Production optimization of a network of multiple wells with each well using a combination of Electrical Submersible Pump and Gas lift

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Abstract

Artificial lift methods such as ESP and GL are commonly used in oil wells around the world, especially in offshore wells. However, these two methods are normally used separately, and this paper therefore aimed to study the possible combination of ESP and GL by analyzing its effects on energy saving using equivalent depth method and on production rate as well as on ESP life cycle using nodal analysis. The paper also performed the production optimization for a network of wells using each well a combination of GL and ESP. The optimization process consists of selecting the appropriate operation frequency for the ESP system and the injection gas lift distributed to each well with the aim of maximizing the total production of the network. In addition, this optimization process was conducted in two cases: unlimited and limited volume of injection gas lift. In case the GL flow is limited, the BST (Binary Search Tree) algorithm was used to determine the suitable gas rates injected into each well to maximize the total network production. The optimization workflow proposed in this study was applied to the field X in Cuu Long basin of Vietnam and was calibrated from the real data of this field. The results demonstrated the advantage of the combination of ESP and GL in energy saving and in application for small diameter wells. In addition, the workflow and source code will allow engineers to replicate the results and to apply this method for future studies in order to determine optimum operating parameters of this hybrid artificial lift to achieve the highest production rate from a network of multiple wells.

Keywords Electrical Submersible Pump · Equivalent depth · Gas lift · Nodal analysis · Optimization

Abbreviations

GL	Gas lift
ESP	Electric submersible pump
IPR	Inflow performance relationship
VLP	Vertical lift performance relationship
BST	Binary search tree
VSD	Variable speed drive
VFG	Variable frequency drive
FBHP	Flow bottomhole pressure
PVT	Pressure volume temperature
LH	Left higher
EH	Equal height
RH	Right higher
AVL	G.M.Adel'son-Vel'skii and E.M.Landis [15]

Nomenclature

Q	Total fluid flow rate, STB/day
GLR	Gas liquid ratio, scf/STB
P_t	Wellhead pressure, psia
P_{wf}	Bottom-hole pressure, psia
HP	motor power, Hp
f	frequency operation, Hz
k	number of wells in the network
$P_{discharge}$	Pressure pump discharge, psia
P_{intake}	Pressure pump intake, psia
Q_{g_av}	Gas lift rate available, MMscf/day
$q_{g_inj_maxoil}$	Gas lift injection rate for maximum production oil, MMscf/day
q_{inj}	Gas lift injection rate, MMscf/day
Δq_{inj}	Variation of injection gas rate, MMscf/day
$q_{o_initial}$	Oil flow rate without GL, STB/day
Q_o	Total production oil rate, STB/day
GOR	Gas oil ratio, scf/STB
PI	Productivity index, STB/day/psi.
MD	Measured depth, m
TVD	True measured depth, m

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Introduction

ESP (Electrical Submersible Pump) and GL (Gas Lift) are the two most popular methods of artificial lift because they both greatly increase oil production and are easily applied to offshore wells. However, these two methods are often used separately because ESP is difficult to apply to wells with high GLR, while GL is very suitable for these wells because the gas solution from the oil will be recirculated to the GL system. Some examples of the use of ESP and GL together were observed in some wells of the world, such as in wells in the Australian North West Shelf since the technology increased production rate there by 40–80%. At the same time, the lifecycle of ESP was also improved by more than 100% (Aitken et al. 2000). The ESP systems in combination with GL were also tested at two fields in Columbia and achieved quite similar results from the Australian North West Shelf, namely that the oil production volume increased to about 3300 STB/day (Hubert Borja et al. 1999). Moreover, the study in Columbia also showed that using the combination of these two artificial lift methods saved 40% more energy than when using only conventional ESP. In addition, when one of the two systems fails, the production well will not be shut down but can still be operated. Therefore, this system can maintain stability and flexibility of the artificial lift system (Samieh et al. 2014). In addition, the combination of ESP and GL created great benefits in terms of fluid dynamics making it easy to produce in deep wells or heavy oil production (Saputelli 1997). In Indonesia, the heavy oil production in wells with depths greater than 3000 feet using ESP alone posed many challenges, so a combination of ESP and GL using only one valve was tested (Prakoso et al. 2010), and the results showed that it prevented the loss of 4,400 barrels of oil in one year. Furthermore, Rohman et al. (Rohman et al. 2015) continued to develop the system using more than one valve, and the production rate of wells increased from 36% to more than 50% compared to the case when ESP was used only with the original GL design. However, the application of this combined method has not been widely disseminated in the world, so further research is still needed on this issue.

The main research method applied in the analysis of combined ESP and GL was “Equivalent depth” (Aitken et al. 2000; Hubert Borja et al. 1999; Tran et al. 2016). The concept of equivalent depth is used to design and evaluate artificial lift systems where there is a combination in a production well. When using ESP in combination with GL, the ESP system is considered the main driving force used to push the column of fluid to the surface. The GL system is used to inject gas into the tubing, and their purpose is to reduce the density of fluid in the tubing from the point of injection to the surface. If the required pump pressure of the ESP does

not change, the combination of the ESP system with the amount of gas injected into the well will increase the wellhead pressure (Tran et al. 2016). On the other hand, if the wellhead pressure does not change, the combination of the ESP system with the amount of air being injected reduces the pump pressure requirement (Hubert Borja et al. 1999). For the equivalent depth method, the energy requirements due to the main system can be calculated while considering other factors such as injection gas pressure, injection point depth and pump gas flow. However, the “Equivalent depth” method has the limitation that it does not give the production rate of the well, but only evaluates the energy benefits of the pump.

Besides using the “Equivalent depth” method, Tran et al. (2016) also used the nodal analysis, but the authors still evaluated only the energy benefits. Their research showed that when using ESP at 49 Hz in combination with 0.2 MMscf/day of gas injected into the well, the wellhead pressure increased from 294 to 448 psia and the required pressure of the ESP pump decreased by 80 psia. Furthermore, the analysis of the performance inflow and performance outflow of the ESP system combined with the GL system also gave similar results with the injection gas rate of 0.28 MMscf/day, the required pump pressure value decreased from 470 to 388 psia. However, this study had some following limitations. First, the exploitation flow has not been given with the change of each pump operating frequency and each pumped gas flow. Second, for a production rate required, the selection of operating frequency and injection gas rate will optimize the profit obtained. Moreover, the study of Tran et al. (2016) was limited to a production well only and not generalized to a network of wells.

In order to solve the remaining limitations in this research field as mentioned above, the contributions of this paper are to use both “Equivalent Depth” and “Nodal Analysis” methods to study the energy savings as well as the optimization of production for a network of wells using a combination of ESP and GL. Firstly, this paper conducted a sensitivity study which showed different production flowrates of each well in function of various ESP operating frequency and injection gas rates. In addition, this study will select the operating frequency of ESP and gas lift rate to be injected into each well to obtain the maximum total production oil at a production network with limited injection gas rate. This optimization of production has important practical significance because the limited total lift gas flow will cause a significant reduction in the production rate when the amount of lift gas in each well does not allow its production rate to reach the maximum value. The procedure was programmed using C++ language based on the BST (Binary Search Tree) algorithm to determine the most suitable injection gas rate of each well (in a production network with any number of wells) so that the total production oil was the highest. The calculation

procedures proposed in this study will then be applied to actual data at field X in Cuu Long basin of Vietnam.

Methodology

Electrical submersible pump

ESP is one of the artificial lift methods for the production of large production rates as well as for deep wells (Bataee et al. 2013). ESP regulates production rate by operating frequency of electricity supplied to pump operation. Frequency of electricity supplied can be changed by VSD (Variable Speed Drive) or VFG (Variable Frequency Drive) devices. Pump rate and pump power are proportional to the operating frequency. When the operating frequency of the pump increases, the pump rate and pump capacity will also increase, which is clearly shown in Eqs. (1) and (2) (Takacs 2018). In addition, the decrease in pressure discharge of the pump means that the pump power is reduced, so the modification of the IPR curve or the VLP curve depends on the selected node position in relation to the selected node intake or discharge of the pump. Each pump will have a certain limit on pump rate and horsepower pump because when the pump horsepower limit is larger, the larger the pump diameter is (Electric Submersible Pumps for the Petroleum Industry. Wood Group ESP Inc 2004). This factor will make it difficult to install pumps in wells which have small diameters but require large pump horsepower (Table 1). Thus, determining the optimal operating frequency is very important in order to produce the maximum oil flow without affecting the lifecycle of the ESP. Our paper will detail the optimization of operating frequency in Sect. **Optimization of ESP operating frequency and injection gas rate**.

Eqs. (1) and (2) mentioned above are expressed as:

$$Q_2 = Q_1 \left(\frac{f_2}{f_1} \right) \quad (1)$$

Table 1 Pump horsepower dependence on motor diameter (Electric Submersible Pumps for the Petroleum Industry. Wood Group ESP Inc 2004)

Horsepowers Available		
Motor size (in)	60 Hz HP	50 Hz HP
3.75	7.5–127.5	6.3–106.5
4.56	10–480	8.3–400
5.44	20–800	16.7–667
5.62	200–1500	167–1250

$$HP_2 = HP_1 \left(\frac{f_2}{f_1} \right) \quad (2)$$

where

- f_1, f_2 : AC (Alternating Current) frequencies, Hz.
- Q_1, Q_2 : Pumping rates at f_1 and f_2 , bpd.
- HP_1, HP_2 : Motor powers available at f_1 and f_2 , HP.

Gas lift

Optimizing the injection gas rate consists of determining the suitable amount of gas injected into each well to obtain a maximum profit. Figure 1 shows the lift efficiency (LPR) curve of a gas lift well. As shown in Fig. 1, the production rate increases rapidly with the initial injection gas speed and then tends to stop when the MAXIMUM OIL PRODUCTION RATE point is reached. From this point, if you continue to inject gas, the flow rate will decrease. The PVPAT (Present Value Profit After Tax) is an approximate operating point for maximum after-tax profit, and the Maximum Current OIC (Operating Cash Increase) is the maximum daily operating cash increase. In Fig. 1, the economic optimal point for the gas lift well is at the point PVPAT. The optimal principle for a production well as well as for a production network depends on total injection gas available to use for the GL. If there is enough lift gas, the GL system can produce the maximum PVPAT at a production well as well as at a production network. However, in case where the injection gas available is less than this optimal, the PVPAT is determined by how the gas is injected to each well.

Table 2 shows the oil rate in function of injection gas rate in Malaysia (Ghazali et al. 2014). According to Boyun Guo et al. (Guo 2017), the decrease in flow rate when injection gas is greater than the rate required for maximum oil

Fig. 1 Relationship between injection gas rate and production rate (Rashid et al. 2012)

Table 2 Effect of injection gas rate on FBHP and oil rate (Ghazali et al. June 2014)

Gas injection rate [MMscf/day]	Frictional loss [psi]	Hydrostatic pressure [psi]	FBHP [psi]	Oil produced [stb/d]
0	2	1927	2090	209
1	133	1075	1471	2352
2	225	802	1322	2866
3	288	663	1263	3071
4	339	574	1235	3169
5	381	511	1222	3215
6	417	44	1216	3233
7	448	429	1216	3234
8	476	401	1219	3224
9	501	378	1224	3208

production rate can be explained by gas friction with the tubing which creates obstruction to the flow inside the tubing. According to Mahmood Bataee et al. (Bataee et al. 2013), the optimization process is to find the right injection gas rate for the well to optimize the oil production. The calculation to find the accuracy of the optimal pumping point depends on many other factors such as installation costs, operating costs, gas treatment costs. To simplify the problem, the solution to the gas lift optimization must be efficient and responsive to the key operational factors that affect production. Therefore, our study will only focus on the maximum production oil when combining ESP and GL for a well and for a network of multiple wells.

Equivalent depth

The combination of ESP and GL aims to act on the hydrostatic pressure column of the fluid column in the tubing. Figure 2 shows the relationship between pressure and depth graphically. When the wellhead pressure is constant, the ESP causes the pressure to increase locally at a point. The gas lift is injected into the tubing, reducing the pressure drop. Figures 2 and 3 show two pressure gradients, in which the first dashed line is the pressure gradient when the GL system has not been established, this line has a smaller slope than the pressure gradient when the GL system is established (the solid line). Based on the graph in Fig. 3, Hubert Borja proposed a method that is “equivalent depth” (Hubert Borja et al. 1999). Based on this method, the production engineer can evaluate the benefits of the system combining ESP and GL. When incorporating GL into the ESP system, the equivalent depth gives less depth for ESP design, which also means less pressure required. Figure 3 showed that the required pressure is large when using only the conventional ESP system, but if ESP and GL are combined, the required pressure of the ESP in the well will be equal to the normal

Fig. 2 Pressure profiles for regular and combined systems (Hubert Borja et al. 1999)

pump required pressure minus the AB pressure range. In our paper, the equivalent depth method will be used to evaluate the energy savings when combining the ESP system with GL in the production well.

Fig. 3 Pump “equivalent depth” by lightening of the column with gas injection (Hubert Borja et al. 1999)

Nodal analysis

In Nodal analysis, the equilibrium rate is defined as the intersection of the IPR and the VLP. The injection gas changes the VLP curve in the nodal analysis. In addition, ESP changes the IPR curve or VLP curve values depending on the chosen node (Bruijnen 2016). Figure 4 shows the performance curve of a system including IPR, VLP and ESP. Based on Fig. 4, if the nodal point is located at the discharge of the pump, the IPR curve will become IPR + ESP curve, but if the nodal point is located at the intake of the pump, the IPR curve will become VLP-ESP curve. Although the flow rate has only one value whatever the nodal point is located at the intake or discharge of the pump, the intake and discharge pressure values are different. The difference of that value is due to the pump pressure generated during operation.

Figure 5 shows how the ESP performance affects the curves in the nodal analysis of a production system. When the nodal point is located at the discharge of the pump, point A will be the first production point of the system. Hence, if the performance of the ESP curve changes, the production point will move as well. Based on Fig. 5, when the ESP performance decreases, the production point will move from point A to point B, causing a decrease in production rate. For the nodal point at the pump intake, when the ESP performance decreases, the VLP-ESP curve will increase,

Fig. 5 System performance diagram for changing ESP performance (Bruijnen 2016)

which in turn will reduce production rate. In this study, the nodal analysis method is used to show the changes in production rate in relation with changes in operating frequency of the pump as well as changes in injection gas rate. From the results obtained for each well, this study will give the operation frequency for the ESP as well as the injection gas rate to each well so that the total production rate of a network of wells is the maximum.

Fig. 4 System performance diagram for IPR, ESP and VLP curve (Bruijnen 2016)

Fig. 6 Hybrid ESP/GL calibration process

Workflow of production optimization for a network of multiple wells with each well using a combination of ESP and Gas lift

Model building

Figure 6 shows the workflow for modeling a system combined ESP and GL. The nodal point is located at the

discharge of the pump, so the IPR curve becomes $IPR + ESP$ and will be used for GL matching. Then, the production rate and bottomhole pressure will be calibrated from the model with the smallest errors compared to the test data selected for sensitivity analysis of ESP and GL. Various production rates can be estimated after realizing a sensitivity study of operating frequency of ESP system and of injection gas rate. This procedure is used for each well in a network of multiple

wells with each well using a combination of ESP and GL. Results obtained from this model will be used to predict the optimized production rates of all wells in the network with corresponding values of ESP operating frequency and GL rate.

Optimization of ESP operating frequency and injection gas rate

The process of optimization of production rate at a network of multiple wells (denoted by letter k for the number of wells) using a combination of ESP and GL is discussed in this section. First, the appropriate operating frequency of ESP for each production well is selected. The next step is to determine the injection gas rate for each well to achieve the maximum total production oil rate at a network of k wells.

Optimization of ESP operating frequency

As mentioned in Sect. [Electrical submersible pump](#), changing the oil production rate can be controlled by changing the operating frequency of the ESP. Figure 7 shows the procedure for selecting the appropriate ESP operating frequency. Based on Eqs. (1) and (2), an augmentation in the operating frequency of the pump leads to an increase in the horsepower pump and hence consequently the production rate will increase. But if the operating frequency is increased beyond the designed standard value of the motor, vibration in the motor and augmentation of the pump temperature will be observed, which leads to a reduction in the ESP's lifecycle. Therefore, this study effectuated a sensitivity study of ESP operating frequency which could not exceed the standard operating frequency of the motor.

Optimization of injection gas rate

For GL optimization of a network of multiple wells, there are two cases to consider:

- Case 1: Total volume of available injection gas is equal or greater than the total volume of injection gas rates

needed to reach maximum oil production rates for every well in the network. For this case, the amount of gas injected into each well corresponds to the amount of gas needed to obtain the maximum oil production rate of each well.

- Case 2: Total volume of available injection gas is less than the total volume of injection gas rates needed to reach maximum oil production rates for every well in the network. For this case, it is necessary to determine the optimum injection gas rates injected into each well so that the total oil rate of the network of k wells is maximized.

The workflow for these two cases is presented in Fig. 8. The oil production rate is called q_o , which depends on the injection gas rate q_{inj} as shown in Eq. (3):

$$q_o = f(q_{inj}) \quad (3)$$

However, it is difficult to find the exact relation between oil production rate and injection gas rate. Hence, this study used a numerical method to overcome this challenge. From Eq. (3), the differential equation of the function q_o depends on the injection gas rate as expressed in Eq. (4):

$$dq_o = \frac{df(q_{inj})}{dq_{inj}} \quad (4)$$

Because the collected data from real case study are discrete values, Eq. (4) needs to be discretized into Eq. (5):

$$\Delta q_o = \frac{\Delta f(q_{inj})}{\Delta q_{inj}} \quad (5)$$

where

- Δq_o : Variation of oil production related to the variation of injection gas (STB/MMscf).
- Δq_{inj} : Variation in injection gas rate (MMscf/day).

Fig. 7 Procedure for optimizing frequency operating of ESP

Fig. 8 Procedure for optimizing injection gas rate

Initially, the oil production rate is $q_{o_initial}$. If the well cannot be produced with natural energy, this value will be zero. After a number of changes in injection gas rate (n changes), the oil production rate can be expressed in the following:

$$\begin{aligned}
 (q_{(o_0)}) &= (q_{(o_initial)}) \\
 (q_{(o_1)}) &= (q_{(o_initial)}) + \Delta f(q_{(inj)})_{(1)} \\
 (q_{(o_2)}) &= (q_{(o_initial)}) + \Delta f(q_{(inj)})_{(1)} + \Delta f(q_{(inj)})_{(2)} \\
 (q_{(o_3)}) &= (q_{(o_initial)}) + \Delta f(q_{(inj)})_{(1)} + \Delta f(q_{(inj)})_{(2)} + \Delta f(q_{(inj)})_{(3)} \\
 &\vdots \\
 (q_{(o_n)}) &= (q_{(o_initial)}) + \Delta f(q_{(inj)})_{(1)} + \Delta f(q_{(inj)})_{(2)} + \Delta f(q_{(inj)})_{(3)} + \dots + \Delta f(q_{(inj)})_{(n)} \\
 (q_{(o_1)}) &= (q_{(o_initial)}) + \sum_{i=1}^n \Delta f(q_{(inj)})_{(i)}
 \end{aligned} \tag{6}$$

or a network of k wells, the total production oil can be calculated in Eq. (7) as follows:

$$Q_o = \sum_{j=1}^k (q_{(o_n_j)}) = \sum_{j=1}^k (q_{(o_initial_j)}) + \sum_{j=1}^k \sum_{i=1}^n \Delta f(q_{(inj)})_{(ij)} \tag{7}$$

The objective in Eq. (7) is to maximize the value of Q_o . The algorithm Binary Search Tree (BST) will be used to solve this problem and is presented in Sect. [Using binary search tree algorithm to optimize injection gas rate](#).

Case study

Table 3 presents data of a network of three offshore wells in Cuu Long Basin in Southern Vietnam. All three wells are located in the Middle Miocene Upper—Lower Con

Son formations. These wells have a measured depth of over 1800 m and a true vertical depth of over 1400 m, with diversity in oil properties (22.1207 API to 34.9902 API and 4.932 scf/STB GOR to 26.451 scf/STB GOR), which posed

a challenge in selecting production technology method in order to enhance wellbore lifting efficiency. Therefore, a

Table 3 Production parameters of the network of three wells

Parameter	Well X1	Well X2	Well X3
Reservoir pressure (psia)	1974.7	2100	2290
Reservoir temperature (°F)	170.6	170.6	176
Tubing head pressure (psia)	548.696	390.152	862.4
Pump measured depth (m)	1473.66	1403.58	1484.6
Oil gravity (API)	22.1207	22.1207	34.9902
Gas gravity (sp.gravity)	0.76	0.76	0.76
Viscosity pump depth (cp)	8.9	8.9	8.9
Total GOR (scf/STB)	4.932	4.932	26.451
Productivity index (bpd/psi)	5.6	5.6	5.6
Water cut (%)	57.7	64	54

combination of ESP and GL was selected to be used for this network of these three wells. The ESP system uses a 5.38-inch (400–2250 RB/day) GE_ESP TE1500_COMP pump and a Reda 456_91_Std 70HP 1235 V 36A motor. This motor is designed to operate with a standard operating frequency of 60 Hz. The test data, including wellhead pressure, wellhead temperature, water cut, liquid rate, GOR, and pressure at the outlet of the pump, were recorded, while ESP was operating with a frequency of 49 Hz, and gas injection rates were 0.3 MMscf/day for well X1, 0.2 MMscf/day for well X2 and 0.1 MMscf/day for well X3. These real data will be used later for the matching of the model.

Model matching and binary research tree algorithm for optimization of injection gas rate

Model matching for each well in the network

For each well, the matching process was based on eight test data. Figure 9 shows eight pairs of IPR and VLP curves of eight data tests after matching for well X1. Table 4 shows the error results of each production rate and bottomhole pressure of models compared to data test. Based on the results

of Table 4, the test data number 8 has the smallest error in production rate and bottomhole pressure. The difference between the production rate and the bottomhole pressure in the model compared to the eighth test data is 0.69474% and 0.66876%. Figure 10 shows the IPR curve and VLP curve of the eighth test data. Therefore, the model of the eighth test data is suitable for sensitivity study of operating frequency and injection gas rates. The procedure is similar for the wells X2 and X3.

Using binary search tree algorithm to optimize injection gas rate

In this study, the Binary Search Tree (BST) algorithm was used to determine the injection gas rate injected into each well so that the total oil production rate extracted from a network of k wells is the maximum. AVL tree is a BST which is a balanced tree used to reduce complexity of the search (Kruse and Ryba 2000). This method uses a single linked list with the input data used as the basis for comparison and sorting to form a BST. Each node contains data and two pointers linking the left node and the right node. If the data of the node to be sorted are less than or equal to data of the parent node, the node will be linked to the left of the parent node. Otherwise, if the data of the node to be sorted are larger than

Fig. 9 The IPR curves and VLP curves of well X1 after matching with available test data

Table 4 The error results of the models of well X1 compared to available test data

	Liquid rate (STB/day)			Bottomhole pressure (psi)		
	Calculate	Measure	%Difference	Calculate	Measure	%Difference
Test 1	1726.42	1754	− 1.57257	2308.04	2311.63	− 0.15635
Test 2	1729.05	1745	− 0.91386	2306.83	2308.9	− 0.090294
Test 3	1722.75	1758	− 2.00512	2309.73	2314.34	− 0.20031
Test 4	1726.41	1767	− 2.29722	2308.05	2313.38	− 0.23216
Test 5	1693.76	1746	− 2.99173	2323.11	2329.99	− 0.29728
Test 6	1689.3	1732	− 2.46554	2325.17	2330.71	− 0.23931
Test 7	1727.37	1741	− 0.78288	2307.6	2309.37	− 0.077136
Test 8	1735.86	1748	− 0.69474	2303.69	2305.28	− 0.069685

Fig. 10 The IPR curve and VLP curve of well X1 after matching using the test data number 8

data of the parent node, the node will be linked to the right of the parent node. After adding a node to the tree, a rotation is performed to ensure the AVL tree remains balanced. Finally, the algorithm returns the node with the largest data in the tree, which constitutes the target value of the maximum oil production rate of a network of k wells and also includes the value of the injection gas rate injected for each well. AVL trees ensure that the complexity of the search is $O(\log_2 n)$. Binary Tree Search (BST) was used because it presents some advantages. The BST does not require allocating memory, which is highly beneficial in case the amount of input data is unknown. Obviously, if the size of an allocated array is less than the size of the total data storage, the array will not have enough memory for the storage. Hence, the allocated array must have a size larger than the total data storage, which will be a waste of memory and consequently increase the processing time. Therefore, without declaring the amount of input data in advance, the BST method saves memory and time. Although this benefit can also be achieved by another method called Linked List, however, the self-balancing Binary Search Tree (AVL) can search and insert the data faster than Linked List method. In addition, the complexity of the search using

Linked List is $O(n)$, while the complexity of AVL tree is $O(\log_2 n)$. This advantage is highly beneficial in case of a large amount of data. The disadvantage of the BST method is that the programming of the process is much more complicated than other methods.

Figure 11 shows an example of the BST algorithm used to determine injection gas rate. The data of node are the total oil production rate $Q_o(0)$ and injection gas rate $q_{inj}(1)$, $q_{inj}(2)$, ..., $q_{inj}(k)$ injected into a network of k wells. The total oil production rate of each node is used to sort the node into BST. Based on Fig. 11, the initial node took a value of $Q_o(0)$. The nodes containing $Q_o(1)$, $Q_o(2)$, $Q_o(3)$, etc., are compared with their previous data before being sorted into the tree. After adding any node to the tree, the algorithm checks the balance of the tree. The heights of the left and right subtrees of the root differ by at most 1. The test procedure is a recursive process starting from the subtree to the initial node, ensuring that the left and right subtrees are AVL trees again. The process is repeated until the node containing the last data is sorted.

AVL tree algorithm which is used to optimize injection gas rate consists of three parts:

Part 1: AVL insert algorithm.

```

if node is null then
    node = newnode
    taller = true
    return node
else
    if newnode -> data.Qo <= node->data.Qo then
        node->left = AVLinsert(node->left, Qo, qinj(1), qinj(2), ...qinj(k), taller)
        if taller==true
            if node is LH then
                node = leftBalance(node, taller)
            else if root is EH then
                node->balance = LH
            else
                node->balance = EH
                taller = false
            end
        end
    else
        root->right = AVLinsert(node->right, Qo, qinj(1), qinj(2), ...qinj(k), taller)
        if taller == true
            if node is LH then
                node->balance = EH
                taller = false
            else if root is EH then
                node->balance = RH
            else
                node = rightBalance(node, taller)
            end
        end
    end
end

```

Part 2: Find the total oil production of the network of k wells and compare the total injection gas with the injection gas available. After that, insert the total oil production and injection gas rates of each well that meets the conditions into AVL tree

Results and discussions

Energy saving analysis using equivalent depth method

Figure 12 shows the pressure gradient varied with TVD in the tubing of well X1. The orange line shows the pressure

```

for all data of well X1
  |
  for all data of well X2
    |
    for all data of well X3
      |
      ⋮
      for all data of well Xk
        |
        if  $q_{inj}(1) + q_{inj}(2) + \dots + q_{inj}(k) \leq Q_{g_{inj}}$ 
          |
          AVLinsert(root,  $Q_o$ ,  $q_{inj}(1)$ ,  $q_{inj}(2)$ , ...,  $q_{inj}(k)$ )
          end
        end
      ⋮
    end
  end
end

```

Part 3: Returns the node with the largest total oil production rate.

```

getmax()
while root->right != null
  |
  return max(root->right)
end
return root
end getmax

```

This is the main algorithm used to optimize the injection gas rate. The algorithm needs data and rotates functions for the program to produce results. The entire algorithm above was programmed using C++ language with the code attached in the appendix.

gradient in the tubing when using the ESP running at 49 Hz. The green line shows the pressure gradient from discharge pump to wellhead. The yellow line shows the pressure gradient when using the GL and injection gas rate is at 0.3 MMscf/day. From the discharge point, the pressure

Fig. 11 The BST algorithm is used to optimize injection gas rate

gradient for GL is steeper than pressure gradient for the ESP, which can be explained by the fact that the density of the fluid in the tubing is decreased after injection of gas lift, the pressure drop in the tubing is reduced by consequence. In Fig. 12, point X is the intersection between pressure gradient for ESP and pressure gradient for GL. The X pressure value extended to point A on the gradient curve of ESP system represents an equivalent point in pressure but at a different depth for the subsurface pump. The point B means the equivalent subsurface pump setting of the combined systems with respect to the conventional ESP. The difference between point X and point A is 379.1216 m. This is equivalent to a beneficial hydraulic fluid level height of 379.1216 m. The results show that the required pump pressure of the combination of ESP and GL is 405.8 psia; hence, the difference between required pump pressure of conventional ESP system and required pump pressure of ESP/GL combination is 228.7 psia. This result indicates that using a combination of ESP and GL allows to reduce the required pressure of the pump to produce required production rate at a certain wellhead pressure. Similar observations were equally obtained from the analysis of the well X2 and X3 as indicated in Figs. 17, 18 in Appendix B.

Influence of operating frequency and injection gas rate on production rate

Figure 13 shows the IPR curves and VLP curves of each operating frequency and each injection gas rate. The production rate is determined by the intersection of the IPR curve and VLP curve. The production rate varies if the operating frequency and injection gas change. Figure 13 shows that if the operating frequency of ESP increases, the pressure value of the IPR curve will increase as well. By contrast, if the injection gas rate increases, the pressure value of the VLP curve will decrease. After the well reaches the maximum oil production point (as mentioned in Sect. Gas lift), if we continue to inject gas lift into the well, the pressure value of the VLP curve will increase, and consequently the production rate will decrease. Table 5 shows the production rate for various values of ESP operating frequency and injection gas rate. These results will be used for the optimization of ESP/GL combination in Sect. Production optimization for a network of multiple wells, which consists of selecting the optimum operating frequency of ESP and optimum injection gas rate for each well. Similar analysis was equally done for the wells X2 and X3 as indicated in Figs. 19, 20 and Tables 8, 9 in Appendix B.

Fig. 12 Pressure gradient for ESP and GL of the well X1

Fig. 13 The IPR curves and VLP curves of well X1

Table 5 The results production rate of well X1

Gas rate injection (MMscf/day)	Operating frequency									
	40 Hz		45 Hz		50 Hz		55 Hz		60 Hz	
	Liquid rate STB/day	Oil rate	Liquid rate STB/day	Oil rate	Liquid rate STB/day	Oil rate	Liquid rate STB/day	Oil rate	Liquid rate STB/day	Oil rate
0	0	0	895	388	1425	619	2261	982	2637	1144
0.3	1105	480	1425	619	1761	762	2435	1058	2748	1193
0.5	1215	527	1520	660	1849	802	2502	1087	2792	1212
1	1314	570	1615	702	1952	847	2598	1135	2866	1622
1.5	1383	597	1664	722	1995	866	2629	1142	2899	1258
2	1423	615	1710	742	2032	887	2648	1149	2924	1269

Production optimization for a network of multiple wells

Optimization of ESP operating frequency

Figure 14 shows the relationship between operating frequency and oil production in the three wells X1, X2, and X3. Based on Fig. 14, if the operating frequency of the pump increases, the oil production rate will increase. This relationship between operating frequency and flow rate can be explained by Eqs. (1) and (2) as they showed that the horsepower of the pump is proportional to the operating frequency. As introduced in Sect. Case study, the ESP system uses a 5.38-inch pump GE_ESP TE1500_COMP (400–2250 RB/day) and a motor Reda 456_91_Std 70HP 1235 V 36A. As the motor was designed at a standard frequency of 60 Hz, therefore, in order to avoid affecting the lifecycle of ESP, the pump should be set at the operating frequency of 60 Hz. This parameter will continue to be used in the following Sect. Optimization of injection gas rate to optimize the injection gas rate.

Optimization of injection gas rate

Unlimited total injection gas rate Figure 15 shows the relationship between gas injection rate and oil production rate

at the ESP's operating frequency of 60 Hz at a network of three wells X1, X2, X3. In case of unlimited resource of gas lift, there is no constraint on the volume of the gas lift rate other than the condition that the gas lift rate should not pass beyond the maximum oil rate point. Table 6 shows the injection gas rate of each well in order to obtain the maximum oil production rate of each well.

Limited total injection gas rate In a network of many wells, the total volume of injection gas rate is usually limited. The total volume of injection gas rate available is less than the total volume of injection gas rate needed to achieve maximum oil production rate as obtained in the previous Sect. Unlimited total injection gas rate. Figure 16 shows the relationship between the increase in average oil production rate per unit of injection gas rate injected into the well. The results show that the oil rate increased significantly at the beginning of gas lift injection, but the increase will slow down with more and more gas lift injected into the wells. At the moment when the maximum oil rate is reached, if we continue to increase gas lift rate, the increase in average oil production rate per unit of injection gas rate will be less than zero, and by consequence, the oil production rate will be reduced.

The AVL tree of BST algorithm was used to determine the injection gas rate for each well at this network of three

Fig. 14 The relationship between pump operating frequency and oil production rate

Fig. 15 The relationship between injection gas rate and oil production rate

Table 6 The results of injection gas rate and maximum oil production rate for each well in a network of three wells using each well a combination of ESP and GL with a total unlimited resource of gas lift

Well No.	Injection gas rate (MMscf/day)	Maximum oil production rate (STB/day)
X1	8.2	1305.2
X2	2.4	1517.1
X3	4.5	1591.4

Table 7 The results of optimized injection gas rate for each well in a network of three wells and each well used a combination of ESP and GL with a total limited resource of gas lift

Well number	Injection gas rate (MMscf/day)	Oil production rate (STB/day)
X1	0.6	1094.1
X2	0.9	1304.47
X3	0.5	1244.8

Fig. 16 The increased in average oil production rate per unit of injection gas rate injected into the wells

wells when the total injection gas rate distribution is limited only at 2 MMscf/day and the ESP system operates at optimized frequency of 60 Hz. The results are presented in

Table 7 which indicates that the maximum total oil production rate achieved at this network of three wells is 3643.37 STB/day. The volume of injection gas rate injected into well X2 is the largest which can be coherently explained based on Fig. 16 as it showed that when injection gas rate in X2 raised from 0.5 to 1 MMscf/day, the line segment created by these two points of the X2 presented the greatest slope among the three. This means that the increase in oil production rate reaches the maximum value in well X2 in comparison with the wells X1 and X3, and this explanation is logically related to the fact that each time we decide to increase the gas lift rate, we must prioritize the well which has the largest increase in oil production, in order to achieve the maximum oil production rate of a network of multiple wells.

Conclusion

This paper proposed a workflow to optimize the production of a network of multiple wells with each well using a combination of ESP and GL. The equivalent depth method

was used to study the energy savings of the combination of ESP and GL compared with the conventional ESP system. In addition, the nodal analysis method was used to predict the production rate of each operating frequency and each injection gas rate. The results allow us to determine the optimum operating parameters of this hybrid artificial lift to achieve the highest production rate in two cases (unlimited and limited gas lift resource), while paying attention to the economic efficiency by avoiding bad effects on the lifecycle of the ESP.

This study therefore combined multiple techniques from equivalent depth method, nodal analysis to the coding of BST algorithm in C++ in order to provide the best optimization of a network of k wells. The results obtained from this study indicated a clear benefit in the energy saved by using ESP and GL together in comparison with using only one of the two methods. In addition, as the maximum horsepower of the pump also depends on the size of the ESP, it is difficult to increase production rate for small diameter wells; hence, the combination of ESP and GL constitutes a feasible option in this case.

Appendix A

Appendix A

AVL code

```
#include <iostream>

#include <math.h>

#include <queue>

using namespace std;

#define SEPARATOR "<#<ab@17943918#@>#"

enum BalanceValue
{
    LH = -1,
    EH = 0,
    RH = 1
};

template<class T>
class AVLTree
{
public:
    class Node;
private:
    Node *root;
protected:
    int getHeightRec(Node *node)
    {
        if (node == NULL)
            return 0;

        int lh = this->getHeightRec(node->pLeft);
        int rh = this->getHeightRec(node->pRight);
        return (lh > rh ? lh : rh) + 1;
    }
};
```

```

    }

public:
    AVLTree() : root(nullptr) {}
    ~AVLTree(){}
    int getHeight()
    {
        return this->getHeightRec(this->root);
    }

    Node* rotateLeft(Node* &node){
        Node* p = node->pRight;
        node->pRight = p->pLeft;
        p->pLeft = node;
        return p;
    }
    /*Exchanges pointers to rotate the tree left.
    Node is pointer to tree to be rotated.
    Post: node rotated and node updated.*/

    Node* rotateRight(Node* &node){
        Node* p = node->pLeft;
        node->pLeft = p->pRight;
        p->pRight=node;
        return p;
    }
    /*Exchanges pointers to rotate the tree right.
    Node is pointer to tree to be rotated.
    Post: node rotated and node updated.*/

    Node* leftBalance(Node* &node,bool &taller){
        if (node->pLeft->balance == LH)

```

```
{
    node->pLeft->balance = EH;
    node->balance = EH;
    node = rotateRight(node);
    taller = false;
}
else
{
    if (node->pLeft->pRight->balance == EH)
    {
        node->balance = EH;
        node->pLeft->balance = EH;
        node->pLeft->pRight->balance = EH;
    }
    else if (node->pLeft->pRight->balance == LH)
    {
        node->balance = RH;
        node->pLeft->balance = EH;
        node->pLeft->pRight->balance = EH;
    }
    else
    {
        node->balance = EH;
        node->pLeft->balance = LH;
        node->pLeft->pRight->balance = EH;
    }
    node->pLeft = rotateLeft(node->pLeft);
    node = rotateRight(node);
    taller = false;
}
return node;
```



```

    }

    /* This algorithm is used when the left subtree is higher than the right subtree.
    Node is a pointer to the root of the subtree
    taller is true.
    Post: node has been updated (if necessary)
    taller has been updated. */

```

```

Node* rightBalance(Node* &node, bool &taller){
    if (node->pRight->balance == RH)
    {
        node->pRight->balance = EH;
        node->balance = EH;
        node = rotateLeft(node);
        taller = false;
    }
    else
    {
        if (node->pRight->pLeft->balance == EH)
        {
            node->balance = EH;
            node->pRight->balance = EH;
            node->pRight->pLeft->balance = EH;
        }
        else if (node->pRight->pLeft->balance == RH)
        {
            node->balance = LH;
            node->pRight->balance = EH;
            node->pRight->pLeft->balance = EH;
        }
        else
        {

```

```

        node->balance = EH;

        node->pRight->balance = RH;

        node->pRight->pLeft->balance = EH;

    }

    node->pRight = rotateRight(node->pRight);
    node = rotateLeft(node);
    taller = false;
}

return node;
}

/* This algorithm is used when the right subtree is higher than the left subtree.
Pre: node is a pointer to the root of the subtree
taller is true
Post: node has been updated (if necessary)
taller has been updated.*/

```

```

Node* insertRec(Node*&node, T Q, T g1, T g2, T g3, T q1, T q2, T q3, bool &taller){
    if (node == nullptr)
    {
        taller = true;
        return new Node(Q,g1,g2,g3,q1,q2,q3);
    }
    else {
        if (Q < node->data)
        {
            node->pLeft = insertRec(node->pLeft, Q,g1,g2,g3,q1,q2,q3, taller);
            if (taller)
            {
                if (node->balance == LH)
                {
                    node = leftBalance(node, taller);

```

```

    }

    else if (node->balance == EH)
    {
        node->balance = LH;
    }

    else
    {
        node->balance = EH;
        taller = false;
    }
}

}

else
{
    node->pRight = insertRec(node->pRight, Q,g1,g2,g3,q1,q2,q3, taller);
    if (taller)
    {
        if (node->balance == LH)
        {
            node->balance = EH;
            taller = false;
        }

        else if (node->balance == EH)
        {
            node->balance = RH;
        }

        else
        {
            node = rightBalance(node, taller);
        }
    }
}

```

```

    }

    return node;

}

}

void insert(T Q,T g1,T g2,T g3, T q1, T q2, T q3)
{
    bool taller = false;

    this->root = insertRec(this->root, Q,g1,g2,g3,q1,q2,q3 ,taller);

}

```

/* Insert node into the AVL tree. The node includes oil production rate and gas injection rate. After inserting the node, the program will proceed to check the height of the left subtree and the height of the right subtree. The result will be a balanced tree.

Using recursion to insert a node into an AVL tree:

- node is a pointer to first node in AVL tree/subtree
- Post: taller is a Boolean: true indicating the subtree height has increased, false indicating same height
- Return node returned recursively up the tree.*/

```

T getMax() {
    Node *p = new Node(0,0,0,0,0,0,0,0);

    p = root;

    while(p->pRight!=NULL){
        p = p->pRight;
    }

    cout << "Total production oil: "<< p->data << "\n";

    cout << "Prodcution oil X1: "<<p->q1<< "\n";

    cout << "Prodcution oil X2: "<<p->q2<< "\n";

    cout << "Prodcution oil X3: "<<p->q3<< "\n";

    cout << "Gas rate injection for each well X1, X2, X3: " << p->g1 << ", " << p->g2 << ",
    "<<p->g3 ;

}

/*Return the maximum values of nodes in the tree.*/

```

```

class Node
{
private:
    T data;

    T g1;

    T g2;

    T g3;

    T q1;

    T q2;

    T q3;

    Node *pLeft, *pRight;

    BalanceValue balance;

    friend class AVLTree<T>;

public:
    Node(T value, T g1, T g2, T g3, T q1, T q2, T q3) : data(value), g1(g1), g2(g2),
g3(g3),q1(q1),q2(q2),q3(q3), pLeft(NULL), pRight(NULL), balance(EH) {}

    ~Node() {}

/*Declare node's data in an AVL. Node includes oil production rate and gas injection rate.*/

};

};

int main()
{
    AVLTree<float> avl;

    float GL[] = {0, 0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1,1.1,1.2,1.3,1.4,1.5,1.6,1.7,1.8,1.9,2};

    float X1[] =
{980,1018.1,1036.5,1054.3,1068.7,1082,1094.1,1104.2,1112.7,1118.5,1123.6,1128.2,1131.1,1133.8,1136
,1138.2,1140.5,1142.5,1145,1147.2,1149.2};

    float X2[] =
{955.558,1045.12,1109.566,1159.476,1198.254,1229.028,1253.868,1274.108,1290.714,1304.468,1315.876,
1323.88,1329.63,1334.368,1338.278,1341.498,1344.212,1346.512,1348.352,1349.778,1350.9418};

```

```

float X3[] =
{1168.4,1186.8,1203.1,1218.4,1232.4,1244.8,1255.8,1265.7,1274.5,1282.4,1289.5,1296,1301.8,1307,131
1.8,1316.1,1320.1,1323.7,1326.9,1329.9,1332.6};

/*Oil production rate of the network of three wells.*/

for (int i = 0;i<21;i++){
    for (int j = 0;j<21;j++){
        for (int k = 0;k<21;k++){
            if (GL[i]+GL[j]+GL[k]<=2){
                float Q = X1[i]+X2[j]+X3[k];
                avl.insert(Q,GL[i],GL[j],GL[k],X1[i],X2[j],X3[k]);
            }
        }
    }
}

avl.getMax();

return 0;
}

```

Appendix B

See (Figs. 17, 18, 19, 20); See (Table 8 and 9).

Fig. 17 Pressure gradient for ESP and GL of the well X2

Fig. 18 Pressure gradient for ESP and GL of the well X3 **Fig. 19** The IPR curves and VLP curves of well X2

Fig. 20 The IPR curves and VLP curves of well X3

Table 8 Production rates of well X2 in function of injection gas rate and operating frequency of ESP

Gas rate injection (MMscf/day)	Operating frequency									
	40 Hz		45 Hz		50 Hz		55 Hz		60 Hz	
	Liquid rate STB/day	Oil rate	Liquid rate STB/day	Oil rate	Liquid rate STB/day	Oil rate	Liquid rate STB/day	Oil rate	Liquid rate STB/day	Oil rate
0	0	0	522	240	1125	517	1622	746	2077	956
0.2	1175	540	1471	677	1764	811	2096	964	2412	1110
0.5	1570	722	1818	836	2105	968	2373	1091	2672	1229
1	1774	816	2036	936	2291	1054	2556	1176	2861	1316
1.5	1842	847	2113	972	2354	1083	2624	1207	2916	1341
2	1860	856	2138	983	2375	1093	2649	1218	2937	1351

Table 9 Production rates of well X3 in function of injection gas rate and operating frequency of ESP

Gas rate injection (MMscf/day)	Operating frequency									
	40 Hz		45 Hz		50 Hz		55 Hz		60 Hz	
	Liquid rate STB/day	Oil rate	Liquid rate STB/day	Oil rate	Liquid rate STB/day	Oil rate	Liquid rate STB/day	Oil rate	Liquid rate STB/day	Oil rate
0	865	398	1334	614	1768	813	2159	993	2540	1168
0.1	1042	479	1469	676	1839	846	2222	1022	2580	1187
0.5	1403	646	1741	801	2050	943	2404	1106	2706	1245
1	1599	736	1880	865	2187	1006	2507	1153	2803	1289
1.5	1673	770	1956	900	2264	1041	2566	1180	2861	1316
2	1716	789	2001	921	2310	1063	2602	1197	2897	1333

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Declarations

Conflict of interest On behalf of all the co-authors, the corresponding author states that there is no conflict of interest.

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References

- Aitken KJ, Allan JC, Brodie AD, Holmes JD (2000) Combined esp/ auto gas lift completions in high gor/high sand wells on the Australian Northwest shelf. In: *SPE Asia Pacific oil and gas conference and exhibition, SPE-64466-MS*, Brisbane, Australia. <https://doi.org/10.2118/64466-MS>
- Borja H, Castano R (1999) Production optimization by combined artificial lift systems and its application in two Colombian fields. In: *Latin American and Caribbean petroleum engineering conference, SPE-53966-MS*, Caracas, Venezuela. <https://doi.org/10.2118/53966-MS>
- Samieh A, Kamel I, Metwally A (2014) Intelligent production application {ESP/GL hybrid system (electro-gas)}". In: *SPE Middle East artificial lift conference and exhibition, SPE-173696-MS*, Manama, Bahrain. <https://doi.org/10.2118/173696-MS>
- Saputelli L (1997) Combine artificial lift system – An innovative approach. In: *Paper presented at the Latin American and Caribbean petroleum engineering conference, SPE-39041-MS*. <https://doi.org/10.2118/39041-MS>
- Prakoso NF (2010) Single string packerless ESP gas lift Hybrid; optimizing production and minimizing loss. In: *SPE oil and gas india conference and exhibition, SPE-128974-MS*, 20 Mumbai, India. <https://doi.org/10.2118/128974-MS>
- Rohman AF, Arseto YI, Hamzah K (2015) Redesign of a single string packerless ESP-gas lift hybrid. In: *SPE/IATMI Asia pacific oil & gas conference and exhibition, SPE-128974-MS*, Nusa Dua, Bali, Indonesia. <https://doi.org/10.2118/176291-MS>
- Tran ST, Vu HV, Le VM Nguyen TN, Nguyen LH (2016) Hybrid system of ESP and gas lift application from conceptual design pilot test to system analysis. In: *SPE Middle East artificial lift conference and exhibition, SPE-184215-MS*, Manama, Kingdom of Bahrain. <https://doi.org/10.2118/184215-MS>
- Bataee M, Irawana S, Reisabadib MZ, Yahyazadehc A (2013) Production optimization using different scenarios of gas lift and ESP installation. *Int J Petrol Geosci Eng (IJPGE)* 1:50–61
- Takacs G (2018) Electrical submersible pumps manual. Gulf Professional Publishing, ISBN: 978–0–12–814570–8. <https://doi.org/10.1016/C2017-0-01308-3>
- Electric submersible pumps for the petroleum industry (2004) Wood Group ESP, Inc. 5500 S.E. 59th St, Oklahoma City, OK, p 73135
- Rashid K, Bailey W, Couet B (2012) A survey of methods for gas-lift optimization. *Model Simul Eng* 2012:1–16. <https://doi.org/10.1155/2012/516807>
- Ghazali NA, Mohd TAT, Alias N, Yahya E, Shahrudin MZ, Azizi A, Fazil AY (2014) Gas lift optimization of an oil field in Malaysia. *Adv Mater Res* 974:367–372
- Guo B, Liu X and Tan X (2017) Petroleum production engineering. Gulf Professional Publishing, ISBN: 978–0–12–809374–0
- Bruijnen PM (2016) Nodal analysis by use of ESP intake and discharge pressure gauges. *SPE Prod Oper* 31(01):76–84
- Kruse RL, Ryba AJ (2000) Data structures and program design in C++. Alan apt, ISBN 0–13–087697–6

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