Approach to shape factor determination in estimating horizontal wells productivity for pressure transient model of Mutalik et al.

Firas A. A. Al-Kabbawi¹

Received: 29 July 2021 / Accepted: 15 October 2021 / Published online: 30 October 2021
 © The Author(s) 2021

Abstract

Horizontal well productivity (J_h) equation at pseudo-steady state (PSS) of Mutalik et al. (in: Presented at SPE 63rd annual technical conference and exhibition, Houston, Texas, 1988) has long and widely been known to estimate well performance. One key parameter of that equation is the so-called shape factor (C_A) that characterizes the effect of the pressure transient behavior at PSS depending on well and reservoir configuration. For limited well and reservoir configurations, C_A tables had been generated decades ago, but at unknown accuracy they are still used widely via tedious correlation or interpolation from those reported C_A tables to estimate J_h for different configurations even today. The innovation points in this study are as follows: (1) The pressure transient model of Mutalik et al. (in: Presented at SPE 63rd annual technical conference and exhibition, Houston, Texas, 1988) (PTM) is improved to generalized pressure transient model (GPTM) and verified by converting several dimensional parameters to dimensionless forms. (2) An efficient algorithm is developed, and accurate C_A and J_h are obtained for any well and reservoir configuration. (3) The accuracy of the reported C_A tables is quantified, and the consequences are determined when they are used to estimate J_h . (4) New C_A tables and their correspondence dimensionless time based on drainage area at PSS ($(T_{DA})_{PSS}$) tables are generated for a wider range of well and reservoir configurations. Although horizontal wells usually have been drilled with high dimensionless well length (L_D) industrially (i.e., $L_D > 10$), it is still a significant error in J_h by ($< 7\%$) that has been determined when using the reported C_A of PTM for rectangular reservoir aspect ratio ($x_e/y_e = 1$) and (2) in the both assumed input data, while for ($x_e/y_e = 5$), significant error in J_h starts from (46.43%) to (1037%) for $L_D = (10)$ to (100), respectively, in the first assumed input data [variable, dimensionless wellbore radius (r_{wD})] and expected the same in the second assumed input data (constant r_{wD}).

Keywords Horizontal well productivity · Shape factor · Pressure transient model · Rectangular bounded reservoir · Instantaneous source functions

Introduction

J_h equation of Mutalik et al. (1988) is well known and presented by many authors (Duda et al. (1991), Economides et al. (1991b), Economides et al. (1991a), Frick and Economides (1993), Thomas et al. (1998), Penmatcha et al. (1999), Saavedra and Joshi (2002), Cho (2003), Jackson et al. (2011), Adesina et al. (2011), Ouyang (2015), Cetkovic et al. (2016), Wu et al. (2018), Luo et al. (2018), and Zhang et al. (2020)). In reservoir simulators, J_h equation of Mutalik et al. (1988) has been used to compute well productivity

index (Krawchuk et al. 2006; Pinzon et al. 2007; Huebsch et al. 2008; Haidar et al. 2008; Brown and Tiwari 2010; Wood et al. 2010; and Leone et al. 2015). The J_h equation is defined as follows:

$$J_h = \frac{0.007078kh/\mu_o B_o}{\ln\left(\frac{\sqrt{A/\pi}}{r_w}\right) - 2.1243 - \ln(L/4r_w) + \ln\sqrt{30.8828/C_A}} \quad (1)$$

$$C_A = \frac{8.9832x_e y_e}{L^2} \exp[2(2\pi(T_{DA})_{PSS} - (P_D T)_{PSS})] \quad (2)$$

where J_h is the horizontal well productivity (STB/day.psi), k is the reservoir permeability (md), h is the reservoir thickness (ft), μ_o is the oil viscosity (cp), B_o is the oil formation volume factor (bbl/STB), A is the drainage area (ft²), r_w is the well radius (ft), L is the length of horizontal well (ft), x_e

✉ Firas A. A. Al-Kabbawi
 fi_im2005@yahoo.com

¹ Iraqi Ministry of Oil, Baghdad, Iraq

is the reservoir length in the X -direction (ft), the horizontal well is placed along the X -direction, y_e is the reservoir length in the Y -direction (ft), and $(P_D T)_{PSS}$ is the total dimensionless pressure at PSS.

The J_h equation [Eq. (1)] is for rectangular bounded reservoir at depletion stage (i.e., PSS). In C_A equation [i.e., Eq. (2)], the $(P_D T)_{PSS}$ was obtained by the PTM:

$$P_D T = 2\pi \int_0^{T_{DA}} \left\{ \left[1 + \frac{8x_e}{\pi L} \sum_{n=1}^{\infty} \frac{1}{n} \exp\left(\frac{-n^2 \pi^2 y_e T_{DA}}{x_e}\right) \cos\left(\frac{n\pi L x_{WD}}{4x_e}\right) \cos\left(\frac{n\pi L (x_{WD} + x_D)}{4x_e}\right) \sin\left(\frac{n\pi L}{4x_e}\right) \right] \times \left[1 + 2 \sum_{n=1}^{\infty} \exp\left(\frac{-n^2 \pi^2 x_e T_{DA}}{y_e}\right) \cos\left(\frac{n\pi L y_{WD}}{4y_e}\right) \cos\left(\frac{n\pi L (y_{WD} + y_D)}{4y_e}\right) \right] \times [1 + 2 \sum_{n=1}^{\infty} \exp\left(-4n^2 \pi^2 \frac{x_e y_e}{h^2} T_{DA}\right) \cos(n\pi z_{WD}) \cos(n\pi z_D)] \right\} dT_{DA} \quad (3)$$

where $P_D T$ is the total dimensionless pressure, T_{DA} is the dimensionless time based on drainage area, x_{WD} , y_{WD} , and z_{WD} are the dimensionless well location coordinates, x_D is the X -coordinate for a point in the porous media. ($x_D=0$) at the horizontal well center, and ($x_D=1$ and -1) at the well tips. y_D is the Y -coordinate for a point in the porous media. ($y_D=r_{WD}$) if the point is at the well wall, z_D is the Z -coordinate for a point in the porous media. ($z_D=z_{WD}$) if the point is at the well wall

PTM [i.e., Eq. (3)] is based on 3D instantaneous source functions with Newman product method under homogenous and isotropic rectangular bounded reservoir at constant well flow rate and uniform flux wellbore condition of Gringarten and Ramey (1973). It is assumed that horizontal well is analogous to a vertical well of a vertical fracture. The equivalent infinite conductivity wellbore condition was obtained by solving $P_D T$ at x_D equal to (0.732) (Mutalik et al. (1988) and Gringarten et al. (1974)). “Appendix 1” presents a summary of PTM.

When $P_D T$ slope with time becomes constant at PSS and is equal to (2π) , both the $(T_{DA})_{PSS}$ and $(P_D T)_{PSS}$ have been obtained to use for C_A calculation. C_A at PSS also becomes constant for a given well and reservoir configuration.

PTM and their reported C_A tables have identified three issues. Firstly, PTM as defined in Eq. (3) contains several dimensional parameters (i.e., x_e , y_e , h , and L) and this makes the solution not only awkward, but also difficult to use PTM to expand the reported C_A tables to any other well and reservoir configurations. Secondly, PTM has not been verified in terms of $P_D T$ which may lead to errors in the reported C_A tables. The third issue is the discrepancy in the assumed input data of the reported C_A tables. The first assumption of the input data

(i.e., $r_w=0.225$ ft and $y_e=933.33$ ft) produces variable r_{WD} where ($r_{WD}=2r_w/L$) depends on penetration ratio (PR) and (x_e/y_e). The second assumption of the input data shows that r_{WD} is constant and is equal to (10^{-4}) for any well and reservoir configurations. These three limitations cast a doubt on the representativeness and usefulness of the reported C_A tables for their respective configurations or other configurations for

which simply interpolation or extrapolation of those tables is required as the correlative models previously presented by Bahadori (2012), Bahadori et al. (2013), and Ahmadi et al. (2015).

The main purposes of this study are in four parts. (1) To overcome the first weakness in PTM (i.e., the dimensional parameters) by recasting PTM into a fully dimensionless form (i.e., GPTM) and to verify GPTM in terms of $P_D T$. (2) To develop efficient algorithm to compute C_A and J_h accurately for any well and reservoir configuration. Because C_A depends on the all-configuration parameters, unlimited tables of C_A can be generated; therefore, the MATLAB code is provided. (3) To quantify the accuracy of the reported C_A tables in their both assumed input data (i.e., constant and variable r_{WD}), and to determine the consequences of any identified errors in the reported C_A tables when they are used to estimate J_h . (4) To generate new C_A tables for a wider spectrum of well and reservoir configurations.

The generalized pressure transient model (GPTM)

The PTM is improved to fully dimensionless form (i.e., to GPTM) by applying the following dimensionless parameters into Eq. (3):

$$L_D = \frac{L}{2h} \quad (4)$$

$$x_{eD} = \frac{2x_e}{L} \quad (5)$$

$$y_{eD} = \frac{2y_e}{L} \quad (6)$$

Note that OCWL is used to indicate the offset of the well relative to the central well location (i.e., OCWL = 1) at a respective coordinate direction for the purpose of clas-

$$P_D T = 2\pi \int_0^{T_{DA}} \left\{ \left[1 + \frac{4x_{eD}}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \exp\left(\frac{-n^2 \pi^2 y_{eD} T_{DA}}{x_{eD}}\right) \cos\left(\frac{n\pi x_{wD}}{2x_{eD}}\right) \cos\left(\frac{n\pi(x_{wD} + x_{eD})}{2x_{eD}}\right) \sin\left(\frac{n\pi x_{wD}}{2x_{eD}}\right) \right] \times \left[1 + 2 \sum_{n=1}^{\infty} \exp\left(\frac{-n^2 \pi^2 x_{eD} T_{DA}}{y_{eD}}\right) \cos\left(\frac{n\pi y_{wD}}{2y_{eD}}\right) \cos\left(\frac{n\pi(y_{wD} + y_{eD})}{2y_{eD}}\right) \right] \times [1 + 2 \sum_{n=1}^{\infty} \exp(-4n^2 \pi^2 x_{eD} y_{eD} L_D^2 T_{DA}) \cos(n\pi z_{wD}) \cos(n\pi z_D)] \right\} dT_{DA} \quad (7)$$

where x_{eD} is the dimensionless reservoir length in the X-direction and y_{eD} is the dimensionless reservoir length in the Y-direction

For anisotropic reservoir solution, the dimensionless parameters are presented in “Appendix 2.”

The following meaningful mathematical relationship should be noted in the well and reservoir configuration:

$\frac{y_{wD}}{y_{eD}} = \frac{y_w}{y_e}$ Used for centrally and off-centrally well location (OCWL) in the Y-direction.

$\frac{x_{wD}}{x_{eD}} = \frac{x_w}{x_e}$ Used for centrally and OCWL in the X-direction.

$z_{wD} = \frac{z_w}{h}$ Used for centrally and OCWL in the Z-direction.

$\frac{x_{eD}}{y_{eD}} = \frac{x_e}{y_e}$ Dimensionless rectangular reservoir aspect ratio.

$PR = \frac{L}{2x_e}$ PR is also equal to $(1/x_{eD})$ [i.e., the inverse of Eq. (5)].

sifying the configurations only.

The efficient algorithm to solve Eq. (7) is developed and implemented in MATLAB code and is given in “Appendix 3.”

$P_D T$ by GPTM of this study [i.e., Eq. (7)] is verified against its counterpart by the horizontal well pressure transient model of Ozkan and Raghavan (1988) for a square bounded reservoir (i.e., $x_e/y_e = 1$). L_D is equal to (10), and $PR = (0.2)$ and (1), for a well that is centered in x -, y -, and z -directions [i.e., $(x_{wD}/x_{eD} = 1)$, $(y_{wD}/y_{eD} = 1)$, and $(z_{wD} = 0.5)$, or (OCWL = 1)]. The wellbore condition is uniform flux ($x_D = 0$), and r_{wD} is equal to (2×10^{-4}) . Figure 1 shows the identical results of $P_D T$ by both the models.

For further verification on GPTM, $P_D T$ of this study is compared with $P_D T$ of the pressure transient model of Gringarten et al. (1974) for vertical fracture well under square bounded reservoir. ($L_D = 10^3$) (i.e., to make the thickness of the fracture to be zero), $PR = (0.5)$ and (1), (OCWL = 1),

Fig. 1 Comparison of $P_D T$ by GPTM and by the pressure transient model of Ozkan and Raghavan (1988) for a centrally well located in a square reservoir

Fig. 2 Comparison of $P_D T$ by GPTM and by the pressure transient model of Gringarten et al. (1974) for vertical fracture well under square reservoir

($x_D = 0$), and ($r_{wD} = 10^{-4}$). Figure 2 shows the identical results of $P_D T$ by both the models.

In the both previous verifications, the time intervals of T_{DA} in Eq. (7) are to be taken: ($0-10^{-9}$), ($10^{-9}-10^{-8}$), ($10^{-8}-10^{-7}$), ($10^{-7}-10^{-6}$), ($10^{-6}-10^{-5}$), ($10^{-5}-10^{-4}$), ($10^{-4}-10^{-3}$), ($10^{-3}-10^{-2}$), ($10^{-2}-10^{-1}$), ($10^{-1}-1$), ($1-10$), and ($10-100$), and within each of these intervals its T_{DA} increment is taken to be (10^{-9}), (10^{-9}), (10^{-9}), (10^{-9}), (10^{-9}), (10^{-8}), (10^{-7}), (10^{-6}), (10^{-5}), (10^{-4}), (10^{-3}), and (10^{-2}), respectively. These T_{DA} intervals and their increments are sufficient to yield accurate $P_D T$. It is found that the first-time logarithmic cycle (FTLC) [i.e., ($0-10^{-9}$)] plays a crucially important role in determining the accuracy of $P_D T$ at any period even at PSS and then in C_A calculation as to be shown in the next sections.

The C_A equation [i.e., Eq. (2)] can be written in general form by substituting Eqs. (5) and (6) into Eq. (2):

$$C_A = 2.2458 x_{eD} y_{eD} \exp[2(2\pi(T_{DA})_{PSS} - (P_D T)_{PSS})] \quad (8)$$

For vertical fracture well (i.e., when ($L_D > 10^3$)), a comparison between the $C_A(s)$ by this study and their

counterparts by Earlougher (1977) and Hagoort (2009) is presented in Table 1. For FTLC equal to ($0-10^{-9}$) or ($0-10^{-5}$), excellent matching is found with $C_A(s)$ by Hagoort (2009) at any PR and with $C_A(s)$ by Earlougher (1977) at just high PR.

In what follows, the reported C_A tables by PTM for selected well and reservoir configurations are examined, and their well and reservoir configurations can be classified into four cases in terms of OCWL. Table 2 lists the four cases of the reported C_A tables. The parameters in Cases (1) to (3) correspond to tables (4), (7), and (8) in Mutalik et al. (1988) paper, respectively. Note that parameters in Case (4) are deduced from Fig. 12 in Mutalik et al. (1988) paper.

The accuracy of the reported C_A tables by PTM (case 1)

Although the accuracy of the reported C_A tables by PTM for other cases has been examined, here only the results for the Case 1 are presented for both assumed input data (i.e., the discrepancy issue). For each C_A by PTM, the counterpart C_A

Table 1 Comparison of C_A by this study, Earlougher (1977) and Hagoort (2009)

PR	$C_A (X_D = 0.732)$			$C_A (X_D = 0.740098)$		
	Earlougher (1977)	FTLC of this study		Hagoort (2009)	FTLC of this study	
		10^{-5}	10^{-9}		10^{-5}	10^{-9}
1	0.7887	0.788619	0.788344	0.7881	0.788619	0.788344
0.7	1.3127	1.309808	1.309011	1.2733	1.322587	1.321782
0.5	1.6620	1.589124	1.587596	1.5680	1.609405	1.607857
0.3	1.9986	1.789103	1.785893	1.7895	1.815019	1.811763
0.2	2.0348	1.855014	1.849753	1.8625	1.882811	1.877470
0.1	2.6541	1.899797	1.888444	1.9068	1.928817	1.917290

Table 2 The four cases of the reported C_A tables by Mutalik et al. (1988)

The parameters	Case 1 Central	Case 2 Y_w/Y_e	Case 3 X_w/X_e	Case 4 Z_w/h
OCWL	1	0.25, 0.5, 1.0	0.5, 0.75, 1.0	0.1, 0.2, 0.3, 0.4, 0.5
x_e/y_e	1, 2, 5	1, 2, 5	1, 2, 5	1
PR	0.2, 0.4, 0.6, 0.8, 1.0	0.1, 0.3, 0.5, 0.7, 1.0	0.5	Not mentioned
L_D	1, 5, 10, 20, 50, 100	100	1, 2, 5, 10, 100	5
r_{wD}	Depends on the assumed input data (the discrepancy issue)			

is calculated by GPTM. The relative errors of the reported C_A tables are determined [i.e., $C_A\% \text{ Err} = ((C_A)_{\text{PTM}} - (C_A)_{\text{GPTM}}) / ((C_A)_{\text{GPTM}} \times 100)$] for the both assumed input data.

Tables 3 and 4 show the relative errors in the reported C_A tables by PTM for the variable r_{wD} and the constant r_{wD} , respectively. For the first assumed input data (variable r_{wD}), Table 3 shows that in $x_e/y_e = (1)$ and (2) , the C_A errors of any PR are in reverse order with L_D , while in $x_e/y_e = (5)$, they are in non-order with L_D . The maximum C_A error is (50%) at (PR = 1), ($L_D = 1$), and ($x_e/y_e = 2$). For the second assumed input data (constant $r_{wD} = 10^{-4}$), Table 4 shows that in $x_e/y_e = (1)$ and (2) , the C_A errors of any PR are in reverse order with L_D , while in $x_e/y_e = (5)$, they are in non-order with L_D . The maximum C_A error is (1215%) at (PR = 0.2), ($L_D = 1$), and ($x_e/y_e = 1$). For any assumed input data, $(T_{DA})_{\text{PSS}}$ by GPTM is noticeably close to its counterpart by PTM and both are independent of L_D .

The errors in the reported C_A tables are also found in the cases of off-center well location in x - and y -directions for both assumed input data but are not reported here.

To determine the consequences of the identified errors in the reported C_A tables of Mutalik et al. (1988) when they are used to estimate J_h , an example problem is applied. The reservoir of ($y_e = 933.33$ ft) and ($r_w = 0.225$ ft) is for the first assumed input data, and the reservoir of ($y_e = 2640$ ft) and ($r_{wD} = 10^{-4}$) is for the second assumed input data, and all are at ($k = 1$ md), ($\mu_o = 0.5$ cp), and ($B_o = 1.2$ rbbl/STB). The J_h is calculated by using Eq. (1), and the $C_A(s)$ by PTM and GPTM is applied by using Tables 3 and 4. For the first assumed input data, PR = (0.4), (1), and (0.8) for $x_e/y_e = (1)$, (2), and (5), respectively. For the second assumed input data, PR = (0.2) and (0.8) for ($x_e/y_e = 1$), and for $x_e/y_e = (2)$ and (5) they are not reported here. (OCWL = 1) and L_D is from (1) to (100). Figures 3 and 4 show a comparison of those $J_h(s)$ and $C_A(s)$ for the both assumed input data, respectively.

Figure 3 shows that the reported C_A tables of PTM lead to underestimation in the J_h for $x_e/y_e = (1)$ and (2) and lead to overestimation in the J_h for $x_e/y_e = (5)$. The relative error in the J_h for each selected $x_e/y_e = (1)$, (2), and (5) follows the same trend as those in C_A table. The maximum underestimation in J_h is (8.29%) corresponding to the error in C_A at ($x_e/y_e = 1$) and ($L_D = 1$), while the maximum overestimation in J_h is (1037%) corresponding to the error in C_A

at ($x_e/y_e = 5$) and ($L_D = 100$). For both $x_e/y_e = (1)$ and (2), although the maximum error in J_h (i.e., underestimation) at ($L_D = 1$), a rather unrealistic case in most real fields, from Fig. 3 the error in J_h can still reach (6.38%) underestimation for even a realistic configuration of L_D equal to (10). For ($x_e/y_e = 5$), the error in J_h (i.e., overestimation) is much more significant and starts from (46.43%) to (1037%) overestimation for $L_D = (10)$ to (100), respectively.

Figure 4 shows that the reported C_A tables of PTM lead to overestimation in the J_h for both PR = (0.2) and (0.8). The relative error in the J_h for each PR follows the same trend as those in C_A table. The maximum overestimation in J_h is (32%) corresponding to the error in C_A at ($L_D = 1$) and (PR = 0.2). Although the maximum error in J_h (i.e., overestimation) is at ($L_D = 1$), a rather unrealistic case in most real fields, from Fig. 4 the error in J_h can still reach (7%) overestimation for even a realistic configuration of L_D equal to (10).

The source of errors in the reported C_A tables

Given that both PTM and GPTM solutions are derived identically, except re-casting some variables into dimensionless forms in the latter model, the source of the errors in the reported C_A tables may be originated from the evaluation of the $P_D T$ which was not verified by Mutalik et al. (1988).

Figure 5 shows the effect of using FTLC = $(0-10^{-9})$ and $(0-10^{-5})$ on the C_A accuracy. GPTM is used to calculate C_A for square reservoir, ($r_{wD} = 10^{-4}$), (OCWL = 1), PR = (0.1), (0.5), and (1), and L_D from (1) to (10^3) . The maximum error in C_A is (3630%) which appears at the lowest PR (i.e., PR = 0.1) and L_D (i.e., $L_D = 1$). Notice that the error in C_A becomes small and not significant at ($L_D > 100$) and suggests that the FTLC of $(0-10^{-5})$ would be sufficient for C_A accuracy when ($L_D > 100$) for any well and reservoir configuration. This has also been shown previously in Table 1 for vertical fracture well case when ($L_D > 10^3$).

The pressure drop that occurs at the very early time in the $P_D T$ solution, especially when L_D is small (i.e., when the reservoir thickness is large), has significant effects on the C_A accuracy. The evaluation of $P_D T$ in that period requires

Table 3 Comparison between C_A and $(T_{DA})_{PSS}$ of this study and Mutalik et al. (1988) (when $r_{wD}=0.225$ ft and $y_e=933.33$ ft) (variable r_{wD})

PR=0.2, Central well, $x_{eD}/y_{eD}=1$, $r_{wD}=0.0012054$						PR=0.4, Central well, $x_{eD}/y_{eD}=1$, $r_{wD}=0.0006026$					
L_D	Mutalik et al. (1988)		This study		C_A % Err	L_D	Mutalik et al. (1988)		This study		C_A % Err
	C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$			C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$	
1	0.0163	0.4	0.0147	0.4075	10.8994	1	0.0043	0.4	0.0067	0.4025	36.28
5	0.9813	0.4	0.9608	0.4075	2.1390	5	0.703	0.4	0.7678	0.4025	8.44
10	1.4438	0.4	1.4286	0.4075	1.0640	10	1.1538	0.4	1.2235	0.4025	5.70
20	1.6917	0.4	1.6827	0.4075	0.5349	20	1.4595	0.4	1.4920	0.4025	2.18
50	1.8178	0.4	1.8138	0.4075	0.2205	50	1.6276	0.4	1.6419	0.4025	0.87
100	1.834	0.4	1.8445	0.4075	0.5693	100	1.6738	0.4	1.6811	0.4025	0.44
PR=0.6, Central well, $x_{eD}/y_{eD}=1$, $r_{wD}=0.000401787$						PR=0.8, Central well, $x_{eD}/y_{eD}=1$, $r_{wD}=0.00030134$					
L_D	Mutalik et al. (1988)		This study		C_A % Err	L_D	Mutalik et al. (1988)		This study		C_A % Err
	C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$			C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$	
1	0.0034	0.4	0.0039	0.3944	11.92	1	0.002	0.4	0.0023	0.3911	12.10
5	0.6268	0.4	0.6075	0.3944	3.17	5	0.4406	0.4	0.4514	0.3911	2.38
10	1.0241	0.4	1.0082	0.3944	1.58	10	0.7616	0.4	0.7709	0.3911	1.21
20	1.2644	0.4	1.2546	0.3944	0.78	20	0.9673	0.4	0.9732	0.3911	0.60
50	1.402	0.4	1.3975	0.3944	0.32	50	1.0909	0.4	1.0934	0.3911	0.23
100	1.439	0.4	1.4367	0.3944	0.16	100	1.126	0.4	1.1273	0.3911	0.12
PR=1, Central well, $x_{eD}/y_{eD}=1$, $r_{wD}=0.000241072$						PR=0.2, Central well, $x_{eD}/y_{eD}=2$, $r_{wD}=0.0006027$					
L_D	Mutalik et al. (1988)		This study		C_A % Err	L_D	Mutalik et al. (1988)		This study		C_A % Err
	C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$			C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$	
1	0.00085	0.4	0.0012	0.3924	28.79	1	0.0044	0.6	0.0054	0.7396	17.86
5	0.2773	0.4	0.2968	0.3924	6.56	5	0.5869	0.6	0.6095	0.7396	3.71
10	0.501	0.4	0.5183	0.3924	3.34	10	0.9531	0.6	0.9714	0.7396	1.88
20	0.6506	0.4	0.6617	0.3924	1.67	20	1.1734	0.6	1.1845	0.7396	0.93
50	0.7435	0.4	0.7485	0.3924	0.66	50	1.2986	0.6	1.3035	0.7396	0.37
100	0.771	0.4	0.7734	0.3924	0.31	100	1.3322	0.6	1.3346	0.7396	0.18
PR=0.4, Central well, $x_{eD}/y_{eD}=2$, $r_{wD}=0.00030134$						PR=0.6, Central well, $x_{eD}/y_{eD}=2$, $r_{wD}=0.000200894$					
L_D	Mutalik et al. (1988)		This study		C_A % Err	L_D	Mutalik et al. (1988)		This study		C_A % Err
	C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$			C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$	
1	0.0033	0.6	0.0026	0.708	25.43	1	0.0013	0.6	0.0016	0.6296	18.82
5	0.544	0.6	0.5212	0.708	4.37	5	0.423	0.6	0.4388	0.6296	3.60
10	0.9095	0.6	0.8902	0.708	2.17	10	0.7664	0.6	0.7805	0.6296	1.81
20	1.1359	0.6	1.1238	0.708	1.08	20	0.9963	0.6	1.0055	0.6296	0.91
50	1.2686	0.6	1.2627	0.708	0.47	50	1.1394	0.6	1.1436	0.6296	0.36
100	1.3046	0.6	1.3018	0.708	0.22	100	1.1817	0.6	1.1838	0.6296	0.18
PR=0.8, Central well, $x_{eD}/y_{eD}=2$, $r_{wD}=0.00015067$						PR=1, Central well, $x_{eD}/y_{eD}=2$, $r_{wD}=0.000120536$					
L_D	Mutalik et al. (1988)		This Study		C_A % Err	L_D	Mutalik et al. (1988)		This study		C_A % Err
	C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$			C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$	
1	0.00061	0.6	0.0010	0.6077	37.08	1	0.00025	0.2	0.0005	0.2138	50.19
5	0.3051	0.6	0.3291	0.0724	7.30	5	0.19	0.2	0.2181	0.2046	12.87
10	0.58525	0.6	0.6025	0.0724	2.86	10	0.381	0.2	0.4082	0.2046	6.66
20	0.7825	0.6	0.7874	0.0724	0.62	20	0.5212	0.2	0.5395	0.2046	3.39
50	0.9109	0.6	0.9033	0.0724	0.84	50	0.6145	0.2	0.6230	0.2046	1.37
100	0.95	0.6	0.9378	0.0724	1.31	100	0.6438	0.2	0.6482	0.2046	0.69

Table 3 (continued)

PR=0.2, Central well, $x_{eD}/y_{eD}=5$, $r_{wD}=0.000241072$						PR=0.4, Central well, $x_{eD}/y_{eD}=5$, $r_{wD}=0.000120536$					
L_D	Mutalik et al. (1988)		This study		C_A % Err	L_D	Mutalik et al. (1988)		This study		C_A % Err
	C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$			C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$	
1	0.0002	2	0.0003	2.029	39.65	1	0.0003	2	0.0003	1.957	0.82
5	0.0859	2	0.0785	2.029	9.44	5	0.149	2	0.1251	1.957	19.13
10	0.1606	2	0.1371	2.029	17.16	10	0.285	2	0.2341	1.957	21.73
20	0.212	2	0.1750	2.029	21.15	20	0.3822	2	0.3094	1.957	23.53
50	0.2442	2	0.1979	2.029	23.37	50	0.4446	2	0.3573	1.957	24.42
100	0.2503	2	0.2045	2.029	22.38	100	0.4636	2	0.3718	1.957	24.69
PR=0.6, Central well, $x_{eD}/y_{eD}=5$, $r_{wD}=0.00008035$						PR=0.8, Central well, $x_{eD}/y_{eD}=5$, $r_{wD}=0.000060268$					
L_D	Mutalik et al. (1988)		This study		C_A % Err	L_D	Mutalik et al. (1988)		This study		C_A % Err
	C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$			C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$	
1	0.00019	2	0.0003	1.75	34.71	1	0.0003	2	0.0002	1.7030	43.92
5	0.1869	2	0.1701	1.75	9.89	5	0.2111	2	0.1617	1.7030	30.55
10	0.3876	2	0.3315	1.75	16.91	10	0.4144	2	0.3244	1.7030	27.74
20	0.5412	2	0.4471	1.75	21.05	20	0.5609	2	0.4439	1.7030	26.37
50	0.645	2	0.5227	1.75	23.40	50	0.6571	2	0.5234	1.7030	25.55
100	0.6782	2	0.5460	1.75	24.20	100	0.687	2	0.5483	1.7030	25.29
PR=1, Central well, $x_{eD}/y_{eD}=5$, $r_{wD}=0.0000482$											
L_D	Mutalik et al. (1988)		This study		C_A % Err	L_D	Mutalik et al. (1988)		This study		C_A % Err
	C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$			C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$	
1	0.00011	0.4	0.0001	0.4832	19.72						
5	0.1241	0.6	0.0994	0.0748	24.81						
10	0.2548	0.6	0.2040	0.0748	24.92						
20	0.3527	0.6	0.2822	0.0748	24.98						
50	0.4187	0.6	0.3350	0.0748	24.98						
100	0.4397	0.6	0.3518	0.0748	19.81						

to use a much shorter FTLC [i.e., $FTLC = (0-10^{-9})$] to gain sufficient accuracy.

Due to the absence of the information about the selected FTLC in the calculation of the reported C_A tables by PTM, C_A tables have errors in reverse order with L_D as shown previously for $x_e/y_e = (1)$ and (2) in Tables 3 and 4. Therefore, the source of errors in the reported C_A tables by PTM could be originated and justified from the using of long FTLC in the $P_D T$ evaluation. About $(x_e/y_e = 5)$, no analysis can be achieved.

Additional characteristics of PTM and GPTM

Figure 6 shows the effect of OCWL in the Z-direction on C_A for square reservoir, ($r_{wD} = 10^{-4}$) and PR is equal to (0.1) and (0.9). The effect of z_{wD} (i.e., $z_D = z_{wD}$) on the C_A that is

calculated by GPTM becomes insignificant when ($L_D > 35$) (C_A % error $< 5\%$) which is inconsistent with their C_A counterparts by PTM [i.e., Fig. 12 in Mutalik et al. (1988) paper] because z_{wD} will be insignificant when ($L_D > 5$). Therefore, z_{wD} should be considered as a major parameter for any correlation.

Table 5 which is presented at the end of the paper shows the effect of the wellbore radius on C_A for square reservoir and PR=(0.1) and (1). When r_{wD} increases two times (i.e., from (5×10^{-5}) to (10^{-4})), a significant error occurs when ($L_D < 15$), and when r_{wD} increases (10) to (20) times [i.e., from (5×10^{-4}) to (10^{-3})], a significant error occurs when ($L_D < 50$). This is also inconsistent with their C_A counterparts by PTM (i.e., table (9) in Mutalik et al. (1988) paper), the significant error will occur when ($L_D < 3$) if r_{wD} increases 10 times. Therefore, r_{wD} should be considered as a major parameter for any correlation.

Table 4 Comparison between C_A and $(T_{DA})_{PSS}$ of this study and Mutalik et al. (1988) (when constant $r_{wD} = 10^{-4}$)

PR=0.2, Central well, $x_{eD}/y_{eD} = 1$, $r_{wD} = 10^{-4}$						PR=0.4, Central well, $x_{eD}/y_{eD} = 1$, $r_{wD} = 10^{-4}$					
L_D	Mutalik et al. (1988)		This study		C_A % Err	L_D	Mutalik et al. (1988)		This study		C_A % Err
	C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$			C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$	
1	0.0163	0.4	0.0012	0.4075	1215.58	1	0.0043	0.4	0.0011	0.4025	284.27
5	0.9813	0.4	0.5858	0.4075	67.50	5	0.703	0.4	0.5361	0.4025	31.14
10	1.4438	0.4	1.1155	0.4075	29.43	10	1.1538	0.4	1.0224	0.4025	12.86
20	1.6917	0.4	1.4869	0.4075	13.77	20	1.4595	0.4	1.3638	0.4025	7.02
50	1.8178	0.4	1.7261	0.4075	5.31	50	1.6276	0.4	1.5839	0.4025	2.76
100	1.834	0.4	1.7990	0.4075	1.95	100	1.6738	0.4	1.6511	0.4025	1.37
PR=0.6, Central well, $x_{eD}/y_{eD} = 1$, $r_{wD} = 10^{-4}$						PR=0.8, Central well, $x_{eD}/y_{eD} = 1$, $r_{wD} = 10^{-4}$					
L_D	Mutalik et al. (1988)		This study		C_A % Err	L_D	Mutalik et al. (1988)		This study		C_A % Err
	C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$			C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$	
1	0.0034	0.4	0.0010	0.3944	253.80	1	0.002	0.4	0.0008	0.3911	164.90
5	0.6268	0.4	0.4600	0.3944	36.26	5	0.4406	0.4	0.3620	0.3911	21.71
10	1.0241	0.4	0.8773	0.3944	16.73	10	0.7616	0.4	0.6904	0.3911	10.32
20	1.2644	0.4	1.1703	0.3944	8.04	20	0.9673	0.4	0.9209	0.3911	5.03
50	1.402	0.4	1.3592	0.3944	3.15	50	1.0909	0.4	1.0696	0.3911	1.99
100	1.439	0.4	1.4168	0.3944	1.57	100	1.126	0.4	1.1149	0.3911	0.99
PR=1, Central well, $x_{eD}/y_{eD} = 1$, $r_{wD} = 10^{-4}$						PR=0.2, Central well, $x_{eD}/y_{eD} = 2$, $r_{wD} = 10^{-4}$					
L_D	Mutalik et al. (1988)		This study		C_A % Err	L_D	Mutalik et al. (1988)		This study		C_A % Err
	C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$			C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$	
1	0.00085	0.4	0.0005	0.3924	71.72	1	0.0044	0.6	0.0009	0.7396	396.05
5	0.2773	0.4	0.2489	0.3924	11.41	5	0.5869	0.6	0.4254	0.7396	37.96
10	0.501	0.4	0.4747	0.3924	5.55	10	0.9531	0.6	0.8115	0.7396	17.45
20	0.6506	0.4	0.6332	0.3924	2.75	20	1.1734	0.6	1.0826	0.7396	8.39
50	0.7435	0.4	0.7354	0.3924	1.10	50	1.2986	0.6	1.2574	0.7396	3.28
100	0.771	0.4	0.7666	0.3924	0.58	100	1.3322	0.6	1.3108	0.7396	1.64
PR=0.4, Central well, $x_{eD}/y_{eD} = 2$, $r_{wD} = 10^{-4}$						PR=0.6, Central well, $x_{eD}/y_{eD} = 2$, $r_{wD} = 10^{-4}$					
L_D	Mutalik et al. (1988)		This study		C_A % Err	L_D	Mutalik et al. (1988)		This study		C_A % Err
	C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$			C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$	
1	0.0033	0.6	0.0009	0.708	278.01	1	0.0013	0.6	0.0008	0.6296	63.11
5	0.544	0.6	0.4180	0.708	30.14	5	0.423	0.6	0.3817	0.6296	10.83
10	0.9095	0.6	0.7972	0.708	14.08	10	0.7664	0.6	0.7279	0.6296	5.29
20	1.1359	0.6	1.0635	0.708	6.81	20	0.9963	0.6	0.9710	0.6296	2.61
50	1.2686	0.6	1.2351	0.708	2.71	50	1.1394	0.6	1.1277	0.6296	1.04
100	1.3046	0.6	1.2875	0.708	1.33	100	1.1817	0.6	1.1755	0.6296	0.52
PR=0.8, Central well, $x_{eD}/y_{eD} = 2$, $r_{wD} = 10^{-4}$						PR=1, Central well, $x_{eD}/y_{eD} = 2$, $r_{wD} = 10^{-4}$					
L_D	Mutalik et al. (1988)		This study		C_A % Err	L_D	Mutalik et al. (1988)		This study		C_A % Err
	C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$			C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$	
1	0.00061	0.6	0.0006	0.6077	-5.13	1	0.00025	0.2	0.0004	0.2138	-39.90
5	0.3051	0.6	0.3087	0.6077	-1.15	5	0.19	0.2	0.2101	0.2046	-9.56
10	0.58525	0.6	0.5887	0.6077	-0.58	10	0.381	0.2	0.4006	0.2046	-4.90
20	0.7825	0.6	0.7852	0.6077	-0.35	20	0.5212	0.2	0.5344	0.2046	-2.48
50	0.9109	0.6	0.9120	0.6077	-0.12	50	0.6145	0.2	0.6207	0.2046	-1.00
100	0.95	0.6	0.9507	0.6077	-0.07	100	0.6438	0.2	0.6470	0.2046	-0.50

Table 4 (continued)

PR=0.2, Central well, $x_{eD}/y_{eD}=5$, $r_{wD}=10^{-4}$						PR=0.4, Central well, $x_{eD}/y_{eD}=5$, $r_{wD}=10^{-4}$					
L_D	Mutalik et al. (1988)		This study		C_A % Err	L_D	Mutalik et al. (1988)		This study		C_A % Err
	C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$			C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$	
1	0.0002	2	0.0001	2.029	44.93	1	0.0003	2	0.0003	1.957	19.52
5	0.0859	2	0.0658	2.029	30.49	5	0.149	2	0.1205	1.957	23.66
10	0.1606	2	0.1255	2.029	27.93	10	0.285	2	0.2298	1.957	24.03
20	0.212	2	0.1675	2.029	26.60	20	0.3822	2	0.3065	1.957	24.69
50	0.2442	2	0.1945	2.029	25.56	50	0.4446	2	0.3560	1.957	24.89
100	0.2503	2	0.2027	2.029	23.46	100	0.4636	2	0.3711	1.957	24.93
PR=0.6, Central well, $x_{eD}/y_{eD}=5$, $r_{wD}=10^{-4}$						PR=0.8, Central well, $x_{eD}/y_{eD}=5$, $r_{wD}=10^{-4}$					
L_D	Mutalik et al. (1988)		This study		C_A % Err	L_D	Mutalik et al. (1988)		This study		C_A % Err
	C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$			C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$	
1	0.00019	2	0.0004	1.75	-47.51	1	0.0003	2	0.0003	1.703	-13.29
5	0.1869	2	0.1777	1.75	5.19	5	0.2111	2	0.1789	1.703	17.97
10	0.3876	2	0.3389	1.75	14.39	10	0.4144	2	0.3413	1.703	21.43
20	0.5412	2	0.4520	1.75	19.73	20	0.5609	2	0.4552	1.703	23.21
50	0.645	2	0.5250	1.75	22.86	50	0.6571	2	0.5287	1.703	24.28
100	0.6782	2	0.5472	1.75	23.93	100	0.687	2	0.5511	1.703	24.65
PR=1, Central well, $x_{eD}/y_{eD}=5$, $r_{wD}=10^{-4}$											
L_D	Mutalik et al. (1988)		This study		C_A % Err	L_D	Mutalik et al. (1988)		This study		C_A % Err
	C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$			C_A	$(T_{DA})_{PSS}$	C_A	$(T_{DA})_{PSS}$	
1	0.00011	0.4	0.0002	0.4832	-42.41						
5	0.1241	0.6	0.1150	0.07477	7.87						
10	0.2548	0.6	0.2194	0.07477	16.13						
20	0.3527	0.6	0.2927	0.07477	20.50						
50	0.4187	0.6	0.3399	0.07477	23.17						
100	0.4397	0.6	0.3543	0.07477	24.09

Fig. 3 The J_h comparison by using the reported C_A tables and this study C_A tables (variable r_{wD})

Fig. 4 The J_h comparison by using the reported C_A tables and this study C_A tables (constant $r_{wD}=10^{-4}$)

Fig. 5 Effect of FTLC=($0-10^{-9}$) and ($0-10^{-5}$) on the C_A accuracy

Conclusions

1. The PTM was improved to GPTM and verified by converting several dimensional parameters to dimensionless forms.
2. An efficient algorithm was developed, and accurate C_A and J_h were obtained for any well and reservoir configuration.
3. The accuracy of the reported C_A tables was quantified, and the consequences were determined when they were used to estimate J_h .
4. New C_A tables and their $(T_{DA})_{PSS}$ tables were generated for a wider range of well and reservoir configurations. The tables are provided in the Supplementary Material.
5. Because C_A depends on the all-configuration parameters, unlimited tables of C_A can be generated; therefore, the MATLAB code was provided.
6. Although horizontal wells usually have been drilled with high L_D industrially (i.e., $L_D > 10$), it is still a significant error in J_h by ($< 7\%$) that has been determined when using the reported C_A of PTM for $(x_e/y_e)=(1)$ and (2) in the both assumed input data, while for $(x_e/y_e=5)$, a significant error in J_h starts from (46.43%) to (1037%) for $L_D=(10)$ to (100) , respectively, in the first assumed

Fig. 6 The effect of OCWL in the Z-direction on C_A **Table 5** The effect of r_{wD} on C_A

PR=0.1, Central well, $x_e/y_e=1$						PR=1, Central well, $x_e/y_e=1$					
r_{wD}	C_A				$(T_{DA})_{PSS}$	r_{wD}	C_A				$(T_{DA})_{PSS}$
L_D	5×10^{-5}	1×10^{-4}	5×10^{-4}	1×10^{-3}		L_D	5×10^{-5}	1×10^{-4}	5×10^{-4}	1×10^{-3}	
1	0.000598	0.001146	0.006227	0.012452	0.4091	1	0.000248	0.000495	0.002476	0.004952	0.3924
3	0.182649	0.226932	0.398949	0.502599	0.4091	3	0.077276	0.097352	0.16647	0.20974	0.3924
5	0.514803	0.58642	0.822663	0.944943	0.4091	5	0.216684	0.248891	0.343403	0.394468	0.3924
7	0.782982	0.859325	1.094378	1.208252	0.4091	7	0.328789	0.362998	0.456834	0.50439	0.3924
10	1.056593	1.127695	1.335671	1.431512	0.4091	10	0.442901	0.474676	0.557565	0.597591	0.3924
15	1.31725	1.375702	1.540035	1.612867	0.4091	15	0.551404	0.577472	0.642882	0.673298	0.3924
20	1.462146	1.510542	1.643952	1.701941	0.4091	20	0.611638	0.633198	0.686265	0.710482	0.3924
25	1.552613	1.593591	1.705232	1.753204	0.4091	25	0.649214	0.667458	0.711848	0.731882	0.3924
35	1.657668	1.688803	1.772501	1.808014	0.4091	35	0.692815	0.706668	0.739932	0.754763	0.3924
50	1.735952	1.758712	1.8193	1.844797	0.4091	50	0.725277	0.735398	0.75947	0.770118	0.3924
75	1.794858	1.810513	1.851889	1.869246	0.4091	75	0.749682	0.756641	0.773076	0.780325	0.3924
100	1.822908	1.83482	1.866212	1.879416	0.4091	100	0.761294	0.766588	0.779056	0.78457	0.3924

input data (i.e., variable r_{wD}) and expected the same in the second assumed input data (i.e., constant r_{wD}).

$$P_D T = \frac{0.007078 k h \Delta P}{q_w B_o \mu_o} \quad (9)$$

Appendix 1

$$t_D = \frac{0.000264 k t}{\phi \mu_o c_t (L/2)^2} \quad (10)$$

Summary of pressure transient model of Mutalik et al. (1988)

$$T_{DA} = \frac{0.000264 k t}{\phi \mu_o c_t 4 x_e y_e} \quad (11)$$

The mathematical expression of the dimensionless parameters of PTM (Eq. (3)) and the geometrical figure of PTM is given as follows:

$$x_D = \frac{2x}{L} \quad (12)$$

$$y_D = \frac{2y}{L} = r_{wD} = \frac{2r_w}{L}$$

$$z_D = \frac{z}{h}$$

$$x_{wD} = \frac{2x_w}{L}$$

$$y_{wD} = \frac{2y_w}{L}$$

$$z_{wD} = \frac{z_w}{h}$$

$$(13) \quad T_{DA} = \frac{0.000264k_{eq}t}{\phi\mu_o c_t 4x_e y_e} \quad (22)$$

$$(14) \quad x_D' = \frac{2x}{L} \sqrt{\frac{k_{eq}}{k_x}} \quad (23)$$

$$(15) \quad y_D' = \frac{2y}{L} \sqrt{\frac{k_{eq}}{k_y}} = r_{wD}' = \frac{2r_w}{L} \sqrt{\frac{k_{eq}}{k_y}} \quad (24)$$

$$(16) \quad L_D' = \frac{L}{2h} \sqrt{\frac{k_z}{k_{eq}}} \quad (25)$$

Geometrical Figure of PTM

(A-10) (18)

Appendix 2

The dimensionless parameters in anisotropic reservoir

The mathematical expression of the dimensionless parameters for anisotropic reservoir by Ozkan et al. (1987), Ozkan and Raghavan (1988) and Besson (1990) in (') symbol is given as follows:

$$\bar{k} = k_{eq} = \sqrt[3]{k_x k_y k_z} \quad (19)$$

$$P_D T' = \frac{0.007078 k_{eq} h \Delta P}{q_w B_o \mu_o} \quad (20)$$

$$t_D' = \frac{0.000264 k_{eq} t}{\phi \mu_o c_t (L/2)^2} \quad (21)$$

$$Z_D' = \frac{z}{h} \quad (26)$$

$$x_{wD}' = \frac{2x_w}{L} \sqrt{\frac{k_{eq}}{k_x}} \quad (27)$$

$$y_{wD}' = L \frac{2y_w}{L} \sqrt{\frac{k_{eq}}{k_y}} \quad (28)$$

$$z_{wD}' = \frac{z_w}{h} \quad (29)$$

Appendix 3

The MATLAB code

```

% SHAPE FACTORS(CA) COMPUTATIONS
% PD and TDA Calculation for Rectangular Bounded Reservoir
% THE GENERALIZED SOLUTION TO MUTALIK, P.N., GODBOLE, AND JOSHI, S.D. 'S, (1988) MODEL
clear
clc
kx=1; ky=1; kz=1; kh=(kx*ky)^0.5; keq=(kx*ky*kz)^(1/3);
PR=0.2; XeD=(1/PR)*((keq/kx)^0.5); YeD = (XeD/1)*((keq/ky)^0.5); XWD=1*XeD *((keq/kx)^0.5);
YWD=1*YeD *((keq/ky)^0.5); LD=1*((kz/keq)^0.5); S=0; A=0; slope=100; slopeTD=100; slopeb=10;
TDA= 0; PDT=0; XD=0.732; YD=0.0001 *((keq/ky)^0.5); ZWD=0.5; ZD=ZWD; INC=1*10^-
9; PDF=0; diff=0; TD=-10; PD=0;
T=[0.9879925; -0.9879925; 0.9372734; -0.9372734; 0.8482066; -0.8482066; 0.7244177; -0.7244177;
0.5709722; -0.5709722; 0.3941513; -0.3941513; 0.2011941; -0.2011941; 0];
U=[0.0307532; 0.0307532; 0.0703660; 0.0703660; 0.1071592; 0.1071592; 0.1395707; 0.1395707;
0.1662692; 0.1662692; 0.1861610; 0.1861610; 0.1984315; 0.1984315; 0.2025782];
while round(slopeb*10^8) ~= round(slope*10^8)
TDA=INC+TDA;
SUM1=0; SUM2=0; SUM3=0; INP1=0; INP2=0; INP3=0; ZZ=0; M=0;
A=(TDA-INC);
M=(TDA-A)/2; C=(A+TDA)/2;
for LL= 1:15
TDi=(M*T(LL))+C; SUM1=0; SUM2=0; SUM3=0;
% CALCULATION OF THE FIRST SUM. TERM , X-direction
RA1=0; RA2=0; RA3=0; PP=0; V1=0; V2=0; V3=0; SUM1=0; NN=0; P1=100;
while P1 ~= 0
NN = NN+1;
RA1=(NN*pi)/(2*XeD); RA2=(NN*pi*XWD)/(2*XeD); RA3=(NN*pi*(XWD+XD))/(2*XeD);
PP=((NN^2)*(pi^2)*YeD*TDi)/XeD;
P1=1/exp(PP); V1=sin(RA1); V2=cos(RA2); V3=cos(RA3);
SUM1=SUM1+(P1*V1*V2*V3/NN);
end
% CALCULATION OF THE SECOND SUM. TERM , Y-direction
RA11=0; RA22=0; PP2=0; V11=0; V22=0; SUM2=0; MM = 0; P2=100;
while P2 ~= 0
MM = MM+1;
RA11=(MM*pi*YWD)/(2*YeD); RA22=(MM*pi*(YWD+YD))/(2*YeD);
PP2=((MM^2)*(pi^2)*XeD*TDi)/YeD;
P2=1/exp(PP2); V11=cos(RA11); V22=cos(RA22);
SUM2=SUM2+(P2*V11*V22);
end
% REM*CALCULATION OF THE THIRD SUM. TERM , Z-direction
RA111=0; RA222=0; P3=100; PP3=0; V111=0; V222=0; SUM3=0; GG=0;
while P3 ~= 0
GG=GG+1;
RA111=(GG*pi*ZWD); PP3=(GG^2)*(pi^2)*4*XeD*YeD*(LD^2)*TDi;
P3=1/exp(PP3); V111=(cos(RA111))^2;
SUM3=SUM3+(P3*V111);
end
INP1=INP1+(U(LL)*(1+(4*XeD*SUM1/pi)))*(1+(2*SUM2))*(1+(2*SUM3));
end
PDF=PDT; TD=4*XeD*YeD*TDA; TDINC=4*XeD*YeD*INC;
PD=(INP1*M*pi^2); PDT=PDT+PD; slopeb=slope;
CA=(2.2458*XeD*YeD)*(exp((4*pi*TDA)-(2*PDT)));
SCA=log((30.8828/CA)^0.5); slope=(PDT-PDF)/INC; diff=(PDT-PDF); slopeTD=(PDT-PDF)/TDINC;
TDA,
TD,
PD,
PDT,
CA,
SCA,
slopeb,
slope,
if TDA>10^-5
INC=10^-8;
end
if TDA>10^-4
INC=10^-7;
end
if TDA>10^-3
INC=10^-6;
end
if TDA>10^-2
INC=10^-5;
end
if TDA>10^-1
INC=10^-4;
end
if TDA>1
INC=10^-3;
end
end
end

```

Supplementary Information The online version contains supplementary material available at <https://doi.org/10.1007/s13202-021-01339-3>.

Funding No funding supported this study.

Declarations

Conflict of interest The authors declare no conflict of interest.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

References

- Adesina FAS, Churchill A, Olugbenga F (2011) Modeling productivity index for long horizontal well. *J Energy Resour Technol*, 133(3)
- Ahmadi MA, Soleimani R, Lee M, Kashiwao TB (2015) Determination of oil well production performance using artificial neural network (ANN) linked to the particle swarm optimization (PSO) tool. *Petroleum* 1:118–132
- Bahadori A, Jamili A, Zendheboudi S (2013) Calculating pseudo-steady-state horizontal oil well productivity in rectangular drainage areas using a simple method. *Chem Eng Commun* 200(2):222–234
- Bahadori A (2012) Prediction of skin factor and pseudo-steady state horizontal wells productivity for various drainage areas. In: Presented at SPE 2012 energy conference and exhibition, Port of Spain, Trinidad. <https://doi.org/10.2118/156278-MS>
- Besson J (1990) Performance of slanted and horizontal wells on an anisotropic medium. In: Presented at Europec 90, Hague. <https://doi.org/10.2118/20965-MS>
- Brown GA, Tiwari P (2010) Using DTS flow measurements below electrical submersible pumps to optimize production from depleted reservoirs by changing injection support around the well. In: Presented at SPE annual technical conference and exhibition, Florence. <https://doi.org/10.2118/135130-MS>
- Cetkovic I, Shammari M, Sager T (2016) A methodology for multilateral wells optimization—field case study. In: Abu Dhabi international petroleum exhibition & conference
- Cho H (2003) Integrated optimization on a long horizontal well length. *SPE Reserv Eval Eng* 6(02):81–88
- Duda JR, Salamy SP, Aminian K, Ameri S (1991) Pressure analysis of an unstimulated horizontal well with type curves. *J Petrol Technol* 43(08):988–993
- Earlougher RCJ (1977) Advances in well test analysis. Society of Petroleum Engineers, Richardson, Texas, USA
- Economides M, Deimbachor FX, Brand CW, Heinemann ZE (1991a) Comprehensive-simulation of horizontal-well performance. *SPE Form Eval* 6(04):418–426
- Economides MJ, Naceur KB, Klem RC (1991b) Matrix stimulation method for horizontal wells. *J Petrol Technol* 43(07):854–861
- Frick TP, Economides MJ (1993) Horizontal well damage characterization and removal. *SPE Prod Facil* 8(01):15–22
- Gringarten AC, Ramey HJ (1973) The use of source and green's functions in solving unsteady-flow problems in reservoirs. *Soc Petrol Eng J*. <https://doi.org/10.2118/3818-PA>
- Gringarten AC, Ramey HJ, Raghavan R (1974) Unsteady-state pressure distributions created by a well with a single infinite-conductivity vertical fracture. *Soc Petrol Eng J*. <https://doi.org/10.2118/4051-PA>
- Hagoort J (2009) The productivity of a well with a vertical infinite-conductivity fracture in a rectangular closed reservoir. *Soc Petrol Eng J*. <https://doi.org/10.2118/112975-PA>
- Haidar A, Vafin I, Borisov G, Dzhabrailov A, Nikitin A (2008) Formation hydrofracturing experience on horizontal wells of N@N NK rosneft. In: Presented at SPE 2008 Russian oil & gas technical conference and exhibition, Moscow. <https://doi.org/10.2118/117418-MS>
- Huebsch H, Moss M, Trilsbeck T, Brown G, Rogers S, Bouchard T (2008) Monitoring inflow distribution in multi-zone, velocity string gas wells using slickline deployed fiber optic distributed temperature measurements. In: Presented at SPE 2008 annual technical conference and exhibition, Denver, Colorado. <https://doi.org/10.2118/115816-MS>
- Jackson GT, Balhoff MT, Huh C, Delshad M (2011) CFD-based representation of non-Newtonian polymer injectivity for a horizontal well with coupled formation-wellbore hydraulics. *J Petrol Sci Eng* 78(1):86–95
- Krawchuk P, Beshry MA, Brown GA, Brough B (2016) Predicting the flow distribution on total E&P Canada's Joslyn Project horizontal SAGD producing wells using permanently installed fiber-optic monitoring. In: Presented at SPE 2006 annual technical conference, San Antonio, Texas. <https://doi.org/10.2118/102159-MS>
- Leone A, Galli G, Drago A, Brown G (2015) Characterizing reservoir thermofacies by using distributed temperature sensing measurements. In: Presented at SPE annual technical conference and exhibition, Houston, Texas. <https://doi.org/10.2118/175080-MS>
- Luo W, Tang C, Feng Y (2018) A semianalytical model for horizontal-well productivity with pressure drop along the wellbore. *SPE J* 23(05):1603–1614
- Mutalik PN, Godbole SP, Joshi SD (1988) Effect of drainage area shapes on the productivity of horizontal wells. In: Presented at SPE 63rd annual technical conference and exhibition, Houston, Texas. <https://doi.org/10.2118/18301-MS>
- Ouyang L-B (2015) Assessing the needs to incorporate completion details in a petroleum reservoir simulation model. *Open Petrol Eng J* 8:16–28
- Ozkan E, Raghavan R (1988) *Some new solutions to solve problems in well test analysis: Part 1 - Analytical considerations*. Paper Soc Petrol Eng 18615
- Ozkan E, Raghavan R, Joshi SD (1988) Horizontal well pressure analysis. In: Presented at SPE California regional meeting, Ventura, California. <https://doi.org/10.2118/16378-MS>
- Penmatcha VR, Arbabi S, Aziz K (1999) Effects of pressure drop in horizontal wells and optimum well length. *SPE J* 4(03):215–223
- Pinzon ID, Davies JE, Mammadkhan F, Brown GA (2007) Monitoring production from gravel-packed sand-screen completions on BP's azeri field wells using permanently installed distributed temperature sensors. In: Presented at SPE 2007 annual technical conference and exhibition, Anaheim, California. <https://doi.org/10.2118/110064-MS>
- Saavedra NF, Joshi SD (2002) Application of horizontal well technology in Colombia. *J Can Petrol Technol* 41(03)
- Thomas LK, Todd BJ, Evans CE, Pierson RG (1998) Horizontal well IPR calculations. *SPE Reserv Eval Eng* 1(05):392–399

- Wood P, Simpson A, Holland B, Brown G, Rogers G, Anderson S, Balster E, Figueroa M (2010) Monitoring flow and completion integrity of a north sea subsea HPHT appraisal well during an extended well test using permanently installed fiber-optic temperature sensors. In: Presented at SPE deep water drilling and completions conference, Galveston, Texas. <https://doi.org/10.2118/137120-MS>
- Wu M, Luo W, Wang X (2018) A new algorithm for computation of horizontal-well pressure in Laplace domain. *Adv Geo-Energy Res* 2(4):393–403
- Zhang L, Zheng W, Zhu G (2020) Analysis of initial productivity evaluation method for horizontal well in offshore block oilfield. Springer, Singapore, pp. 1–13

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.