the conventional bubble point pressure determination is evident.

There are other methods to estimate bubble point parameters. Al-Yousef and Al-Marhoun (1995) presented a nonlinear parameter estimation approach to estimate bubble point parameters. It smooths the PV data points above and below the bubble point pressure and simultaneously determines the volume and pressure of the bubble point. The authors introduced x-function to model the PV data above the bubble point and the conventional y-function to describe the PV data below it. Hoang et al. (2017) presented a scheme to estimate the bubble point of oils from PV data. The proposed technique relies on an iterative process utilizing the Tait equation, described by Dymond and Malhotra (1988), to model the liquid data. Y-function describes the two-phase data. Table 1 presents a summary of the literature review for the determination of bubble point pressure.

Several US patents presented procedures for the determination of bubble point pressure of hydrocarbon systems. Shwe, et al. (2001), DiFoggio, et al. (2006) and Angelescu, et al. (2015), in their inventions, were concerned with finding a method to determine unique bubble point pressure. Their methods are not simple, and their solution is not unique.

The proposed new approach is a derivative-based procedure. The new procedure utilizes the PV data to determine bubble point pressure and volume of hydrocarbon systems directly. A new model based on the exponential-power function proves to be a reliable fluid model that accurately describes the PV data. This method leads to a unique solution of bubble point pressure and volume. Accuracy and repeatability in the new procedure are guaranteed regardless of who performs the calculation.

Data acquisition

Experimental PV data of two oil samples, black and volatile, were collected from laboratory CCE tests. The two oil samples are studied because the black oil PV data behave differently from the volatile sample at bubble point pressure. In the CCE experiment, reservoir liquid is maintained at reservoir temperature and a pressure higher than the bubble point pressure. Then the pressure is gradually reduced in steps, and the fluid volume is recorded at each step. Columns 2 and 3 of Table 2 present the experimental PV data of the black oil sample, while columns 2 and 3 of Table 3 present the experimental PV data of the volatile oil sample. Figure 1 shows the plots of the experimental PV data of black and volatile oil samples from the CCE test.

The conventional procedure of the determination of bubble point pressure

The conventional industry method of determining bubble point pressure is two step procedure utilizing PV data obtained from the CCE test:

The estimation of bubble point pressure by visual graphical analysis

The PV data is plotted on a rectangular coordinate system and shown in Fig. 2. The plotted curve shows an inflection for black oils where the behavior traverses into

Table 1 Literature review for the determination of bubble point pressure

Year	Authors	Description
1952	Standing	The procedure involves estimating initial bubble point pressure by visual graphical analysis and smoothing the two-phase data utilizing y-function to refine the initial estimate
1995	Al-Yousef and Al-Marhoun	The method is a nonlinear parameter estimation approach to estimate bubble point pressure and volume. It uses a new x-function to model the liquid data and y-function to describe the two-phase data
2017	Hoang, Baylaucq, and Galliero	The procedure is an iterative process based on the Tait equation to model the liquid data and the y-function to describe the two-phase data. It consists of bracketing the bubble point pressure interval and locating the bubble point within this interval

Table 2 Experimental PV data and analysis of the black oil sample

No	Pressure psi	Experimental volume cm ³	Derivative f'	Derivative ratio f' _{ratio}	Peak
1	2874	105.75	− 0.0006	0.90	
2	2469	106.01	− 0.0006	0.90	
3	1638	106.49	− 0.0007	1.16	1.16
4	1054	106.88	− 0.0008	1.15	
5	767	107.10	− 0.0007	0.94	
6	530	107.27	− 0.0053	7.40	
7	368	108.13	− 0.0985	18.55	18.55
8	348	110.10	− 0.1268	1.29	
9	329	112.51	− 0.1600	1.26	
10	309	115.71	− 0.2106	1.32	
11	262	125.61	− 0.2973	1.41	
12	229	135.42	− 0.4391	1.48	1.48
13	206	145.52	− 0.5828	1.33	
14	181	160.09	− 0.7905	1.36	1.36
15	162	175.11	− 0.9462	1.20	
16	141	194.98			

Table 3 Experimental PV data and analysis of the volatile oil sample

No	Pressure psi	Experimental volume cm ³	Derivative f'	Derivative ratio f' _{ratio}	Peak
1	7489	90.58	− 0.0025	1.12	
2	7085	91.59	− 0.0028	1.12	
3	6681	92.72	− 0.0030	1.09	
4	6277	93.95	− 0.0035	1.13	
5	5873	95.34	− 0.0040	1.17	1.17
6	5469	96.98	− 0.0046	1.14	
7	5065	98.85	− 0.0063	1.36	
8	4646	101.49	− 0.0133	2.11	2.11
9	4040	109.56	− 0.0185	1.39	
10	3535	118.92	− 0.0261	1.41	
11	3030	132.10	− 0.0384	1.47	
12	2525	151.48	− 0.0609	1.59	
13	2020	182.21	− 0.1072	1.76	
14	1515	236.36			

two-phase oil and gas regions. For some volatile oils, the pressure–volume curve is smooth, and the inflection is not very clear, and it becomes difficult to determine the bubble point. In general, at the high-pressure end, the pressure–volume follows almost a straight line reflecting single-phase behavior. Below bubble point pressure, the PV data is curved and far from behaving like a straight line reflecting two-phase behavior. However, with a few points after the inflection point, a straight line is drawn. The intersection of the two straight lines is the first estimate of bubble point pressure and volume.

Smoothing of PV data below bubble point pressure

The two-phase PV data are the points below the estimated bubble point pressure. Y-function is used to smooth the PV data below the bubble point to refine the value of bubble point pressure. The first estimated bubble point volume is not refined but taken as the final value. The dimensionless y-function is defined as:

$$y = \frac{(P_b/P - 1)}{(V_t/V_b - 1)} \quad (1)$$

The smoothing method is a trial-and-error. It starts by taking the first estimate of the bubble point pressure from the visual graphical analysis. Y-function is calculated as a function of pressure. Change the value of bubble point pressure until a straight line of y-function versus pressure is obtained; otherwise, a new value of bubble point pressure is assumed, and the process is repeated. If a straight line is obtained, the assumed value of bubble point pressure is correct. The straight line is fitted as:

$$y_i = a_1 + a_2 p_i \quad (2)$$

Combining Eq. 1 and Eq. 2 yield

$$\frac{V_t}{V_b} = 1 + \frac{P_b/P - 1}{y} \quad (3)$$

Applications of Eq. 3 calculate smoothed relative two-phase volumes v_t/v_b . A plot of y-function versus pressure is constructed, as shown in Fig. 3.

Methodology

The proposed new approach is a derivative-based technique that utilizes the PV data obtained from the CCE test. The new technique determines bubble point pressure and volume of hydrocarbon systems directly. The proposed new approach is summarized in the following three consecutive steps:

Step 1. Locate the inflection point of PV data

The classical approach to bracket the bubble point pressure is a trial-and-error approach or graphical and visual judgment. In this study, a new approach is developed to directly determine the inflection point of experimental PV data where the bubble point is.

Fig. 1 Experimental PV data from laboratory CCE test

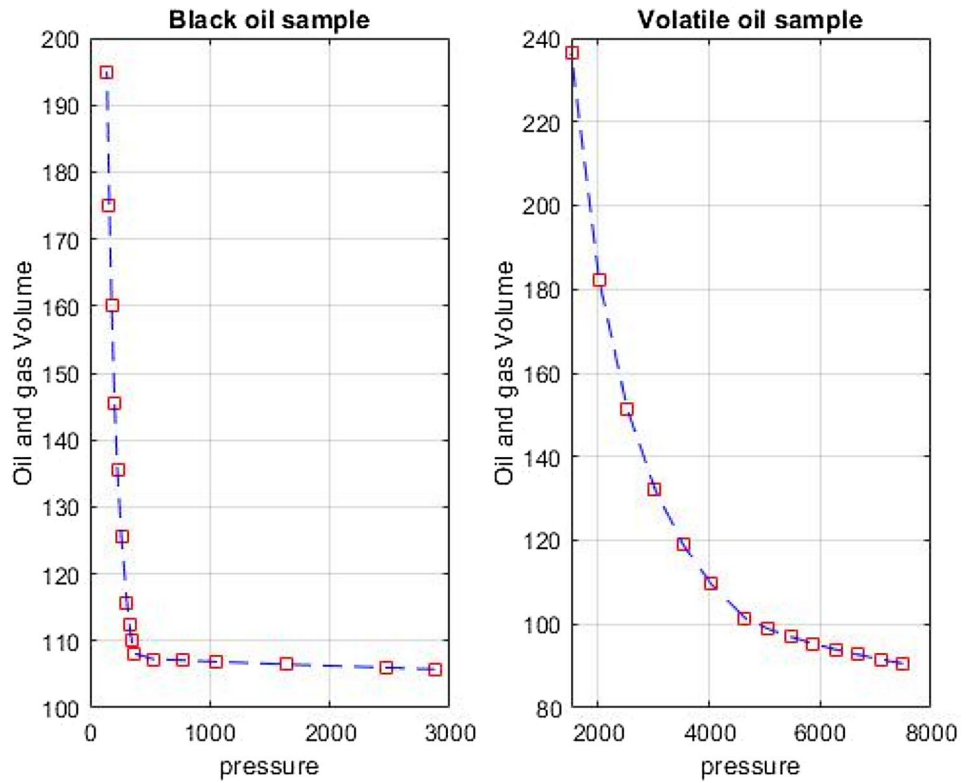


Fig. 2 Visual graphical analysis to obtain the first estimate of bubble point pressure

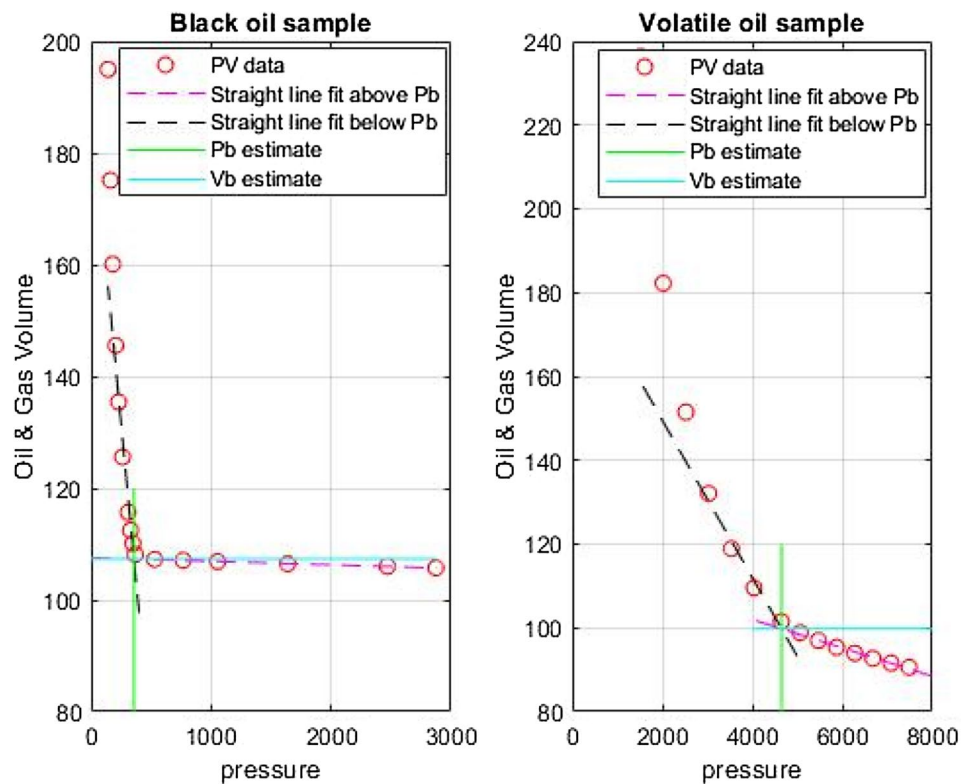
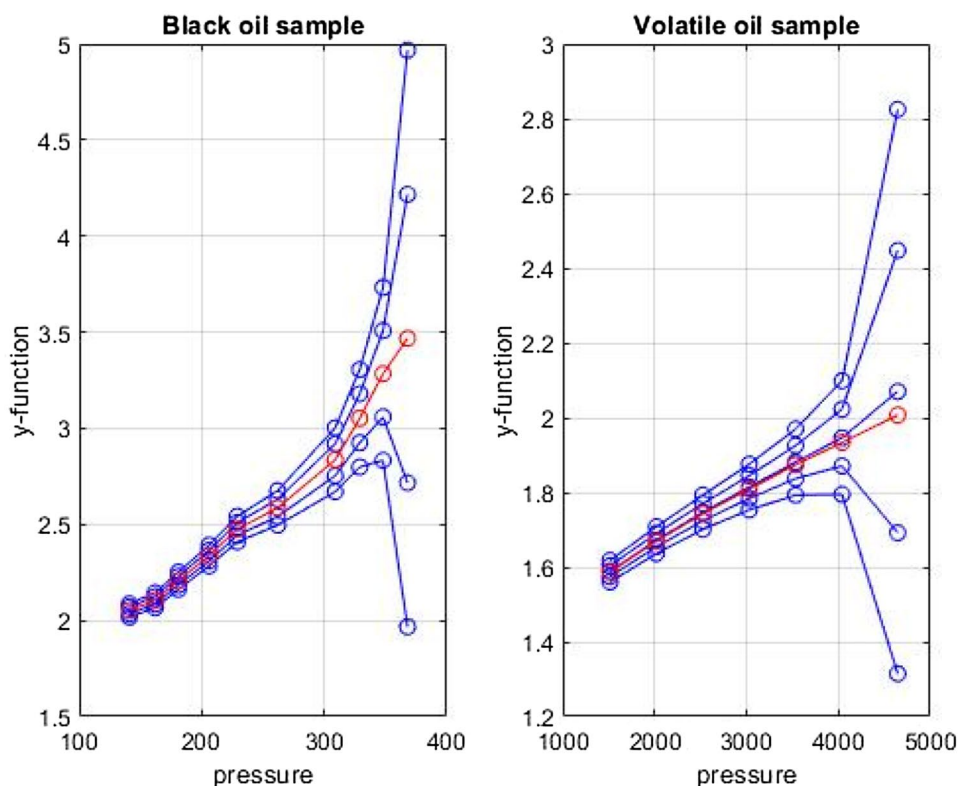


Fig. 3 Y-function versus pressure for several bubble point assumptions



For a given temperature, the fluid volume is a function of pressure.

$$v = f(p) \tag{4}$$

The CCE test provides a set of PV data points (p, v) in descending order of pressure corresponding to ascending order of fluid volume, as shown in Tables 2 and 3. For discrete PV data, the volume of fluid at pressure i is

$$v_i = f(p_i), i = 1, 2, \dots \dots n \tag{5}$$

Finite differences approximate the function derivative for discrete data points. The numerical derivative of a function f at a point p_i is defined by the limit as follows:

$$f'(p_i) = \lim_{p_i \rightarrow p_{i+1}} \left(\frac{v_i - v_{i+1}}{p_i - p_{i+1}} \right), i = 1, 2, \dots \dots n - 1 \tag{6}$$

The numerical estimates of the first derivative of all PV data are taken sequentially. Introducing the concept of derivative ratio as follows:and

$$f'_{ratio}(p_i) = \frac{f'(p_i)}{f'(p_{i-1})}, i = 2, 3, \dots \dots n - 1$$

$$f'_{ratio}(p_1) = f'_{ratio}(p_2) \tag{7}$$

The consecutive derivative ratios form several peaks and troughs. The highest peak always exists at the inflection of PV data to traverse into the two-phase region. The inflection point separates the PV data into two groups. The PV data above the pressure of the highest peak are the PV data above bubble point pressure. The PV data at and below the highest peak pressure are the PV data below bubble point pressure.

Step 2. Develop mathematical models to describe PV data

The PV data does not follow a power function, nor does it follow an exponential one. The PV data follows a combination of the two. Let us call this new mathematical model an exponential-power function. The exponential-power function model accurately describes the PV data above and below the bubble point.

Above the bubble point pressure, the new exponential-power function model is introduced to describe the volume of a liquid, v_o , in terms of pressure along an isotherm as

$$v_o = e^{a_1 + a_2 p_o} p_o^{a_3} \tag{8}$$

where a 's are the least square fitted numerical parameters.

Similarly, below the bubble point pressure, a new exponential-power function model is introduced to describe the

volume of a two-phase fluid, v_t , in terms of pressure along an isotherm as

$$v_t = e^{b_1 + b_2 p_t} p_t^{b_3} \quad (9)$$

where b 's are the least square fitted numerical parameters.

Step 3. Determine bubble point pressure and volume

Mathematical manipulations of the new mathematical model, the exponential-power function, leads to direct determination of both bubble point pressure and volume simultaneously.

Taking the logarithm of both sides of Eq. 8 and Eq. 9

$$\ln v_o = a_1 + a_2 p_o + a_3 \ln p_o \quad (10)$$

$$\ln v_t = b_1 + ab_2 p_t + b_3 \ln p_t \quad (11)$$

The intersection of the two curves is the bubble point volume, v_b , therefore

$$\ln v_b = a_1 + a_2 p_b + a_3 \ln p_b \quad (12)$$

$$\ln v_b = b_1 + b_2 p_b + b_3 \ln p_b \quad (13)$$

Combining the two equations yield

$$g(p_b) = (b_1 + b_2 p_b + b_3 \ln p_b) - (a_1 + a_2 p_b + a_3 \ln p_b) = 0 \quad (14)$$

or

$$g(p_b) = (b_3 - a_3) \ln p_b + (b_2 - a_2) p_b + (b_1 - a_1) = 0 \quad (15)$$

The general form of the intersection function and its derivative at x is

$$g(x) = (b_3 - a_3) \ln x + (b_2 - a_2)x + (b_1 - a_1) \quad (16)$$

$$g'(x) = \frac{(b_3 - a_3)}{x} + (b_2 - a_2) \quad (17)$$

$$x_{\max} = -\frac{b_3 - a_3}{b_2 - a_2}, \text{ when } g'(x) = 0 \quad (18)$$

x_{\max} corresponds to the maximum point of Eq. 15. Therefore, the conditions of finding a solution to Eq. 15 are as follows:

If $g'(x_{\max}) > 0$, then the two PV curves intersect, and there exist two solutions. The bubble point is the smallest value.

If $g'(x_{\max}) = 0$, the two PV curves intersect at a single point, and the $p_b = x_{\max}$.

If $g'(x_{\max}) < 0$, the two PV curves do not intersect, and Eq. 15 has no solution.

Eq. 15 is a nonlinear equation with one unknown. There are several methods of solving such an equation, like Newton's method or the bisection method.

After the bubble point pressure is determined, the bubble point volume is evaluated by Eq. 12 or Eq. 13. Both equations must yield the same value.

$$v_b = \exp(a_1 + a_2 p_b + a_3 \ln p_b) \quad (19)$$

or

$$v_b = \exp(b_1 + b_2 p_b + b_3 \ln p_b) \quad (20)$$

Applications of the proposed scheme

Experimental PV data of two oil samples, black and volatile, were collected from laboratory CCE tests. The PV data are presented in columns 1 and 2 of Tables 2 and 3. The application of the proposed new scheme, including detailed data analysis, is done in three consecutive steps:

Step 1. Locate the inflection point of PV data.

The CCE test provides a set of PV data points (p_i, v_i) in descending order of pressure corresponding to an ascending order of fluid volume, where i varies from 1 to n .

The first derivative of discrete data points is approximated by finite differences described by Eq. 6. The numerical estimate of the first derivative of all PV data is taken sequentially and presented in column 4 of Tables 2 and 3 and shown in Fig. 4.

The concept of the derivative ratio is defined by Eq. 7. The calculated derivative ratio of all PV data is taken sequentially and presented in column 5 of Tables 2 and 3.

The consecutive derivative ratios form several peaks and troughs. The peaks are presented in column 6 of Tables 2 and 3. The highest peak exists at the inflection of PV data, as shown in Fig. 4.

Step 2. Develop mathematical models to describe PV data.

The inflection point separates the PV data into two groups, one above and one below the bubble point. Then, the mathematical models are developed for the PV data above and below bubble point pressure. Linearized least square regression analysis is used to find the best fit.

The PV data above the pressure of the highest peak are the PV data above bubble point pressure. Eq. 10 describes the PV data above bubble point pressure. A least-square fitting of the linearized exponential-power function yields predicted oil volume:

Fig. 4 Finite difference derivatives across the upper half and derivative ratios span the lower half

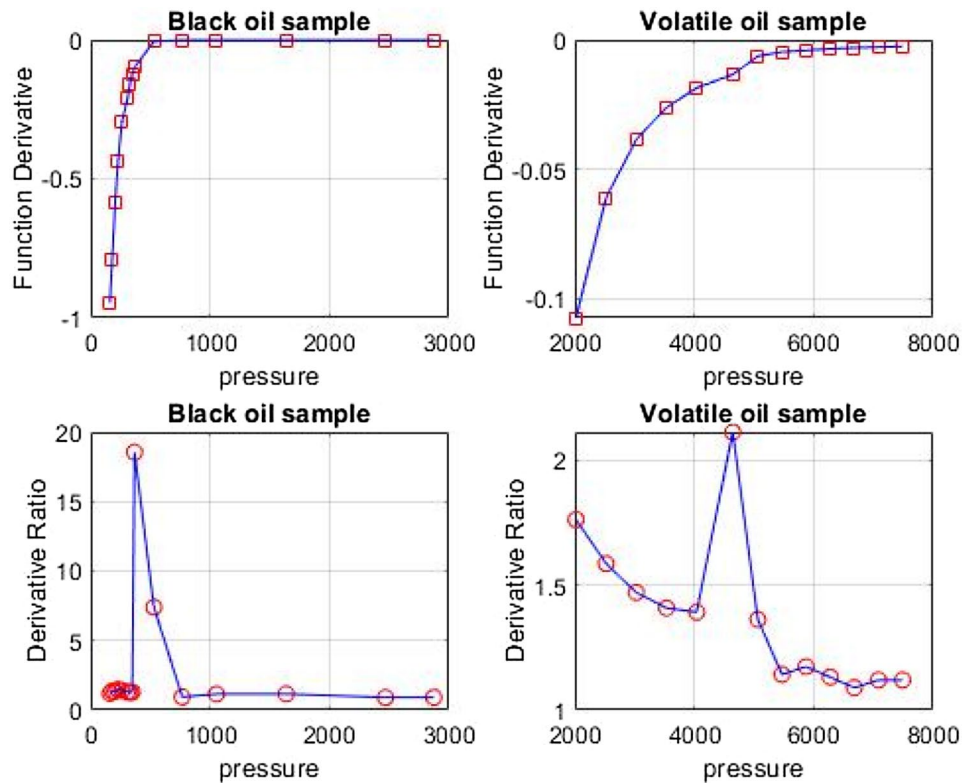


Table 4 Fitting parameters and errors of PV data above the bubble point

Eq. 21 coefficients	Black oil sample	Volatile oil sample
a_1	4.6878	7.6872
a_2	-4.9596×10^{-6}	2.4973×10^{-5}
a_3	-0.0015522	-0.37751
Average absolute relative error, Eq. 23	8.3832×10^{-5}	0.00014669

$$\ln v_o^\wedge = a_1 + a_2 p_o + a_3 \ln p_o \tag{21}$$

The relative errors or differences between measured and predicted values are

$$\epsilon_{oi} = (v_{oi} - \hat{v}_{oi}) / v_{oi}, 1, 2, \dots, n_o \tag{22}$$

The average absolute relative error of all PV data points above bubble point pressure is defined as

$$E_a = \frac{1}{n_o} \sum_i |\epsilon_{oi}| \tag{23}$$

The fitting parameters for the PV data above bubble point pressure and the average absolute relative error are presented in Table 4.

The PV data at and below the highest peak pressure are the PV data below bubble point pressure. Eq. 11 fits

the PV data below bubble point pressure. A least-square fitting of the linearized exponential-power function yields the predicted two-phase volume:

$$\ln v_t^\wedge = b_1 + ab_2 p_t + b_3 \ln p_t \tag{24}$$

The relative errors or differences between measured and predicted values are

$$\epsilon_{ti} = (v_{ti} - v_{ti}^\wedge) / v_{ti}, 1, 2, \dots, n_t \tag{25}$$

The average absolute relative error of all PV data points below bubble point pressure is defined as

$$E_b = \frac{1}{n_t} \sum_i |\epsilon_{ti}| \tag{26}$$

The fitting parameters for the PV data below bubble point pressure and the average absolute relative error are presented in Table 5.

Figure 5 shows the comparison of experimental PV data and smoothed exponential-power function fit.

Step 3. Determine bubble point pressure and volume.

The exponential-power function curves describing PV data above and below the bubble point intersect at the bubble point pressure and volume. The intersection defined in Eq. 15 is solved for bubble point pressure with Newton's method or bisection method. The bubble point pressure is determined directly from Eq. 15, while the bubble point

Table 5 Fitting parameters and errors of PV data below the bubble point

Eq. 24 coefficients	Black oil sample	Volatile oil sample
b_1	10.805	13.651
b_2	0.0024078	0.00014101
b_3	- 1.1861	- 1.1471
Average absolute relative error, Eq. 26	0.0017168	0.00069634

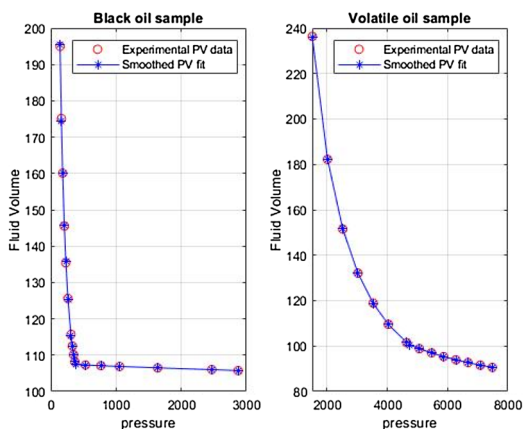


Fig. 5 Comparison of experimental PV data and smoothed exponential-power function fit

Table 6 Bubble point pressure and volume

Sample	Bubble point pressure solution of Eq. 15	Bubble point volume Eq. 19 or Eq. 20
Black oil	377.30	107.4133
Volatile oil	4756.05	100.4347

volume is evaluated by Eq. 19 or Eq. 20. The bubble point pressure and volume are presented in Table 6. Figure 6 shows the graphical solution and behavior of the bubble point intersection function, Eq. 15.

Smoothed PV relationship is usually tabulated in fluid analysis laboratory reports for reservoir properties calculations. Smoothed volume above the bubble point pressure

is calculated by Eq. 21, while smoothed volume below the bubble point pressure is calculated by Eq. 24. The relative volume of fluid is evaluated as follows:

$$v_r = \frac{\hat{v}_{oi}}{v_b}, \text{ for PV data above } p_b$$

$$v_r = \frac{\hat{v}_{ti}}{v_b}, \text{ for PV data below } p_b \tag{27}$$

Smoothed volume, relative error, and relative volume are presented in columns 4–7 of Tables 7 and 8.

Conclusions

Based on the results of this study, the following conclusions are drawn:

1. A new method for the determination of bubble point pressure and volume of hydrocarbon systems is presented.
2. The concept of derivative ratio clearly shows that the single-phase fluid traverses into a two-phase region. The highest peak of the derivative ratio separates the PV data into two groups, one above bubble point pressure and the other below bubble point pressure.
3. Bubble point pressure and volume are determined directly and simultaneously, while the current method goes through visual graphical judgment and a trial-and-error procedure.
4. The exponential-power function model accurately describes the PV data above and below the bubble point.
5. The new non-iterative technique utilizes all data points above and below the bubble point to obtain unique values for the bubble point pressure and volume. While conventional y-function method utilizes the PV data below bubble point only.
6. The new method yields a unique bubble point pressure and volume. Therefore, the procedure is repeatable regardless of who performs the calculation, while the conventional procedure for determining bubble point pressure is not reproducible.

Fig. 6 Solution of bubble point intersection function, Eq. 15

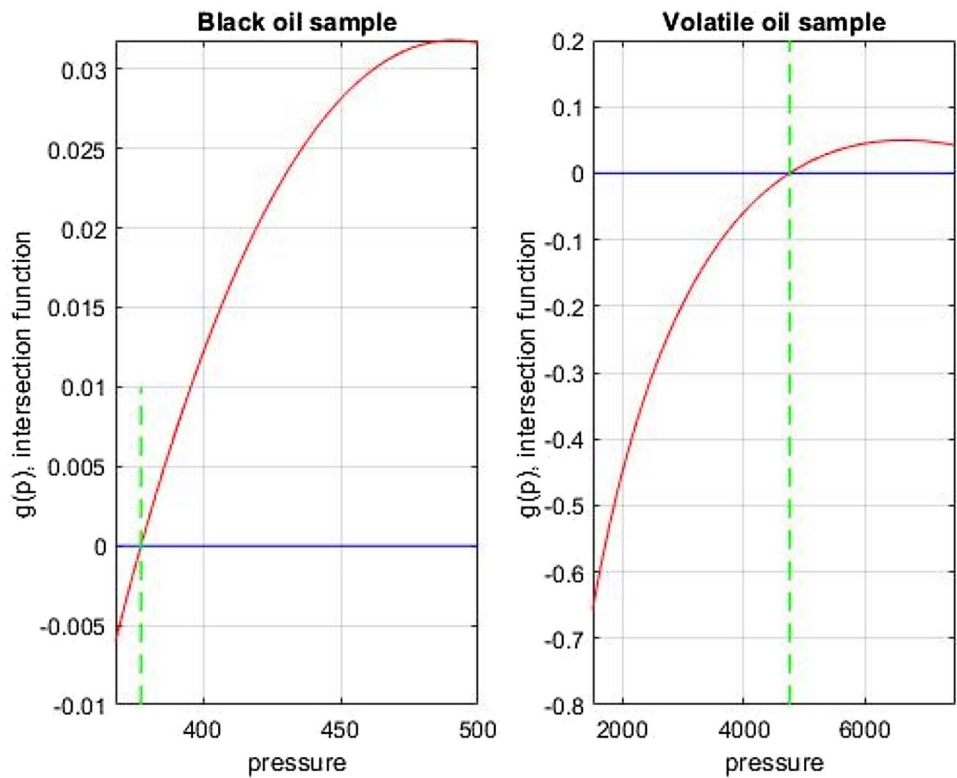


Table 7 Smoothed PV data of the black oil sample

No	Pressure psi	Experimental volume cm ³	Smoothed volume Eq. 21	Smoothed volume Eq. 24	Relative error Eqs. 22 and 25	Relative volume Eq. 27
1	2874	105.75	105.76		- 7.22E- 05	0.9846
2	2469	106.01	106.00		1.39E- 04	0.9868
3	1638	106.49	106.50		- 1.02E- 04	0.9915
4	1054	106.88	106.88		- 2.69E- 05	0.9951
5	767	107.10	107.09		1.13E- 04	0.9970
6	530	107.27	107.28		- 5.06E- 05	0.9987
pb	377.30		107.41	107.41		1.0000
7	368	108.13		108.19	- 5.58E- 04	1.0072
8	348	110.10		110.17	- 6.26E- 04	1.0257
9	329	112.51		112.49	1.84E- 04	1.0473
10	309	115.71		115.48	2.00E- 03	1.0751
11	262	125.61		125.41	1.56E- 03	1.1676
12	229	135.42		135.89	- 3.46E- 03	1.2651
13	206	145.52		145.77	- 1.69E- 03	1.3571
14	181	160.09		160.01	4.77E- 04	1.4897
15	162	175.11		174.35	4.36E- 03	1.6231
16	141	194.98		195.42	- 2.26E- 03	1.8193

Table 8 Smoothed PV data of the volatile oil sample

No	Pressure psi	Experimental volume cm ³	Smoothed volume Eq. 21	Smoothed volume Eq. 24	Relative error Eqs. 22 and 25	Relative volume Eq. 27
1	7489	90.58	90.59		− 1.61E− 04	0.9020
2	7085	91.59	91.58		8.19E− 05	0.9118
3	6681	92.72	92.69		2.80E− 04	0.9229
4	6277	93.95	93.95		2.18E− 05	0.9354
5	5873	95.34	95.37		− 2.75E− 04	0.9496
6	5469	96.98	96.99		− 7.73E− 05	0.9657
7	5065	98.85	98.84		1.30E− 04	0.9841
pb	4756.05		100.43	100.43		1.0000
8	4646	101.49		101.58	− 8.83E− 04	1.0114
9	4040	109.56		109.48	7.64E− 04	1.0900
10	3535	118.92		118.83	7.82E− 04	1.1831
11	3030	132.10		132.06	2.78E− 04	1.3149
12	2525	151.48		151.59	− 7.54E− 04	1.5094
13	2020	182.21		182.36	− 8.00E− 04	1.8157
14	1515	236.36		236.22	6.13E− 04	2.3519

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DiFoggio R, Michaels J, and Lee J. (2006). Downhole PV tests for bubble point pressure (U.S. Patent No. 7,346,460 B2).

Dymond JH, Malhotra R (1988) The tait equation: 100 years on. *Int J Thermophys* 9(6):941–951

Hoang H, Baylaucq A, Galliero G (2017) Accurate determination of bubble-point of oils from PV data using a combination of Y-function and Tait equation. *J Petrol Sci Eng* 149:801–810

Shwe, T, Flecker M, Thompson S, and Torrance R. (2001). Determining fluid bubble point pressure using an adjustable choke (U.S. Patent No. 6,250,138 B2).

Standing MB (1952). *Volumetric and phase behavior of oil field hydrocarbon systems*: Reinhold Publishing Corp. New York.

Williams John M. "Why Y?." Paper presented at the SPE Annual technical conference and exhibition, Denver, Colorado, USA, October 2011. doi: <https://doi.org/10.2118/146394-MS>

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References

- Al-Yousef HY, Al-Marhoun MA (1995) Automated method for smoothing experimental flash liberation PVT data. *J Can Pet Technol* 34(03):4
- Angelescu D, Sullivan M, and Harrigan, E. (2015). Thermal bubble point measurement system and method (U.S. Patent No. 8,950,246 B2).